

# Speakers-and-Listeners

May 25, 2022

## 1 Preliminaries

```
[1]: import json
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import itertools
import numpy as np
from scipy import stats

%load_ext autoreload
%autoreload 2
```

```
[2]: from literal_listener import StatelessLiteralListener

from configuration import TRUE_REWARDS
```

```
[3]: TEST_CONTEXT = [{'color': 'red', 'shape': 'circle'},
                     {'color': 'red', 'shape': 'triangle'},
                     {'color': 'blue', 'shape': 'square'}]
```

## 2 Literal Listener

### 2.1 Acting from Instructions

```
[4]: listener = StatelessLiteralListener(alphaL=3)
```

```
[5]: instruction_present = {"type": "instruction", "color": "red", "shape": "circle"}
print("\nUtterance: {}".format(instruction_present))
for a in TEST_CONTEXT:
    print("\t{} : {}".format(a, listener._prob_action_from_instruction(a,
↪TEST_CONTEXT, instruction_present)))

instruction_absent = {"type": "instruction", "color": "green", "shape": "
↪circle"}
print("\nUtterance: {}".format(instruction_absent))
for a in TEST_CONTEXT:
```

```
print("\t{} : {}".format(a, listener._prob_action_from_instruction(a,
↪TEST_CONTEXT, instruction_absent)))
```

```
Utterance: {'type': 'instruction', 'color': 'red', 'shape': 'circle'}
          {'color': 'red', 'shape': 'circle'} : 1
          {'color': 'red', 'shape': 'triangle'} : 0
          {'color': 'blue', 'shape': 'square'} : 0
```

```
Utterance: {'type': 'instruction', 'color': 'green', 'shape': 'circle'}
          {'color': 'red', 'shape': 'circle'} : 0.3333333333333333
          {'color': 'red', 'shape': 'triangle'} : 0.3333333333333333
          {'color': 'blue', 'shape': 'square'} : 0.3333333333333333
```

## 2.2 Acting from Descriptions

Given a message  $u$  which consists of a feature-value tuple  $(\phi, \mathbb{R})$ , condition worlds  $w$  and return only those which are literally consistent with that utterance. Choose actions w.r.t. the posterior beliefs.

```
[6]: description = {"type": "description", "feature": "red", "value": 1}
print("Utterance: {}".format(description))
for a in TEST_CONTEXT:
    print("\t{} : {}".format(a, listener._prob_action_from_description(a,
↪TEST_CONTEXT, description)))

description_circle = {"type": "description", "feature": "circle", "value": 1}
print("\nUtterance: {}".format(description_circle))
for a in TEST_CONTEXT:
    print("\t{} : {}".format(a, listener._prob_action_from_description(a,
↪TEST_CONTEXT, description_circle)))
```

```
Utterance: {'type': 'description', 'feature': 'red', 'value': 1}
          {'color': 'red', 'shape': 'circle'} : 0.4878555511603684
          {'color': 'red', 'shape': 'triangle'} : 0.4878555511603684
          {'color': 'blue', 'shape': 'square'} : 0.024288897679263205
```

```
Utterance: {'type': 'description', 'feature': 'circle', 'value': 1}
          {'color': 'red', 'shape': 'circle'} : 0.9094429985127419
          {'color': 'red', 'shape': 'triangle'} : 0.04527850074362907
          {'color': 'blue', 'shape': 'square'} : 0.04527850074362907
```

### 2.2.1 Present Rewards

```
[7]: print("Instructions - Present Rewards")
print("\tGood Instruction: {}".format(listener.
    ↳present_rewards(instruction_present, TEST_CONTEXT, TRUE_REWARDS)))
print("\tNot Present Instruction: {}".format(listener.
    ↳present_rewards(instruction_absent, TEST_CONTEXT, TRUE_REWARDS)))

print("\nDescriptions - Present Rewards")
print("\tPresent Rewards: {}".format(listener.present_rewards(description,
    ↳TEST_CONTEXT, TRUE_REWARDS)))
print("\tPresent Rewards: {}".format(listener.
    ↳present_rewards(description_circle, TEST_CONTEXT, TRUE_REWARDS)))
```

Instructions - Present Rewards  
Good Instruction: 1  
Not Present Instruction: -0.6666666666666666

Descriptions - Present Rewards  
Present Rewards: 0.4149888581225788  
Present Rewards: 0.7736074962818548

### 2.2.2 Future Feature Counts / Rewards

```
[8]: listener = StatelessLiteralListener(alphaL=3)
avg_features_advice = listener.future_feature_counts({"type": "description",
    ↳"feature": "green", "value": 2})

expected_rewards_advice = listener.feature_count_rewards(avg_features_advice,
    ↳TRUE_REWARDS)

string_feature_counts = [{"{}: {:.2f}"].format(k, v) for k, v in
    ↳avg_features_advice.items()]

print("Generalizability of \"Green is +2\" description:\n")
print("Future Features:")
for feature in string_feature_counts:
    print("\t{}".format(feature))
print("Future Rewards: {}".format(expected_rewards_advice))
```

Generalizability of "Green is +2" description:

Future Features:  
red:0.12  
square:0.33  
circle:0.33  
triangle:0.33  
green:0.76

```
blue:0.12
Future Rewards: 1.2769904161327992
```

## 3 Literal Speaker

### 3.1 Utterance Preferences

#### 3.1.1 Description-only feature shift

```
[9]: from literal_speaker import LiteralSpeaker
     from configuration import DESCRIPTIONS, ALL_UTTERANCES, EXP_UTTERANCES
```

```
[10]: results_list = []
      description_only_speaker = LiteralSpeaker(listener, alphaS=10,
      ↪utterances="descriptions")

      for horizon in range(1, 11):

          probs = description_only_speaker.all_utterance_probabilities(TEST_CONTEXT,
          ↪horizon=horizon)

          prob_df = pd.DataFrame(DESCRIPTIONS)
          prob_df["prob"] = probs
          prob_df["horizon"] = horizon
          results_list.append(prob_df)

      all_horizon_results = pd.concat(results_list)
```

#### 3.1.2 Instruction to description shift

```
[11]: import copy

      results_list = []
      utterances = ALL_UTTERANCES
      all_utterances_speaker = LiteralSpeaker(listener, alphaS=10, utterances="all")

      for horizon in [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]:

          probs = all_utterances_speaker.all_utterance_probabilities(TEST_CONTEXT,
          ↪horizon=horizon)

          prob_df = pd.DataFrame(utterances)
          prob_df["prob"] = probs
          prob_df["horizon"] = horizon
          results_list.append(prob_df)

      all_horizon_results = pd.concat(results_list)
```

```

[12]: green = all_horizon_results[all_horizon_results.feature == "green"]
blue = all_horizon_results[all_horizon_results.feature == "blue"]
circle = all_horizon_results[all_horizon_results.feature == "circle"]
instruction = all_horizon_results[(all_horizon_results.color == "red") &
    ↳(all_horizon_results["shape"] == 'circle')]

green_utt = "Description - Green"
blue_utt = "Description - Blue"
spotted_utt = "Description - Spotted"
instruct = "Instruction - Red Spotted"

green["Utterance"] = green_utt
blue["Utterance"] = blue_utt
circle["Utterance"] = spotted_utt
instruction["Utterance"] = instruct

ordering = [instruct, spotted_utt, blue_utt, green_utt]

all_utts = pd.concat([green, blue, circle, instruction])

type_by_horizon = all_utts.groupby(["horizon", "Utterance"]).sum().
    ↳reset_index()[["horizon", "Utterance", "prob"]]

plt.figure(figsize=(8, 4))
ax = sns.lineplot(data=type_by_horizon, y='prob', x='horizon', linewidth=4,
    ↳hue='Utterance',
                    palette=['#ee2f24', 'dimgray', '#5580c1', '#69a94f'],
    ↳hue_order=ordering,
                    style='Utterance', dashes=['', '', '', (1,1)])

plt.ylabel("Probability of Utterance", size=18)
plt.legend(loc='best', fontsize=12)

for x in [0, .25, .5, .75, 1]:
    plt.axhline(x, alpha=.2, c='k', linestyle='--', zorder=0)

plt.yticks([0, .25, .5, .75, 1], fontsize=15);
plt.xticks(range(1, 11), fontsize=15);
plt.xlabel("Speaker Horizon $$$", fontsize=18)
plt.xlim(1, 10)

```

```

/var/folders/gv/42lb0z1j4dx3f3wsk74nrwx80000gn/T/ipykernel_60994/908865511.py:11
: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

```

See the caveats in the documentation: <https://pandas.pydata.org/pandas->

```
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
green["Utterance"] = green_utt
/var/folders/gv/42lb0z1j4dxf3wsk74nrwx80000gn/T/ipykernel_60994/908865511.py:12
: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead
```

See the caveats in the documentation: [https://pandas.pydata.org/pandas-docs/stable/user\\_guide/indexing.html#returning-a-view-versus-a-copy](https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy)

```
blue["Utterance"] = blue_utt
/var/folders/gv/42lb0z1j4dxf3wsk74nrwx80000gn/T/ipykernel_60994/908865511.py:13
: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead
```

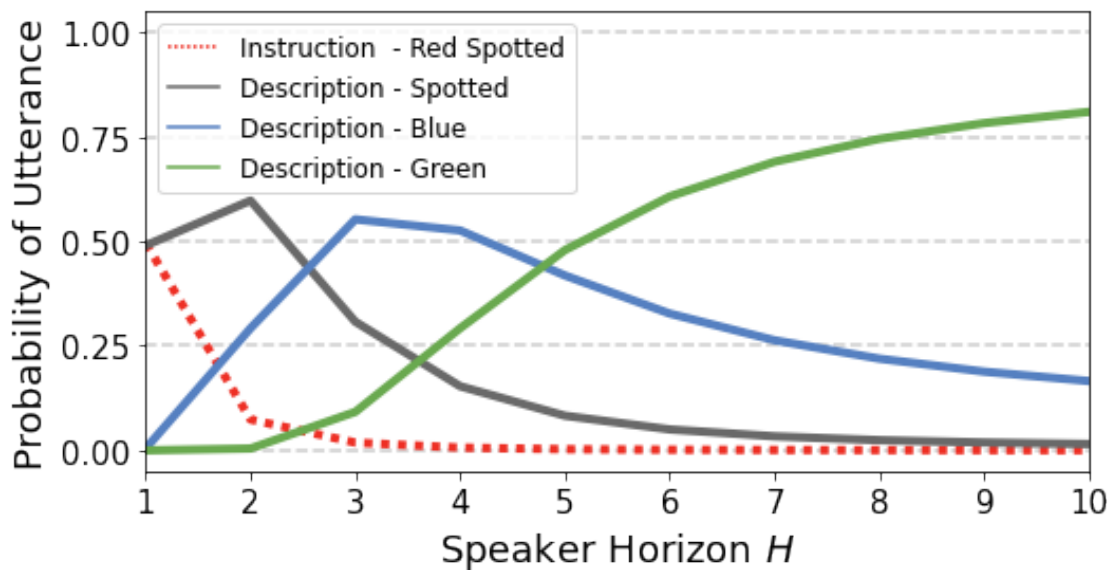
See the caveats in the documentation: [https://pandas.pydata.org/pandas-docs/stable/user\\_guide/indexing.html#returning-a-view-versus-a-copy](https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy)

```
circle["Utterance"] = spotted_utt
/var/folders/gv/42lb0z1j4dxf3wsk74nrwx80000gn/T/ipykernel_60994/908865511.py:14
: SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead
```

See the caveats in the documentation: [https://pandas.pydata.org/pandas-docs/stable/user\\_guide/indexing.html#returning-a-view-versus-a-copy](https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy)

```
instruction["Utterance"] = instruct
```

[12]: (1.0, 10.0)



### 3.1.3 Multi-context: instruction-to-description

```
[13]: from configuration import UTTERANCES, ALL_STATES

from literal_listener import StatelessLiteralListener
from literal_speaker import LiteralSpeaker

def uttProbabilityS1(speaker_horizons, utterance_sets, contexts=ALL_STATES,
    ↪alphaS=10):

    results = []
    listener = StatelessLiteralListener()
    for utt_set in utterance_sets:

        for h in speaker_horizons:

            utt_probabilities = [0] * len(UTTERANCES[utt_set])
            speaker = LiteralSpeaker(listener=listener, utterances=utt_set,
    ↪alphaS=alphaS)

            for c in contexts:
                context_utt_probabilities = speaker.
    ↪all_utterance_probabilities(c, horizon=h)
                for i, u in enumerate(context_utt_probabilities):
                    utt_probabilities[i] += u

            df = pd.DataFrame(UTTERANCES[utt_set])
            df["horizon"] = h
            df["utterances"] = utt_set
            df["probs"] = utt_probabilities
            df["probs"] = df["probs"] / len(contexts)
            results.append(df)

    return pd.concat(results)
```

### 3.1.4 Increasing Context Size

```
[14]: from configuration import ACTIONS

action_context_string = "Action Context Size |S|"

res = []
for size in range(2, 9):
    contexts = [list(l) for l in itertools.combinations(ACTIONS, size)]
    new_data = uttProbabilityS1(range(1, 11), ["all"], contexts=contexts)
    new_data[action_context_string] = size
    res.append(new_data)
```

```

df = pd.concat(res)

# type_by_horizon = df.groupby(["horizon", "Action Context Size", "type"]).
#   ↳ probs.sum().reset_index()
# sns.lineplot(data=type_by_horizon, x='horizon', y='probs', hue='Action_
#   ↳ Context Size', style='type')

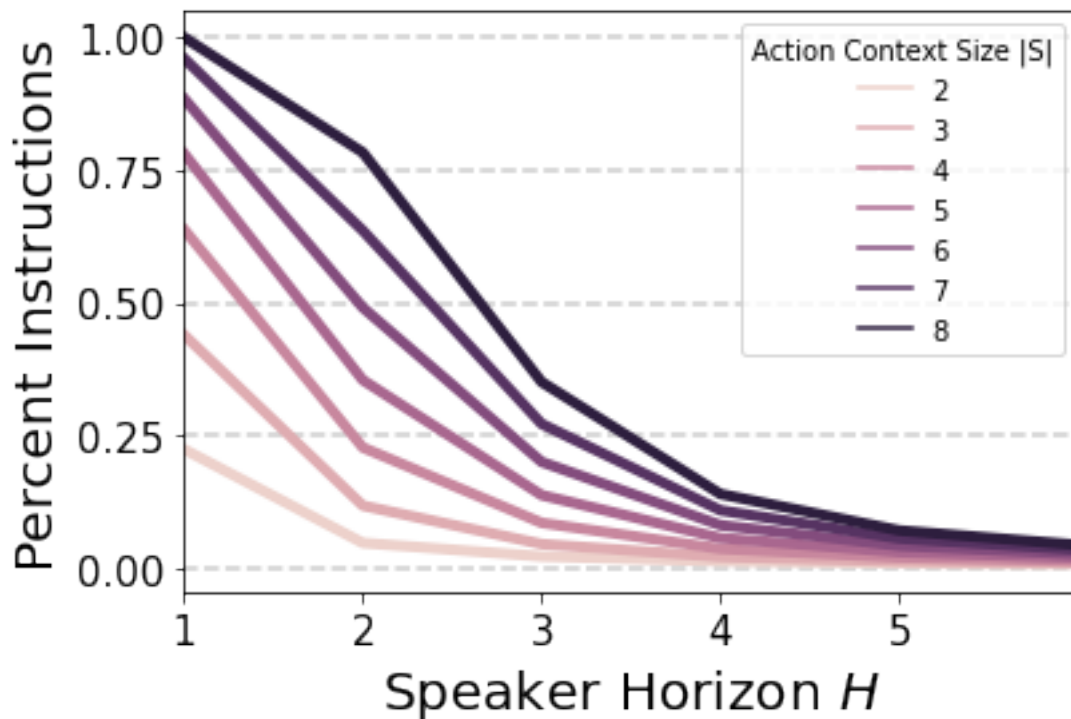
type_by_horizon = df[df["type"] == "instruction"].groupby(["horizon",
#   ↳ action_context_string]).probs.sum().reset_index()
sns.lineplot(data=type_by_horizon, x='horizon', y='probs',
#   ↳ hue=action_context_string, linewidth=4)

plt.xticks(range(0, 6), fontsize=15);
plt.ylabel("Percent Instructions", fontsize=20)
plt.xlabel("Speaker Horizon  $H$ ", fontsize=20)
# plt.legend(fontsize=15)

plt.xlim(1, 6)

ys = [0, .25, .5, .75, 1.0]
for y in ys:
    plt.axhline(y, alpha=.2, c='k', linestyle='--', zorder=0)
plt.yticks(ys, fontsize=15);

```





## 3.2 Utterance Utility

```
[15]: def literalUtilityFromS1(speaker_horizons, utterance_sets, contexts=ALL_STATES):

    results = []
    for utt_set in utterance_sets:

        for h in speaker_horizons:

            global_lit_rewards = 0
            local_lit_rewards = 0
            pct_instructions = 0
            pct_lies = 0

            speaker = LiteralSpeaker(listener=listener, utterances=utt_set)

            for c in contexts:
                utt_probabilities = speaker.all_utterance_probabilities(c,
↪horizon=h)

                for u, p in zip(UTTERANCES[utt_set], utt_probabilities):
                    if u["type"] == "instruction":
                        pct_instructions += p
                    if u["type"] == "description" and
↪TRUE_REWARDS[u["feature"]] != u["value"]:
                        pct_lies += p
                        global_lit_rewards += p * listener.future_rewards(u, None,
↪TRUE_REWARDS)
                        local_lit_rewards += p * listener.present_rewards(u, c,
↪TRUE_REWARDS)

                results.append({"global": global_lit_rewards/len(contexts),
                              "local": local_lit_rewards/len(contexts),
                              "pct_instructions": pct_instructions /
↪len(contexts),
                              "pct_lies": pct_lies / len(contexts),
                              "Utterance Set": utt_set,
                              "horizon": h})

    return pd.DataFrame(results)

[16]: size_three_contexts = [list(l) for l in itertools.combinations(ACTIONS, 3)]
size_five_contexts = [list(l) for l in itertools.combinations(ACTIONS, 5)]

contexts_to_use = size_three_contexts
```

```
[17]: res = literalUtilityFromS1(range(1, 11), ["instructions", "descriptions",
    ↪ "all"], contexts=contexts_to_use)
res["objective_utility"] = ((res["horizon"] - 1) * res["global"] +
    ↪ res["local"])/res["horizon"]

[18]: rename_dict = {"instructions": "Instructions", "descriptions": "Descriptions",
    ↪ "all": "All Utterances"}

res["Utterance Set"] = res["Utterance Set"].apply(lambda x: rename_dict[x])

[19]: grilled_cheese = res.melt(id_vars=["Utterance Set", "horizon"],
    value_vars=["global", "objective_utility", "local"],
    var_name="Reward Type", value_name="rewards")

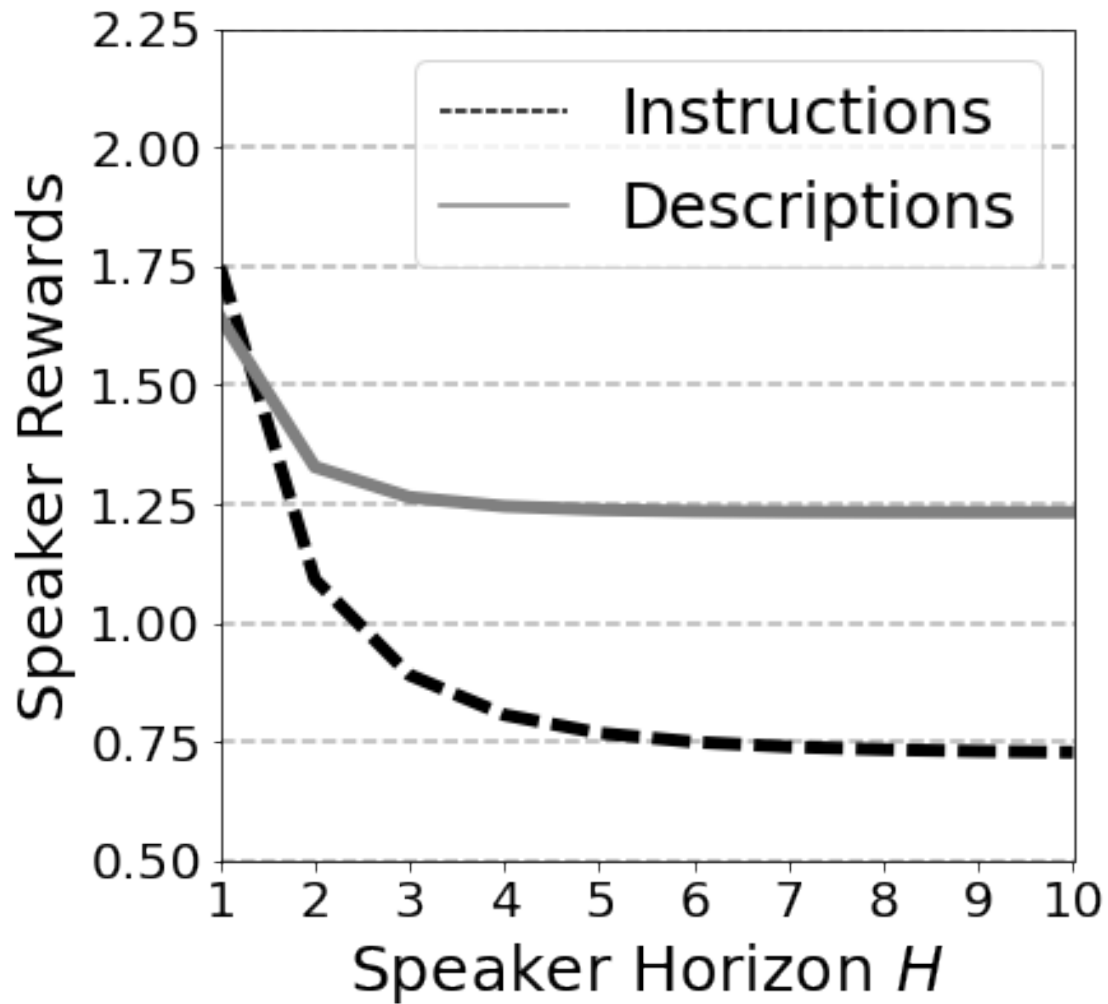
[20]: to_plot = grilled_cheese[grilled_cheese["Utterance Set"] != "All Utterances"]

plt.figure(figsize=(6, 6))
g = sns.lineplot(data=to_plot[to_plot['Reward Type'] == "objective_utility"],
    ↪ x='horizon', y='rewards', hue="Utterance Set",
    alpha=1,
    linewidth=5,
    hue_order=["Instructions", "Descriptions"],
    palette=["k", "gray"],
    style="Utterance Set", dashes=[(3,1), ''])

# plt.ylim(.4, 2)
ys = [.5, .75, 1, 1.25, 1.5, 1.75, 2, 2.25]

plt.yticks(ys, fontsize=20)
plt.xticks(fontsize=20)
# plt.tick_params(axis='both', which='major', labelsize=15)
plt.xlim(1, 10)
for y in ys:
    plt.axhline(y, c='k', alpha=.3, linestyle='--', zorder=0)
plt.legend(loc='best', fontsize=25)
plt.ylabel("Speaker Rewards", fontsize=25)
plt.xlabel("Speaker Horizon $H$", fontsize=25)

[20]: Text(0.5, 0, 'Speaker Horizon $H$')
```



```
[21]: to_plot = grilled_cheese[grilled_cheese["Utterance Set"] != "All Utterances"]

g = sns.relplot(data=to_plot, x='horizon', y='rewards', hue="Utterance Set",
                col='Reward Type', col_order=["objective_utility", "local", "global"], kind='line',
                alpha=1,
                linewidth=5,
                hue_order=["Instructions", "Descriptions"],
                palette=["k", "gray"],
                style="Utterance Set", dashes=[(3,1), ''])

for i, ax in enumerate(g.axes.flat): # set every-other axis for testing purposes
    if i == 0:
```

```

ax.set_ylabel("Speaker Rewards", fontsize=20)
ax.set_title("Horizon-Weighted Rewards (Eq. 7)", fontsize=18)

if i == 1:
    ax.set_title("Present Rewards (Eq. 5)", fontsize=18)

if i == 2:
    ax.set_title("Future Rewards (Eq. 6)", fontsize=18)

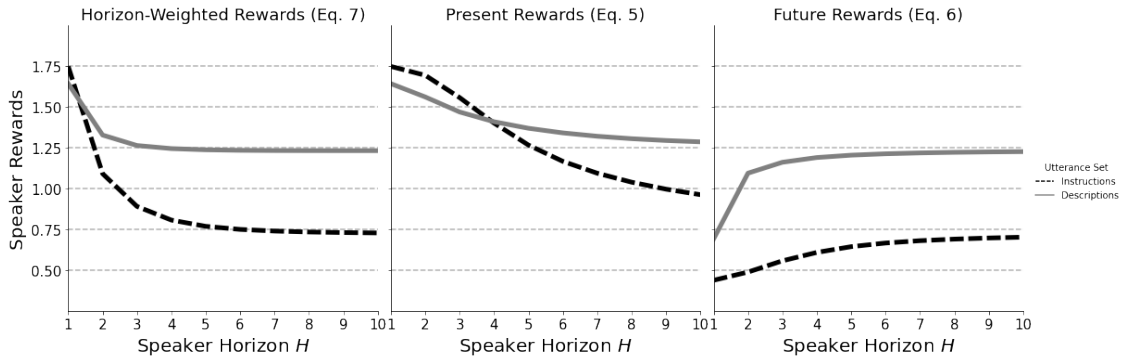
ax.set_xlabel("Speaker Horizon  $H$ ", fontsize=20)
ax.set_xticks(range(1,11))

if contexts_to_use == size_three_contexts:
    ax.set_ylim(.25, 2)
    ys = [.5, .75, 1, 1.25, 1.5, 1.75]

else:
    ax.set_ylim(.25, 2.5)
    ys = [.5, .75, 1, 1.25, 1.5, 1.75, 2, 2.25]

ax.set_yticks(ys)
ax.tick_params(axis='both', which='major', labelsize=15)
ax.set_xlim(1, 10)
for y in ys:
    ax.axhline(y, c='k', alpha=.3, linestyle='--', zorder=0)

```



```

[22]: all_utterances = res[res["Utterance Set"] == "All Utterances"]
all_utterances["pct_descriptions"] = 1 - all_utterances.pct_instructions

by_utt_type = all_utterances.melt(id_vars=["horizon"],
                                value_vars=["pct_descriptions",
                                ↪ "pct_instructions"],
                                var_name="Utterance Type",
                                ↪ value_name="Percent of Utterances")

```

```

rename_dict = {"pct_descriptions": "Descriptions", "pct_instructions": "Instructions"}
by_utt_type["Utterance Type"] = by_utt_type["Utterance Type"].apply(lambda x: rename_dict[x])

plt.figure(figsize=(6, 6))
g = sns.lineplot(data=by_utt_type, x='horizon', y='Percent of Utterances', hue="Utterance Type",
                  alpha=1,
                  linewidth=5,
                  hue_order=["Instructions", "Descriptions"],
                  palette=["gray", "k"],
                  style="Utterance Type", dashes=[(3,1), ''])

ys = [.25, .5, .75, 1]

plt.yticks(ys, fontsize=20)
plt.xticks(fontsize=20)

for y in ys:
    plt.axhline(y, c='k', alpha=.3, linestyle='--', zorder=0)
plt.legend(loc='best', fontsize=16)
plt.ylabel("Percent of Utterances", fontsize=20)
plt.xlabel("Speaker Horizon $$", fontsize=20)

```

/var/folders/gv/42lb0z1j4dx3wsk74nrwx80000gn/T/ipykernel\_60994/2820617515.py:2

: SettingWithCopyWarning:

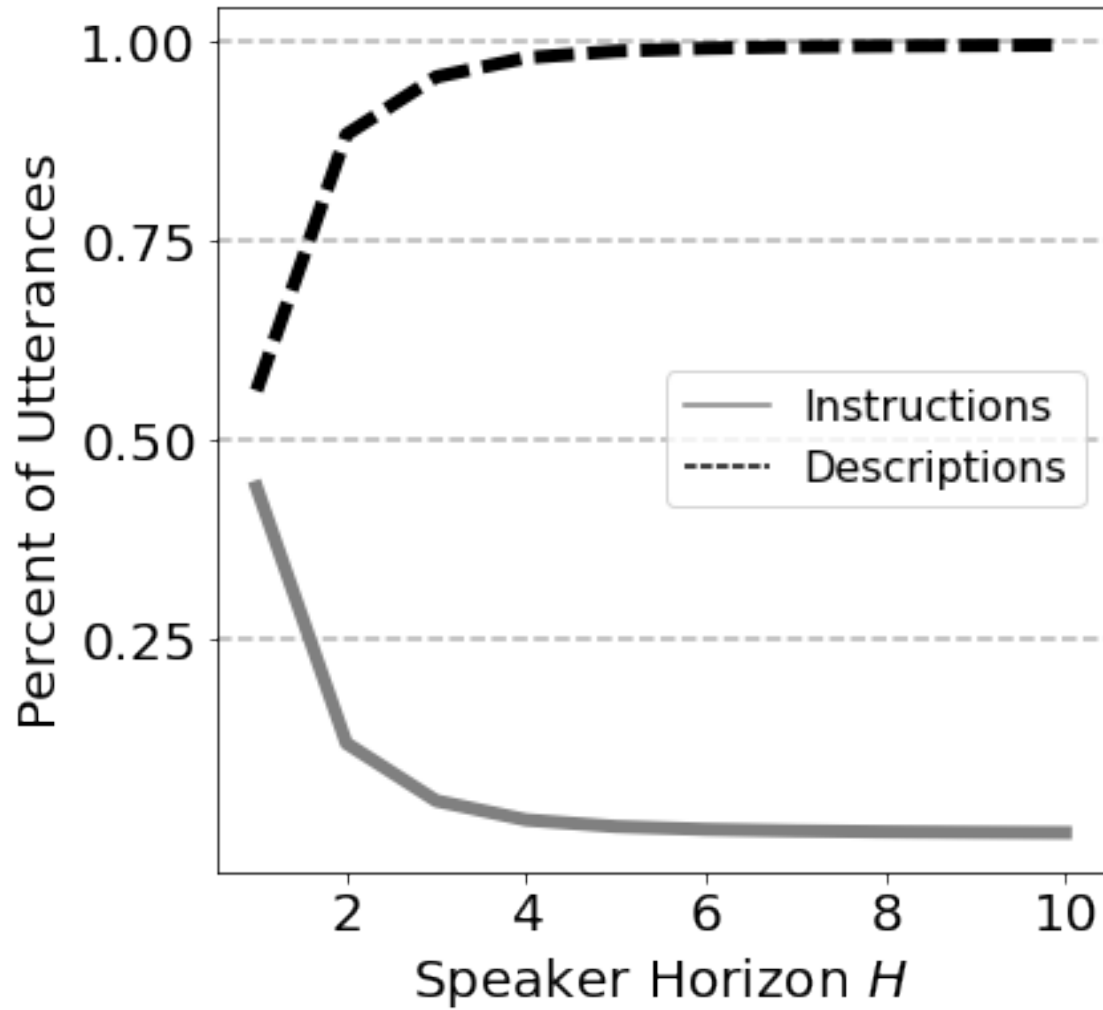
A value is trying to be set on a copy of a slice from a DataFrame.

Try using .loc[row\_indexer,col\_indexer] = value instead

See the caveats in the documentation: [https://pandas.pydata.org/pandas-docs/stable/user\\_guide/indexing.html#returning-a-view-versus-a-copy](https://pandas.pydata.org/pandas-docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy)

```
all_utterances["pct_descriptions"] = 1 - all_utterances.pct_instructions
```

[22]: Text(0.5, 0, 'Speaker Horizon \$\$')



## 4 Pragmatic Listener

### 4.1 Beliefs from Utterances

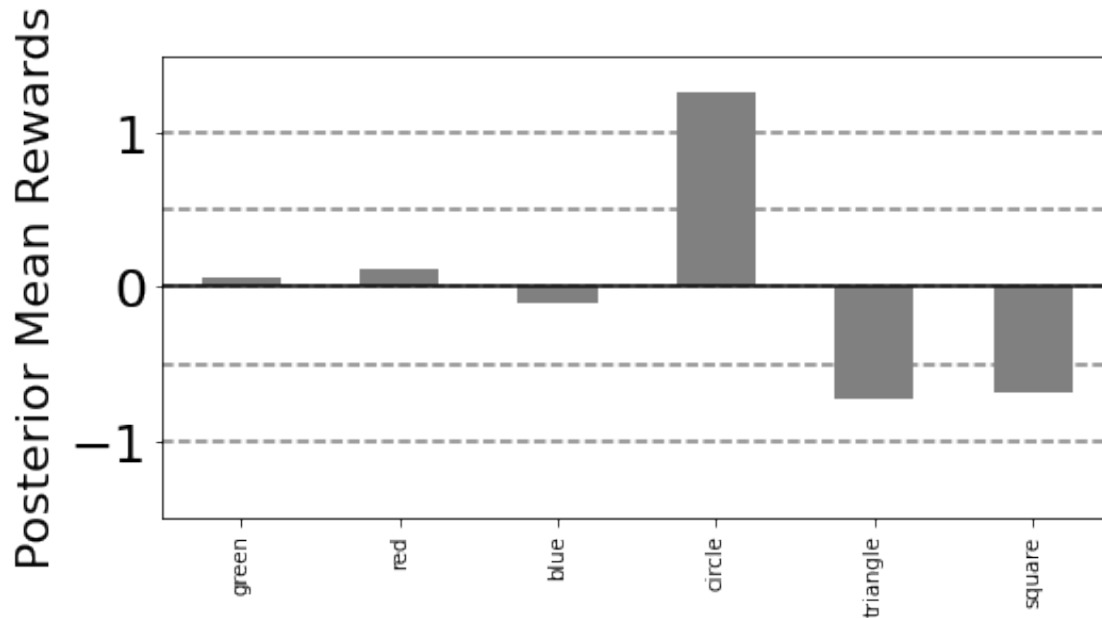
```
[23]: from pragmatic_listener import PragmaticListener
      from visualizations import plot_point_estimate, plot_full_posterior,
      ↪ plot_horizon_estimate

      base_listener = StatelessLiteralListener(alphaL=3)
      base_speaker = LiteralSpeaker(listener=base_listener, alphaS=10,
      ↪ utterances="all")

      pragmatic_listener = PragmaticListener(base_speaker)
```

```
[24]: description_to_use = {"type": "description", "feature": "circle", "value": 1}

description_posterior = pragmatic_listener.inference(description_to_use,
↳TEST_CONTEXT, horizon=[1, 2, 3, 4, 5, 10])
point_estimate = pragmatic_listener.
↳point_estimate_from_posterior(description_posterior)
plot_point_estimate(point_estimate)
```



#### 4.1.1 Inference over horizon

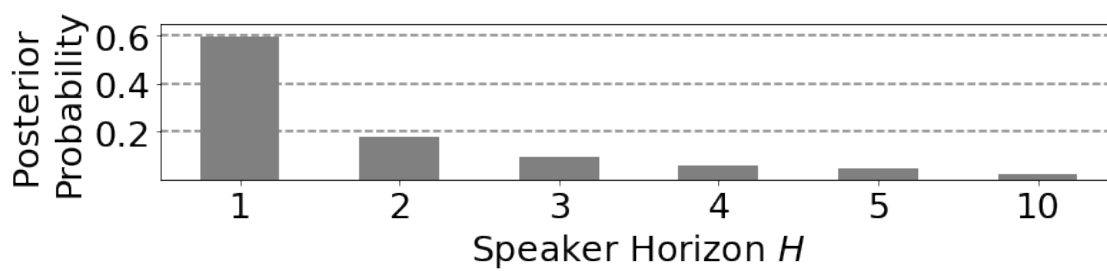
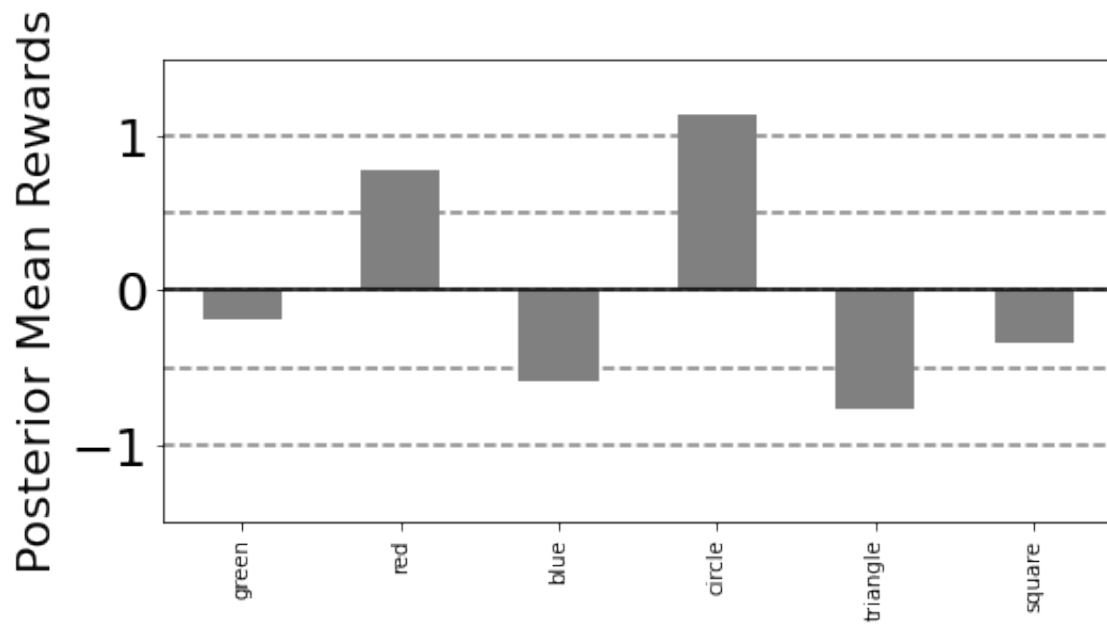
```
[25]: from configuration import FEATURES

theory_to_mushrooms = {"red": "Red", "green": "Green", "blue": "Blue",
↳"circle": "Spotted", "triangle": "Solid", "square":
↳"Striped"}

MUSHROOM_FEATURES = [theory_to_mushrooms.get(k) for k in FEATURES]
```

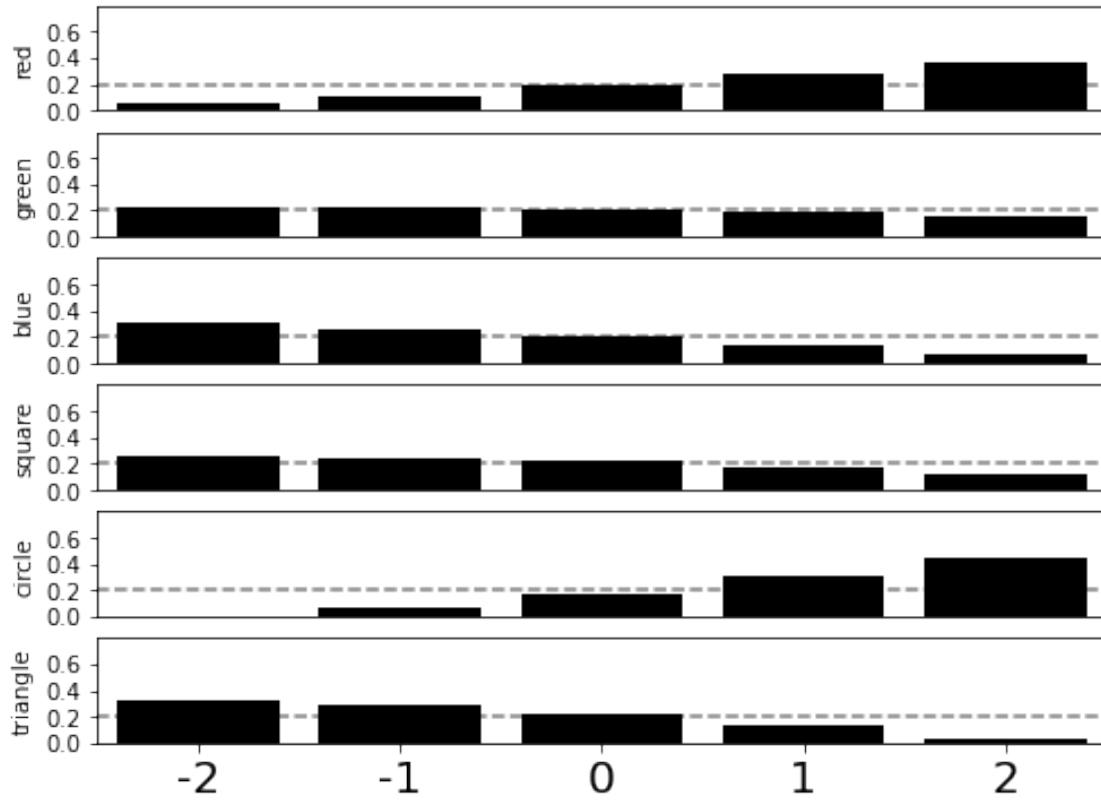
```
[26]: instruction = {"type": "instruction", "color": "red", "shape": "circle"}
reward_belief_df = pragmatic_listener.inference(instruction, TEST_CONTEXT,
↳horizon=[1, 2, 3, 4, 5, 10])

point_estimate = pragmatic_listener.
↳point_estimate_from_posterior(reward_belief_df)
plot_point_estimate(point_estimate)
plot_horizon_estimate(reward_belief_df)
```



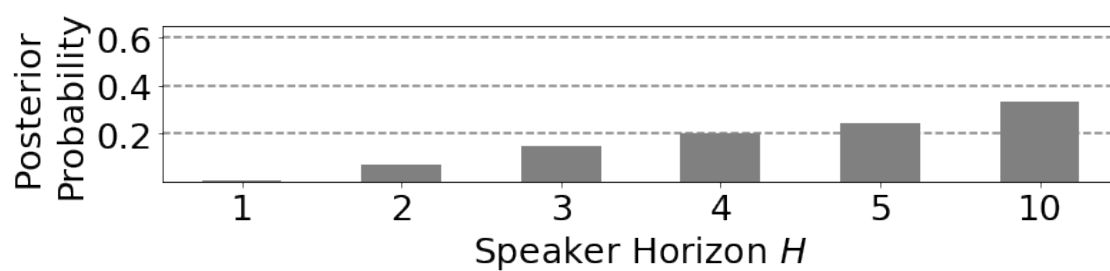
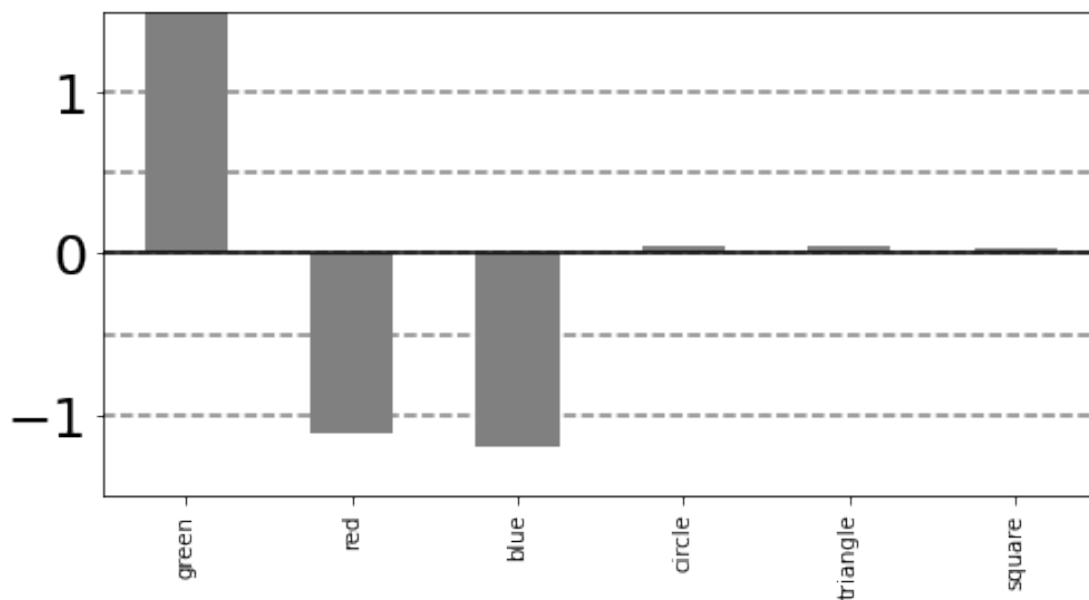
```
[27]: plot_full_posterior(reward_belief_df, ylabel=True)
```



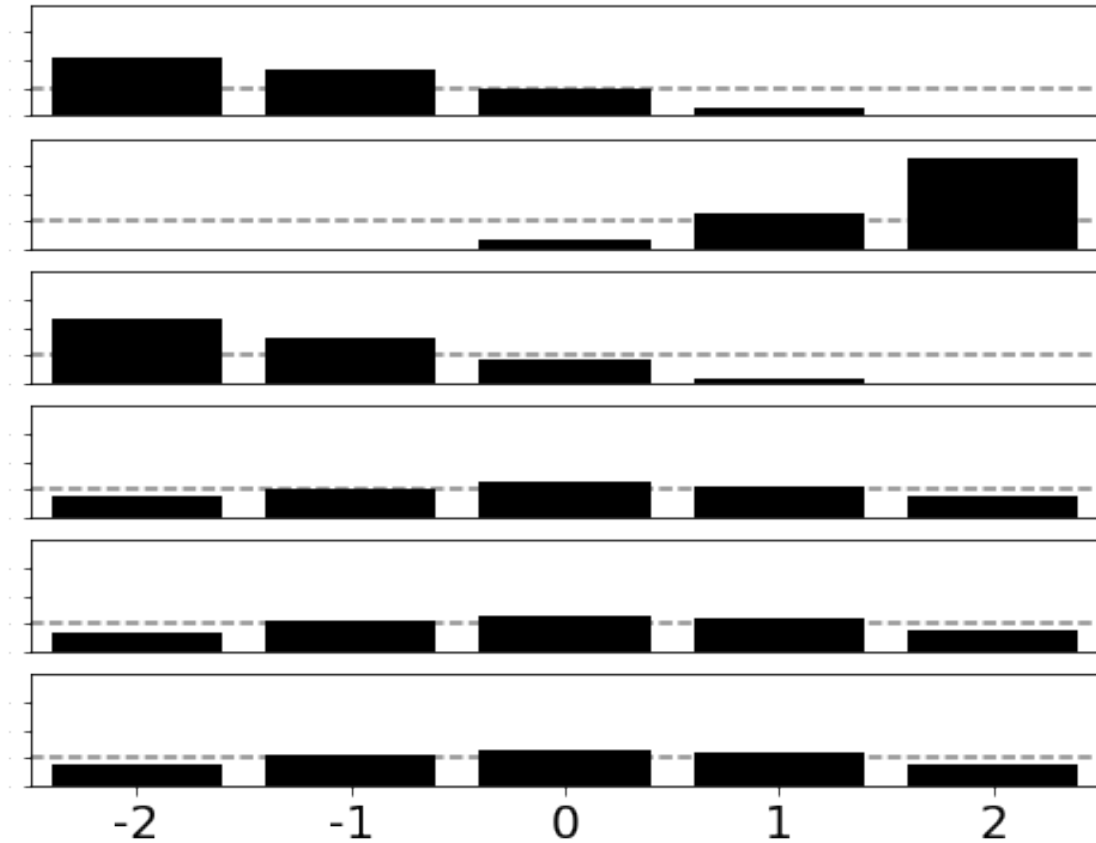


```
[28]: description = {"type": "description", "feature": "green", "value": 2}
reward_belief_df = pragmatic_listener.inference(description, TEST_CONTEXT,
↳ horizon=[1, 2, 3, 4, 5, 10])

point_estimate = pragmatic_listener.
↳ point_estimate_from_posterior(reward_belief_df)
plot_point_estimate(point_estimate, include_text=False)
plot_horizon_estimate(reward_belief_df, include_text=True, include_ticks=True)
```



```
[29]: plot_full_posterior(reward_belief_df, ylabel=False)
```



## 4.2 Cache Pragmatic Inference

```
[30]: import time
from os.path import exists

from configuration import ALL_STATES, UTTERANCES

##### Experimental settings #####
utterances_to_cache = "exp"
horizons = [1, 2, 4]
alphaSes = [3]

##### Theoretical settings #####
# utterances_to_cache = "all"
# horizons = [1, 2, 3, 4, 5, 10]
# alphaSes = [10]

for alphaS in alphaSes:
```

```

to_cache_listener = StatelessLiteralListener()
to_cache_speaker = LiteralSpeaker(listener=to_cache_listener,
↪utterances=utterances_to_cache, alphaS=alphaS)
to_cache_pragmatic_listener = PragmaticListener(speaker=to_cache_speaker)

for h in horizons:

    n_completed = 0
    n_in_horizon = len(ALL_STATES) * len(UTTERANCES[utterances_to_cache])
    print(f'Horizon {h}: {n_in_horizon} utterance-context pairs.')
    for c in ALL_STATES:
        for u in UTTERANCES[utterances_to_cache]:

            start_time_ms = round(time.time() * 1000)
            to_cache_pragmatic_listener.inference(u, c, h)
            end_time_ms = round(time.time() * 1000)

            n_seconds = (end_time_ms - start_time_ms)/1000
            if n_completed % 10 == 0 and n_seconds > 1:
                print("\tRan #{} in {:.2f} seconds.".format(n_completed,
↪n_seconds))
                n_completed += 1

```

Horizon 1: 2100 utterance-context pairs.  
Horizon 2: 2100 utterance-context pairs.  
Horizon 4: 2100 utterance-context pairs.

## 5 Simulations for Paper

```

[31]: def futureRewardsLiteralPragmatic(pragmatic_listener, speaker_horizon,
↪listener_horizons, utterance_set_name, contexts):
    """Given speaker / listener horizon(s) and utterances, return L0/L1 rewards.
    ↪"""

    pragmatic_rewards = 0
    literal_rewards = 0

    literal_listener = StatelessLiteralListener()

    for i, c in enumerate(contexts):

        utt_set = UTTERANCES[utterance_set_name]
        utt_probabilities = pragmatic_listener.speaker.
↪all_utterance_probabilities(c, horizon=speaker_horizon)

        for u, p in zip(utt_set, utt_probabilities):

```

```

        pragmatic_rewards += p * pragmatic_listener.future_rewards(u, c,
↪TRUE_REWARDS, listener_horizons)
        literal_rewards += p * literal_listener.future_rewards(u,
↪context=None, rewards=TRUE_REWARDS)

    return literal_rewards / len(contexts), pragmatic_rewards / len(contexts)

```

## 5.1 Simulation of literal / pragmatic listeners

### Config: Theoretical

- all utterances
- $\alpha_S = 10$
- horizon = [1-10]

```

[32]: # utterances_for_plot = "all"
      # horizons_for_plot = [1, 2, 3, 4, 5, 10]
      # alpha_S = 10

```

### Config: Experimental

- exp utterances
- $\alpha_S = 3$
- horizon = [1,2,4]

```

[33]: utterances_for_plot = "exp"
      horizons_for_plot = [1, 2, 4]
      alpha_S = 3

```

### Run various simulations

```

[34]: listener = StatelessLiteralListener()
      speaker = LiteralSpeaker(listener, utterances=utterances_for_plot,
↪alpha_S=alpha_S)
      pragmatic_listener = PragmaticListener(speaker)

```

### Known Horizon

```

[35]: results = []
      for h in horizons_for_plot:
          print("Running horizon {}".format(h))
          literal, pragmatic = futureRewardsLiteralPragmatic(pragmatic_listener, h,
↪h, utterances_for_plot, contexts=ALL_STATES)
          results.append({"horizon":h,
                        "alpha": a,
                        "literal": literal,
                        "pragmatic": pragmatic,

```

```

        "pragmatic_diff": pragmatic-literal})

aligned_df = pd.DataFrame(results)

```

Running horizon 1.  
Running horizon 2.  
Running horizon 4.

### Pedagogic Assumption ( $H = 4$ )

```

[36]: results = []
      for h in horizons_for_plot:
          literal, pragmatic = futureRewardsLiteralPragmatic(pragmatic_listener, h,
          ↪max(horizons_for_plot), utterances_for_plot, contexts=ALL_STATES)
          results.append({"horizon":h,
                          "literal": literal,
                          "pragmatic": pragmatic,
                          "pragmatic_diff": pragmatic-literal})

pedagogic_assumption_df = pd.DataFrame(results)

```

### Locally-optimal assumption ( $H = 1$ )

```

[37]: results = []
      for h in horizons_for_plot:
          literal, pragmatic = futureRewardsLiteralPragmatic(pragmatic_listener, h,
          ↪1, utterances_for_plot, contexts=ALL_STATES)
          results.append({"horizon":h,
                          "literal": literal,
                          "pragmatic": pragmatic,
                          "pragmatic_diff": pragmatic-literal})

conservative_listener = pd.DataFrame(results)

```

#### 5.1.1 Uncertain Pragmatic Listener

```

[38]: results = []
      for h in horizons_for_plot:
          literal, pragmatic = futureRewardsLiteralPragmatic(pragmatic_listener, h,
          ↪horizons_for_plot, utterances_for_plot, contexts=ALL_STATES)
          results.append({"horizon":h,
                          "literal": literal,
                          "pragmatic": pragmatic,
                          "pragmatic_diff": pragmatic-literal})

joint_inference_df = pd.DataFrame(results)

```

```

[39]: aligned_df["calibration"] = "Known Horizon"
pedagogic_assumption_df["calibration"] = "Pedagogic Assumption (H=4)"
conservative_listener["calibration"] = "Conservative Assumption (H=1)"
joint_inference_df["calibration"] = "Joint Inference"

full_df = pd.concat([aligned_df, pedagogic_assumption_df,
    ↪ conservative_listener, joint_inference_df])

[40]: full_df["horizon"] = full_df["horizon"].apply(lambda x: 7 if x == 10 else x)

plt.figure(figsize=(6,6))

known_horizon = full_df[full_df["calibration"] == "Known Horizon"]
plt.plot(known_horizon.horizon, known_horizon.literal, c='k', linewidth=4,
    ↪ alpha=1, label="Literal Listener")

plt.plot(known_horizon.horizon, known_horizon.pragmatic, c='k', linestyle='--',
    ↪ linewidth=4, alpha=.5, label="Pragmatic - Known $H$")

conservative = full_df[full_df["calibration"] == "Conservative Assumption
    ↪ (H=1)"]
plt.plot(conservative.horizon, conservative.pragmatic, c='r', linestyle='--',
    ↪ linewidth=4, alpha=.4, label="Pragmatic - $H$=1")

myopic_speaker = full_df[full_df["calibration"] == "Pedagogic Assumption (H=4)"]
plt.plot(myopic_speaker.horizon, myopic_speaker.pragmatic, c='orange',
    ↪ linestyle='--', linewidth=4, alpha=.4, label=f'Pragmatic -
    ↪ $H$={max(horizons_for_plot)}')

joint = full_df[full_df["calibration"] == "Joint Inference"]
plt.plot(joint.horizon, joint.pragmatic, c='g', linewidth=4, alpha=1,
    ↪ label="Pragmatic - Latent $H$")

plt.ylabel("Future Rewards", size=25)
plt.xlabel("Speaker Horizon $H$", fontsize=25)

if horizons_for_plot == [1, 2, 3, 4, 5, 10]:

    plt.xticks([1, 2, 3, 4, 5, 7], labels=[1, 2, 3, 4, 5, 10], size=20);
    plt.ylim(.45, 1.30)
    yticks = [.5, .75, 1, 1.25]
    plt.yticks(yticks, size=20)

else:

```

```

plt.xticks([1, 2, 4], size=20)
plt.ylim(.4, 1.1)
yticks = [.5, .75, 1]
plt.yticks(yticks, size=20)

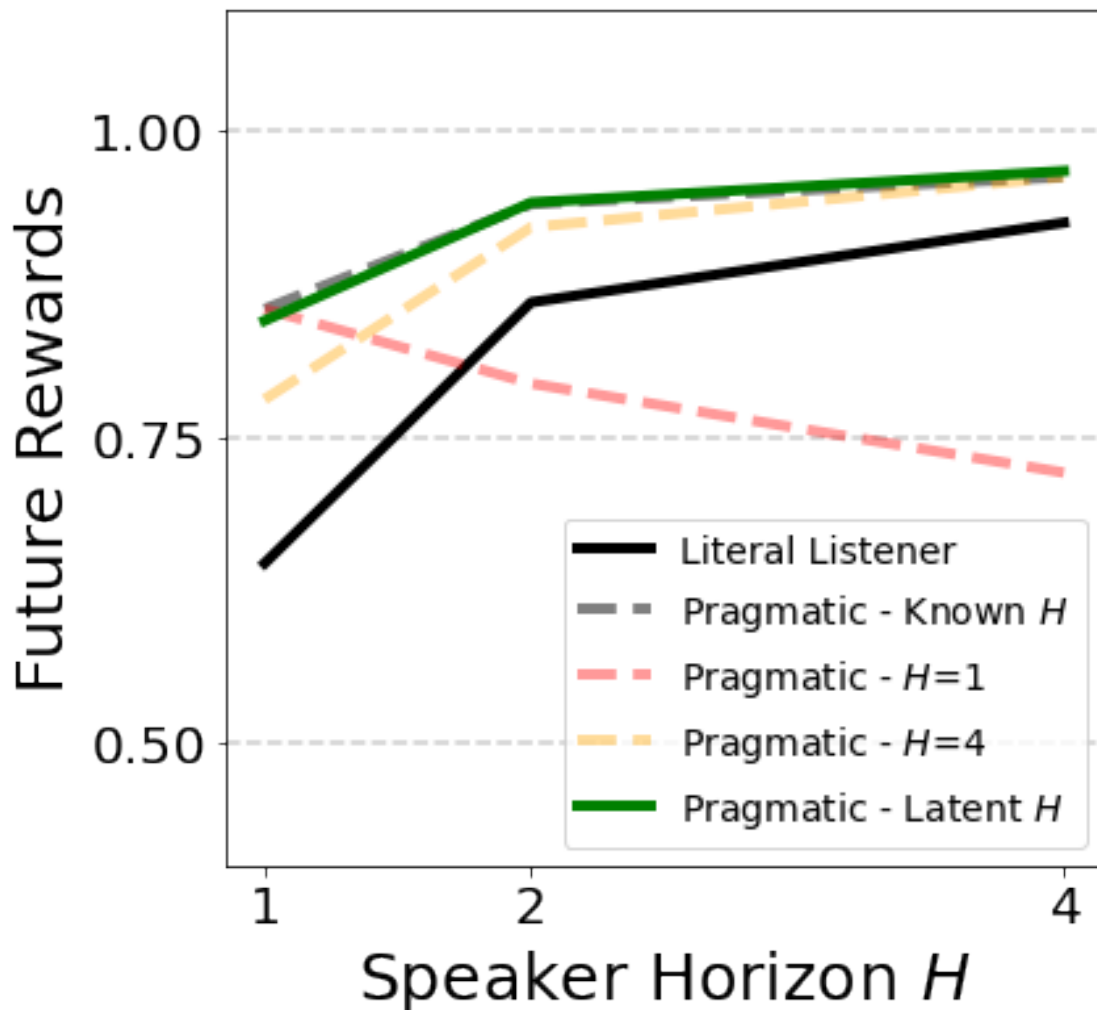
for y in yticks:
    plt.axhline(y, linestyle='--', c='k', alpha=.2, zorder=0)

# plt.show()

plt.legend(loc='best', fontsize=14)

```

[40]: <matplotlib.legend.Legend at 0x7f977bd4b880>



[41]: `full_df.groupby(['calibration', 'horizon']).pragmatic.mean()`



```
[41]: calibration
      Conservative Assumption (H=1)  1      0.854550
                                         2      0.793714
                                         4      0.720938
      Joint Inference                1      0.845070
                                         2      0.941477
                                         4      0.967242
      Known Horizon                  1      0.854550
                                         2      0.940189
                                         4      0.962383
      Pedagogic Assumption (H=4)     1      0.780738
                                         2      0.921340
                                         4      0.962383
      Name: pragmatic, dtype: float64
```

## 6 Behavioral Data

```
[42]: human_utterances = json.load(open("data/exp_utterances.json"))
```

### 6.1 Analysis of chosen utterances

```
[43]: def literalUtilityFromUtterances(human_utterances, contexts=ALL_STATES):

    results = []
    horizons = list(set([u["horizon"] for u in human_utterances]))

    for h in horizons:

        h_utterances = [u for u in human_utterances if u["horizon"] == h]
        global_lit_rewards = 0
        local_lit_rewards = 0
        instructions = 0
        lies = 0

        utterances = [d["utt"] for d in h_utterances]
        contexts = [d["action_context"] for d in h_utterances]
        for u, c in zip(utterances, contexts):

            if u["type"] == "instruction":
                instructions += 1
            if u["type"] == "description" and TRUE_REWARDS[u["feature"]] != 0:
                ↪u["value"]:
                    lies += 1

        global_lit_rewards += listener.future_rewards(u, None, TRUE_REWARDS)
        local_lit_rewards += listener.present_rewards(u, c, TRUE_REWARDS)
```

```

        results.append({"global": global_lit_rewards/len(h_utterances),
                        "local": local_lit_rewards/len(h_utterances),
                        "pct_instructions": instructions / len(h_utterances),
                        "pct_lies": lies / len(h_utterances),
                        "n_utterances": len(h_utterances),
                        "horizon": h})

    return pd.DataFrame(results)

```

```

[44]: res = literalUtilityFromUtterances(human_utterances)
res["objective_utility"] = (res["local"] + (res["horizon"]-1) * res["global"])/
    ↪res["horizon"]

```

```

[45]: grilled_cheese = res.melt(id_vars=["horizon"],
                               value_vars=["global", "objective_utility", "local"],
                               var_name="Reward Type", value_name="rewards")

```

## 6.2 Analysis of pragmatics

```

[46]: def futureRewardsFromExperiment(pragmatic_listener, human_utterances,
    ↪horizons=None):
    """Given speaker / listener horizon(s) and utterances, return L0/L1 rewards.
    ↪"""

    results = []
    literal_listener = StatelessLiteralListener()

    if horizons is None:
        horizons = list(set([u["horizon"] for u in human_utterances]))

    for h in horizons:

        h_utterances = [u for u in human_utterances if u["horizon"] == h]

        print(f"Horizon {h}: {len(h_utterances)} utterances.")

        for i, u in enumerate(h_utterances):

            u = copy.deepcopy(u)

            literal = literal_listener.future_rewards(u["utt"], context=None,
    ↪rewards=TRUE_REWARDS)

            pragmatic_aligned = pragmatic_listener.future_rewards(u["utt"],
    ↪u["action_context"], TRUE_REWARDS, h)

```

```

        pragmatic_conservative = pragmatic_listener.
↪future_rewards(u["utt"], u["action_context"], TRUE_REWARDS, 1)
        pragmatic_long_horizon = pragmatic_listener.
↪future_rewards(u["utt"], u["action_context"], TRUE_REWARDS, 4)
        pragmatic_uncertain = pragmatic_listener.future_rewards(u["utt"],
↪u["action_context"], TRUE_REWARDS, horizons)

        uncertain_posterior = pragmatic_listener.inference(u["utt"],
↪u["action_context"], horizons)
        horizon_estimate = uncertain_posterior.
↪multiply(uncertain_posterior["probability"], axis='index').apply(np.
↪sum)["horizon"]
        point_estimate = pragmatic_listener.
↪point_estimate_from_posterior(uncertain_posterior)

        u["literal"] = literal
        u["pragmatic_aligned"] = pragmatic_aligned
        u["pragmatic_uncertain"] = pragmatic_uncertain

        u["pragmatic_conservative"] = pragmatic_conservative
        u["pragmatic_pedagogic"] = pragmatic_long_horizon

        u["horizon_estimate"] = horizon_estimate
        u["point_estimate"] = point_estimate

        results.append(u)

    return results

```

```

[47]: alphaS = 3

all_results = []

literal = StatelessLiteralListener()
speaker = LiteralSpeaker(literal, utterances="exp", alphaS=alphaS)
pragmatic_listener = PragmaticListener(speaker)

results = futureRewardsFromExperiment(pragmatic_listener, human_utterances)

res = pd.DataFrame(results)
res["alphaS"] = alphaS
all_results.append(res)

res = pd.concat(all_results)

```

Horizon 1: 939 utterances.  
Horizon 2: 917 utterances.

Horizon 4: 916 utterances.

```
[48]: rename_dict = {"pragmatic_aligned": "Pragmatic - Known $$$",
                    "pragmatic_uncertain": "Pragmatic - Latent $$$",
                    "pragmatic_conservative": "Pragmatic - $$$=1",
                    "pragmatic_pedagogic": "Pragmatic - $$$=4",
                    "literal": "Literal Listener"}

to_plot_human_data = res.melt(id_vars="horizon", var_name="listener",
    ↪value_name="rewards",
                             value_vars=["literal", "pragmatic_aligned",
    ↪"pragmatic_uncertain", "pragmatic_conservative", "pragmatic_pedagogic"])

to_plot_human_data["listener"] = to_plot_human_data.listener.apply(lambda x:
    ↪rename_dict[x])

[49]: plt.figure(figsize=(6,6))

sns.lineplot(data=to_plot_human_data, x='horizon', y="rewards", hue="listener",
             ci=95, err_style='bars', err_kws={"capsize": 5},
             hue_order=["Literal Listener", "Pragmatic - Known $$$", "Pragmatic
    ↪- $$$=1", "Pragmatic - $$$=4", "Pragmatic - Latent $$$"],
             palette=["k", "k", "r", 'orange', "g"], linewidth=4, alpha=1,
    ↪style='listener', dashes=['', (3,1), '', (3,1), (3,1)])

plt.xticks([1, 2, 4], size=20)
plt.xlabel("Speaker Horizon $$$", fontsize=25)

yticks = [.5, .75, 1, 1.25]
plt.yticks(yticks, size=0)
for y in yticks:
    plt.axhline(y, linestyle='--', c='k', alpha=.2, zorder=0)

plt.ylabel("Future Rewards", fontsize=0)

ax_children = plt.gca().get_children()
print()

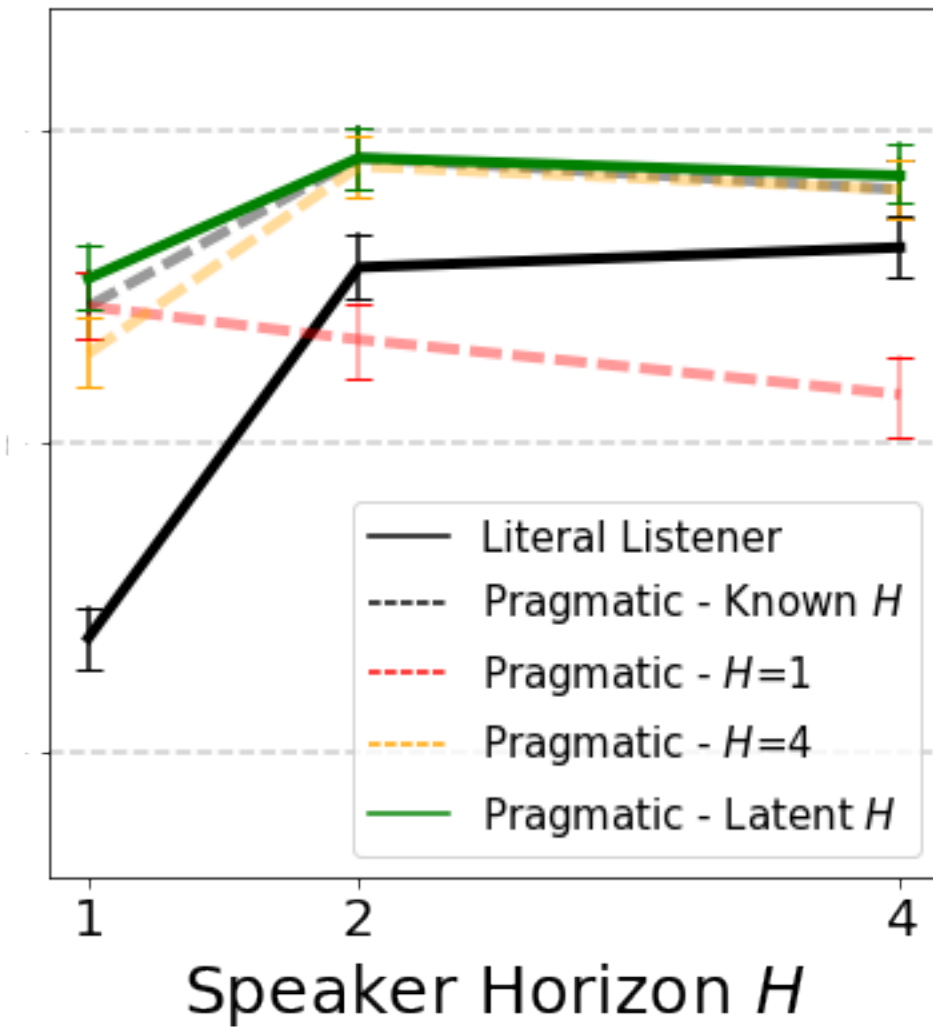
# pragmatic - known h
plt.setp([ax_children[9]],alpha=.4)

# pragmatic- h = 1
plt.setp([ax_children[2]],alpha=.4)
plt.setp([ax_children[13]],alpha=.4)

# pragmatic- h = 4
plt.setp([ax_children[17]],alpha=.4)
```

```
plt.legend(fontsize=15)
plt.ylim(.4, 1.1)
```

[49]: (0.4, 1.1)



```
[50]: from configuration import utt_to_string, context_to_string

res["item_key"] = res.apply(lambda x:
    ↳ f'{utt_to_string(x["utt"])}|{context_to_string(x["action_context"])}|{x["workerid"]}',
    ↳ axis=1)
```

### 6.2.1 Calculate means / export to R

```
[51]: res_to_export = res.drop(['action_context', 'workerid', 'horizon', 'utt',  
    ↪ 'horizon_estimate', 'point_estimate', 'alphaS', "item_key"], axis=1)  
res_to_export.to_csv("utterance_posterior_rewards.csv", index=False)
```

```
[52]: res[['literal', 'pragmatic_uncertain', 'pragmatic_aligned',  
    ↪ 'pragmatic_conservative', 'pragmatic_pedagogic']].describe().round(2)
```

```
[52]:
```

	literal	pragmatic_uncertain	pragmatic_aligned \
count	2772.00	2772.00	2772.00
mean	0.79	0.94	0.93
std	0.41	0.39	0.40
min	-0.75	-1.34	-1.34
25%	0.50	0.64	0.65
50%	0.75	1.02	1.03
75%	1.28	1.25	1.26
max	1.28	1.58	1.60

	pragmatic_conservative	pragmatic_pedagogic
count	2772.00	2772.00
mean	0.83	0.91
std	0.46	0.40
min	-1.36	-1.32
25%	0.47	0.63
50%	0.90	1.05
75%	1.20	1.25
max	1.60	1.48

### 6.3 Appendix E: Pragmatic Inference Details

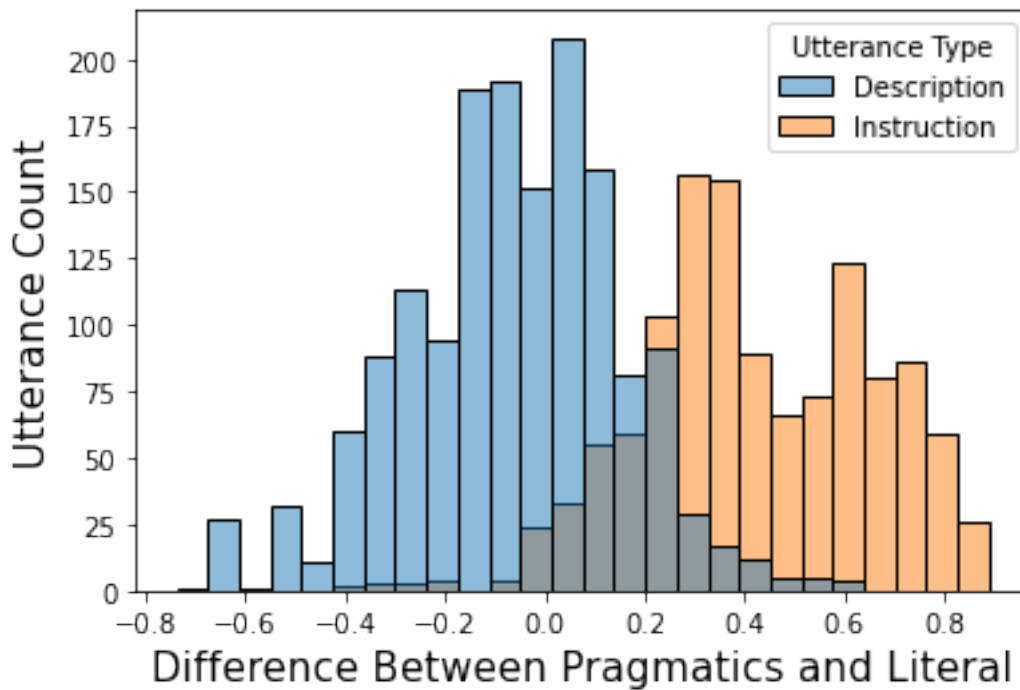
```
[53]: res["instruct_color"] = res.utt.apply(lambda x: x.get("color"))  
res["instruct_shape"] = res.utt.apply(lambda x: x.get("shape"))  
res["descript_feature"] = res.utt.apply(lambda x: x.get("feature",  
    ↪ "instruction"))  
res["descript_value"] = res.utt.apply(lambda x: x.get("value"))  
res["instruction"] = res.utt.apply(lambda x: x.get("type") == "instruction")
```

```
[54]: res["pragmatic_aligned_diff"] = res.pragmatic_aligned - res.literal  
res["pragmatic_uncertain_diff"] = res.pragmatic_uncertain - res.literal
```

```
[55]: res["Utterance Type"] = res.instruction.apply(lambda x: "Instruction" if x else  
    ↪ "Description")
```

```
[56]: sns.histplot(data=res, hue="Utterance Type", x='pragmatic_uncertain_diff')  
plt.xlabel("Difference Between Pragmatics and Literal", fontsize=15)  
plt.ylabel("Utterance Count", fontsize=15)
```

```
[56]: Text(0, 0.5, 'Utterance Count')
```



```
[57]: res.groupby("Utterance Type")[["pragmatic_uncertain_diff"]].describe().round(3)
```

```
[57]:
```

	pragmatic_uncertain_diff	
	count	mean
Utterance Type		
Description	1569.0	-0.067
Instruction	1203.0	0.423
		std
		min
		25%
		50%
		75%
		max
Utterance Type		
Description	0.076	0.617
Instruction	0.616	0.890

```
[58]: pragmatic_difference_descriptions = res[~res.instruction].
      ↪pragmatic_uncertain_diff
      stats.ttest_1samp(pragmatic_difference_descriptions, 0)
```

```
[58]: Ttest_1sampResult(statistic=-12.288703528317031, pvalue=3.32739264533416e-33)
```

```
[59]: pragmatic_difference_instructions = res[res.instruction].
      ↪pragmatic_uncertain_diff
```

```
stats.ttest_1samp(pragmatic_difference_instructions, 0)
```

```
[59]: Ttest_1sampResult(statistic=63.34314897797516, pvalue=0.0)
```

```
[60]: res.groupby("descript_feature").pragmatic_uncertain_diff.describe().round(3)
```

```
[60]:
```

	count	mean	std	min	25%	50%	75%	max
descript_feature								
blue	389.0	-0.196	0.271	-0.644	-0.353	-0.236	-0.016	0.520
circle	213.0	0.083	0.147	-0.360	-0.003	0.070	0.188	0.468
green	860.0	-0.050	0.156	-0.402	-0.151	-0.049	0.050	0.378
instruction	1203.0	0.423	0.232	-0.594	0.268	0.395	0.616	0.890
square	107.0	-0.023	0.246	-0.739	-0.134	-0.043	0.076	0.617

```
[61]: res.groupby(["instruct_color", "instruct_shape"]).pragmatic_aligned_diff.  
      ↪describe()
```

```
[61]:
```

		count	mean	std	min	25%	\
instruct_color	instruct_shape						
blue	circle	16.0	0.464016	0.405353	-0.239676	0.109121	
	square	2.0	-0.519293	0.100403	-0.590288	-0.554790	
	triangle	4.0	-0.283082	0.171797	-0.526259	-0.337772	
green	circle	514.0	0.470636	0.275594	-1.213894	0.377254	
	square	152.0	0.431725	0.309199	-0.318704	0.200191	
	triangle	266.0	0.388986	0.280168	-0.500000	0.152600	
red	circle	142.0	0.397844	0.168106	-0.045301	0.275930	
	square	24.0	0.324693	0.194851	0.014863	0.147987	
	triangle	83.0	0.326271	0.177794	-0.235731	0.205801	

		50%	75%	max
instruct_color	instruct_shape			
blue	circle	0.557623	0.886528	0.886528
	square	-0.519293	-0.483795	-0.448297
	triangle	-0.234915	-0.180225	-0.136238
green	circle	0.492556	0.639280	0.851747
	square	0.385381	0.645880	1.023056
	triangle	0.337304	0.586262	0.948830
red	circle	0.397385	0.481486	0.687240
	square	0.326866	0.541763	0.541763
	triangle	0.303651	0.407212	0.625669

## 6.4 Appendix C: Choosing $\beta_{S_1}$

```
[62]: alphaTestResults = []  
      horizons = [1, 2, 4]  
      alphas = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
```



```

for alpha in alphas:

    print(f'AlphaS: {alpha}.')

    literal = StatelessLiteralListener()
    speaker = LiteralSpeaker(literal, utterances="exp", alphaS=alpha)
    pragmatic_listener = PragmaticListener(speaker)

    results = futureRewardsFromExperiment(pragmatic_listener, human_utterances)
    res = pd.DataFrame(results)
    res["alphaS"] = alpha
    alphaTestResults.append(res)

alphaTest = pd.concat(alphaTestResults)

```

```

AlphaS: 1.
Horizon 1: 939 utterances.
Horizon 2: 917 utterances.
Horizon 4: 916 utterances.
AlphaS: 2.
Horizon 1: 939 utterances.
Horizon 2: 917 utterances.
Horizon 4: 916 utterances.
AlphaS: 3.
Horizon 1: 939 utterances.
Horizon 2: 917 utterances.
Horizon 4: 916 utterances.
AlphaS: 4.
Horizon 1: 939 utterances.
Horizon 2: 917 utterances.
Horizon 4: 916 utterances.
AlphaS: 5.
Horizon 1: 939 utterances.
Horizon 2: 917 utterances.
Horizon 4: 916 utterances.
AlphaS: 6.
Horizon 1: 939 utterances.
Horizon 2: 917 utterances.
Horizon 4: 916 utterances.
AlphaS: 7.
Horizon 1: 939 utterances.
Horizon 2: 917 utterances.
Horizon 4: 916 utterances.
AlphaS: 8.
Horizon 1: 939 utterances.
Horizon 2: 917 utterances.
Horizon 4: 916 utterances.

```

AlphaS: 9.  
Horizon 1: 939 utterances.  
Horizon 2: 917 utterances.  
Horizon 4: 916 utterances.  
AlphaS: 10.  
Horizon 1: 939 utterances.  
Horizon 2: 917 utterances.  
Horizon 4: 916 utterances.

```
[63]: alphaTestToPlot = alphaTest

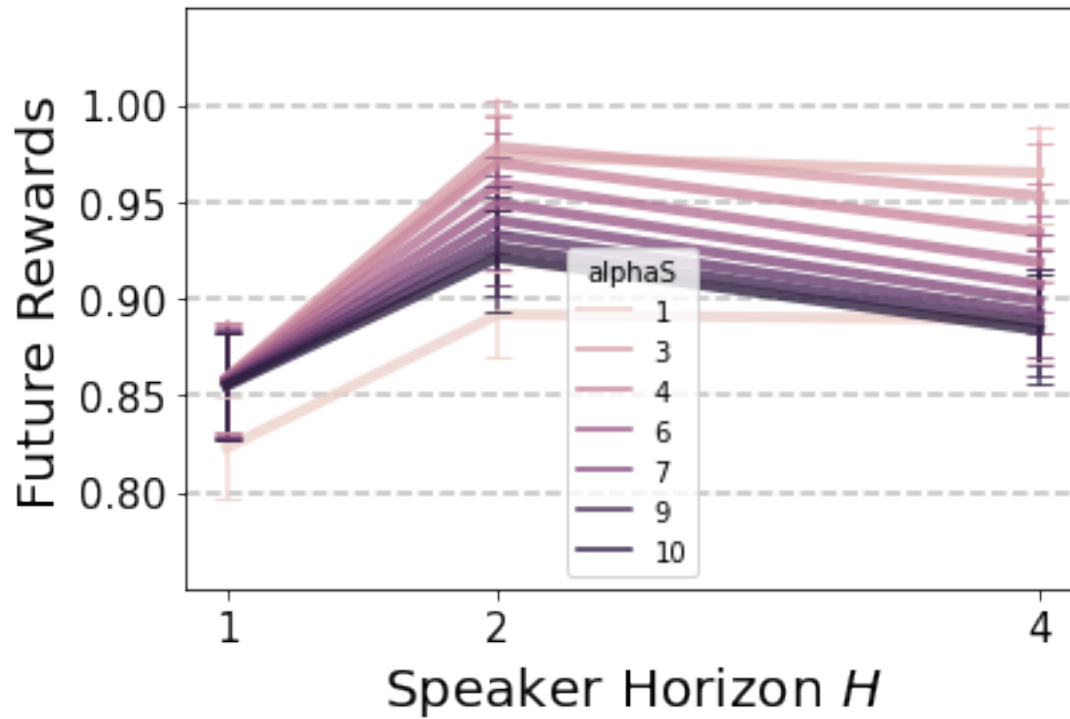
sns.lineplot(data=alphaTestToPlot, x='horizon', y='pragmatic_aligned',
             hue='alphaS', err_style='bars', err_kws={"capsize": 5},
             linewidth=4, alpha=.8)

ys = [.8, .85, .9, .95, 1]
for y in ys:
    plt.axhline(y, alpha=.25, linestyle='--', zorder=0, c='k')
plt.yticks(ys, fontsize=15)
plt.xticks([1, 2, 4], fontsize=15)

plt.ylim(.75, 1.05)

plt.xlabel("Speaker Horizon  $H$ ", fontsize=20)
plt.ylabel("Future Rewards", fontsize=20)
```

```
[63]: Text(0, 0.5, 'Future Rewards')
```



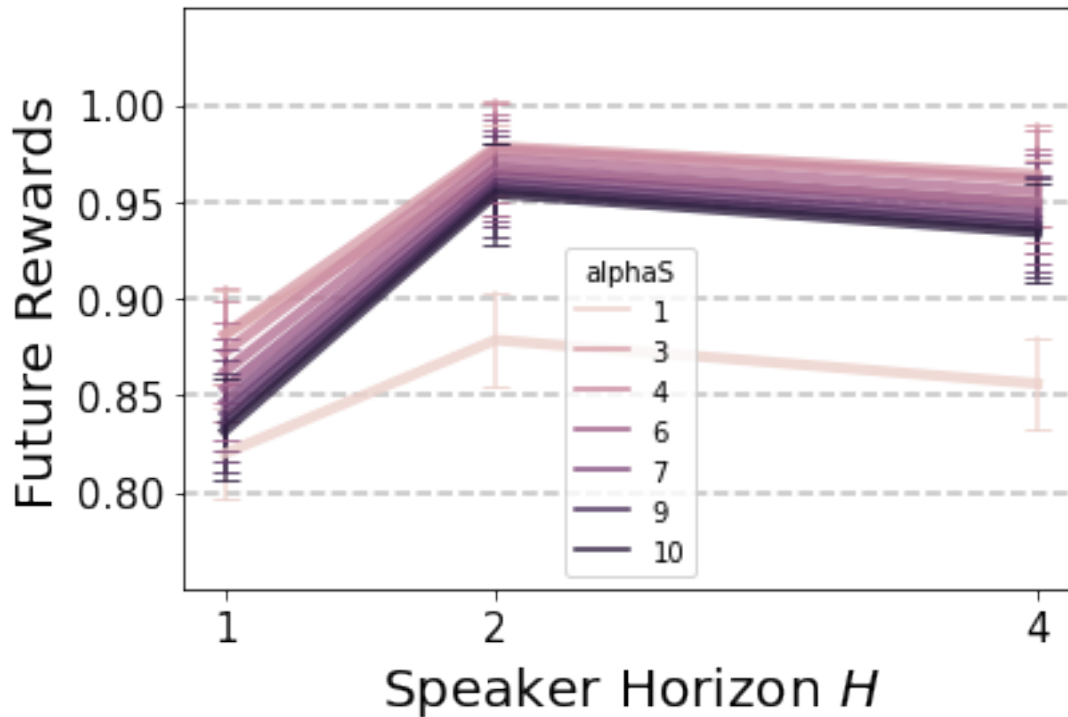
```
[64]: sns.lineplot(data=alphaTestToPlot, x='horizon', y='pragmatic_uncertain',
    ↪ hue='alphaS', err_style='bars', err_kws={"capsize": 5},
        linewidth=4, alpha=.8)

ys = [.8, .85, .9, .95, 1]
for y in ys:
    plt.axhline(y, alpha=.25, linestyle='--', zorder=0, c='k')
plt.yticks(ys, fontsize=15)
plt.xticks([1, 2, 4], fontsize=15)

plt.ylim(.75, 1.05)

plt.xlabel("Speaker Horizon  $H$ ", fontsize=20)
plt.ylabel("Future Rewards", fontsize=20)
```

```
[64]: Text(0, 0.5, 'Future Rewards')
```



```
[65]: melted = alphaTestToPlot.melt(id_vars=["alphaS"],
                                     value_vars=["pragmatic_aligned",
                                     ↪ "pragmatic_uncertain"],
                                     var_name="speaker", value_name="rewards")

rename_dict = {"pragmatic_aligned": "Known  $\$H\$$ ", "pragmatic_uncertain": "Latent
                                     ↪  $\$H\$$ "}
melted["speaker"] = melted.speaker.apply(lambda x: rename_dict[x])
```

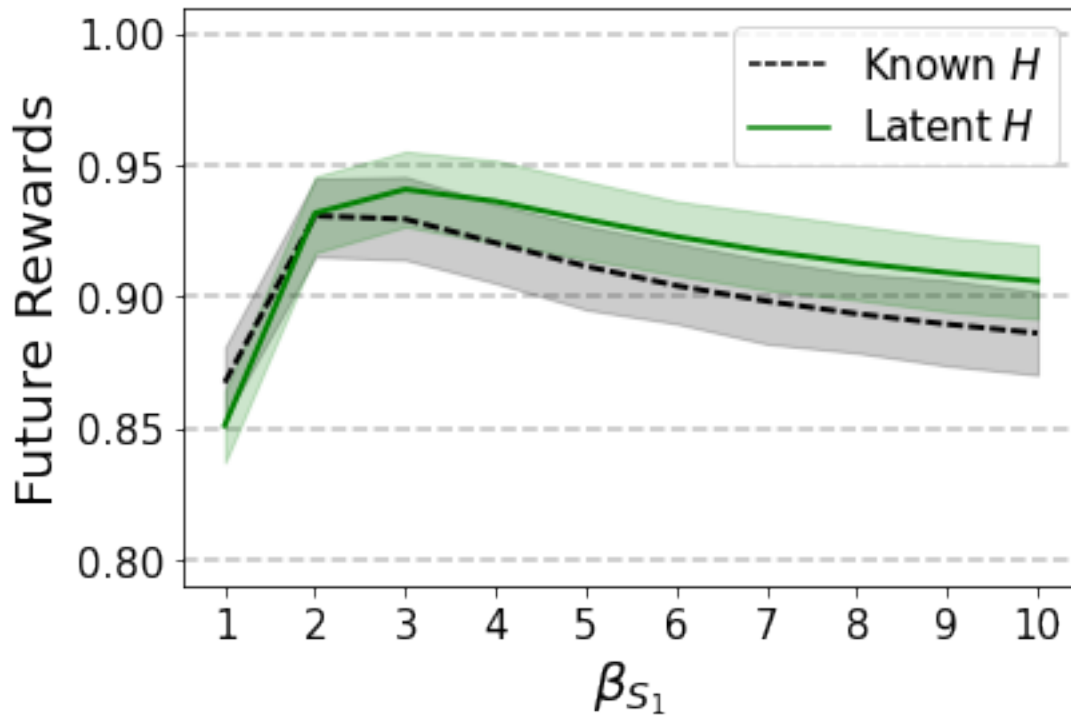
```
[66]: sns.lineplot(data=melted, x='alphaS', y='rewards', hue='speaker',
                   hue_order=["Known  $\$H\$$ ", "Latent  $\$H\$$ "],
                   palette=['k', 'g'],
                   linewidth=2, style='speaker', dashes=[(3,1), ''])

plt.legend(loc='best', fontsize=15)

ys = [.8, .85, .9, .95, 1]
for y in ys:
    plt.axhline(y, alpha=.25, linestyle='--', zorder=0, c='k')
plt.yticks(ys, fontsize=15)
plt.xticks(range(1, 11), fontsize=15);

plt.ylabel("Future Rewards", fontsize=20)
```

```
plt.xlabel(r"$\beta_{S_1}$", fontsize=20);
```



```
[67]: print(alphaTestToPlot.groupby(["alphaS"])[["pragmatic_aligned",
↪ "pragmatic_uncertain"]].mean().round(4).style.to_latex())
```

```
\begin{tabular}{lrr}
& pragmatic_aligned & pragmatic_uncertain \\
alphaS & & \\
1 & 0.867500 & 0.851200 \\
2 & 0.930800 & 0.931800 \\
3 & 0.929500 & 0.940800 \\
4 & 0.920500 & 0.936300 \\
5 & 0.911600 & 0.929400 \\
6 & 0.904300 & 0.922900 \\
7 & 0.898400 & 0.917400 \\
8 & 0.893600 & 0.912900 \\
9 & 0.889700 & 0.909200 \\
10 & 0.886300 & 0.906000 \\
\end{tabular}
```

```
[68]: alpha_three_known_h = alphaTestToPlot[alphaTestToPlot.alphaS ==
↪ 3][["pragmatic_aligned"]]
```

```
alpha_two_known_h = alphaTestToPlot[alphaTestToPlot.alphaS ==  
↪2]["pragmatic_aligned"]  
  
stats.ttest_rel(alpha_three_known_h, alpha_two_known_h)
```

```
[68]: Ttest_relResult(statistic=-1.696996750381101, pvalue=0.0898095767898488)
```