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# Towards Training GNNs using Explanation Directed Message Passing

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## Abstract

1  
2 With the increasing use of Graph Neural Networks (GNNs) in critical real-world ap-  
3 plications, several post hoc explanation methods have been proposed to understand  
4 their predictions. However, there has been no work in generating explanations on  
5 the fly during model training and utilizing them to improve the expressive power  
6 of the underlying GNN models. In this work, we introduce a novel explanation-  
7 directed neural message passing framework for GNNs, EXPASS (EXplainable  
8 message PASSing), which aggregates only embeddings from nodes and edges  
9 identified as important by a GNN explanation method. EXPASS can be used with  
10 any existing GNN architecture and subgraph-optimizing explainer to learn accurate  
11 graph embeddings. We theoretically show that EXPASS alleviates the oversmooth-  
12 ing problem in GNNs by slowing the layer-wise loss of Dirichlet energy and that  
13 the embedding difference between the vanilla message passing and EXPASS frame-  
14 work can be upper bounded by the difference of their respective model weights.  
15 Our empirical results show that graph embeddings learned using EXPASS improve  
16 the predictive performance and alleviate the oversmoothing problems of GNNs,  
17 opening up new frontiers in graph machine learning to develop explanation-based  
18 training frameworks.

## 19 1 Introduction

20 Graph Neural Networks (GNNs) are increasingly used as powerful tools for representing graph-  
21 structured data, such as social, information, chemical, and biological networks [1, 2]. With  
22 the deployment of GNN models in critical applications (e.g., financial systems and crime forecasting [3,  
23 4]), it becomes essential to ensure that the relevant stakeholders understand and trust their decisions.  
24 To this end, several approaches [5–13] have been proposed in recent literature to generate *post hoc*  
25 explanations for predictions of GNN models.

26 In contrast to other modalities like images and texts, generating instance-level explanations for  
27 graphs is non-trivial. In particular, it is more challenging since individual node embeddings in GNNs  
28 aggregate information using the entire graph structure, and, therefore, explanations can be on different  
29 levels (i.e., node attributes, nodes, and edges). While several categories of GNN explanation methods  
30 have been proposed: gradient-based [5, 10, 14], perturbation-based [8, 9, 11, 13, 15], and surrogate-  
31 based [7, 12], their utility is limited to generating post hoc node- and edge-level explanations for a  
32 given pre-trained GNN model. Thus, the capability of GNN explainers to improve the predictive  
33 performance of a GNN model lacks understanding as there is very little work on systematically  
34 analyzing the reliability of state-of-the-art GNN explanation methods on model performance [16].

35 To address this, recent works have explored the joint optimization of machine learning models and  
36 explanation methods to improve the reliability of explanations [17, 18]. Zhou et al. [18] proposed  
37 DropEdge as a technique to drop random edges (similar to generating random edge explanations)  
38 during training to reduce overfitting in GNNs. More recently, Spinelli et al. [17] used meta-learning  
39 frameworks to generate GNN explanations and show an improvement in the performance of specific  
40 GNN explanation methods. While these works make an initial attempt at jointly optimizing explainers  
41 and predictive models, they are neither generalizable nor exhaustive. They fail to show improvement

42 in the downstream GNN performance [17] and degree of explainability [18] across diverse GNN  
 43 architectures and explainers. Further, there is little to no work done on either theoretically analyzing  
 44 the effect of GNN explanations on the neural message framework in GNNs or on important GNN  
 45 properties like oversmoothing [19].

46 **Present work.** In this work, we introduce a novel explanation-directed neural message passing  
 47 framework, EXPASS, which can be used with any GNN model and subgraph-optimizing explainer to  
 48 learn accurate graph representations. In particular, EXPASS utilizes GNN explanations to steer the  
 49 underlying GNN model to learn graph embeddings using only important nodes and edges. EXPASS  
 50 aims to define local neighborhoods for neural message passing, i.e., identify the most important  
 51 edges and nodes, using explanation weights, in the  $k$ -hop local neighborhood of every node in the  
 52 graph. Formally, we augment existing message passing architectures to allow information flow along  
 53 important edges while blocking information along irrelevant edges.

54 We present an extensive theoretical and empirical analysis to show the effectiveness of EXPASS on  
 55 the predictive, explainability, and oversmoothing performance of GNNs. Our theoretical results show  
 56 that the embedding difference between vanilla message passing and EXPASS frameworks is upper-  
 57 bounded by the difference between their model weights. Further, we show that embeddings learned  
 58 using EXPASS relieve the oversmoothing problem in GNNs as they reduce information propagation  
 59 by slowing the layer-wise loss of Dirichlet energy (Section 4.2). For our empirical analysis, we  
 60 integrate EXPASS into state-of-the-art GNN models and evaluate their predictive, oversmoothing,  
 61 and explainability performance on real-world graph datasets (Section 5). Our results show that, on  
 62 average, across five GNN models, EXPASS improves the degree of explainability of the underlying  
 63 GNNs by 39.68%. Our ablation studies show that for an increasing number of GNN layers, EXPASS  
 64 achieves 34.4% better oversmoothing performance than its vanilla counterpart. Finally, our results  
 65 demonstrate the effectiveness of using explanations during training, paving the way for new frontiers  
 66 in GraphXAI research to develop explanation-based training algorithms.

## 67 2 Related works

68 **Graph Neural Networks.** Graph Neural Networks (GNNs) are complex non-linear functions that  
 69 transform input graph structures into a lower dimensional embedding space. The main goal of  
 70 GNNs is to learn embeddings that reflect the underlying input graph structure, i.e., neighboring  
 71 nodes in the graph are mapped to neighboring points in the embedding space. Prior works have  
 72 proposed several GNN models using spectral and non-spectral approaches. Spectral models [20–24]  
 73 leverage Fourier transform and graph Laplacian to define convolution approaches for GNN models.  
 74 However, non-spectral approaches [25–29] define the convolution operation by leveraging the local  
 75 neighborhood of individual nodes in the graph. Most modern non-spectral models are message  
 76 passing frameworks [30, 31], where nodes update their embedding by aggregating information from  
 77  $k$ -hop neighboring nodes.

78 **Post hoc Explanations.** With the increasing development of complex high-performing GNN mod-  
 79 els [25–29], it becomes critical to understand their decisions. Prior works have focused on developing  
 80 several post hoc explanation methods to explain the decisions of GNN models [5, 7, 9, 11–13, 32].  
 81 More specifically, these explanation methods can be broadly categorized into i) gradient-based meth-  
 82 ods [5] that leverage the gradients of the GNN model to generate explanations; ii) perturbation-based  
 83 methods [9, 11, 13] that aim to generate explanations by calculating the change in GNN predictions  
 84 upon perturbations of the input graph structure (nodes, edges, or subgraphs); and iii) surrogate-based  
 85 methods [7, 12] that fit a simple interpretable model to approximate the predictive behavior of the  
 86 given GNN model. Finally, recent works have introduced frameworks to theoretically and empirically  
 87 analyze the behavior of state-of-the-art GNN explanation methods with respect to several desirable  
 88 properties [16, 33].

## 89 3 Preliminaries

90 **Notations.** Let  $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathbf{X})$  denote an undirected graph comprising of a set of nodes  $\mathcal{V}$  and a  
 91 set of edges  $\mathcal{E}$ . Let  $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\}$  denote the set of node feature vectors for all nodes in  $\mathcal{V}$ ,  
 92 where  $\mathbf{x}_v \in \mathbb{R}^d$  captures the attribute values of a node  $v$  and  $N = |\mathcal{V}|$  denotes the number of nodes in  
 93 the graph. Let  $\mathbf{A} \in \mathbb{R}^{N \times N}$  be the graph adjacency matrix, where element  $\mathbf{A}_{uv} = 1$  if there exists  
 94 an edge  $e \in \mathcal{E}$  between nodes  $u$  and  $v$  and  $\mathbf{A}_{uv} = 0$  otherwise. We use  $\mathcal{N}_u$  to denote the set of

95 immediate neighbors of node  $u$ , i.e.,  $\mathcal{N}_u = \{v \in \mathcal{V} | A_{uv} = 1\}$ . Finally, the function  $\text{deg} : \mathcal{V} \mapsto \mathbb{Z}_{>0}$   
 96 is defined as  $\text{deg}(v) = |\mathcal{N}_v|$  and outputs the degree of a node  $v \in \mathcal{V}$

97 **Graph Neural Networks (GNNs).** Formally, GNNs can be formulated as message passing net-  
 98 works [30] specified by three key operators MSG, AGG, and UPD. These operators are recur-  
 99 sively applied on a given graph  $\mathcal{G}$  for a  $L$ -layer GNN model defining how neural messages are  
 100 shared, aggregated, and updated between nodes to learn the final node representations in the  $L^{\text{th}}$   
 101 layer of the GNN. Commonly, a message between a pair of nodes  $(u, v)$  in layer  $l$  is charac-  
 102 terized as a function of their hidden representations  $\mathbf{h}_u^{(l-1)}$  and  $\mathbf{h}_v^{(l-1)}$  from the previous layer:  
 103  $\mathbf{m}_{uv}^{(l)} = \text{MSG}(\mathbf{h}_u^{(l-1)}, \mathbf{h}_v^{(l-1)})$ . The AGG operator retrieves the messages from the neighborhood of  
 104 node  $u$  and aggregates them as:  $\mathbf{m}_u^{(l)} = \text{AGG}(\mathbf{m}_{uv}^{(l)} | v \in \mathcal{N}_u)$ . Next, the UPD operator takes the aggre-  
 105 gated message  $\mathbf{m}_u^{(l)}$  at layer  $l$  and combines it with  $\mathbf{h}_u^{(l-1)}$  to produce node  $u$ 's representation for layer  
 106  $l$  as  $\mathbf{h}_u^{(l)} = \text{UPD}(\mathbf{m}_u^{(l)}, \mathbf{h}_u^{(l-1)})$ . Lastly, the final node representation for node  $u$  is given as  $\mathbf{z}_u = \mathbf{h}_u^{(L)}$ .

107 **Graph Explanations.** In contrast to other modalities like images and texts, an explanation method  
 108 for graphs can formally generate multi-level explanations. For instance, in a graph classification task,  
 109 the explanations for a given graph prediction can be with respect to node attributes  $\mathbf{M}_x \in \mathbb{R}^d$ , nodes  
 110  $\mathbf{M}_n \in \mathbb{R}^N$ , or edges  $\mathbf{M}_e \in \mathbb{R}^{N \times N}$ . Note that these explanation masks are continuous but can be  
 111 discretized using specific thresholding strategies [33].

112 **Oversmoothing.** Cai et al. [34] and Zhou et al. [35] defined bounds for analyzing oversmoothing  
 113 for a GNN using Dirichlet Energy. For a graph  $\mathcal{G}$  with adjacency matrix  $\mathbf{A}$  and degree matrix  $\mathbf{D}$ , we  
 114 define  $\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{I}_N$  and  $\tilde{\mathbf{D}} = \mathbf{D} + \mathbf{I}_N$  as the adjacency and degree matrices respectively of the graph  $\mathcal{G}$   
 115 with self-loops. We also define the augmented normalized Laplacian of  $\mathcal{G}$  as  $\tilde{\Delta} = \mathbf{I}_N - \tilde{\mathbf{D}}^{-\frac{1}{2}} \tilde{\mathbf{A}} \tilde{\mathbf{D}}^{-\frac{1}{2}}$ ,  
 116 and  $\mathbf{P} = \mathbf{I}_N - \tilde{\Delta}$ .

## 117 4 Our Framework: EXPASS

118 Here, we describe EXPASS, our proposed explainable message passing framework that aims to learn  
 119 accurate and interpretable graph embeddings. In particular, EXPASS incorporates explanations into  
 120 the message passing framework of GNN models by only aggregating embeddings from key nodes  
 121 and edges as identified using an explanation method.

122 **Problem formulation (Explanation Directed Message Passing).** Given a graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathbf{X})$ ,  
 123 EXPASS aims to generate a  $d$ -dimensional embedding  $\mathbf{z}_u \in \mathbb{R}^d$  for each node  $u \in \mathcal{V}$  using an  
 124 explanation-directed message passing framework that filters out the noise from unimportant edges  
 125 and improves the expressive power of GNNs.

### 126 4.1 Explanation Directed Message Passing

127 The central idea of EXPASS is to propose a novel method for improving the neural message passing  
 128 scheme of GNN models by utilizing explanations during model training and aggregating important  
 129 neural messages along edges in graph neighborhoods. Next, we describe the existing message passing  
 130 scheme in GNNs and our explainable counterpart.

131 **Message Passing.** As described in Section 3, each GNN layer can be described using the MSG, AGG,  
 132 and UPD operators. For each node  $u \in \mathcal{V}$ , the  $(l+1)^{\text{th}}$  layer embeddings  $\mathbf{h}_u^{(l+1)}$  is computed using a  
 133 GNN operating on the node's neighboring attributes. Formally, the GNN layer can be formulated as:

$$\mathbf{h}_u^{(l+1)} = \phi \left( \mathbf{h}_u^{(l)}, \bigoplus_{v \in \mathcal{N}_u} \psi(\mathbf{h}_u^{(l)}, \mathbf{h}_v^{(l)}) \right)$$

UPDATE                      MESSAGE                      AGGREGATE

134 where  $\mathbf{h}_u^{(l+1)}$  represents the updated embedding of node  $u$ ,  $\psi$  is the MSG operator,  $\bigoplus$  is the AGG  
 135 operator (e.g., summation),  $\phi$  is an UPD function (e.g., any non-linear activation function), and  $\mathbf{h}_u^{(l)}$   
 136 represents the embedding of node  $u$  from the previous layer. We obtain an embedding  $\mathbf{z}_u$  for node  
 137  $u$  by stacking  $L$  GNN layers. Finally, the node embeddings  $\mathbf{Z} \in \mathbb{R}$  are then passed to a READOUT  
 138 function to obtain an embedding for the graph.

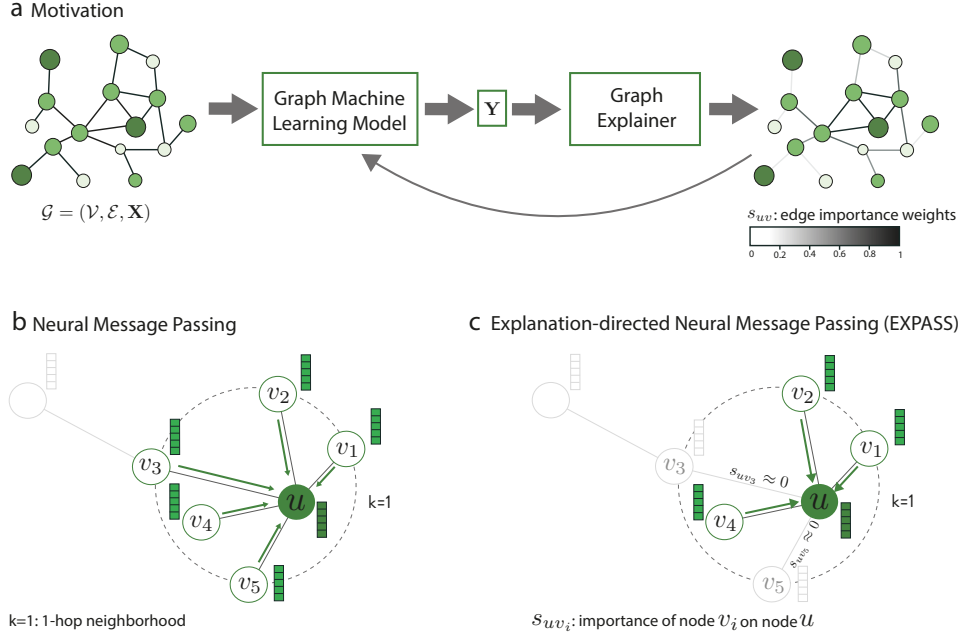


Figure 1: **Overview of EXPASS:** **a)** EXPASS investigates the problem of injecting explanations into the message passing framework to increase the expressive power and performance of GNNs. **b)** Shown is the general message passing scheme where, for node  $u$ , messages are aggregated from nodes  $v_i \in \mathcal{N}_u$  in the 1-hop neighborhood of  $u$ . **c)** EXPASS injects explanations into the message passing framework translates by masking out messages from neighboring nodes  $v_i \in \mathcal{N}_u$  with explanation scores  $s_{uv_i} \approx 0$  when  $u$  is correctly classified.

139 **EXPASS.** Here, we describe our proposed explainable message passing scheme that incorporates  
 140 explanations into the message passing step in individual GNN layers on the fly during the training  
 141 process. Given an explanation method, which generates an importance score  $s_{uv} \in \mathbb{M}_u^e$  for every  
 edge  $e_{uv} \in \mathcal{E}$ , we can weight the edge contribution in the neighborhood  $\mathcal{N}_u$  of node  $u$  as:

$$\mathbf{h}'_u{}^{(l+1)} = \phi \left( \mathbf{h}_u^{(l)}, \bigoplus_{v \in \mathcal{N}_u} s_{uv} \psi(\mathbf{h}_u^{(l)}, \mathbf{h}_v^{(l)}) \right)$$

142

143 Note that EXPASS is agnostic to explanation types and can also incorporate explanations on node  
 144 attributes and node level. For instance, the importance scores for individual nodes can be computed  
 145 by averaging the outgoing scores  $s_{uv}$  for all  $v \in \mathcal{N}_u$ . Subsequently, we can replace the  $s_{uv}$  score  
 146 by using the average score  $s_u$  to weight edges in the EXPASS layers, and for node attributes, we can  
 147 multiply the node attribute explanation  $\mathbf{M}_u^a$  to the original node attribute vector.

148 To enable explainable message passing and only retain the important embeddings for node  $u$ , EXPASS  
 149 removes the knowledge of irrelevant nodes and edges from the local neighborhood  $\mathcal{N}_u$  of node  $u$   
 150 using its explanations. For instance, if node  $v$  is considered important to node  $u$ , EXPASS transforms  
 151 the aggregated messages of node  $u$  using the node importance scores  $s_{uv}$ . Note that since the  
 152 explanations of node  $u$  include important nodes and edges in the  $L$ -hop neighborhood of node  $u$ , even  
 153 though node  $u$  is only locally modified, the change will spread through all the nodes in every GNN  
 154 layer. Furthermore, to avoid spurious correlations, we ensure that explanations are only generated  
 155 for correctly classified nodes and graphs. Explanation weights infuse information from higher-order  
 156 neighborhoods into each layer of the GNN model, specifically, from as many  $L$ -hop neighbors  
 157 because explanation weights within each layer are computed using the  $L$ -layer GNN model. To  
 158 illustrate this, we next show the weight computations for a GNN explanation method.

159 Without loss of generality, let us consider GNNExplainer as our explanation method whose mask  
 160 for selected graph is formulated as:  $\mathcal{G}_{\text{mask}} = (\mathbf{X}', \mathbf{A}') = (\mathbf{X} \odot \sigma(\mathbf{M}^x), \mathbf{A} \odot \sigma(\mathbf{M}^e))$ , where  
 161  $W = [\mathbf{M}^x, \mathbf{M}^e]$  are the explainers parameters,  $\sigma$  is the sigmoid function, and  $\odot$  denotes element-

wise multiplication. Here,  $s_{uv}$  represents the element in row  $v$  and column  $u$  of  $\mathbf{M}^c$ . Gradient descent-based optimization is used to find the optimal values for the masks minimizing the following objective:  $L_e = -\sum_{c=1}^C 1[y=c] \log f_\theta(Y=y|\mathcal{G}_{\text{mask}})$ , where  $f_\theta$  is the  $L$ -layer GNN model and  $C$  is the total number of classes. This shows that a  $L$ -hop neighborhood is used to compute  $s_{uv}$ . Formally, it minimizes the uncertainty of the predictive model when the GNN computation is limited to the explanation subgraph. This uncertainty is minimized as a proxy of the maximization of the mutual information between the prediction with the unmasked graph and masked graph.

## 4.2 Theoretical Analysis

Here, we provide a detailed theoretical analysis of our proposed EXPASS framework. In particular, we (i) provide a theoretical upper bound on the embedding difference obtained from a vanilla message passing and EXPASS framework and (ii) show that graph embeddings learned using EXPASS relieves the oversmoothing problem in GNNs by reducing information propagation.

**Theorem 1** (Differences between EXPASS and Vanilla Message Passing). *Given a non-linear activation function  $\sigma$  that is Lipschitz continuous, the difference between the node embeddings between a vanilla message passing and EXPASS framework can be bounded by the difference in their individual weights, i.e.,*

$$\|\mathbf{h}_u^{(l)} - \mathbf{h}'_u^{(l)}\|_2 \leq \|\mathbf{W}_a^{(l)} - \mathbf{W}'_a^{(l)}\|_2 \|\mathbf{h}_u^{(l-1)}\|_2 + \|\mathbf{W}_n^{(l)} - \mathbf{W}'_n^{(l)}\|_2 \sum_{v \in \mathcal{N}_u \cap s_v=1} \|\mathbf{h}_v^{(l-1)}\|_2, \quad (1)$$

where  $\mathbf{W}_a^{(l)}$  and  $\mathbf{W}'_a^{(l)}$  are the weights for node  $u$  in layer  $l$  of the vanilla message passing and EXPASS framework and  $\mathbf{W}_n^{(l)}$  and  $\mathbf{W}'_n^{(l)}$  are their respective weight matrix with the neighbors of node  $u$  at layer  $l$ .

*Proof Sketch.* In Theorem 1, we prove that the  $\ell_2$ -norm of the differences between the embeddings of vanilla message passing and EXPASS framework at layer  $l$  is upper bounded by the difference between their weights and the embeddings of node  $u$  and its subgraph. See Appendix A for more details.  $\square$

**Definition 1** (Dirichlet Energy for a Node Embedding Matrix [35]). *Given a node embedding matrix  $\mathbf{H}^{(l)} = [\mathbf{h}_1^{(l)}, \dots, \mathbf{h}_n^{(l)}]^T$  learned from the GNN model at the  $l^{\text{th}}$  layer, the Dirichlet Energy  $E(\mathbf{H}^{(l)})$  is defined as:*

$$E(\mathbf{H}^{(l)}) = \text{tr}(\mathbf{H}^{(l)T} \tilde{\Delta} \mathbf{H}^{(l)}) = \frac{1}{2} \sum_{i,j \in \mathcal{V}} a_{ij} \left\| \frac{\mathbf{H}_i^{(l)}}{\sqrt{1 + \text{deg}_i}} - \frac{\mathbf{H}_j^{(l)}}{\sqrt{1 + \text{deg}_j}} \right\|_2^2 \quad (2)$$

where  $a_{ij}$  are elements in the adjacency matrix  $\tilde{\mathbf{A}}$  and  $\text{deg}_i, \text{deg}_j$  is the degree of node  $i$  and  $j$ , respectively.

Cai et al. [34] extensively show that higher Dirichlet energies correspond to lower oversmoothing. Furthermore, they show that the removal of edges or, similarly, the reduction of edge weights on graphs helps alleviate oversmoothing.

**Proposition 1** (EXPASS relieves Oversmoothing). *EXPASS alleviates oversmoothing by slowing the layer-wise loss of Dirichlet energy.*

The complete proof is provided in Appendix A.

## 5 Experiments

Next, we present experimental results for our EXPASS framework. More specifically, we address the following questions: **Q1**) Does EXPASS enable GNNs to learn more accurate embeddings and improve their degree of explainability? **Q2**) How does EXPASS affect the oversmoothing and predictive performance of GNNs with an increasing number of layers? **Q3**) Does EXPASS depend on the quality of explanations for improving the predictive and oversmoothing performance of GNNs and are they better than attention weights?



## 203 5.1 Datasets and Experimental setup

204 We first describe the datasets used to study the utility of our proposed EXPASS framework and then  
205 outline the experimental setup.

206 **Datasets.** We use real-world molecular chemistry datasets to evaluate the effectiveness of EXPASS  
207 w.r.t. the performance of the underlying GNN model and understand the trade-off between explain-  
208 ability and accuracy for a graph classification task. We consider four benchmark datasets, which  
209 includes Mutag [36], Alkane-Carbonyl [37], DD [38], and Proteins [39]. See Appendix B.1 for a  
210 detailed overview of the datasets.

211 **GNN Architectures and Explainers.** To investigate the flexibility of EXPASS, we incorporate  
212 it into five different GNN models: GCN [40], GraphConv [41], LEConv [42], GraphSAGE [28],  
213 and GIN [27]. We use GNNExplainer [13] as our baseline GNN explanation method to generate  
214 edge-level explanations for most of our experiments. In addition, we use Integrated Gradients [43], a  
215 node-level explanation method, to demonstrate EXPASS’s sensitivity to the choice of explainers.

216 **Implementation details.** We consider DropEdge [44] as our baseline method for comparing the  
217 oversmoothing performance of EXPASS as DropEdge randomly removes edges from the input graph  
218 at each training epoch, acting like a message passing reducer. Across all experiments, we use topK  
219 ( $k=40\%$ ) node features/edges, and use them to generate explanations for all explanation methods.  
220 All other hyperparameters of the explanation and baseline methods were set following the author’s  
221 guidelines. For all our experiments (unless mentioned otherwise), we use the baseline architectures  
222 with three GNN layers followed by ReLU layers and set the hidden dimensionality to 32. Finally,  
223 we use a single linear layer to transform the graph embeddings to their respective classes. See  
224 Appendix B.2 for more details.

225 **Performance metrics for GNN Explainers.** To measure the reliability of GNN explanation methods,  
226 we use the graph explanation faithfulness metric [16]:  $\text{GEF}(\hat{y}_u, \hat{y}_{u'}) = 1 - \exp^{-\text{KL}(\hat{y}_u || \hat{y}_{u'})}$ , where  
227  $\hat{y}_u$  is predicted probability vector using the whole subgraph and  $\hat{y}_{u'}$  is the predicted probability  
228 vector using the masked subgraph, where we generate the masked subgraph by only using the topK  
229 features identified by an explanation and the Kullback-Leibler (KL) divergence score (denoted by “||”  
230 operator) quantifies the distance between two probability distributions. Note that GEF is a measure  
231 of the unfaithfulness of the explanation. So, higher values indicate a higher degree of unfaithfulness.

232 **Performance metrics for Oversmoothing.** Zhou et al. [18] introduced the Group Distance Ratio  
233 (GDR) metric to quantify oversmoothing in GNNs. It measures the ratio between the average of  
234 pairwise representation distances between graphs belonging to different (inter) and same (intra)  
235 groups. Formally, one would prefer to reduce the intra-group class representations and increase the  
236 inter-group distance to relieve the over-smoothing issue. Hence, lower GDR values denote higher  
237 oversmoothing in GNNs.

238 **Burn-in period.** We defined the *burn-in period* as a number  $n$  of epochs during training in which no  
239 explanations are used. The burn-in period is necessary to avoid feeding spurious explanations to the  
240 model. The length of the burn-in period, e.g. the number of epochs, was treated as a hyperparameter  
241 and fine-tuned during the model fine-tuning phase. At the end of the burn-in period, a predefined  
242 percentage of correctly predicted graphs per batch is randomly sampled and their explanations are  
243 used in the model training. The percentage of correctly predicted graphs sampled in each batch was  
244 treated as an hyperparameter and was set to 0.4 for all our experiments.

## 245 5.2 Results

246 **Q1) EXPASS improves the predictive performance and explainability of GNNs.** To measure the  
247 predictive performance and degree of explainability of GNNs trained using EXPASS, we compute  
248 their average predictive performance (using AUROC and F1-score) and fidelity (using Graph Explan-  
249 ation Faithfulness) using different GNN models and datasets. Across four datasets and five GNN  
250 architectures, we find that EXPASS-augmented GNNs learn graph embeddings that are more accurate  
251 (higher AUROC and F1-score) and result in more faithful explanations (lower Graph Explanation  
252 Faithfulness score) than their vanilla counterparts. On average, EXPASS improves the AUROC  
253 and F1-score by 1.51% and 1.05%, respectively. In particular, we observe that EXPASS improves  
254 the predictive behavior of high-performing models like GIN (+2.06% in AUROC and +2.50% in  
255 F1-score) but shows little to no improvement in the case of LeConv, which utilizes a node-scoring

Table 1: Results of EXPASS for five GNNs and four graph datasets. Shown is average performance across three independent runs. Arrows ( $\uparrow$ ,  $\downarrow$ ) indicate the direction of better performance. EXPASS improves the predictive power (AUROC and F1-score) and degree of explainability (Graph Explanation Faithfulness) of original GNNs across multiple datasets (shaded area). Values corresponding to best performance are bolded.

Dataset	Method	AUROC ( $\uparrow$ )	F1-score ( $\uparrow$ )	GEF ( $\downarrow$ )
ALKANE-CARBONYL	GCN	0.97 $\pm$ 0.01	0.95 $\pm$ 0.01	0.33 $\pm$ 0.02
	EXPASS-GCN	<b>0.98</b> $\pm$ 0.00	<b>0.96</b> $\pm$ 0.01	<b>0.23</b> $\pm$ 0.02
	GraphConv	0.97 $\pm$ 0.01	0.94 $\pm$ 0.00	0.38 $\pm$ 0.05
	EXPASS-GraphConv	<b>0.98</b> $\pm$ 0.00	<b>0.97</b> $\pm$ 0.00	<b>0.22</b> $\pm$ 0.03
	LeConv	0.98 $\pm$ 0.01	0.96 $\pm$ 0.00	0.37 $\pm$ 0.03
	EXPASS-LeConv	0.98 $\pm$ 0.00	0.96 $\pm$ 0.01	<b>0.24</b> $\pm$ 0.03
	GraphSAGE	0.98 $\pm$ 0.00	0.96 $\pm$ 0.00	0.40 $\pm$ 0.12
	EXPASS-GraphSAGE	<b>0.99</b> $\pm$ 0.00	<b>0.97</b> $\pm$ 0.01	<b>0.18</b> $\pm$ 0.06
	GIN	0.96 $\pm$ 0.01	0.94 $\pm$ 0.02	0.35 $\pm$ 0.06
EXPASS-GIN	<b>0.98</b> $\pm$ 0.01	<b>0.96</b> $\pm$ 0.02	<b>0.11</b> $\pm$ 0.04	
DD	GCN	0.73 $\pm$ 0.02	0.70 $\pm$ 0.02	0.49 $\pm$ 0.04
	EXPASS-GCN	<b>0.74</b> $\pm$ 0.01	0.70 $\pm$ 0.02	<b>0.30</b> $\pm$ 0.09
	GraphConv	0.75 $\pm$ 0.03	0.73 $\pm$ 0.03	0.25 $\pm$ 0.10
	EXPASS-GraphConv	<b>0.77</b> $\pm$ 0.03	0.73 $\pm$ 0.03	<b>0.19</b> $\pm$ 0.04
	LeConv	0.76 $\pm$ 0.03	<b>0.74</b> $\pm$ 0.02	<b>0.17</b> $\pm$ 0.03
	EXPASS-LeConv	<b>0.77</b> $\pm$ 0.03	0.73 $\pm$ 0.04	0.31 $\pm$ 0.10
	GraphSAGE	0.74 $\pm$ 0.02	0.70 $\pm$ 0.02	0.21 $\pm$ 0.04
	EXPASS-GraphSAGE	<b>0.76</b> $\pm$ 0.03	<b>0.71</b> $\pm$ 0.02	<b>0.20</b> $\pm$ 0.03
	GIN	0.74 $\pm$ 0.01	0.70 $\pm$ 0.01	0.37 $\pm$ 0.03
EXPASS-GIN	<b>0.76</b> $\pm$ 0.01	<b>0.74</b> $\pm$ 0.01	<b>0.35</b> $\pm$ 0.05	
MUTAG	GCN	0.71 $\pm$ 0.11	0.87 $\pm$ 0.01	0.09 $\pm$ 0.03
	EXPASS-GCN	<b>0.77</b> $\pm$ 0.02	<b>0.89</b> $\pm$ 0.00	<b>0.04</b> $\pm$ 0.01
	GraphConv	0.91 $\pm$ 0.02	0.94 $\pm$ 0.02	0.66 $\pm$ 0.03
	EXPASS-GraphConv	<b>0.93</b> $\pm$ 0.01	<b>0.94</b> $\pm$ 0.01	<b>0.24</b> $\pm$ 0.03
	LeConv	0.92 $\pm$ 0.03	0.94 $\pm$ 0.02	0.65 $\pm$ 0.05
	EXPASS-LeConv	0.92 $\pm$ 0.03	<b>0.96</b> $\pm$ 0.01	<b>0.30</b> $\pm$ 0.06
	GraphSAGE	0.76 $\pm$ 0.02	0.86 $\pm$ 0.03	0.24 $\pm$ 0.08
	EXPASS-GraphSAGE	0.76 $\pm$ 0.02	<b>0.87</b> $\pm$ 0.03	<b>0.11</b> $\pm$ 0.03
	GIN	0.92 $\pm$ 0.02	0.93 $\pm$ 0.01	0.61 $\pm$ 0.05
EXPASS-GIN	<b>0.94</b> $\pm$ 0.02	<b>0.95</b> $\pm$ 0.01	<b>0.32</b> $\pm$ 0.04	
PROTEINS	GCN	0.73 $\pm$ 0.05	0.68 $\pm$ 0.04	0.19 $\pm$ 0.02
	EXPASS-GCN	<b>0.74</b> $\pm$ 0.03	<b>0.69</b> $\pm$ 0.03	<b>0.08</b> $\pm$ 0.02
	GraphConv	0.75 $\pm$ 0.03	0.70 $\pm$ 0.03	0.49 $\pm$ 0.06
	EXPASS-GraphConv	0.75 $\pm$ 0.03	0.70 $\pm$ 0.04	<b>0.10</b> $\pm$ 0.03
	LeConv	<b>0.77</b> $\pm$ 0.03	<b>0.72</b> $\pm$ 0.04	0.51 $\pm$ 0.01
	EXPASS-LeConv	0.76 $\pm$ 0.02	0.71 $\pm$ 0.03	<b>0.15</b> $\pm$ 0.07
	GraphSAGE	0.73 $\pm$ 0.04	0.69 $\pm$ 0.04	0.17 $\pm$ 0.07
	EXPASS-GraphSAGE	0.73 $\pm$ 0.04	0.69 $\pm$ 0.04	<b>0.06</b> $\pm$ 0.01
	GIN	0.77 $\pm$ 0.04	0.73 $\pm$ 0.05	0.20 $\pm$ 0.07
EXPASS-GIN	<b>0.78</b> $\pm$ 0.03	0.73 $\pm$ 0.04	<b>0.19</b> $\pm$ 0.01	

256 mechanism through the similarity between a node and its neighbors’ embeddings. Finally, we find  
 257 that EXPASS-augmented GNNs significantly improve the explainability of a GNN and achieve a  
 258 39.68% better faithfulness score as compared to vanilla GNNs (Table 1).

259 **Q2) EXPASS relieves Oversmoothing in GNNs.** We examine the oversmoothing (using the Group  
 260 Distance Ratio metric [18]) and predictive performance of GNNs trained using EXPASS with their  
 261 vanilla counterparts. The oversmoothing problem in GNNs shows that the representations of nodes  
 262 converge to similar vectors as the number of layers increases. Therefore, we analyze the oversmooth-  
 263 ing of the GNNs for an increasing number of layers and find that, on average, across two architectures,  
 264 EXPASS improves the group distance ratio by 34.4% (Figure 2). Further, our results indicate an inher-  
 265 ent trade-off between oversmoothing and predictive performance of GNNs, as shown in Figures 4-6.

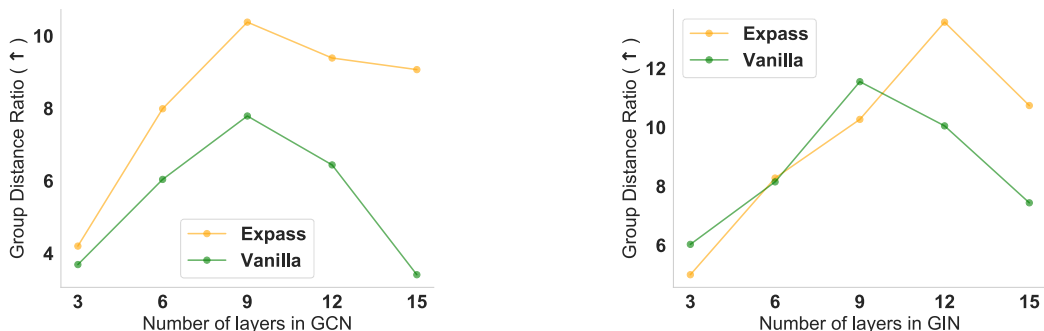


Figure 2: The effects of the number of GNN layers on the oversmoothing performance of EXPASS (orange) and Vanilla (green) GCN (left column) and GIN (right column) models trained on Alkane-Carbonyl dataset. Across models with increasing number of layers, EXPASS achieves higher GDR performance without sacrificing the predictive performance of the GCN model. See Figs. 4-6 for predictive performance results.

266 **Q3) Ablation studies.** We conduct ablations on several components of EXPASS with respect to its  
 267 oversmoothing and predictive performance.

268 **EXPASS for different TopK Explanations.** We investigate the oversmoothing and predictive perfor-  
 269 mance of GNNs for different topK explanations (i.e., topK edges identified by a GNN explanation)  
 270 chosen in the message passing. Results show that EXPASS alleviates oversmoothing by using only  
 271 the topK edges to learn graph embeddings and explicitly filter out the noise from unimportant edges.  
 272 In particular, we observe that the GDR values decrease (denoting higher oversmoothing) with the  
 273 increase in the use of topK edges (Figure 3). More specifically, we find that the GDR value at  
 274 topK=0.1 is 11.92% higher than vanilla message passing (i.e., using all edges in the graph).

275 **EXPASS vs. DropEdge.** We compare the predictive and oversmoothing and predictive performance  
 276 of EXPASS and DropEdge. Here, we show that message passing using optimized explanation-directed  
 277 information outperforms random edge removal. We find that EXPASS outperforms DropEdge across  
 278 both oversmoothing and accuracy metrics. In particular, on average, across different topK values,  
 279 EXPASS improves the oversmoothing, AUROC, and F1-score performance of vanilla message passing  
 280 by 71.16%, 9.53%, and 12.63%, respectively (Figure 3).

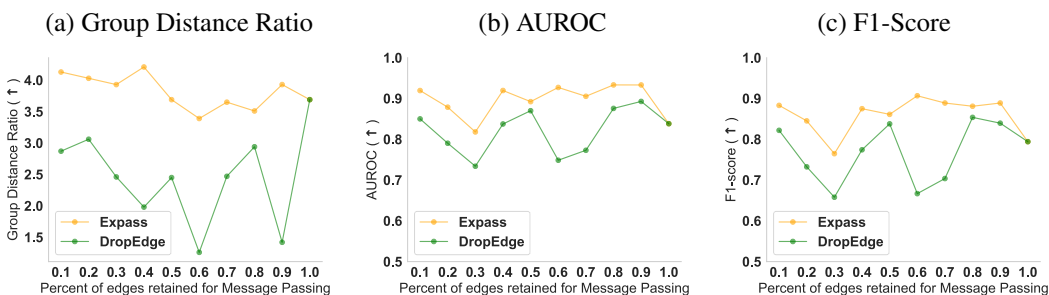


Figure 3: The effects of choosing only the topK percent of important edges on the (a) oversmoothing, (b) AUROC, and (c) F1-score performance of GCN model trained on Alkane-Carbonyl dataset. Over a wide range of topK values ( $0.1 < \text{topK} < 1.0$ ), EXPASS outperforms DropEdge [44] on all the three metrics. Note that their performance converges for topK = 1.0 as that denotes using all the edges in the graph.

281 **EXPASS using Node Explanations.** We investigate the effect of the choice of the baseline  
 282 explanation method on the performance of EXPASS with respect to the vanilla message passing  
 283 framework. More specifically, we evaluate the predictive and explainability performance of  
 284 EXPASS-augmented GNNs when trained using node explanations generated using Integrated  
 285 Gradients (IG) [43]. Similar to the results of EXPASS with GNNExplainer as the baseline explanation  
 286 method (Table 1), we find that EXPASS trained using IG explanations also improves the AUROC  
 287 (+2.80%), F1-score (+1.11%), and GEF (+23.67%) of the vanilla GNN model. Our results show that  
 288 the choice of explainer can make a difference in the EXPASS performance, depending on the dataset.  
 289 For instance, IG is a node-masking explainer that is not considered a strong explanation method



Table 2: Results of EXPASS for GCN using the node explanations from Integrated Gradients [43] for message passing for various datasets. Shown is average performance across three independent runs. Arrows ( $\uparrow$ ,  $\downarrow$ ) indicate the direction of better performance. EXPASS improves the predictive power (AUROC and F1-score) and degree of explainability (Graph Explanation Faithfulness) of original GNNs across multiple datasets (shaded area).

Dataset	Method	AUROC ( $\uparrow$ )	F1-score ( $\uparrow$ )	GEF ( $\downarrow$ )
DD	GCN	0.73 $\pm$ 0.02	0.70 $\pm$ 0.02	0.25 $\pm$ 0.03
	EXPASS-GCN	<b>0.75</b> $\pm$ 0.01	<b>0.71</b> $\pm$ 0.03	<b>0.23</b> $\pm$ 0.04
ALKANE	GCN	0.97 $\pm$ 0.01	0.95 $\pm$ 0.01	<b>0.09</b> $\pm$ 0.01
	EXPASS-GCN	0.97 $\pm$ 0.01	0.95 $\pm$ 0.01	0.1 $\pm$ 0.01
MUTAG	GCN	0.71 $\pm$ 0.11	0.87 $\pm$ 0.01	0.09 $\pm$ 0.02
	EXPASS-GCN	<b>0.77</b> $\pm$ 0.02	<b>0.88</b> $\pm$ 0.01	<b>0.04</b> $\pm$ 0.02
PROTEINS	GCN	0.73 $\pm$ 0.04	<b>0.68</b> $\pm$ 0.04	0.05 $\pm$ 0.01
	EXPASS-GCN	0.73 $\pm$ 0.04	0.67 $\pm$ 0.05	<b>0.04</b> $\pm$ 0.01

and its effects are variable across datasets [33]. We recommend using graph-specific explainers that optimize for fidelity and sparsity on the edges of the input graph, which would be a best fit to increase the performance of the network. Further, our results show that EXPASS is a model- and explainer-agnostic framework that can improve the downstream task and explainability performance across different GNN architectures using diverse GNN explainers.

## 6 Conclusion

In this work, we propose the problem of learning graph embeddings using explanation-directed message passing in GNNs. To this end, we introduce EXPASS, a novel message passing framework that can be used with any existing GNN model and subgraph-optimizing explainer to learn accurate embeddings by aggregating only embeddings from nodes and edges identified as important by a GNN explainer. We perform an extensive theoretical analysis to show that EXPASS relieves the oversmoothing problem in GNNs, and the embedding difference between the vanilla message passing framework and EXPASS can be upper bounded by the difference of their respective layer weights. Our empirical results on benchmark datasets show that EXPASS improves the explainability of the underlying GNN model without sacrificing its predictive performance. Our proposed method and findings open up exciting new avenues to generate graph embeddings by jointly training models and explanation methods. We anticipate that EXPASS could open new frontiers in graph machine learning for developing explanation-based training frameworks.

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