A Appendix

A.1 One-step Denoising Experiments

We have included additional figures for our one-step denoiser experiments. Figures 5 and 6 show the one step denoiser performance of the various methods at the measurement noise level and a lower test noise level for the AFHQ and NBU datasets respectively. We observe that our method performs better at denoising below the measurement noise level compared to other self-supervised denoising techniques.

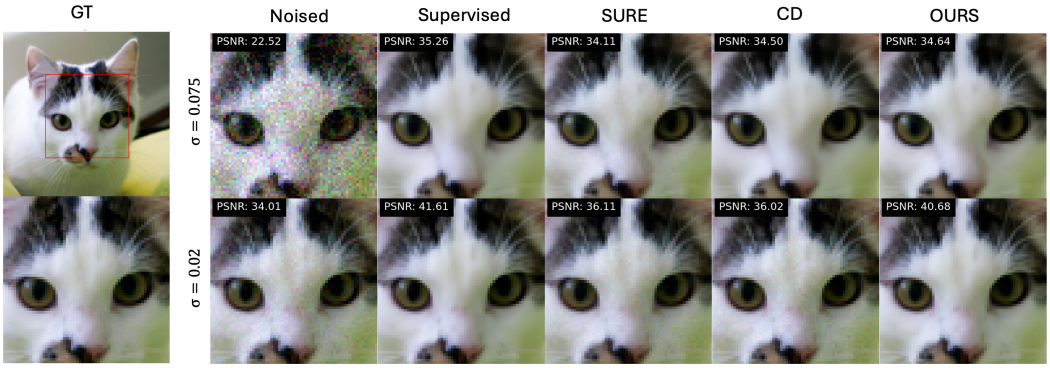

Figure 5: Example restorations of various denoisers on AFHQ dataset at test noise levels $\sigma_t = 0.075$ (top) and $\sigma_t = 0.02$ (bottom). All models, except for supervised were trained on only noisy data with $\sigma_n = 0.075$.

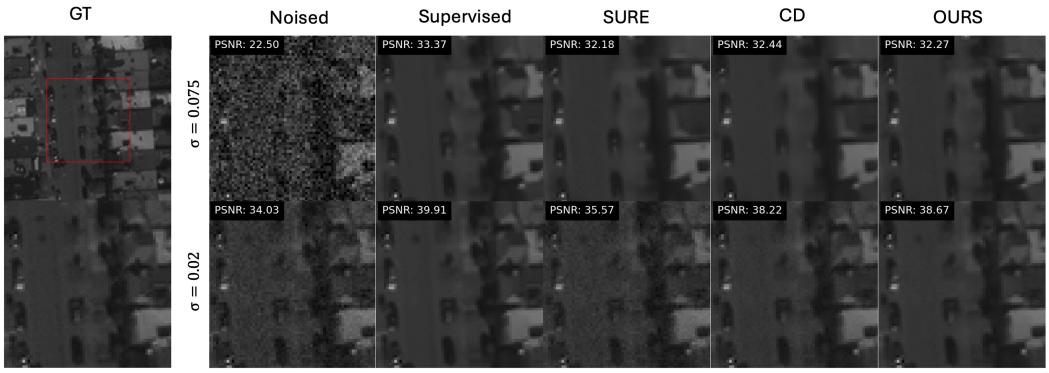

Figure 6: Example restorations of various denoisers on NBU dataset at test noise levels $\sigma_t = 0.075$ (top) and $\sigma_t = 0.02$ (bottom). All models, except for supervised were trained on only noisy data with $\sigma_n = 0.075$.

A.2 Diffusion Sampling

Additional figures have been provided for diffusion sampling experiments on the AFHQ and NBU datasets. Figures 7 and 8 show example diffusion samples for supervised and self-supervised approaches discussed in the paper on AFHQ and NBU respectively. Figure 9 shows diffusion samples for different training + inference noise levels with accompanying radial spectrum plots in Figure 10. Here we see that while one-step supervised and self-supervised MMSE denoisers tend to reduce high frequency features, our method retains higher frequencies lending to our method providing better perceptual images.

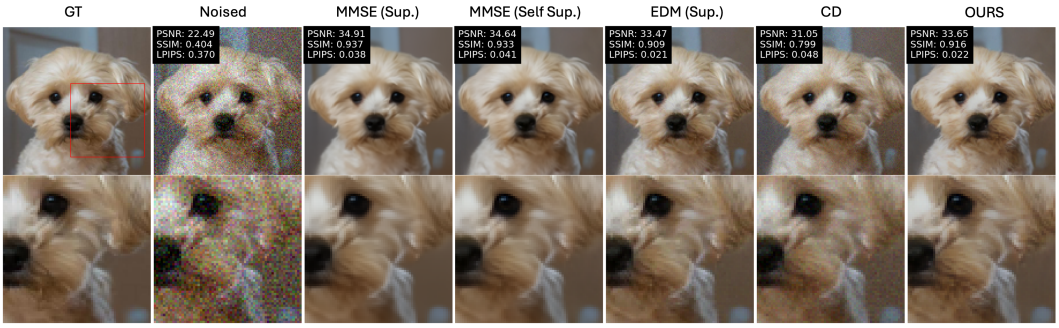

Figure 7: Example of various denoisers using diffusion sampling (except MMSE(Self Sup.) and MMSE(Sup.) columns) on AFHQ dataset with training and test noise level $\sigma_n = \sigma_t = 0.075$.

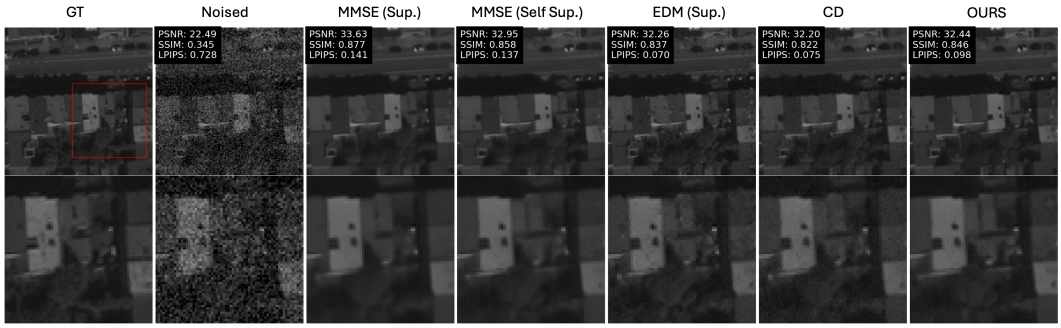

Figure 8: Example of various denoisers using diffusion sampling (except MMSE(Self Sup.) and MMSE(Sup.) columns) on NBU dataset with training and test noise level $\sigma_n = \sigma_t = 0.075$.

A.3 Extension to Linear Inverse Problems

We provide example reconstructions for the inpainting and demosaicing tasks in Figures 11 and 12 respectively.

A.4 Inference Procedure

For diffusion sampling in our experiments we use a slightly modified version of the samplers proposed in Karras et al. (2022) by conducting sampling in measurement space. We show the inference procedure in Algorithm 1.

Algorithm 1 Equivariant Sampling Inference

Require: $\mathrm{D}_{\boldsymbol{\theta}}(\cdot, \sigma)$, $\{\sigma_K = \sigma_n, \ldots, \sigma_1 = \sigma_{\min}\}, \mathbf{y}$

$\mathbf{y}_{\text{next}} = \mathbf{y}$
for $i \in \{K, \ldots, 1\}$ **do**
  $\mathbf{y}_{\text{cur}} = \mathbf{y}_{\text{next}}$
  $\hat{\mathbf{x}} = \left(\mathbf{x}_{\text{cur}} - \mathrm{D}_{\boldsymbol{\theta}}(\mathbf{A}^\top \mathbf{y}_{\text{cur}}, \sigma_i)\right)/\sigma_i$
  $\mathbf{y}_{\text{next}} = \mathbf{y}_{\text{cur}} + 2(\sigma_{i+1} - \sigma_i)\mathbf{A}\hat{\mathbf{x}}$
  $\boldsymbol{\eta} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
  $\mathbf{y}_{\text{next}} = \mathbf{y}_{\text{next}} + \sqrt{2(\sigma_{i+1} - \sigma_i)\sigma_i}\,\boldsymbol{\eta}$
  if $i > 1$ **then**
    $\hat{\mathbf{x}}' = (\mathbf{x}_{\text{next}} - \mathrm{D}_{\boldsymbol{\theta}}(\mathbf{A}^\top \mathbf{y}_{\text{next}}, \sigma_{i+1}))/\sigma_{i+1}$
    $\mathbf{y}_{\text{next}} = \mathbf{y}_{\text{cur}} + \frac{1}{2}(\sigma_{i+1} - \sigma_i)\mathbf{A}(\hat{\mathbf{x}} + \hat{\mathbf{x}}')$
return $\hat{\mathbf{x}}$

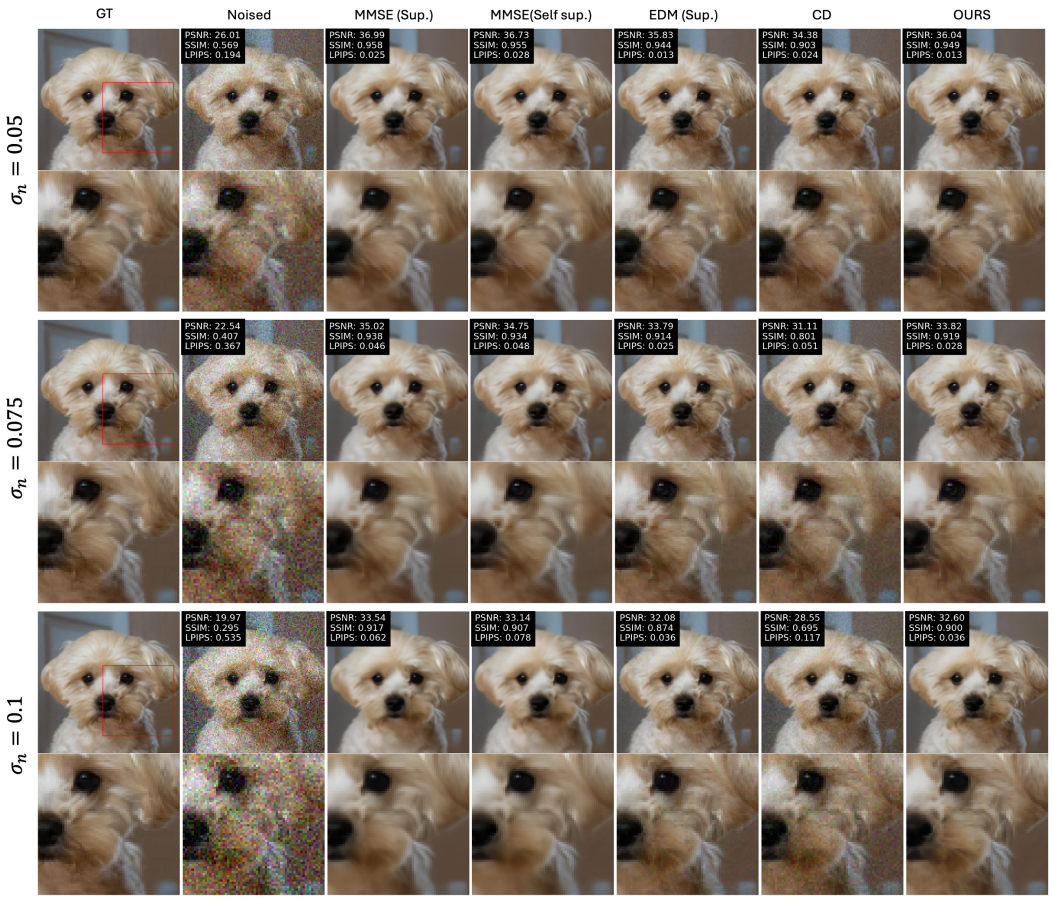

Figure 9: Example reconstructions for different training + inference noise levels on AFHQ.

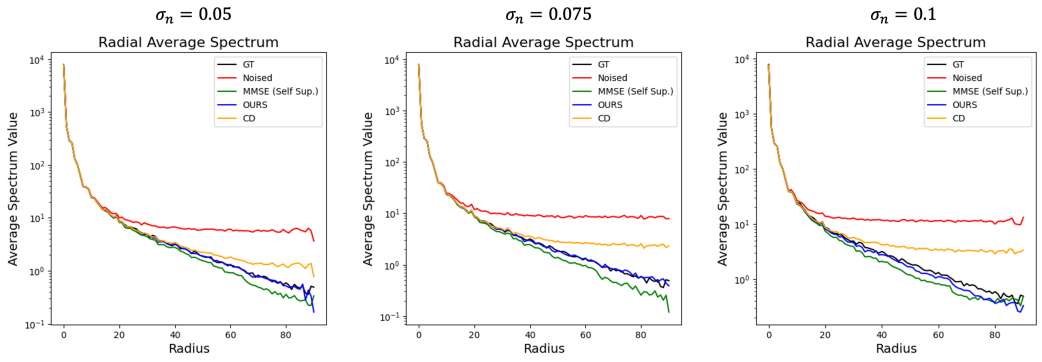

Figure 10: Radial Spectrum of images in Figure 9.

A.5 Patch Norms

To investigate the assumption that many real image distributions are approximately scale-invariant we plot the histogram of patch-wise norms for several image distributions using various patch sizes in Figure 13. We see that, in fact, we have a spread in energy within each dataset which implies we may be observing a dataset that exhibits weak invariance.

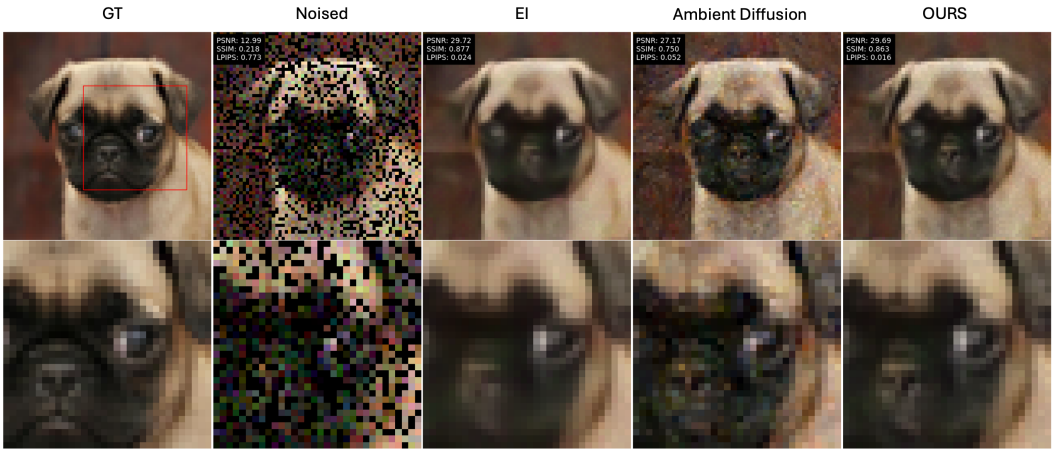

Figure 11: AFHQ inpainting example where the models are all trained in a self-supervised fashion.

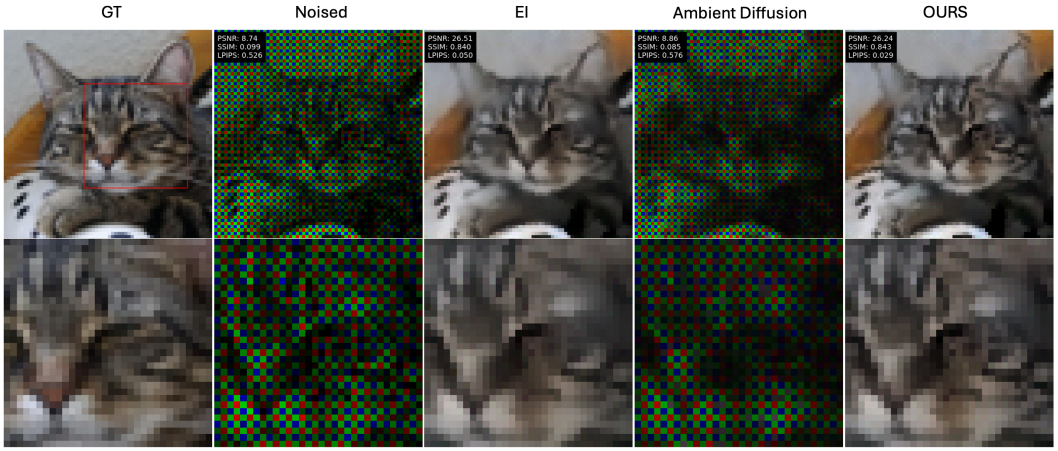

Figure 12: AFHQ demosaic example where the models are all trained in a self-supervised fashion.

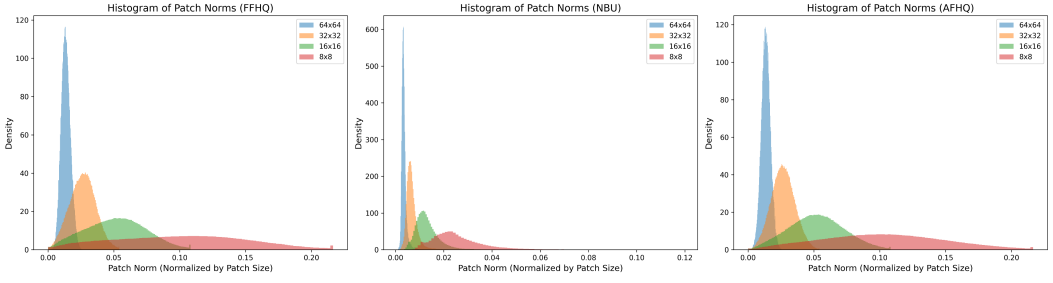

Figure 13: Histogram of image patches for each image distribution.