

# Supplementary Materials: Improving the Training of GANs with Limited Data via Dual Adaptive Noise Injection

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## 1 PROOFS

**Theorem 1.** Let us choose  $S$  such that the optimization of  $G$  in Eq.(15) (main paper) can be written as

$$\min_G \text{KLD}((P_G + P_G * P_\epsilon) || (P_R + P_R * P_\epsilon)) - \text{JSD}(\frac{1}{2}(P_R + P_R * P_\epsilon) || \frac{1}{2}(P_G + P_G * P_\epsilon)), \quad (\text{S1})$$

where  $P_\epsilon$  is a zero-mean Gaussian function produced by DANI with a positive definite covariance  $\Sigma$ . Let us also assume that the domain of  $P_G$  is restricted to  $\Omega$  (and thus  $P_G \in L^2(\Omega)$ ). Then, the global optimum of Eq.(S1) is  $P_G(x) = P_R(x), \forall x \in \Omega$ .

*Proof.* According to [1, 3, 4], the global minimum of the KL-2JS divergence is achieved if and only if

$$P_R + P_R * P_\epsilon = P_G + P_G * P_\epsilon. \quad (\text{S2})$$

Following the proofs of Theorem 1 as in [4], let  $P_G = P_R + \Delta$ . Then, we can obtain the  $\int |\Delta(x)|^2 dx < \infty$ . By substituting the  $P_G$  with  $P_R + \Delta$  in Eq.(S2), we have that  $\Delta * P_\epsilon = -\Delta$ . Based on our definition in the §3.2 in the main paper, both  $\Delta$  and  $P_\epsilon$  are in  $L^2(\Omega)$ . Then, we take the Fourier transform of both sides in the reformulated Eq.(S2) above, compute the absolute value and obtain

$$|\hat{\Delta}(\omega)| |\hat{P}_\epsilon(\omega)| = |\hat{\Delta}(\omega)|, \quad \forall \omega \in \hat{\Omega}. \quad (\text{S3})$$

Because  $P_G$  and  $P_R$  integrate to 1,  $\int \Delta(x) dx = 0$  and  $\hat{\Delta}(0) = 0$ . Suppose  $\exists \omega \neq 0$  such that  $\hat{\Delta}(\omega) \neq 0$ . Since  $P_\epsilon$  is under the Gaussian distribution, then  $|\hat{P}_\epsilon(\omega)| = |e^{-\frac{1}{2}\omega^\top \Sigma^{-1} \omega}| < 1$ , which contradicts the optimality condition in Eq.(S3). Thus  $\hat{\Delta}(x) = 0, \forall x \in \Omega$  and we can conclude that  $P_G(x) = P_R(x), \forall x \in \Omega$ .

**Theorem 2.** Let us choose  $S$  such that the optimization of  $G$  in Eq.(18) (main paper) can be written as

$$\min_G \text{KLD}((P_G + P_G * P_\epsilon) || (P_R + P_R * P_\epsilon)), \quad (\text{S4})$$

where  $P_\epsilon$  is a zero-mean Gaussian function produced by DANI with a positive definite covariance  $\Sigma$ . Let us also assume that the domain of  $P_G$  is restricted to  $\Omega$  (and thus  $P_G \in L^2(\Omega)$ ). Then, the global optimum of Eq.(S4) is  $P_G(x) = P_R(x), \forall x \in \Omega$ .

*Proof.* Based on the [4–6, 9] and the proofs in Theorem 1 above, the global minimum of the KL divergence is achieved if and only if

$$P_R + P_R * P_\epsilon = P_G + P_G * P_\epsilon, \quad (\text{S5})$$

where formulation of Eq.(S5) is the same as the formulation of Eq.(S2). Then, based on the detailed proofs in Theorem 1 above, we can obtain the same conclusion in Theorem 1 that  $P_G(x) = P_R(x), \forall x \in \Omega$ .

## 2 MORE DETAILS ABOUT EXPERIMENTS

### 2.1 More Experiments requirements and Pre-trained model with test code

**Experiments requirements:** The results in the main paper are trained by a workstation with CPU i9-10980XE, 128G ECC memory and four TITAN RTX GPUs ( $4 \times 24\text{G}$ ). The operating system is Ubuntu 18.04. To replicate our experimental environment, we recommend referring to the official open-source codes of InsGen<sup>1</sup> and Projected GAN<sup>2</sup> for instructions on obtaining the necessary software and Python libraries.

**Pre-trained models with test codes:** The pre-trained model with test code can be found in the anonymous Google drive link<sup>3</sup>. We will release all training codes once the paper is accepted.

### 2.2 More Experiments Results with Inception Score (IS)

To further show the superiority of the proposed DANI, we also show the experiment results on other commonly-used GANs evaluation metrics, i.e., Inception Score (IS) [7]. The results on low-shot datasets compared with the baseline Projected GAN are shown in Table S1. Projected GAN + DANI can achieve higher IS, demonstrating the superiority of the proposed DANI.

### 2.3 More Ablation Studies with Diffusion-GAN on the FFHQ Dataset

To further demonstrate that the noise injection form in the proposed DANI is better than Diffusion-GAN [10], we perform more ablation studies with Diffusion-GAN on the FFHQ dataset, and the results are shown in Table S2. Projected GAN + DANI can achieve better performance compared with Diffusion-Projected GAN on the FFHQ dataset.

### 2.4 More generated results on Projected GAN + DANI

To further show the superiority of the proposed DANI, we also show the interpolation videos of Projected GAN + DANI on low-shot datasets. Please check the folder “More generated results of interpolation videos on low-shot datasets” for the results.

## 3 ETHICS IMPACT

This paper proposes a novel noise injection method for the GANs with limited data called Dual Adaptive Noise Injection (DANI) that can benefit the practical deployment of training GANs with limited data with negligible computational cost. The technical contributions of this paper do not raise any particular ethical challenges. However,

<sup>1</sup><https://github.com/genforce/insgen>

<sup>2</sup><https://github.com/autonomousvision/projected-gan>

<sup>3</sup><https://drive.google.com/file/d/1txf67sw33M7wkXsnQcZ3WRuQDzsCT6UV/view?usp=sharing>

Method	100-shot			Animal-Face	
	Obama	Grumpy Cat	Panda	Cat	Dog
Projected GAN [8]	1.67	1.47	1.00	2.22	15.01
<b>Projected GAN + DANI</b>	<b>1.68</b>	<b>1.48</b>	<b>1.02</b>	<b>2.29</b>	<b>15.46</b>

**Table S1: Inception score (higher is better) on several low-shot datasets ( $256 \times 256$ ). Massive Augmentation [2] is applied to all of the methods. For a fair comparison, the Inception Scores are averaged over three runs; all standard deviations are less than 1% relatively.**

Method	MA	Backbone	FFHQ			
			100	1K	2K	5K
Diffusion-Projected GAN	Yes	FastGAN	25.47	10.97	7.99	6.59
<b>Projected GAN + DANI</b>	<b>Yes</b>	<b>FastGAN</b>	<b>23.98</b>	<b>10.81</b>	<b>7.73</b>	<b>6.20</b>

**Table S2: FID score (lower is better) on the  $256 \times 256$  FFHQ dataset. We follow the setting as in [11]. MA means Massive Augmentation, i.e., xflipping, which has the same meaning as in [2]. For a fair comparison, the FIDs are averaged over five runs; all standard deviations are less than 1% relatively. The results of the Diffusion-Projected GAN are run by ourselves based on the official open-source codes.**

because technology is usually a double-edged sword, our work may also bring potential social risks when applying GANs with limited data. For example, it may ease the fake media synthesis using only limited data, which may cause more fake media in daily life.

REFERENCES

[1] Martin Arjovsky and Léon Bottou. 2017. Towards principled methods for training generative adversarial networks. In *ICLR*.

[2] Kaiwen Cui, Jiaying Huang, Zhipeng Luo, Gongjie Zhang, Fangneng Zhan, and Shijian Lu. 2022. GenCo: Generative Co-training for Generative Adversarial Networks with Limited Data. In *AAAI*.

[3] Ian Goodfellow, Jean Pouget-Abadie, Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair, Aaron Courville, and Yoshua Bengio. 2014. Generative adversarial nets. In *NeurIPS*.

[4] Simon Jenni and Paolo Favaro. 2019. On stabilizing generative adversarial training with noise. In *CVPR*.

[5] Jae Hyun Lim and Jong Chul Ye. 2017. Geometric gan. *arXiv preprint arXiv:1705.02894* (2017).

[6] Takeru Miyato, Toshiaki Kataoka, Masanori Koyama, and Yuichi Yoshida. 2018. Spectral normalization for generative adversarial networks. In *ICLR*.

[7] Tim Salimans, Ian Goodfellow, Wojciech Zaremba, Vicki Cheung, Alec Radford, and Xi Chen. 2016. Improved techniques for training gans. In *NeurIPS*.

[8] Axel Sauer, Kashyap Chitta, Jens Müller, and Andreas Geiger. 2021. Projected gans converge faster. In *NeurIPS*.

[9] Dustin Tran, Rajesh Ranganath, and David M Blei. 2017. Deep and hierarchical implicit models. *arXiv preprint arXiv:1702.08896* 7, 3 (2017), 13.

[10] Zhendong Wang, Huangjie Zheng, Pengcheng He, Weizhu Chen, and Mingyuan Zhou. 2023. Diffusion-GAN: Training GANs with Diffusion. In *ICLR*.

[11] Shengyu Zhao, Zhijian Liu, Ji Lin, Jun-Yan Zhu, and Song Han. 2020. Differentiable augmentation for data-efficient gan training. In *NeurIPS*.