

## A CALCULATION OF THE EXPECTATION ON THE STYLE INFORMATION

We provide details of calculating  $\mathbb{E}_{\hat{s}(\tilde{X}) \sim \mathcal{N}(\mu(\tilde{X}), \sigma^2 \mathbf{I})} CE(g(\hat{s}(\tilde{X}); W_g), Y)$ . We assume a normal distribution for the styles, i.e.,  $\hat{s}(\tilde{X}) \sim \mathcal{N}(\mu(\tilde{X}), \sigma^2 \mathbf{I})$ . According to the definition of the cross-entropy loss, for a input pair  $(x, y)$  we have:

$$\begin{aligned}
& \mathbb{E}_{\hat{s}(x) \sim \mathcal{N}(\mu(x), \sigma^2 \mathbf{I})} CE(g(\hat{s}(x); W_g), y) \\
&= \mathbb{E}_{\hat{s}(x) \sim \mathcal{N}(\mu(x), \sigma^2 \mathbf{I})} \log \frac{1}{P(Y = y | g(\hat{s}(x); W_g))} \\
&\leq \log \frac{1}{\mathbb{E}_{\hat{s}(x) \sim \mathcal{N}(\mu(x), \sigma^2 \mathbf{I})} P(Y = y | g(\hat{s}(x); W_g))} \\
&= \log \frac{1}{\mathbb{E}_{\hat{s}(x) \sim \mathcal{N}(\mu(x), \sigma^2 \mathbf{I})} \frac{e^{W_{g,y}^\top \hat{s}(x)}}{\sum_j e^{W_{g,j}^\top \hat{s}(x)}}} \\
&= \log \mathbb{E}_{\hat{s}(x) \sim \mathcal{N}(\mu(x), \sigma^2 \mathbf{I})} \sum_j e^{(W_{g,j} - W_{g,y})^\top \hat{s}(x)} \\
&= \log \sum_j e^{(W_{g,j} - W_{g,y})^\top \hat{s}(x) + \frac{1}{2} (W_{g,j} - W_{g,y})^\top \sigma^2 \mathbf{I} (W_{g,j} - W_{g,y})} \\
&= \log \frac{\sum_j e^{W_{g,j}^\top \hat{s}(x) + \frac{\sigma^2}{2} (W_{g,j} - W_{g,y})^\top (W_{g,j} - W_{g,y})}}{W_{g,y}^\top \hat{s}(x)} \\
&\triangleq \log \frac{1}{P((Y = y | \bar{g}(\hat{s}(x); W_g))} \\
&\triangleq CE(\bar{g}(\hat{s}(x); W_g), y),
\end{aligned} \tag{1}$$

where the inequality follows from the Jensen's inequality:  $\mathbb{E} \log(X) \leq \log \mathbb{E} X$ , the expectation is calculated by leveraging the moment-generating function:

$$\mathbb{E} e^{tX} = e^{t\mu + \frac{1}{2} \sigma^2 t^2}, X \sim \mathcal{N}(\mu, \sigma^2). \tag{2}$$

Note that, we define the function  $\bar{g}(\hat{s}(x); W_g)$  for simplicity:

$$P((Y = y | \bar{g}(\hat{s}(x); W_g)) \triangleq \frac{W_{g,y}^\top \hat{s}(x)}{\sum_{j=1} e^{W_{g,j}^\top \hat{s}(x) + \frac{\sigma^2}{2} (W_{g,j} - W_{g,y})^\top (W_{g,j} - W_{g,y})}}. \tag{3}$$

## B RELATIONSHIP BETWEEN ORTHOGONALITY AND STATISTICAL INDEPENDENCE

We give the proof for the following lemma in Sec. 3.3. Note that, we use  $R$  to present the learned representation of  $X$ , and replace  $X$  with  $R$  for simplicity.

**Lemma 1.**  $R \in \mathbb{R}^d$  is the learned representation, where  $d$  is the number of dimension of  $R$ . Assume that  $R$  is a normal distribution with mean  $m$  and covariance matrix  $M$ . The content used for predicting labels, i.e., logits, is obtained by applying a linear functions to  $R$ , i.e.,  $\hat{c}(R) = W_c R$ , where  $W_c$  are parameters used for mapping  $R$  to logits. The style is modeled by a normal distribution, i.e.,  $\hat{s}(R) = \mu(R; W_s) + \Sigma(Y)^{\frac{1}{2}} \mathbf{n}$ , where  $W_s$  presents parameters for modeling the mean of styles, and  $\mathbf{n}$  is sampled from a standard normal distribution. Assume that  $\mu(R; W_s)$  is a linear function, i.e.,  $\hat{s}(R) = W_s R + \Sigma(Y)^{\frac{1}{2}} \mathbf{n}$ . Then, setting  $W_s$  as an instantiate of the orthogonal complement of  $W_c$  leads to statistical independence, i.e.,  $\hat{c}(R) \perp\!\!\!\perp \hat{s}(R)$ . Here,  $\perp\!\!\!\perp$  denotes the statistical independence, and we define  $\langle a, b \rangle_M = \langle a, Mb \rangle$  for a given semi-definite matrix  $M$ . The orthogonality  $A \perp_M B$  of two subspaces  $A$  and  $B$  is defined likewise.

*Proof.* Under the assumption in Lemma 1, setting  $W_s$  as an instantiate of the orthogonal complement of  $W_c$ , we have:

$$\begin{aligned}
\ker(W_s)^\perp \perp_M \ker(W_c)^\perp &\iff \text{im}(W_s^\top) \perp_M \text{im}(W_c^\top) \iff \langle W_s^\top \mathbf{a}, W_c^\top \mathbf{b} \rangle_M = 0 \forall \mathbf{a}, \mathbf{b} \\
&\iff \langle W_s^\top \mathbf{a}, MW_c^\top \mathbf{b} \rangle = 0 \forall \mathbf{a}, \mathbf{b} \iff W_s MW_c^\top = 0 \iff \mathbb{E}_R W_s (R - m) (R - m)^\top W_c^\top = 0 \\
&\iff \mathbb{E}_{R, \mathbf{n}} W_s \left( R + \Sigma^{\frac{1}{2}} \mathbf{n} - m \right) (R - m)^\top W_c^\top = 0 \iff \text{Cov}(\hat{s}(R), \hat{c}(R)) = 0 \\
&\iff \hat{c}(R) \perp \hat{s}(R)
\end{aligned} \tag{4}$$

□

## C MORE DETAILS ABOUT EVALUATION METRICS AND TRAINING DETAILS

**Evaluation metrics.** For MNIST dataset, we set the maximum perturbation bound  $\epsilon = 0.3$ , perturbation step size  $\eta = 0.01$ , and the number of iterations  $K = 40$  for PGD and C&W attacks, which keeps the same as (Zhang et al., 2019). Following (Rice et al., 2020), we set perturbation bound  $\epsilon = 8/255$ , perturbation step size  $\eta = \epsilon/10$ , and the number of iterations  $K = 20$  for CIFAR10 dataset.

**training details.** For MNIST, we use the same CNN architecture as (Carlini & Wagner, 2017; Zhang et al., 2019). Following (Zhang et al., 2019), the network is trained using SGD with 0.9 momentum for 50 epochs with an initial learning rate 0.01, and the batch size is set to 128. Hyper-parameters used to craft adversarial examples for training are the same as those used for evaluation. These two networks share the same hyper-parameters: we use SGD with 0.9 momentum, weight decay  $2 \times 10^{-4}$ , batch size 128, and an initial learning rate of 0.1. The maximum epoch is 120, and the learning rate is divided by 10 at epoch 60 and 90, respectively. To generate adversarial examples for training, we set the maximal perturbation  $\epsilon = 8/255$ , the perturbation step size  $\eta = 2/255$ , and the number of iterations  $K = 10$ , which is the same as (Rice et al., 2020).

## D EXPERIMENTS OF WRN-34-10 ON CIFAR10

Table 1: Classification accuracy (%) of WRN-34-10 on CIFAR-10 under the white-box threat model. The best-performance model and the corresponding accuracy are highlighted.

Method	Best checkpoint				Last checkpoint			
	Natural	FGSM	PGD-20	CW-20	Natural	FGSM	PGD-20	CW-20
Madry	<b>86.63</b>	59.48	53.65	53.58	<b>86.60</b>	57.07	49.23	49.46
ADA-M	85.24	<b>61.22</b>	<b>55.17</b>	<b>55.68</b>	85.61	<b>60.08</b>	<b>51.76</b>	<b>52.59</b>
TRADES	<b>84.32</b>	60.94	56.69	54.87	<b>84.86</b>	59.94	52.04	52.39
ADA-T	84.19	<b>61.62</b>	<b>57.36</b>	<b>55.75</b>	84.35	<b>61.57</b>	<b>55.15</b>	<b>55.23</b>

In Table 1, we report the accuracy of WRN-34-10 (Zagoruyko & Komodakis, 2016) of Madry, TRADES, and the proposed method on CIFAR10 against various attacks, i.e., FGSM, PGD, and C&W attacks, which are widely used in the literature. Here, “Natural” denotes the accuracy of natural test images. We denote by PGD-20 the PGD attack with 20 iterations for generating adversarial examples, which also applies to the C&W attack. We can see that the proposed method achieves the best robustness against all three types of attacks, demonstrating that taking into account the spurious correlation can significantly improve the adversarial robustness. Note that the standard deviations of 5 runs are omitted, because they hardly affect the results.

## E ABLATION STUDY

We implicitly conducted ablation studies when designing Table 1, Table 2, and Table 3. To further understand the comparative effects of different terms of the proposed method, we reorganize the robust accuracy of the best checkpoint trained on CIFAR-10 and CIFAR-100 in Table 2. Comparing Madry, TRADES, and ADA-M, we find that introducing the second ( $t_2$ ) and the third term ( $t_3$ )

Table 2: Robust accuracy (%) of ResNet-18 on CIFAR-10 and CIFAR-100 under the white-box threat model. For simplicity, we use  $t_1$ ,  $t_2$ , and  $t_3$  to represent the first, second, and third terms in Eq. 11, respectively. The best-performance model and the corresponding accuracy are highlighted.

Method	$t_1$	$t_2$	$t_3$	CIFAR-10			CIFAR-100		
				FGSM	PGD-20	CW-20	FGSM	PGD-20	CW-20
Madry	✓			56.69	51.92	51.00	56.69	51.92	51.00
ADA-M	✓		✓	57.98	54.44	52.51	57.98	54.44	52.51
TRADES	✓	✓		57.25	53.64	51.39	57.25	53.64	51.39
ADA-T	✓	✓	✓	<b>58.97</b>	<b>54.55</b>	<b>52.95</b>	<b>58.97</b>	<b>54.55</b>	<b>52.95</b>

can improve the robustness and that the effect of these two terms is close. Similarly, comparing TRADES and ADA-T, we see that introducing the third term ( $t_3$ ) can further improve the robustness.

## F MORE DETAILS ABOUT ADVERSARIAL LEARNING

Recent work on improving adversarial robustness mainly falls into two categories: certified defense and empirical methods.

Certified defense (Raghunathan et al., 2018; Wong & Kolter, 2018; Singla & Feizi, 2020) aims to endow the model with provably adversarial robustness against norm-bounded perturbations. Although the certified defense strategy is promising, the empirical defense (Goodfellow et al., 2014; Madry et al., 2017; Zhang et al., 2019; Wang et al., 2019; Pang et al., 2020; Wong & Kolter, 2018; Xie et al., 2019; Yang et al., 2019), especially the adversarial training method (Goodfellow et al., 2014; Madry et al., 2017; Zhang et al., 2019), is currently the most effective strategy. Empirical defense firstly generates adversarial examples using a certain adversarial attack, then incorporates the generated adversarial examples into the training process.

Recently, various efforts (Najafi et al., 2019; Carmon et al., 2019; Shafahi et al., 2019; Wong et al., 2020; Wang et al., 2019; Pang et al., 2020; Zhang et al., 2020b; Rice et al., 2020) have been devoted to improving adversarial training. One line of work focuses on accelerating the training procedure (Shafahi et al., 2019; Wong et al., 2020). Another line of research (Najafi et al., 2019; Carmon et al., 2019) shows a promising direction that unlabeled training data can significantly mitigate the adversarial vulnerability. Lastly, recent work (Wang et al., 2019; Pang et al., 2020; Zhang et al., 2020b; Rice et al., 2020) provides an interesting direction where these methods rethink the adversarial training from different aspects, containing rethinking the misclassified examples (Wang et al., 2019), rethinking the importance weight of each example (Zhang et al., 2020b) and rethinking the role of normalization (Pang et al., 2020) and basic training strategies (Rice et al., 2020). However, all these methods overlook the spurious correlation between labels and the style information.

Another related work is (Ilyas et al., 2019), which provides an interesting viewpoint, i.e., adversarial examples can be viewed as a human phenomenon because the model’s reliance on useful but not robust features leads to adversarial vulnerability. Our work gives a new causal perspective of adversarial vulnerability. Specifically, a) (Ilyas et al., 2019) found some features were useful but not robust, while our work explores the phenomenon’s fundamental cause and provides a clear explanation of why some features are useful but not robust: Given  $X$ , labels  $Y$  are spuriously correlated with the style variables, so fitting the spurious correlation can predict labels. Thus, the style variables can be viewed as ‘features’; b) (Ilyas et al., 2019) claimed that adversarial examples could be viewed as a human phenomenon, while our work shows that adversarial examples can be viewed as a model phenomenon rather than merely a human phenomenon. Specifically, the adversarial vulnerability results from fitting the correlation between labels and style variables and failing to fit the causal relations, i.e., DNNs fail to extract content variables.

## G MORE DETAILS ABOUT CAUSAL REASONING

The most relevant work is CAMA (Zhang et al., 2020a) that aims to improve the robustness of DNNs on unseen perturbation via explicitly modeling the perturbation from a causal view. The main difference between our method and CAMA is that we focus on the adversarial vulnerability

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while CAMA aims to improve the robustness of unseen perturbations. In addition, CAMA assumes a hard intervention on a latent variable. It promotes robustness via modeling the perturbation in the latent space. In this paper, we employ a soft intervention and propose to penalize DNNs when the adversarial distribution is different from the natural distribution. Another related work is RELIC (Mitrovic et al., 2020), a regularizer used in self-supervised learning that uses the independence of mechanisms (Peters et al., 2017) and encourages DNNs to be invariant to different augmentations of the same instance. The self-supervised learning method (Mitrovic et al., 2020) also constructs a causal graph to model the data generation process, but the focus of RELIC is on the content invariant property, overlooking the importance of style information.

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