Motivation
Deep learning (DL) has proven effective in medical image reconstruction. However, little is known about how robust these reconstructions are. Thus, this work analyzes the uncertainty of DL methods to improve risk quantification in a medical setting.

Background
- MRI is very effective but suffers from long acquisition times [1]
- To make scanning faster, less data is acquired, leading to low-quality images that must be “reconstructed” for a radiologist
- The equation $y = \phi x$ links measurements in the frequency domain to recovered images (i.e. linear inverse problem) [2]

SURE
- Stein’s Unbiased Risk Estimator (SURE) is a surrogate for MSE when ground truth is unknown
  $$ SURE = -n\sigma^2 + \|\hat{x} - x_0\|^2 + \sigma^2 tr\left(\frac{\partial^2}{\partial x_0^2}\right) $$
- Depends on following assumption for residuals (difference between input and ground truth):
  $$ r \sim N(0, \sigma^2 I) $$
- Density compensation modifies inputs to satisfy SURE requirements (zero mean / Gaussian residuals) by weighting undersampling masks by inverse of density at each point
  $$ \hat{x}_f = F^{-1} D^{-1} \Omega F x_0 $$
  $$ \hat{x}_f = x_0 + (F^{-1} D^{-1} \Omega F - I) x_0 $$
- Residuals are zero-mean because:
  $$ E[D^{-1} \Omega] = I $$

Problem Statement
- Compressive medical image recovery is an ambiguous problem, with many realistic reconstructions that are consistent with real measurement (see Fig. 1)
- What is the uncertainty associated with recovering the true image $x_0$?

Model
- **VAE model**: 4 convolutional layers in encoder, 4 transpose convolution layers for decoder, fully connected layers for latent space $\mu$ and $\sigma$ (Fig 2)
- **Data consistency**: affine projection based on undersampling mask
  $$ \min_{\hat{x}} \|\hat{x} - x_0\|^2 + \eta KL(N(\mu, \sigma)||N(0, I)) $$

Discussion
- SURE results are strongly correlated with MSE even when the ground truth is not known, showing its value as a general-purpose uncertainty metric (can be used across model architectures)
- Density compensation enforces zero-mean property of SURE residuals, in addition to improving normality
- Radiologists can use these uncertainty metrics as a guide when making diagnoses

Future Work
- Improve variance estimates for SURE (at both global and pixel-wise levels)
- Develop regularization schemes to improve the robustness of deep learning models in medical image recovery via training
- Data uncertainty: Explore effects of acquisition method and quantity of training data on uncertainty

References