Risk Quantification in Deep MRI Reconstruction

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Motivation

Deep learning (DL) has **proven effective** in medical image reconstruction. However, little is known about how **robust** these reconstructions are. Thus, this work **analyzes the uncertainty** of DL methods to improve risk quantification in a medical setting.

Background

- MRI is very effective but suffers from **long** acquisition times [1]
- To make scanning faster, less data is acquired, leading to **low-quality** images that must be "reconstructed" for a radiologist
- The equation y = ϕx links measurements in the frequency domain to recovered images (i.e. linear inverse problem) [2]

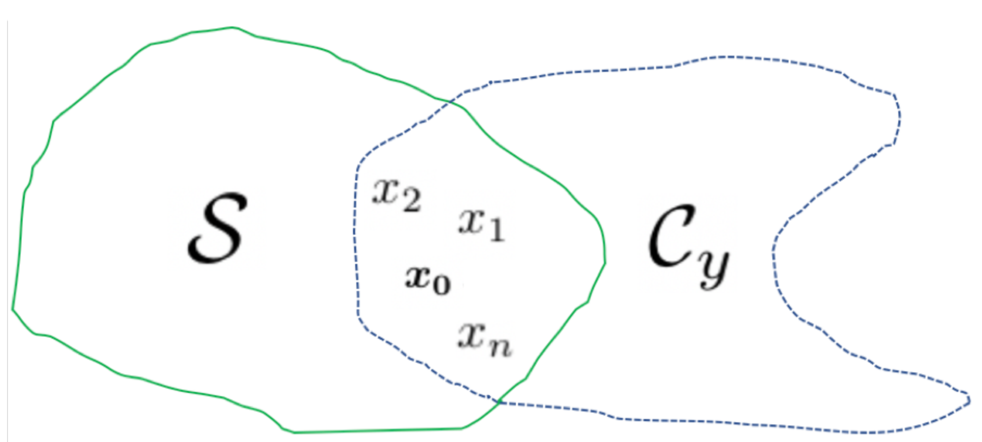


Figure 1 *Admissible solutions (x_0 is the true image we wish to recover)*

Problem Statement

- Compressive medical image recovery is an **ambiguous** problem, with many realistic reconstructions that are consistent with real measurement (see Fig. 1)
- What is the uncertainty associated with recovering the true image x_0?

Model

- **VAE model:** 4 convolutional layers in encoder, 4 transpose convolution layers for decoder, fully connected layers for latent space μ and σ (Fig 2)
- **Data consistency:** affine projection based on undersampling mask

$$\min \mathbb{E}_{x,y}\left[\|\hat{x}-x_0\|_2^2\right]+\eta KL\big(\mathcal{N}(\mu_z,\sigma_z)\|\mathcal{N}(0,1)\big)$$

SURE

- Stein's Unbiased Risk Estimator (SURE) is surrogate for MSE when ground truth is **unknown**

$$SURE = \underbrace{-n\sigma^2+\|\hat{x}-x_{zf}\|^2}_{\text{RSS}}+\sigma^2\underbrace{tr\Big(\frac{\partial\hat{x}}{\partial x_{zf}}\Big)}_{\text{DOF}}$$

- Depends on following assumption for residuals (difference between input and ground truth:

$$r\sim\mathcal{N}(0,\sigma^2 I)$$

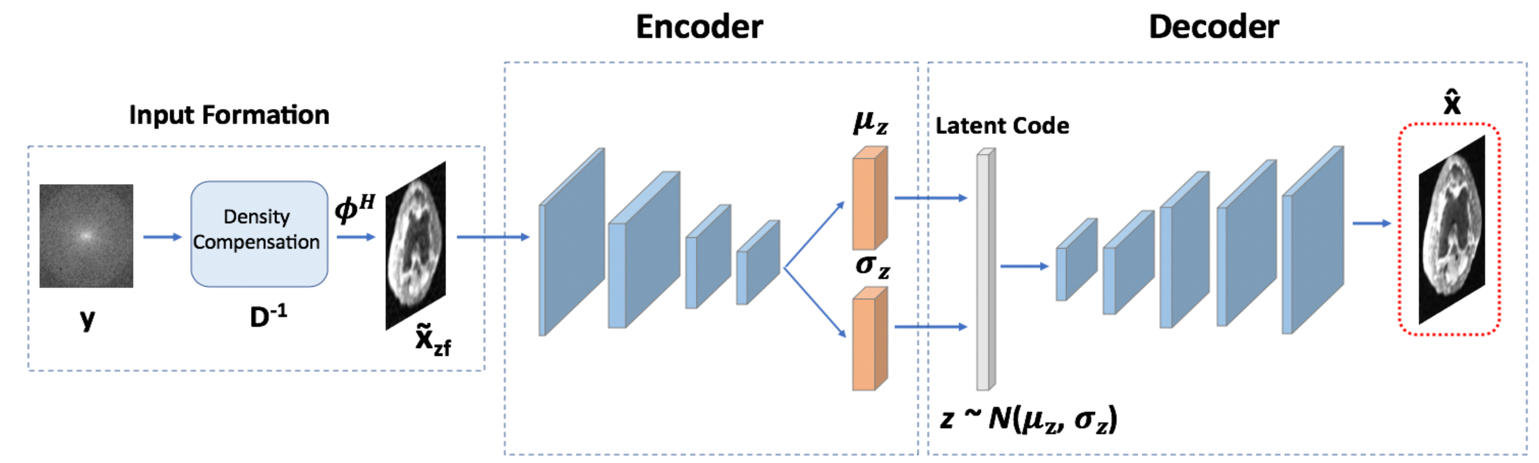
- Density compensation modifies inputs to satisfy SURE requirements (zero mean / Gaussian residuals) by weighting undersampling masks by inverse of density at each point

$$\tilde{x}_{zf}=F^{-1}D^{-1}\Omega F x_0$$

$$\tilde{x}_{zf}=x_0+\underbrace{(F^{-1}D^{-1}\Omega F-I)x_0}_{:=r}$$

- Residuals are zero-mean because:

$$\mathbb{E}[D^{-1}\Omega]=I$$



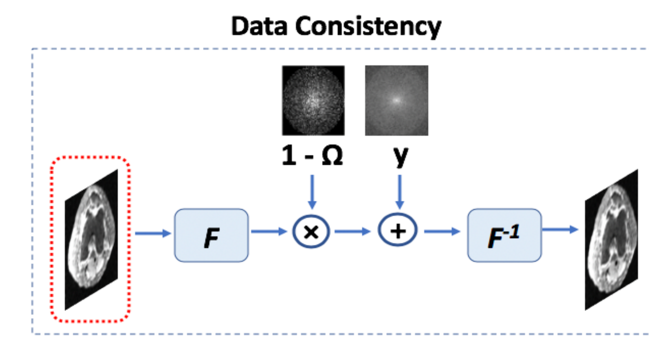


Figure 2 *Model architecture*

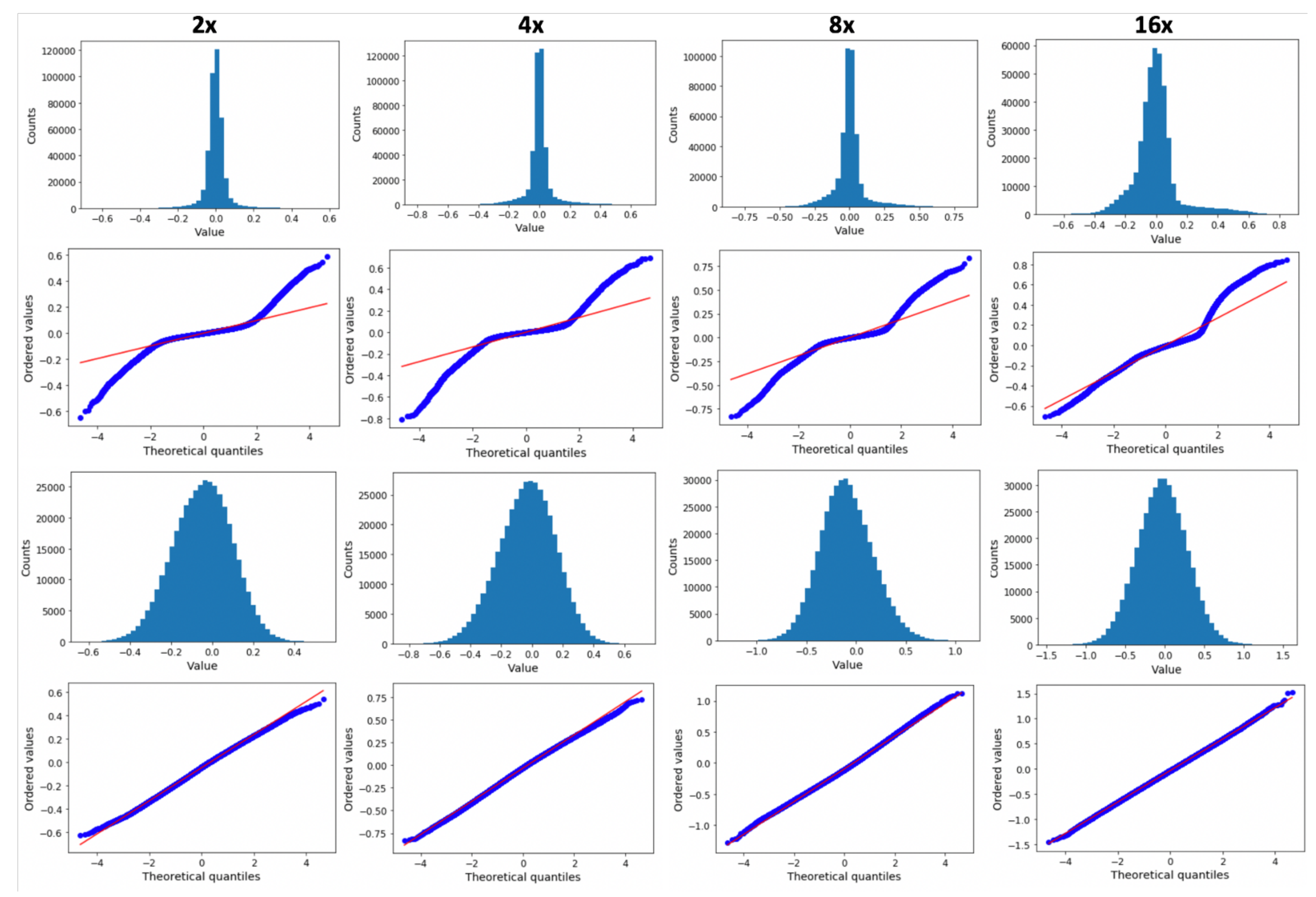


Figure 3 *Histograms and QQ plots before density compensation (top two rows) and after (bottom two rows).*

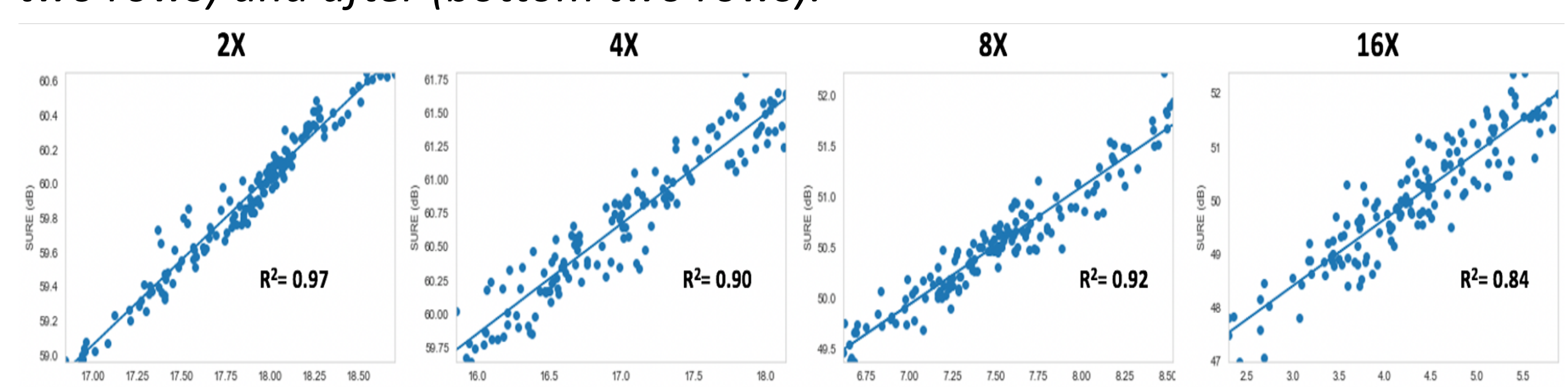


Figure 4 *Correlations between SURE and MSE at various undersampling rates*

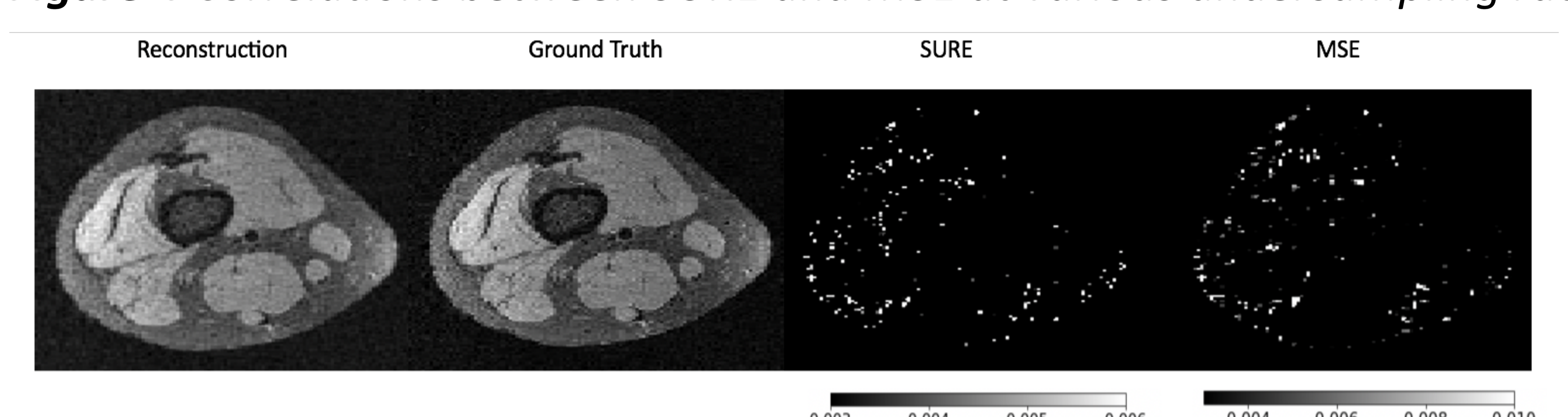


Figure 5 *Pixel-wise SURE and MSE for a given reference slice*

Discussion

- SURE results are strongly correlated with MSE even when the ground truth **is not known**, showing its value as a general-purpose uncertainty metric (can be used across model architectures)
- Density compensation enforces zero-mean property of SURE residuals, in addition to improving normality
- Radiologists can use these uncertainty metrics as a guide when making diagnoses

Future Work

- Improve variance estimates for SURE (at both global and pixel-wise levels)
- Develop **regularization** schemes to improve the robustness of deep learning models in medical image recovery via training
- **Data uncertainty:** Explore effects of acquisition method and quantity of training data on uncertainty

References

1. Tempany, C. M., Stewart, E. A., McDannold, N., Quade, B. J., Jolesz, F. A., & Hynynen, K. (2003). MR imaging–guided focused ultrasound surgery of uterine leiomyomas: a feasibility study. *Radiology*, *226*(3), 897-905.
2. Mardani, M., Gong, E., Cheng, J. Y., Vasanawala, S., Zaharchuk, G., Alley, M., ... & Xing, L. (2017). Deep generative adversarial networks for compressed sensing automates MRI. *arXiv preprint arXiv:1706.00051*.