Saliency Thresholds in Neural Code and its Relation to the Power-Law, Gaussian, and Lambert W Function

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Power-law Response Properties

- V1-like orientation energy models produce invariant heavy-tail, power-law-like response distributions across diverse images.

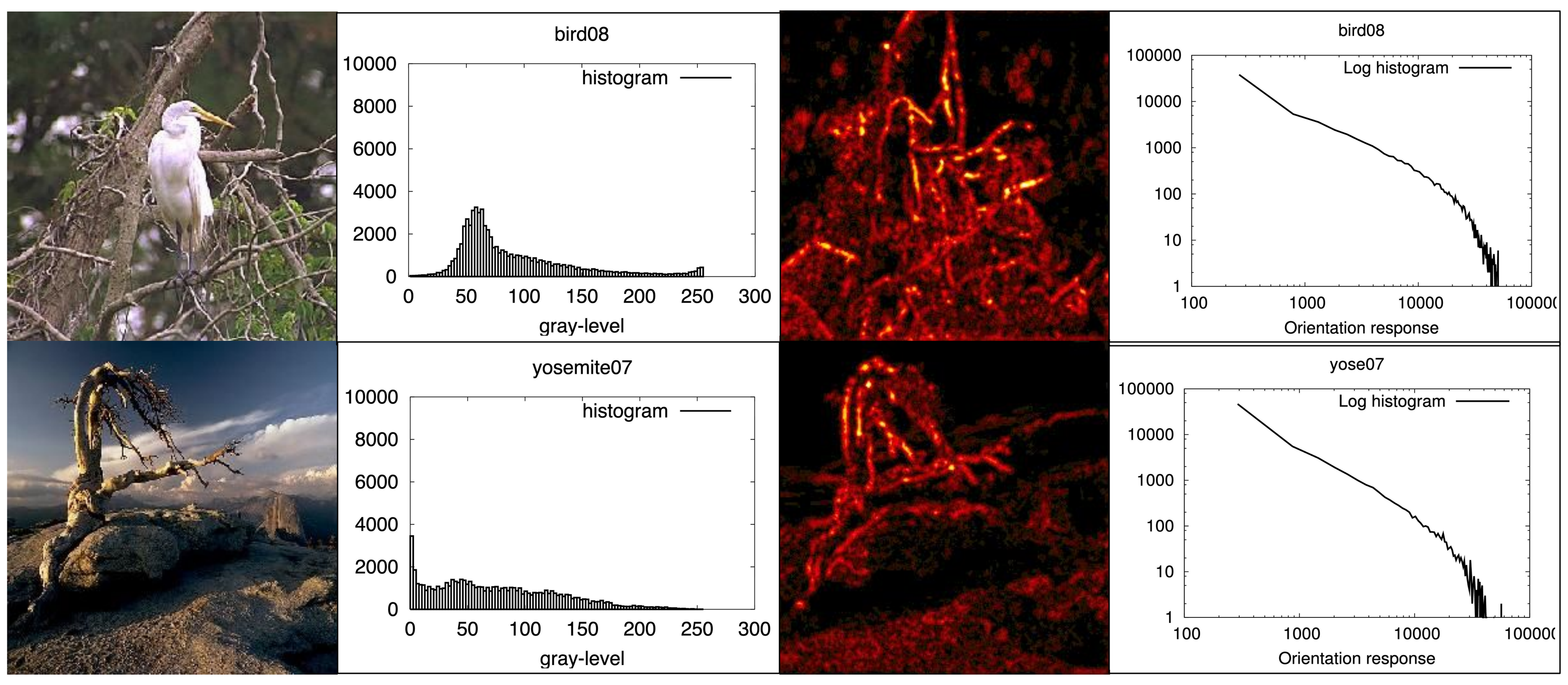


Figure 1. From left to right: natural image, grayscale intensity histogram, V1-like orientation energy map, log-log histogram of orientation responses showing power-law behavior.

Saliency Threshold & the Gaussian Baseline

- Responses to white-noise are Gaussian-like and a Gaussian with matched standard deviation serves as a baseline.

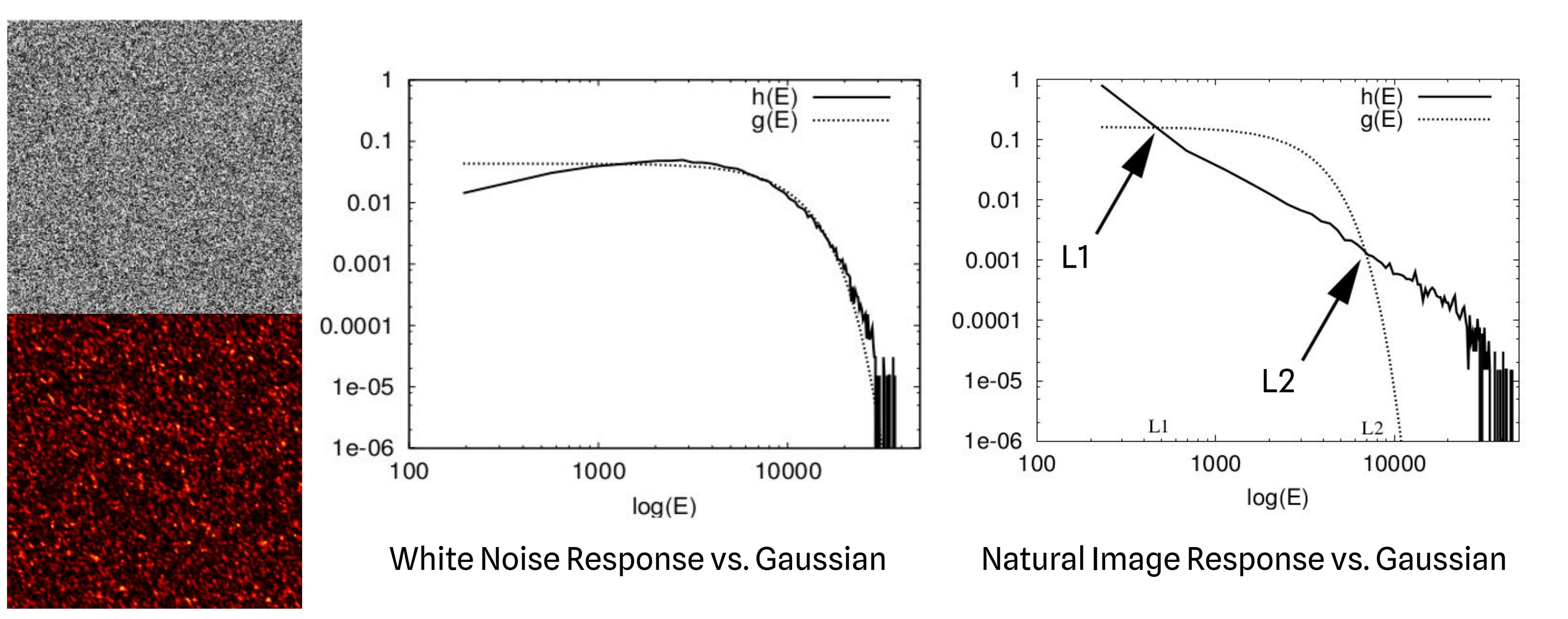
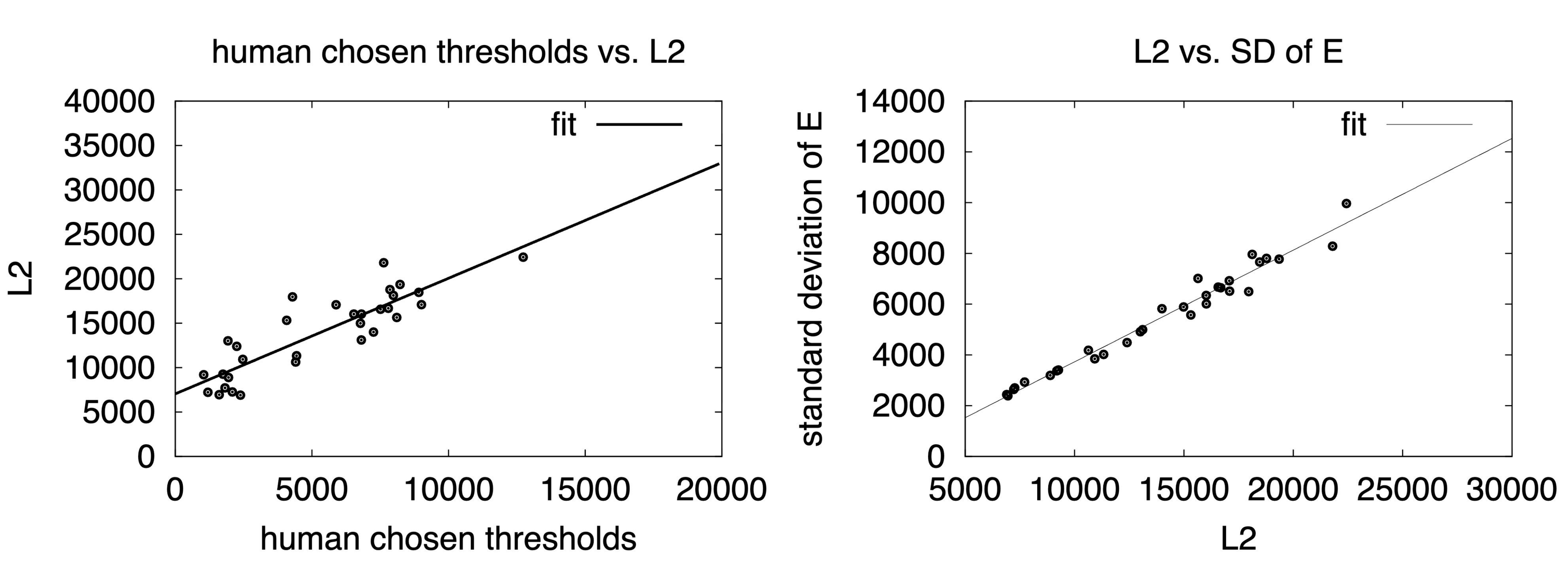


Figure 2. Left: white-noise image and its V1-like orientation-energy response. Middle: log-log histogram of white-noise responses vs. Gaussian fit. Right: log-log histogram of natural image responses compared with the same Gaussian prediction.

- The intersection (L2) between the power-law response and the matched Gaussian is linearly correlated with human-selected saliency thresholds.
- L2 is linearly correlated with the response standard deviation σ, yielding θ ≈ 1.37σ − 2176.69 and suggesting θ/σ ≈ constant (invariance).



- The standard deviation σ is readily computable in neural circuits (e.g., with quadratic and square-root activations), enabling simple and biologically-plausible saliency threshold estimation.

LC-CNN

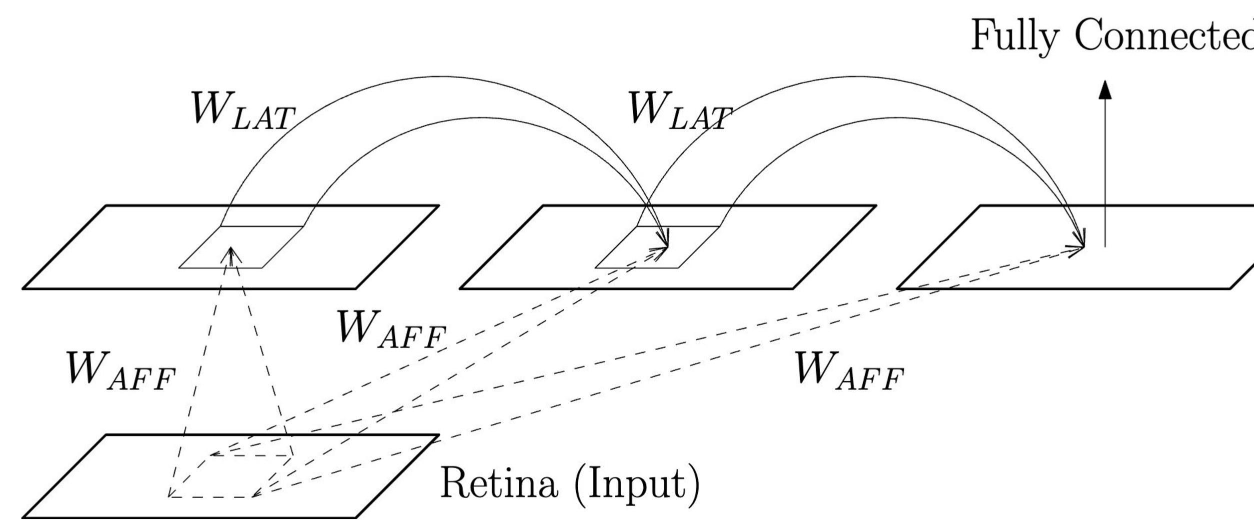
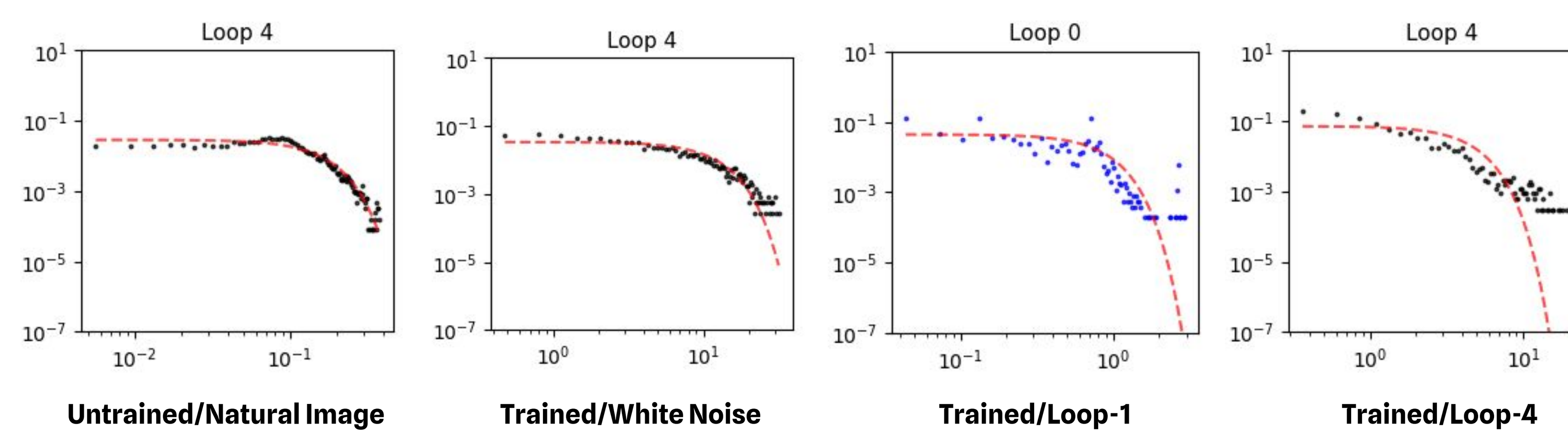


Figure 3. Unfolding of a CNN with lateral connections (LC-CNN Loop-1).

- Recurrent CNNs with shared lateral weights (LC-CNN) transition from Gaussian-like to power-law-like activity as recurrent loops increase.
- Trained LC-CNNs on natural images develop heavy tails, while untrained networks and white-noise inputs remain Gaussian across loops.



Lambert W?

- The Lambert W function is defined as the inverse of xe^x, satisfying $W(z)e^{W(z)} = z$. It naturally arises when solving equations where a variable appears both algebraically and inside an exponential such as enzyme kinetics, population models, and feedback control circuits.
- L2 can be computed analytically via the Lambert W function, providing a closed-form solution for the intersection point.

$$c\frac{1}{x^a} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \text{ gives } x = \pm\sqrt{-a\sigma^2 W\left(-\frac{(c\sigma\sqrt{2\pi})^{2/a}}{a\sigma^2}\right)}$$

Where c is a normalization factor, a is the power-law exponent, and σ = sd.

- This establishes a mathematical nexus linking power law, Gaussian, and Lambert W in neural coding and thresholding:
  - **Power law:** saliency threshold.
  - **Gaussian:** white noise response/baseline.
  - **Lambert W:** analytic bridge between the two.

Discussion and Conclusion

- LC-CNN activity starts Gaussian-like and progressively becomes power-law-like, suggesting recurrent processing is a key mechanism driving saliency extraction, efficient coding, and movement toward criticality.
- The proportionality θ/σ=c suggests invariance and a biologically plausible mechanism for saliency computation in the brain.
- Results point towards a unifying mathematical principle behind neural coding, criticality, and self-organization.

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