A Related Work

Table 1: Comparison of Diffusion-based Recommenders

	Modeling Target	*Forward Perturbation*	*Negative Sampling*	*Representative Work*
Continuous	Score Function	Gaussian on Embedding	✗	DreamRec [1]
	Score Function	Gaussian on Embedding	✓	PreferDiff [2]
	Preference Scores	Gaussian on One-hot	✓	DiffRec [3]
Discrete	Preference Scores	Bernoulli on One-hot	✓	RecFusion [4]
	Preference Scores	Categorical on One-hot	✓	DDSR [5])
	Preference Ratios	Fading on Items	✓	**PreferGrow** (Ours)

Diffusion-based recommenders [1, 6–22, 2, 3, 23–28, 5, 29, 30] utilize forward perturbation in diffusion models to address data sparsity [31], thereby better adapting to sparse recommendation scenarios. They typically consist of three core components: the modeling objective, the forward noise addition, and the corresponding backward generation process, which are summarized in Table 1. Current research on diffusion-based recommenders can be classified into two primary approaches: one involves adding noise to dense item embeddings at the item level, and the other focuses on perturbing the one-hot interaction probability vector for all items, which is referred to as preference scores. The first research line, pioneered by DreamRec [1] and DiffuRec [6], encodes user-preferred items as dense embeddings and adds Gaussian noise to these item embeddings. DreamRec [1] models the score functions (the gradient of the log-likelihood of the perturbed distribution) without negative sampling, whereas DiffuRec [6] incorporates a recommendation loss that includes negative sampling. Building on them, PreferDiff [2] introduced an optimization objective derived from BPR loss [32], which integrates multiple negative samples into the generative modeling framework. However, the application of continuous Gaussian noise to positive preferred items, in contrast to the discrete nature of negative samples, creates an inherent mismatch, making it difficult to optimize both simultaneously during training, leading to a trade-off [6, 33, 2]. [21] Additionally, other works have introduced more sophisticated module designs [7, 8, 10, 12–14, 16, 19, 21, 22] or applied them to different recommendation tasks [9–18, 20]. For instance, DimeRec [33] incorporates multi-interest models, DiQDiff [19] introduces semantic vector quantization, and DiffuASR [9] applies item-level diffusion to sequential recommendation data augmentation. The second research line [3, 23–28, 5, 29, 30] involves converting user preference data into one-hot vectors, which are then mapped to preference scores within the probability simplex. DiffRec [3] add continuous Gaussian noise to the preference scores, and then learn to recover the clean preference scores from the perturbed ones. Consecutively, LD4MRec [24] refines the design for efficient multimedia recommendation, and D3Rec [28] introduces targeted category preferences to control diversity during inference. However, the constraints of the probability simplex—non-negativity and normalization—pose significant challenges in accurately estimating the preference scores [34]. To address these challenges while considering the probability simplex constraints, RecFusion [4] assumes a Bernoulli noise prior, completing the binomial diffusion process and subsequently modeling the parameters of the reverse binomial distribution to facilitate the reverse generation of preference scores. On the other hand, DDSR [5] adopts a categorical noise prior, as proposed in [35], and directly recovers clean preference scores from the perturbed ones. Nonetheless, the constraints of the probability simplex may limit the effectiveness of preference score modeling. Moreover, both diffusion-based recommenders rely on prior noise assumptions, such as Gaussian [1] or Bernoulli noise [23], which may not be optimal for recommendation scenarios where user preference data is inherently discrete.

Discrete Diffusion Models [35–38, 34, 39, 40] have made substantial advances recently. Initially, D3PM [35] proposed a discrete diffusion framework based on an arbitrary probability transition matrix, trained with the evidence lower bound of the log-likelihood. Subsequently, LDR [36] extended this framework to a continuous-time setting using the Kolmogorov forward and backward equations. However, modeling score functions in such models presents challenges, as the gradient of the data distribution is undefined. To address this, CSM [37] introduced Concrete Score—discretization of score functions and the ratios of data distributions—as modeling objectives for discrete diffusion models. Building upon these advances, SEDD [34] further bridges discrete diffusion models and ConcreteScore by introducing the score entropy loss. Expanding on these developments, we present PreferGrow, a matrix-based discrete diffusion framework which perturbing data by retaining or replacing items within a discrete corpus. The idempotent property of the replacement matrix (or fading matrix) is central to PreferGrow, and we demonstrate that it satisfies the Kolmogorov forward and backward equations as LDR [36], aligning it with prior works. Additionally, we introduce a design paradigm for the idempotent replacement matrix, which unifies previous approaches, including absorbing and uniform settings.

B Proofs of Main Results

Proof of Theorem **??**: $\alpha_t : [0, T] \to [0, 1]$ is a strictly decreasing function with $\alpha_0 = 1$ and $\alpha_T = 0$ and the preference fading discrete diffusion process is denoted as $\mathbf{P}_{t|0} = \alpha_t \mathbf{I} + (1 - \alpha_t)\mathbf{E}, \forall t \in [0, T]$. Then we can rewrite preference fading discrete diffusion process as for $\alpha_0 = 1$:

$$\mathbf{P}_{t|0} = \frac{\alpha_t}{\alpha_0}\mathbf{I} + (1 - \frac{\alpha_t}{\alpha_0})\mathbf{E}, \forall t \in [0, T]. \tag{1}$$

For any well-defined $\mathbf{P}_{t|s,0}$ with $0 < s < t$, the following limiting distribution constraint must hold:

$$p_{t|0}(x_t|x_0) = \sum_{x_s \in \mathcal{X}} p_{t|s,0}(x_t|x_s, x_0) p_{s|0}(x_s|x_0), \forall x_t, x_0 \in \mathcal{X}. \tag{2}$$

In matrix form, this constraint reduces to $\mathbf{P}_{t|0} = \mathbf{P}_{t|s,0}\, \mathbf{P}_{s|0}$ for all $0 < s < t$. Note that one possible particular solution of this equation is:

$$\mathbf{P}_{t|s,0} := \mathbf{P}_{t|s} = \frac{\alpha_t}{\alpha_s}\mathbf{I} + (1 - \frac{\alpha_t}{\alpha_s})\mathbf{E}, \forall 0 \le s \le t \le T. \tag{3}$$

We will show that this indeed satisfies the constraint under the condition that $\mathbf{E}$ is idempotent.

$$\begin{aligned}
\mathbf{P}_{t|s}\mathbf{P}_{s|0} &= \left[\frac{\alpha_t}{\alpha_s}\mathbf{I} + (1 - \frac{\alpha_t}{\alpha_s})\mathbf{E}\right] \cdot \left[\frac{\alpha_s}{\alpha_0}\mathbf{I} + (1 - \frac{\alpha_s}{\alpha_0})\mathbf{E}\right] \\
&= \frac{\alpha_t}{\alpha_0}\mathbf{I} + \left(\frac{\alpha_t}{\alpha_s} - 2\frac{\alpha_t}{\alpha_0} + \frac{\alpha_s}{\alpha_0}\right)\mathbf{E} + \left(1 + \frac{\alpha_t}{\alpha_0} - \frac{\alpha_s}{\alpha_0} - \frac{\alpha_t}{\alpha_s}\right)\mathbf{E}^2 \\
&= \frac{\alpha_t}{\alpha_0}\mathbf{I} + \mathbf{E}^2 - \frac{\alpha_t}{\alpha_0}\mathbf{E} + \left(\frac{\alpha_t}{\alpha_s} - \frac{\alpha_t}{\alpha_0} + \frac{\alpha_s}{\alpha_0}\right)(\mathbf{E} - \mathbf{E}^2) \\
&\quad \text{the fading matrix is idempotent} \Rightarrow \mathbf{E}^2 = \mathbf{E} \\
&= \frac{\alpha_t}{\alpha_0}\mathbf{I} + (1 - \frac{\alpha_t}{\alpha_0})\mathbf{E} \\
&= \mathbf{P}_{t|0}.
\end{aligned} \tag{25}$$

Similarly, for all $0 \le r \le s \le t \le T$, there holds the Chapman-Kolmogorov equation:

$$\begin{aligned}
\mathbf{P}_{t|s}\mathbf{P}_{s|r} &= \left[\frac{\alpha_t}{\alpha_s}\mathbf{I} + (1 - \frac{\alpha_t}{\alpha_s})\mathbf{E}\right] \cdot \left[\frac{\alpha_s}{\alpha_r}\mathbf{I} + (1 - \frac{\alpha_s}{\alpha_r})\mathbf{E}\right] \\
&= \frac{\alpha_t}{\alpha_r}\mathbf{I} + \mathbf{E}^2 - \frac{\alpha_t}{\alpha_r}\mathbf{E} + \left(\frac{\alpha_t}{\alpha_s} - \frac{\alpha_t}{\alpha_r} + \frac{\alpha_s}{\alpha_r}\right)(\mathbf{E} - \mathbf{E}^2) \\
&\quad \text{the fading matrix is idempotent} \Rightarrow \mathbf{E}^2 = \mathbf{E} \\
&= \frac{\alpha_t}{\alpha_r}\mathbf{I} + (1 - \frac{\alpha_t}{\alpha_r})\mathbf{E} \\
&= \mathbf{P}_{t|r}.
\end{aligned} \tag{26}$$

We further note that $\mathbf{P}_{t|s}, \forall 0 \le s \le t \le T$ is invertible, with inverse given by:

$$\mathbf{P}_{t|s}^{-1} = \frac{\alpha_s}{\alpha_t}\mathbf{I} + (1 - \frac{\alpha_s}{\alpha_t})\mathbf{E}, \forall 0 \le s \le t \le T. \tag{27}$$

$$\begin{aligned}
\mathbf{P}_{t|s}^{-1}\mathbf{P}_{t|s} &= \left[\frac{\alpha_s}{\alpha_t}\mathbf{I} + (1 - \frac{\alpha_s}{\alpha_t})\mathbf{E}\right] \cdot \left[\frac{\alpha_t}{\alpha_s}\mathbf{I} + (1 - \frac{\alpha_t}{\alpha_s})\mathbf{E}\right] \\
&= \mathbf{I} + \left(\frac{\alpha_t}{\alpha_s} + \frac{\alpha_s}{\alpha_t} - 2\right) \cdot (\mathbf{E} - \mathbf{E}^2) \\
&\quad \text{the fading matrix is idempotent} \Rightarrow \mathbf{E}^2 = \mathbf{E} \\
&= \mathbf{I}.
\end{aligned} \tag{28}$$

$\forall 0 \le r \le s \le t \le T$, combined with limiting distribution constraint $\mathbf{P}_{t|r} = \mathbf{P}_{t|s,r}\mathbf{P}_{s|r}$, there are:

$$\begin{aligned}
\mathbf{P}_{t|s,r} &= \mathbf{P}_{t|r}\mathbf{P}_{s|r}^{-1} \\
&= \left[\frac{\alpha_t}{\alpha_r}\mathbf{I} + (1 - \frac{\alpha_t}{\alpha_r})\mathbf{E}\right] \cdot \left[\frac{\alpha_r}{\alpha_s}\mathbf{I} + (1 - \frac{\alpha_r}{\alpha_s})\mathbf{E}\right] \\
&= \frac{\alpha_t}{\alpha_s}\mathbf{I} + \mathbf{E}^2 - \frac{\alpha_t}{\alpha_s}\mathbf{E} + \left(\frac{\alpha_t}{\alpha_r} - \frac{\alpha_t}{\alpha_s} + \frac{\alpha_r}{\alpha_s}\right)(\mathbf{E} - \mathbf{E}^2) \\
&\quad \text{the fading matrix is idempotent} \Rightarrow \mathbf{E}^2 = \mathbf{E} \\
&= \frac{\alpha_t}{\alpha_s}\mathbf{I} + (1 - \frac{\alpha_t}{\alpha_s})\mathbf{E} \\
&= \mathbf{P}_{t|s}.
\end{aligned} \tag{29}$$

$\mathbf{P}_{t|s,r} = \mathbf{P}_{t|s}$ indicates $p_{t|s,r}(x_t|x_s, x_r) = p_{t|s}(x_t|x_s), \forall 0 \le r \le s \le t \le T$. In summary, the preference fading discrete diffusion process is Markovian but not time-homogeneous, satisfies the Chapman–Kolmogorov equation, and is reversible. □

Proof of Proposition **??***:* $\alpha_t : [0, T] \to [0, 1]$ is a strictly decreasing function with $\alpha_0 = 1$ and $\alpha_T = 0$. α_t is further defined as $e^{-\int_0^t \beta(\tau)\mathrm{d}\tau}$ with $\beta(\tau) > 0$. The preference fading discrete Markov diffusion process is denoted as $\mathbf{P}_{t|s} = \frac{\alpha_t}{\alpha_s}\mathbf{I} + (1 - \frac{\alpha_t}{\alpha_s})\mathbf{E}, \forall 0 \le s \le t \le T$. We first show that the preference fading discrete Markov diffusion process converges to a unified non-preference state $\vec{p}_T$ is well-defined. The transition rate matrix $\mathbf{Q}_t$ is computed as follows:

$$\begin{aligned}
\mathbf{Q}_t &= \lim_{s \to t} \frac{\partial \mathbf{P}_{t|s}}{\partial t} \\
&= \lim_{s \to t} \frac{\partial}{\partial t}\left(\frac{\alpha_t}{\alpha_s}\mathbf{I} + (1 - \frac{\alpha_t}{\alpha_s})\mathbf{E}\right) \\
&= \lim_{s \to t} \frac{\partial}{\partial \alpha_t}\left(\frac{\alpha_t}{\alpha_s}\mathbf{I} + (1 - \frac{\alpha_t}{\alpha_s})\mathbf{E}\right) \cdot \frac{\partial \alpha_t}{\partial t} \\
&= \lim_{s \to t} \left(\frac{1}{\alpha_s}\mathbf{I} - \frac{1}{\alpha_s}\mathbf{E}\right) \cdot (-\beta(t)) \cdot \alpha_t \\
&= \beta(t) \cdot (\mathbf{E} - \mathbf{I}).
\end{aligned} \tag{30}$$

The rate matrix $\mathbf{Q}_t$ characterizes the velocity of probability transitions at time t, encompassing both the direction and rate of transition. As shown in Equation (31), $\mathbf{Q}_t$ at any time $t \in [0, T]$ shares a consistent transition direction $\mathbf{I} - \mathbf{E}$, while the transition rate $\beta(t)$ varies over time. This time-dependent rate results in a non-homogeneous preference fading process. However, the shared transition direction ensures that all diffusion paths converge to the same non-preference state, making the process well-defined. Specifically, the stationary distribution $\vec{\pi}_t$ at time t satisfies the equilibrium condition $\mathbf{Q}_t\vec{\pi}_t = \vec{0}$. Since different $\mathbf{Q}_t$ matrices differ only by a scalar factor $\beta(t)$, they yield the same stationary solution $\vec{\pi}$. Consequently, the Markov process converges to a common steady-state distribution $\vec{\pi}$, *i.e.,* the non-preference state $\vec{p}_T$:

$$\mathbf{Q}_t\vec{\pi} = \vec{0} \Rightarrow (\mathbf{I} - \mathbf{E})\,\vec{p}_T = \vec{0}. \tag{31}$$

Given that $(\mathbf{I}-\mathbf{E})\mathbf{E}=\mathbf{0}$, each column of $\mathbf{E}$ satisfies the non-preference state equation $(\mathbf{I}-\mathbf{E})\,\vec{p}_T = \vec{0}$. To ensure a unique solution $\vec{p}_T$, we therefore assume that all columns of $\mathbf{E}$ are identical, i.e. $\mathbf{E} \propto \vec{p}_T \cdot \vec{1}^\top$. Considering the idempotence constraint $\mathbf{E}^2=\mathbf{E}$, we then have:

$$\mathbf{E} = \frac{\vec{p}_T \cdot \vec{1}^\top}{\vec{1}^\top \vec{p}_T}. \tag{32}$$

$$\begin{aligned}\mathbf{E}^2 &= \frac{\vec{p}_T \cdot \vec{1}^\top \cdot \vec{p}_T \cdot \vec{1}^\top}{(\vec{1}^\top \vec{p}_T)^2} \\ &= \frac{\vec{p}_T \cdot (\vec{1}^\top \vec{p}_T) \cdot \vec{1}^\top}{(\vec{1}^\top \vec{p}_T)^2} \\ &= \frac{\vec{p}_T \cdot \vec{1}^\top}{\vec{1}^\top \vec{p}_T} \\ &= \mathbf{E}.\end{aligned} \tag{33}$$

□

Proof of Proposition **??***:* We have $\mathbf{P}_{t|s} = \frac{\alpha_t}{\alpha_s}\mathbf{I} + (1-\frac{\alpha_t}{\alpha_s})\mathbf{E}$ and $\mathbf{Q}_t = \beta(t)\cdot(\mathbf{E}-\mathbf{I})$. Then we compute $\frac{\partial \mathbf{P}_{t|s}}{\partial t}$ as follows:

$$\begin{aligned}\frac{\partial \mathbf{P}_{t|s}}{\partial t} &= \frac{\partial}{\partial t}\left(\frac{\alpha_t}{\alpha_s}\mathbf{I} + (1-\frac{\alpha_t}{\alpha_s})\mathbf{E}\right) \\ &= \frac{\partial}{\partial \alpha_t}\left(\frac{\alpha_t}{\alpha_s}\mathbf{I} + (1-\frac{\alpha_t}{\alpha_s})\mathbf{E}\right)\cdot\frac{\partial \alpha_t}{\partial t} \\ &= \left(\frac{1}{\alpha_s}\mathbf{I} - \frac{1}{\alpha_s}\mathbf{E}\right)\cdot(-\beta(t))\cdot\alpha_t \\ &= \beta(t)\cdot\frac{\alpha_t}{\alpha_s}\cdot(\mathbf{E}-\mathbf{I}).\end{aligned} \tag{34}$$

There holds the Kolmogorov forward equation:

$$\begin{aligned}\mathbf{Q}_t\mathbf{P}_{t|s} &= \beta(t)\cdot(\mathbf{E}-\mathbf{I})\cdot\left[\frac{\alpha_t}{\alpha_s}\mathbf{I} + (1-\frac{\alpha_t}{\alpha_s})\mathbf{E}\right] \\ &= \beta(t)\cdot\left[-\frac{\alpha_t}{\alpha_s}\mathbf{I} + (2\frac{\alpha_t}{\alpha_s}-1)\mathbf{E} + (1-\frac{\alpha_t}{\alpha_s})\mathbf{E}^2\right] \\ &\qquad \text{the fading matrix is idempotent} \Rightarrow \mathbf{E}^2=\mathbf{E} \\ &= \beta(t)\cdot\frac{\alpha_t}{\alpha_s}\cdot(\mathbf{E}-\mathbf{I}) \\ &= \frac{\partial \mathbf{P}_{t|s}}{\partial t}.\end{aligned} \tag{35}$$

□

Proof of Theorem **??***:* The reverse preference growing process is denoted as $\Omega = \{\mathbf{P}_{s|T}\}_{s=T}^{0}$. A collection $\mathcal{F}$ of subsets of Ω is called a σ-algebra on Ω if it satisfies: 1) $\Omega \in \mathcal{F}$. 2) If $A \in \mathcal{F}$ then its complement $A^c = \Omega \setminus A$ also belongs to $\mathcal{F}$. 3) If $\{A_n\}_{n=1}^{\infty} \subseteq \mathcal{F}$, then the countable union $\bigcup_{n=1}^{\infty} A_n \in \mathcal{F}$. We first show that the preference growing process satisfies the Markov property. $\mathcal{F}_t$ is a σ-algebra of the preference growing process $\Omega_t = \{\mathbf{P}_{s|T}\}_{s=T}^{t}$. $\forall A \in \mathcal{F}_t$ and $0 \le s \le t \le T$:

$$\begin{aligned}p_{s|\ge t}(x_s|x_t,A) &= \frac{p_{s,t|>t}(x_s,x_t|A)}{p_{t|>t}(x_t|A)}\cdot\frac{p(A)}{p(A)} \\ &= \frac{p_{s,\ge t}(x_s,x_t,A)}{p_{\ge t}(x_t,A)} \\ &= \frac{p_{>t|t,s}(A|x_t,x_s)}{p_{>t|t}(A|x_t)}\cdot\frac{p_{t|s}(x_t|x_s)p_s(x_s)}{p_t(x_t)}\end{aligned} \tag{36}$$

$$\text{the preference fading process is Markovian} \Rightarrow \frac{p_{>t|t,s}(A|x_t, x_s)}{p_{>t|t}(A|x_t)} = 1$$

$$= \frac{p_s(x_s)}{p_t(x_t)} p_{t|s}(x_t|x_s)$$

$$\text{the Bayes' theorem} \Rightarrow \frac{p_s(x_s)}{p_t(x_t)} p_{t|s}(x_t|x_s) = p_{s|t}(x_s|x_t)$$

$$= p_{s|t}(x_s|x_t).$$

We rewrite the equation $p_{s|t}(x_s|x_t) = \frac{p_s(x_s)}{p_t(x_t)} \cdot p_{t|s}(x_t|x_s)$ in matrix form $\forall 0 \leq s \leq t \leq T$:

$$p_{s|t}(x_s|x_t) = \frac{p_s(x_s)}{p_t(x_t)} \cdot p_{t|s}(x_t|x_s). \tag{37}$$

$$\vec{p}_s \cdot \left(\frac{1}{\vec{p}_t}\right)^{\top} \text{ denotes the matrix with entries} \frac{p_s(x_s = y)}{p_t(x_t = x)}, \forall x, y \in \mathcal{X}$$

$$\mathbf{P}_{s|t} = \left[\vec{p}_s \cdot \left(\frac{1}{\vec{p}_t}\right)^{\top}\right] \odot \mathbf{P}_{t|s}^{\top}$$

$$\vec{p}_t = \mathbf{P}_{t|s} \cdot \vec{p}_s \Rightarrow \vec{p}_s = \mathbf{P}_{s|t}^{-1} \cdot \vec{p}_t$$

$$= \mathbf{P}_{t|s}^{-1} \cdot \left[\vec{p}_t \cdot \left(\frac{1}{\vec{p}_t}\right)^{\top}\right] \odot \mathbf{P}_{t|s}^{\top}.$$

The reverse-time preference growing process progresses from $s = T$ to $s = 0$, and thus α_s is an increasing function, transitioning from $\alpha_T = 0$ to $\alpha_0 = 1$. We then compute the reverse transition rate matrix $\mathbf{R}_s$ as follows:

$$\mathbf{R}_s = \lim_{t \to s} \frac{\partial \mathbf{P}_{s|t}}{\partial s}$$

$$= \lim_{t \to s} \frac{\partial \mathbf{P}_{s|t}}{\partial \alpha_s} \cdot \frac{\partial \alpha_s}{\partial s} \tag{38}$$

$$= \lim_{t \to s} \frac{\partial}{\partial \alpha_s} \left(\mathbf{P}_{t|s}^{-1} \cdot \left[\vec{p}_t \cdot \left(\frac{1}{\vec{p}_t}\right)^{\top}\right] \odot \mathbf{P}_{t|s}^{\top}\right) \cdot \frac{\partial \alpha_s}{\partial s}$$

$$= \lim_{t \to s} \left(\frac{\partial \mathbf{P}_{t|s}^{-1}}{\partial \alpha_s} \cdot \left[\vec{p}_t \cdot \left(\frac{1}{\vec{p}_t}\right)^{\top}\right] \odot \mathbf{P}_{t|s}^{\top} + \mathbf{P}_{t|s}^{-1} \cdot \left[\vec{p}_t \cdot \left(\frac{1}{\vec{p}_t}\right)^{\top}\right] \odot \frac{\partial \mathbf{P}_{t|s}^{\top}}{\partial \alpha_s}\right) \cdot \frac{\partial \alpha_s}{\partial s}$$

$$\frac{\partial \mathbf{P}_{t|s}^{-1}}{\partial \alpha_s} = \frac{\partial}{\partial \alpha_s} \left(\frac{\alpha_s}{\alpha_t}\mathbf{I} + (1 - \frac{\alpha_s}{\alpha_t})\mathbf{E}\right) = \frac{1}{\alpha_t}(\mathbf{I} - \mathbf{E})$$

$$\lim_{t \to s} \mathbf{P}_{t|s}^{-1} = \lim_{t \to s} \left(\frac{\alpha_s}{\alpha_t}\mathbf{I} + (1 - \frac{\alpha_s}{\alpha_t})\mathbf{E}\right) = \mathbf{I}$$

$$\frac{\partial \mathbf{P}_{t|s}^{\top}}{\partial \alpha_s} = \frac{\partial}{\partial \alpha_s} \left(\frac{\alpha_t}{\alpha_s}\mathbf{I} + (1 - \frac{\alpha_t}{\alpha_s})\mathbf{E}^{\top}\right) = -\frac{\alpha_t}{\alpha_s^2}(\mathbf{I} - \mathbf{E}^{\top})$$

$$\lim_{t \to s} \mathbf{P}_{t|s}^{\top} = \lim_{t \to s} \left(\frac{\alpha_t}{\alpha_s}\mathbf{I} + (1 - \frac{\alpha_t}{\alpha_s})\mathbf{E}^{\top}\right) = \mathbf{I}$$

$$= \lim_{t \to s} \left(\frac{1}{\alpha_t}(\mathbf{I} - \mathbf{E}) \cdot \left[\vec{p}_t \cdot \left(\frac{1}{\vec{p}_t}\right)^{\top}\right] \odot \mathbf{I} - \left[\vec{p}_t \cdot \left(\frac{1}{\vec{p}_t}\right)^{\top}\right] \odot \frac{\alpha_t}{\alpha_s^2}(\mathbf{I} - \mathbf{E}^{\top})\right) \cdot \frac{\partial \alpha_s}{\partial s}$$

$$\mathbf{Q}_t = \beta(t) \cdot (\mathbf{E} - \mathbf{I}), \mathbf{Q}_t^{\top} = \beta(t) \cdot \left(\mathbf{E}^{\top} - \mathbf{I}\right)$$

$$\text{Note that from time } t \text{ to time } s < t, \alpha_s \text{ is increasing.} \Rightarrow \frac{\partial \alpha_s}{\partial s} = \alpha_s \beta(s) > 0$$

$$= \beta(s) \cdot (\mathbf{E}^{\top} - \mathbf{I}) \odot \left[\vec{p}_s \cdot \left(\frac{1}{\vec{p}_s}\right)^{\top}\right] - \beta(s) \cdot (\mathbf{E} - \mathbf{I}) \cdot \left[\vec{p}_s \cdot \left(\frac{1}{\vec{p}_s}\right)^{\top}\right] \odot \mathbf{I}$$

$$= \mathbf{Q}_s^\top \odot \left[\vec{p}_s \cdot \left(\frac{1}{\vec{p}_s}\right)^\top\right] - \mathbf{Q}_s \cdot \left[\vec{p}_t \cdot \left(\frac{1}{\vec{p}_t}\right)^\top\right] \odot \mathbf{I}.$$

97 Moreover, we compute $\frac{\partial \mathbf{P}_{t|s}}{\partial s}$ and $\frac{\partial \mathbf{P}_{s|t}}{\partial s}$ as follows:

$$\frac{\partial \mathbf{P}_{t|s}}{\partial s} = \frac{\partial}{\partial \alpha_s}\left(\frac{\alpha_t}{\alpha_s}\mathbf{I} + (1 - \frac{\alpha_t}{\alpha_s})\mathbf{E}\right) \cdot \frac{\partial \alpha_s}{\partial s} \tag{39}$$

Note that from time t to time $s < t$, α_s is increasing. $\Rightarrow \dfrac{\partial \alpha_s}{\partial s} = \alpha_s \beta(s) > 0$

$$= \left(-\frac{\alpha_t}{\alpha_s^2}\mathbf{I} + \frac{\alpha_t}{\alpha_s^2}\mathbf{E}\right) \cdot \alpha_s \beta(s)$$

$$= \beta(s) \cdot \frac{\alpha_t}{\alpha_s} \cdot (\mathbf{E} - \mathbf{I})$$

$(\mathbf{E} - \mathbf{I})^2 = \mathbf{E}^2 - 2\mathbf{E} + \mathbf{I} = -(\mathbf{E} - \mathbf{I}), (\mathbf{E} - \mathbf{I})\mathbf{E} = \mathbf{0}$

$$= \beta(s) \cdot \left[-\frac{\alpha_t}{\alpha_s} \cdot (\mathbf{E} - \mathbf{I})^2\right] + \beta(s) \cdot (\mathbf{E} - \mathbf{I})\mathbf{E}$$

$$= \left[\frac{\alpha_t}{\alpha_s}\mathbf{I} + (1 - \frac{\alpha_t}{\alpha_s})\mathbf{E}\right] \cdot [\beta(s) \cdot (\mathbf{E} - \mathbf{I})]$$

$$= \mathbf{P}_{t|s}\mathbf{Q}_s.$$

$$\frac{\partial \mathbf{P}_{s|t}}{\partial s} = \frac{\partial}{\partial s}\left(\mathbf{P}_{t|s}^{-1} \cdot \left[\vec{p}_t \cdot \left(\frac{1}{\vec{p}_t}\right)^\top\right] \odot \mathbf{P}_{t|s}^\top\right) \tag{40}$$

$$= \frac{\partial}{\partial s}\left(\left[\vec{p}_s \cdot \left(\frac{1}{\vec{p}_t}\right)^\top\right] \odot \mathbf{P}_{t|s}^\top\right)$$

$$= \left[\frac{\partial \vec{p}_s}{\partial s} \cdot \left(\frac{1}{\vec{p}_t}\right)^\top\right] \odot \mathbf{P}_{t|s}^\top + \left[\vec{p}_s \cdot \left(\frac{1}{\vec{p}_t}\right)^\top\right] \odot \frac{\partial \mathbf{P}_{t|s}^\top}{\partial s}$$

The reverse time s begins with $s = T$. $\Rightarrow \dfrac{\partial \mathbf{P}_{s|0}}{\partial s} = -\mathbf{Q}_s \cdot \mathbf{P}_{s|0}$

$\dfrac{\partial \vec{p}_s}{\partial s} = \dfrac{\partial \mathbf{P}_{s|0} \cdot \vec{p}_0}{\partial s} = -\mathbf{Q}_s \cdot \mathbf{P}_{s|0} \cdot \vec{p}_0 = -\mathbf{Q}_s \cdot \vec{p}_s$

$\dfrac{\partial \mathbf{P}_{t|s}}{\partial s} = \mathbf{P}_{t|s} \cdot \mathbf{Q}_s \Rightarrow \dfrac{\partial \mathbf{P}_{t|s}^\top}{\partial s} = (\mathbf{P}_{t|s} \cdot \mathbf{Q}_s)^\top = \mathbf{Q}_s^\top \cdot \mathbf{P}_{t|s}^\top$

$$= \left[\vec{p}_s \cdot \left(\frac{1}{\vec{p}_t}\right)^\top\right] \odot (\mathbf{Q}_s^\top \cdot \mathbf{P}_{t|s}^\top) - \left[\mathbf{Q}_s \cdot \vec{p}_s \cdot \left(\frac{1}{\vec{p}_t}\right)^\top\right] \odot \mathbf{P}_{t|s}^\top.$$

98 Finally, we prove that the preference growing process satisfies the Kolmogorov backward equation:

$$\mathbf{R}_s\mathbf{P}_{s|t} = \left\{\mathbf{Q}_s^\top \odot \left[\vec{p}_s \cdot \left(\frac{1}{\vec{p}_s}\right)^\top\right] - \mathbf{Q}_s\left[\vec{p}_t \cdot \left(\frac{1}{\vec{p}_t}\right)^\top\right] \odot \mathbf{I}\right\} \cdot \left\{\left[\vec{p}_s \cdot \left(\frac{1}{\vec{p}_t}\right)^\top\right] \odot \mathbf{P}_{t|s}^\top\right\} \tag{41}$$

The idea behind this step is to prove that the elements at each position of the matrices are identical, thereby establishing their equality.
The details are provided in Equation (42), (43) and (44).

$$= \left\{\left[\vec{p}_s \cdot \left(\frac{1}{\vec{p}_t}\right)^\top\right] \odot (\mathbf{Q}_s^\top \cdot \mathbf{P}_{t|s}^\top)\right\} - \left\{\left[\mathbf{Q}_s \cdot \vec{p}_s \cdot \left(\frac{1}{\vec{p}_t}\right)^\top\right] \odot \mathbf{P}_{t|s}^\top\right\}$$

$$= \frac{\partial \mathbf{P}_{s|t}}{\partial s}.$$

$$
\begin{aligned}
&\left\{\mathbf{Q}_s^\top \odot \left[\vec{p}_s \cdot \left(\frac{1}{\vec{p}_s}\right)^\top\right]\right\} \cdot \left\{\left[\vec{p}_s \cdot \left(\frac{1}{\vec{p}_t}\right)^\top\right] \odot \mathbf{P}_{t|s}^\top\right\}(x,y) \\
&= \sum_{z\in\mathcal{X}} q_s(z,x)\frac{p_s(x)}{p_s(z)} \cdot \frac{p_s(z)}{p_t(y)} p_{t|s}(y|z) \\
&= \frac{p_s(x)}{p_t(y)} \cdot \sum_{z\in\mathcal{X}} q_s(z,x) p_{t|s}(y|z) \\
&= \left\{\left[\vec{p}_s \cdot \left(\frac{1}{\vec{p}_t}\right)^\top\right] \odot (\mathbf{Q}_s^\top \cdot \mathbf{P}_{t|s}^\top)\right\}(x,y), \forall x,y \in \mathcal{X}.
\end{aligned}
\tag{42}
$$

$$
\begin{aligned}
&\left\{\mathbf{Q}_s \cdot \left[\vec{p}_s \cdot \left(\frac{1}{\vec{p}_s}\right)^\top\right] \odot \mathbf{I}\right\}(x,y) \\
&= \sum_{z\in\mathcal{X}} q_s(x,z)\frac{p_s(z)}{p_s(y)} \cdot \delta_x(y) \\
&= \delta_x(y) \cdot \sum_{l\in\mathcal{X}} q_s(x,l)\frac{p_s(l)}{p_s(x)}.
\end{aligned}
\tag{43}
$$

$$
\begin{aligned}
&\left\{\mathbf{Q}_s \cdot \left[\vec{p}_s \cdot \left(\frac{1}{\vec{p}_s}\right)^\top\right] \odot \mathbf{I}\right\} \cdot \left\{\left[\vec{p}_s \cdot \left(\frac{1}{\vec{p}_t}\right)^\top\right] \odot \mathbf{P}_{t|s}^\top\right\}(x,y) \\
&= \sum_{z\in\mathcal{X}} \delta_x(z) \cdot \sum_{l\in\mathcal{X}} q_s(x,l)\frac{p_s(l)}{p_s(x)} \cdot \frac{p_s(z)}{p_t(y)} p_{t|s}(y|z) \\
&= \sum_{l\in\mathcal{X}} q_s(x,l)\frac{p_s(l)}{p_s(x)} \cdot \frac{p_s(x)}{p_t(y)} p_{t|s}(y|x) \\
&= p_{t|s}(y|x) \cdot \sum_{l\in\mathcal{X}} q_s(x,l)\frac{p_t(l)}{p_t(y)} \\
&= p_{t|s}(y|x) \cdot \sum_{l\in\mathcal{X}} q_s(x,l)\frac{p_s(l)}{p_t(y)} \\
&= \left\{\left[\mathbf{Q}_s \cdot \vec{p}_s \cdot \left(\frac{1}{\vec{p}_t}\right)^\top\right] \odot \mathbf{P}_{t|s}^\top\right\}(x,y), \forall x,y \in \mathcal{X}.
\end{aligned}
\tag{44}
$$

99 $\square$

100 C Details of Different Fading Matrix Setting

101 C.1 Point-Wise Setting

Let $\vec{p}_1 = \vec{e}_{-1} \in \mathbb{R}^{N+1}$, we have fading matrix $\mathbf{E}$ as follows:

$$
\mathbf{E} = \begin{pmatrix} 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 \\ 1 & \cdots & 1 & 1 \end{pmatrix}
$$

In this case, the reference ratios $r_t(x_0, x_t \in \{x_0, x_{-1}\}, y \in \{x_0, x_{-1}\})$ model only the ratios between the general hard negative x_{-1} and real items $\mathcal{X}$.

$$r_t(x_0, x_t, y) = \begin{cases} r_t(x_0, x_t, x_t) = 0 & \text{if } x_t = y \\ r_t(x_0, x_0, x_{-1}) = \log \frac{1-\alpha_t}{\alpha_t} & \text{if } y = x_{-1} \text{ and } x_t = x_0 \\ r_t(x_0, x_{-1}, x_0) = \log \frac{\alpha_t}{1-\alpha_t} & \text{if } y = x_0 \text{ and } x_t = x_{-1} \end{cases}$$

$$\mathbf{Q}_t(x, y) = \beta(t) \cdot (\mathbf{E} - \mathbf{I}) = \begin{cases} -\beta(t) & \text{if } x = y \neq x_{-1} \\ \beta(t) & \text{if } x = x_{-1} \text{ and } x_t \neq x_{-1} \\ 0 & \text{otherwise} \end{cases}$$

We eastimates the preference ratios $\log \frac{p_t(y|u)}{p_t(x_t|u)}$ with $s_\Theta(x_t, t, u)_y$:

$$l_{SE}(x_0, x_t, y|u) = e^{s_\Theta(x_t,t,u)_y} - e^{r_t(x_0,x_t,y)} s_\Theta(x_t, t, u)_y + e^{r_t(x_0,x_t,y)}[r_t(x_0, x_t, y) - 1].$$

For one user preference data (u, x_0), with faded item x_t, we compute $\mathcal{L}_{SE}(x_0, x_t, y|u)$:

$$\begin{aligned} \mathcal{L}_{SE} &= \sum_{y \in \{x_0, x_{-1}\}} \mathbf{Q}_t(x_t, y) \cdot l_{SE}(x_0, x_t, y|u) \\ &= \mathbf{Q}_t(x_t, x_t) \cdot l_{SE}(x_0, x_t, x_t|u) + \sum_{y \in \{x_0, x_{-1}\} \setminus \{x_t\}} \mathbf{Q}_t(x_t, y) \cdot l_{SE}(x_0, x_t, y|u) \\ &\quad s_\Theta(x_t, t, u)_{x_t} = r_t(x_0, x_t, x_t) = 0 \Rightarrow l_{SE}(x_0, x_t, x_t|u) = 0 \\ &= \sum_{y \in \{x_0, x_{-1}\} \setminus \{x_t\}} \mathbf{Q}_t(x_t, y) \cdot l_{SE}(x_0, x_t, y|u) \\ &\quad \mathbf{Q}_t(x_t \neq x_{-1}, y \neq x_t) = 0 \Rightarrow \mathcal{L}_{SE}(x_0, x_0, y|u) = 0 \Rightarrow x_t = x_{-1} \\ &= \mathbf{Q}_t(x_{-1}, x_0) \cdot l_{SE}(x_0, x_{-1}, x_0|u) \\ &= \beta(t) \cdot \left[e^{s_\Theta(x_t,t,u)_y} - \frac{\alpha_t}{1-\alpha_t} \cdot s_\Theta(x_t, t, u)_y + \frac{\alpha_t}{1-\alpha_t} [\log \frac{\alpha_t}{1-\alpha_t} - 1] \right]. \end{aligned}$$

C.2 Pair-Wise Setting

Let $\vec{p}_T = \vec{1} \in \mathbb{R}^N$, we have fading matrix $\mathbf{E}$ as follows:

$$\mathbf{E} = \begin{pmatrix} \frac{1}{N} & \cdots & \frac{1}{N} \\ \vdots & \ddots & \vdots \\ \frac{1}{N} & \cdots & \frac{1}{N} \end{pmatrix}$$

In this case, the reference ratios $r_t(x_0, x_t \in \mathcal{X}, y \in \mathcal{X})$ model the ratios of all item pairs.

$$r_t(x_0, x_t, y) = \begin{cases} r_t(x_0, x_t, x_t) = 0 & \text{if } x_t = y \\ r_t(x_0, x_0, y \neq x_0) = -\log(1 + N \cdot \frac{\alpha_t}{1-\alpha_t}) & \text{if } y \neq x_0 \text{ and } x_t = x_0 \\ r_t(x_0, x_t \neq x_0, x_0) = \log(1 + N \cdot \frac{\alpha_t}{1-\alpha_t}) & \text{if } y = x_0 \text{ and } x_t \neq x_0 \\ r_t(x_0, x_t \neq x_0, y \neq x_0) = 0 & \text{otherwise} \end{cases}$$

$$\mathbf{Q}_t(x, y) = \beta(t) \cdot (\mathbf{E} - \mathbf{I}) = \begin{cases} \beta(t)(\frac{1}{N} - 1) & \text{if } x = y \\ \beta(t) \cdot \frac{1}{N} & \text{otherwise} \end{cases}$$

We eastimates the preference ratios $\log \frac{p_t(y|u)}{p_t(x_t|u)}$ with $s_\Theta(x_t, t, u)_y$:

$$l_{SE}(x_0, x_t, y|u) = e^{s_\Theta(x_t,t,u)_y} - e^{r_t(x_0,x_t,y)} s_\Theta(x_t, t, u)_y + e^{r_t(x_0,x_t,y)}[r_t(x_0, x_t, y) - 1].$$

For one user preference data (u, x_0), with faded item x_t, we compute $\mathcal{L}_{SE}(x_0, x_t, y|u)$:

$$\mathcal{L}_{SE} = \sum_{y \in \mathcal{X}} \mathbf{Q}_t(x_t, y) \cdot l_{SE}(x_0, x_t, y|u)$$

$$= \mathbf{Q}_t(x_t, x_t) \cdot l_{SE}(x_0, x_t, x_t|u) + \sum_{y \in \mathcal{X} \setminus \{x_t\}} \mathbf{Q}_t(x_t, y) \cdot l_{SE}(x_0, x_t, y|u)$$

$$s_\Theta(x_t, t, u)_{x_t} = r_t(x_0, x_t, x_t) = 0 \Rightarrow l_{SE}(x_0, x_t, x_t|u) = 0$$

$$= \sum_{y \in \mathcal{X} \setminus \{x_t\}} \mathbf{Q}_t(x_t, y) \cdot l_{SE}(x_0, x_t, y|u)$$

$$\mathbf{Q}_t(x_t, y \neq x_t) = \beta(t) \cdot \frac{1}{N}$$

$$= \beta(t) \cdot \frac{1}{N} \cdot \sum_{y \in \mathcal{X} \setminus \{x_t\}} l_{SE}(x_0, x_{-1}, x_0|u)$$

$$e^{\bar{\mathbf{s}}_\Theta(x_t, t, u)} = \frac{1}{N} \cdot \sum_{y \in \mathcal{X} \setminus \{x_t\}} e^{s_\Theta(x_t, t, u)_y} = \frac{1}{N} \cdot \left[\sum_{y \in \mathcal{X}} e^{s_\Theta(x_t, t, u)_y} - 1 \right]$$

$$\bar{\mathbf{s}}_\Theta(x_t, t, u) = \frac{1}{N} \cdot \sum_{y \in \mathcal{X} \setminus \{x_t\}} s_\Theta(x_t, t, u)_y = \frac{1}{N} \cdot \sum_{y \in \mathcal{X}} s_\Theta(x_t, t, u)_y$$

if $x_t = x_0$, denote$\Delta = N \cdot \dfrac{\alpha_t}{1 - \alpha_t}$:

$$= \beta(t) \cdot \left[e^{\bar{\mathbf{s}}_\Theta(x_t, t, u)} - \frac{1}{1 + \Delta} \cdot \bar{\mathbf{s}}_\Theta(x_t, t, u) - \frac{1}{1 + \Delta} (\log(1 + \Delta) + 1) \right]$$

if $x_t \neq x_0$, denote$\Delta = N \cdot \dfrac{\alpha_t}{1 - \alpha_t}$:

$$= \beta(t) \cdot \left[e^{\bar{\mathbf{s}}_\Theta(x_t, t, u)} - \bar{\mathbf{s}}_\Theta(x_t, t, u) - \Delta \cdot s_\Theta(x_t, t, u)_{x_0} + (1 + \Delta)(\log(1 + \Delta) - 1) \right].$$

105 ### C.3 Hybrid-Wise Setting

Let $\vec{p}_1 = \lambda(\vec{1}, 0) + (1 - \lambda)\vec{e}_{-1} \in \mathbb{R}^{n+1}$, we have fading matrix $\mathbf{E}$ as follows:

$$\mathbf{E} = \begin{pmatrix} \frac{\lambda}{N} & \cdots & \frac{\lambda}{N} & \frac{\lambda}{N} \\ \vdots & \ddots & \vdots & \vdots \\ \frac{\lambda}{N} & \cdots & \frac{\lambda}{N} & \frac{\lambda}{N} \\ 1 - \lambda & \cdots & 1 - \lambda & 1 - \lambda \end{pmatrix}$$

In this case, the reference ratios $r_t(x_0, x_t \in \mathcal{X}, y \in \mathcal{X})$ model the ratios of all item pairs.

$$r_t(x_0, x_t, y) = \begin{cases} 0 & \text{if } x_t = y \\ \log \frac{\alpha_t + (1-\alpha_t)\frac{\lambda}{N}}{(1-\alpha_t)(1-\lambda)} & \text{else if } y = x_0 \text{ and } x_t = x_{-1} \\ \log \frac{\frac{\lambda}{N}}{(1-\lambda)} & \text{else if } y \neq x_0 \text{ and } x_t = x_{-1} \\ \log \frac{(1-\alpha_t)(1-\lambda)}{\alpha_t + (1-\alpha_t)\frac{\lambda}{N}} & \text{else if } y = x_{-1} \text{ and } x_t = x_0 \\ \log \frac{(1-\alpha_t)\frac{\lambda}{N}}{\alpha_t + (1-\alpha_t)\frac{\lambda}{N}} & \text{else if } y \neq x_{-1} \text{ and } x_t = x_0 \\ \log \frac{(1-\alpha_t)(1-\lambda)}{(1-\alpha_t)\frac{\lambda}{N}} & \text{else if } y = x_{-1} \text{ and } x_t \neq \{x_{-1}, x_0\} \\ \log \frac{\alpha_t + (1-\alpha_t)\frac{\lambda}{N}}{(1-\alpha_t)\frac{\lambda}{N}} & \text{else if } y = x_0 \text{ and } x_t \neq \{x_{-1}, x_0\} \\ 0 & \text{else if } y \neq \{x_{-1}, x_0\} \text{ and } x_t \neq \{x_{-1}, x_0\} \end{cases}$$

$$\mathbf{Q}_t(x, y) = \beta(t) \cdot (\mathbf{E} - \mathbf{I}) = \begin{cases} \beta(t)(-\lambda) & \text{if } x = y = x_{-1} \\ \beta(t)(\frac{\lambda}{N} - 1) & \text{if } x = y \neq x_{-1} \\ \beta(t)(1 - \lambda) & \text{if } x \neq y \text{ and } x = x_{-1} \\ \beta(t)(\frac{\lambda}{N}) & \text{if } x \neq y \text{ and } x \neq x_{-1} \end{cases}$$

We estimate the preference ratios $\log \frac{p_t(y|u)}{p_t(x_t|u)}$ with $s_\Theta(x_t, t, u)_y$:

$$l_{SE}(x_0, x_t, y|u) = e^{s_\Theta(x_t, t, u)_y} - e^{r_t(x_0, x_t, y)} s_\Theta(x_t, t, u)_y + e^{r_t(x_0, x_t, y)}[r_t(x_0, x_t, y) - 1].$$

For one user preference data (u, x_0), with faded item x_t, we compute $\mathcal{L}_{SE}(x_0, x_t, y|u)$:

$$
\begin{aligned}
\mathcal{L}_{SE} &= \sum_{y\in\mathcal{X}} \mathbf{Q}_t(x_t, y)\cdot l_{SE}(x_0, x_t, y|u) \\
&= \mathbf{Q}_t(x_t, x_t)\cdot l_{SE}(x_0, x_t, x_t|u) + \sum_{y\in\mathcal{X}\backslash\{x_t\}} \mathbf{Q}_t(x_t, y)\cdot l_{SE}(x_0, x_t, y|u) \\
&\quad s_\Theta(x_t, t, u)_{x_t} = r_t(x_0, x_t, x_t) = 0 \Rightarrow l_{SE}(x_0, x_t, x_t|u) = 0 \\
&= \sum_{y\in\mathcal{X}\backslash\{x_t\}} \mathbf{Q}_t(x_t, y)\cdot l_{SE}(x_0, x_t, y|u) \\
&\quad \mathbf{Q}_t(x_t = x_{-1}, y \neq x_t) = \beta(t)\cdot(1-\lambda), \mathbf{Q}_t(x_t \neq x_{-1}, y \neq x_t) = \beta(t)\cdot(\frac{\lambda}{N}) \\
&= \mathbf{Q}_t(x_t, y\neq x_t)\cdot \sum_{y\in\mathcal{X}\backslash\{x_t\}} l_{SE}(x_0, x_{-1}, x_0|u) \\
&\quad e^{\bar{\mathbf{s}}_\Theta(x_t,t,u)} = \frac{1}{N}\cdot\sum_{y\in\mathcal{X}\backslash\{x_t\}} e^{s_\Theta(x_t,t,u)_y} = \frac{1}{N}\cdot\left[\sum_{y\in\mathcal{X}} e^{s_\Theta(x_t,t,u)_y} - 1\right] \\
&\quad \bar{\mathbf{s}}_\Theta(x_t, t, u) = \frac{1}{N}\cdot\sum_{y\in\mathcal{X}\backslash\{x_t\}} s_\Theta(x_t, t, u)_y = \frac{1}{N}\cdot\sum_{y\in\mathcal{X}} s_\Theta(x_t, t, u)_y \\
&\text{if } x_t = x_{-1}\text{, denote } \Delta = N\cdot\frac{\alpha_t}{1-\alpha_t} \text{ and } \Lambda = \frac{\lambda\cdot\Delta}{\lambda\cdot\Delta + N}: \\
&= \beta(t)\cdot\left[N(1-\lambda)\cdot e^{\bar{\mathbf{s}}_\Theta(x_t,t,u)} - \lambda\cdot\bar{\mathbf{s}}_\Theta(x_t,t,u) - \frac{\frac{1}{\Lambda}-\lambda}{N}\cdot s_\Theta(x_t,t,u)_{x_0}\right] \\
&+\beta(t)\cdot\left\{\lambda\cdot\left[1+\frac{1}{N}\left(\frac{1}{\Lambda}-1\right)\right]\cdot\left[\log\left(\frac{1}{N}\cdot\frac{\lambda}{1-\lambda}\right)-1\right] - \frac{\log\Lambda}{\Lambda}\right\} \\
&\text{if } x_t = x_0\text{, denote } \Delta = N\cdot\frac{\alpha_t}{1-\alpha_t} \text{ and } \Lambda = \frac{\lambda\cdot\Delta}{\lambda\cdot\Delta + N}: \\
&= \beta(t)\cdot\left[\lambda\cdot e^{\bar{\mathbf{s}}_\Theta(x_t,t,u)} - \lambda\cdot\Lambda\cdot\bar{\mathbf{s}}_\Theta(x_t,t,u) - (1-\lambda-\frac{\lambda}{N})\cdot\Lambda\cdot s_\Theta(x_t,t,u)_{x_{-1}}\right] \\
&+\beta(t)\cdot\left\{(1-\frac{\lambda}{N})\cdot\Lambda\cdot(\log\Lambda - 1) - (1-\lambda)\cdot\Lambda\cdot(\log(\frac{1}{N}\frac{\lambda}{1-\lambda}))\right\} \\
&\text{if } x_t \neq \{x_0, x_{-1}\}\text{, denote } \Delta = N\cdot\frac{\alpha_t}{1-\alpha_t} \text{ and } \Lambda = \frac{\lambda\cdot\Delta}{\lambda\cdot\Delta + N}: \\
&= \beta(t)\cdot\left[\lambda\cdot e^{\bar{\mathbf{s}}_\Theta(x_t,t,u)} - \lambda\cdot\bar{\mathbf{s}}_\Theta(x_t,t,u)\right] \\
&-\beta(t)\cdot\left[(\frac{1}{N\cdot\Lambda}-\frac{\lambda}{N})\cdot\Lambda\cdot s_\Theta(x_t,t,u)_{x_0} + (1-\lambda-\frac{\lambda}{N})\cdot s_\Theta(x_t,t,u)_{x_0}\right] \\
&+\beta(t)\cdot\left\{\frac{1}{N\cdot\Lambda}\cdot(\log\Lambda + \log\lambda - 1) - (1-\lambda)\cdot(1+\log(\frac{1}{N}\frac{\lambda}{1-\lambda})) - (1-\frac{2}{N})\cdot\lambda\right\}.
\end{aligned}
$$

C.4 Adative Setting

Let $\vec{p}_T = \vec{\mu} \in \mathbb{R}^N$ or $\mathbb{R}^{N+1}$, with $\sum_{x\in\mathcal{X}} \mu_x = 1$, we have fading matrix $\mathbf{E}$ as follows:

$$
\mathbf{E} = \begin{pmatrix} \mu_1 & \cdots & \mu_1 \\ \vdots & \ddots & \vdots \\ \mu_{|\mathcal{X}|} & \cdots & \mu_{|\mathcal{X}|} \end{pmatrix}
$$

By arbitrarily specifying a distribution $\vec{\mu}$ satisfying $\sum_{x\in\mathcal{X}} \mu_x = 1$, we can instantiate any desired non-preference state, which is physically associated with different negative sampling strategies. In this case, the reference ratios $r_t(x_0, x_t \in \mathcal{X}, y \in \mathcal{X})$ are computed as follows:

$$r_t(x_0,x_t,y) = \begin{cases} r_t(x_0,x_t,x_t) = 0 & \text{if } x_t = y \\ r_t(x_0,x_0,y \neq x_0) = -\log(\frac{\alpha_t+(1-\alpha_t)\cdot\mu_{x_t}}{(1-\alpha_t)\cdot\mu_y}) & \text{if } y \neq x_0 \text{ and } x_t = x_0 \\ r_t(x_0,x_t \neq x_0,x_0) = \log(\frac{\alpha_t+(1-\alpha_t)\cdot\mu_{x_t}}{(1-\alpha_t)\cdot\mu_y}) & \text{if } y = x_0 \text{ and } x_t \neq x_0 \\ r_t(x_0,x_t \neq x_0,y \neq x_0) = \log\frac{\mu_y}{\mu_{x_t}} & \text{otherwise} \end{cases}$$

$$\mathbf{Q}_t(x,y) = \beta(t)\cdot(\mathbf{E}-\mathbf{I}) = \begin{cases} \beta(t)(\frac{1}{N}-1) & \text{if } x = y \\ \beta(t)\cdot\frac{1}{N} & \text{otherwise} \end{cases}$$

We eastimates the preference ratios $\log\frac{p_t(y|u)}{p_t(x_t|u)}$ with $s_\Theta(x_t,t,u)_y$:

$$l_{SE}(x_0,x_t,y|u) = e^{s_\Theta(x_t,t,u)_y} - e^{r_t(x_0,x_t,y)}s_\Theta(x_t,t,u)_y + e^{r_t(x_0,x_t,y)}[r_t(x_0,x_t,y)-1].$$

For one user preference data (u,x_0), with faded item x_t, we compute $\mathcal{L}_{SE}(x_0,x_t,y|u)$:

$$\begin{aligned}
\mathcal{L}_{SE} &= \sum_{y\in\mathcal{X}} \mathbf{Q}_t(x_t,y)\cdot l_{SE}(x_0,x_t,y|u) \\
&= \mathbf{Q}_t(x_t,x_t)\cdot l_{SE}(x_0,x_t,x_t|u) + \sum_{y\in\mathcal{X}\setminus\{x_t\}} \mathbf{Q}_t(x_t,y)\cdot l_{SE}(x_0,x_t,y|u) \\
&\quad s_\Theta(x_t,t,u)_{x_t} = r_t(x_0,x_t,x_t) = 0 \Rightarrow l_{SE}(x_0,x_t,x_t|u) = 0 \\
&= \sum_{y\in\mathcal{X}\setminus\{x_t\}} \mathbf{Q}_t(x_t,y)\cdot l_{SE}(x_0,x_t,y|u) \\
&\quad \mathbf{Q}_t(x_t,y\neq x_t) = \beta(t)\cdot\mu_{x_t} \\
&= \beta(t)\cdot\frac{1}{N}\cdot\sum_{y\in\mathcal{X}\setminus\{x_t\}} l_{SE}(x_0,x_{-1},x_0|u) \\
&\quad e^{\bar{\mathbf{s}}_\Theta(x_t,t,u)} = \frac{1}{N}\cdot\sum_{y\in\mathcal{X}\setminus\{x_t\}} e^{s_\Theta(x_t,t,u)_y} = \frac{1}{N}\cdot\left[\sum_{y\in\mathcal{X}} e^{s_\Theta(x_t,t,u)_y} - 1\right] \\
&\quad \tilde{\mathbf{s}}_\Theta(x_t,t,u) = \sum_{y\in\mathcal{X}\setminus\{x_t\}} \mu_y\cdot s_\Theta(x_t,t,u)_y = \sum_{y\in\mathcal{X}} \mu_y\cdot s_\Theta(x_t,t,u)_y \\
&\quad \hat{\mu} = \sum_{y\in\mathcal{X}} \mu_y\cdot\log\mu_y \\
&\text{if } x_t = x_0\text{, denote } \Sigma = \frac{\alpha_t}{1-\alpha_t}: \\
&= \beta(t)\cdot\left[\mu_{x_t}\cdot e^{\bar{\mathbf{s}}_\Theta(x_t,t,u)} - \frac{1}{\Sigma+\mu_{x_t}}\cdot\tilde{\mathbf{s}}_\Theta(x_t,t,u)\right] \\
&+\beta(t)\cdot\frac{\mu_{x_t}}{\mu_{x_t}+\Sigma}\cdot[\hat{\mu}+(\mu_{x_t}-1)\cdot(\log(\mu_{x_t}+\Sigma)-1)-\mu_{x_t}\cdot\log\mu_{x_t}] \\
&\text{if } x_t \neq x_0\text{, denote } \Sigma = \frac{\alpha_t}{1-\alpha_t}: \\
&= \beta(t)\cdot\left[\mu_{x_t}\cdot e^{\bar{\mathbf{s}}_\Theta(x_t,t,u)} - \tilde{\mathbf{s}}_\Theta(x_t,t,u) - \Sigma\cdot s_\Theta(x_t,t,u)_{x_0}\right] \\
&+\beta(t)\cdot[\hat{\mu}+\mu_{x_t}-(1+\Sigma)(1+\log(\mu_{x_t}))+(\Sigma+\mu_{x_0})\log(\Sigma+\mu_{x_0})-(\mu_{x_0})\cdot\log(\mu_{x_0})].
\end{aligned}$$

D Broader Impacts

Theoretically, PreferGrow introduces a well-defined discrete diffusion model building upon prior work. While designed for recommendation, PreferGrow is also applicable to other discrete domains, such as molecular design in chemistry and protein structure prediction. There are many potential societal consequences of our work, none of which we believe warrant specific attention at this time.

E Algorithms for Training and Inference

Algorithm 1 Training Algorithm of PreferGrow

Input: user preference data $\mathcal{D} = \{(u, x_0)\}$, non-preference user ratio p, retention probability $\alpha_t = e^{-\int_0^t \beta(\tau)\mathrm{d}\tau}$ with $\int_0^t \beta(\tau)\mathrm{d}\tau = (\beta_{\min})^{1-t}(\beta_{\max})^t$ or $\int_0^t \beta(\tau)\mathrm{d}\tau = \log(1-(1-\beta_{\text{scale}}\cdot t))$, and the non-preference state $\vec{p}_T$ for preference fading.
Output: estimated *Preference Ratios* $\mathbf{s}_\Theta(x_t, t, u)$ and the non-preference user ϕ.

repeat
- $(u, x_0) \sim \mathcal{D}$ ▷ preference user-item pair.
- $t \sim$ Uniform$(\{1, \ldots, T\})$ ▷ sampling timestep t uniformly.
- $u = \phi$ with probability p ▷ non-preference user modeling.
- $x_t \sim p_{t|0}(\cdot|x_0) = \alpha_t \vec{e}_{x_0} + (1-\alpha_t)\vec{p}_T, x_t \in \mathcal{X}$ ▷ retain or replace for preference fading.
- Compute the score entropy loss $\mathcal{L}_{SE} = \sum_{y\in\mathcal{X}} \mathbf{Q}_t(x_t, y) \cdot l_{SE}(x_0, x_t, y|u)$ as Appendix C.
- Take gradient descent step on $\nabla_\theta \mathcal{L}_{SE}$ and update parameters.

until converged

Algorithm 2 Inference Algorithm of PreferGrow

Input: user condition u, sampling timesteps $S_\tau = \{\tau_i\}_{i=0}^S$ with $\tau_S = T$ and $\tau_0 = 0$, personalization strength w, estimated *Preference Ratios* $\mathbf{s}_\theta(x_t, t, u)$ and the non-preference user ϕ.
Output: grown preference scores $p(x_0|u), x_0 \in \mathcal{X}$ of user u.

$x_T = x_{\tau_S} \sim \vec{p}_T, x_T \in \mathcal{X}$ ▷ the non-preference state.

for $s = S$ to 1 **do**
- $\hat{\mathbf{s}}_\Theta(x_{\tau_s}, \tau_s, u) = (1+w)\mathbf{s}_\Theta(x_{\tau_s}, \tau_s, u) - w\cdot \mathbf{s}_\Theta(x_{\tau_s}, \tau_s, \phi)$ ▷ personalization enhancement.
- $p_{\tau_{s-1}|\tau_s}(x_{\tau_{s-1}}|x_{\tau_s}, u) = p_{\tau_s|\tau_{s-1}}(x_{\tau_s}|x_{\tau_{s-1}}) \cdot \sum_{z\in\mathcal{X}} p^{-1}_{\tau_s|\tau_{s-1}}(x_{\tau_{s-1}}|z) \cdot e^{\hat{\mathbf{s}}_\Theta(x_{\tau_s}, \tau_s, u)_z}$
  ▷ $p_{\tau_s|\tau_{s-1}}(x_{\tau_s}|x_{\tau_{s-1}}) = \frac{\alpha_{\tau_s}}{\alpha_{\tau_{s-1}}}\delta_{x_{\tau_s}}(x_{\tau_{s-1}}) + (1 - \frac{\alpha_{\tau_s}}{\alpha_{\tau_{s-1}}}) \cdot \vec{p}_T(x_{\tau_s})$ as Equation (**??**).
  ▷ $p^{-1}_{\tau_s|\tau_{s-1}}(x_{\tau_{s-1}}|z) = \frac{\alpha_{\tau_{s-1}}}{\alpha_{\tau_s}}\delta_{x_{\tau_{s-1}}}(z) + (1 - \frac{\alpha_{\tau_{s-1}}}{\alpha_{\tau_s}}) \cdot \vec{p}_T(x_{\tau_{s-1}})$ as Equation (**??**).
  ▷ $\sum_{z\in\mathcal{X}} p^{-1}_{\tau_s|\tau_{s-1}}(x_{\tau_{s-1}}|z) \cdot e^{\hat{\mathbf{s}}_\Theta(x_{\tau_s}, \tau_s, u)_z} = \frac{\alpha_{\tau_{s-1}}}{\alpha_{\tau_s}} e^{\hat{\mathbf{s}}_\Theta(x_{\tau_s}, \tau_s, u)_{x_{\tau_{s-1}}}}$
  $+(1 - \frac{\alpha_{\tau_{s-1}}}{\alpha_{\tau_s}}) \cdot \vec{p}_T(x_{\tau_{s-1}}) \cdot \sum_{z\in\mathcal{X}} e^{\hat{\mathbf{s}}_\Theta(x_{\tau_s}, \tau_s, u)_z}$.
- $x_{\tau_{s-1}} \sim p_{\tau_{s-1}|\tau_s}(x_{\tau_{s-1}}|x_{\tau_s}, u), x_{\tau_{s-1}} \in \mathcal{X}$. ▷ reverse preference growing.

end for
return $p(x_0|u) = p_{\tau_0|\tau_1}(x_{\tau_0}|x_{\tau_1}, u), x_0 \in \mathcal{X}$ ▷ grown preference scores of user u.

F Experiments Details

F.1 Datasets

We evaluate PreferGrow on five real-world benchmark datasets:

- **MoviesLens** [41] is a commonly used movie recommendation dataset that contains user ratings, movie titles, and movie genres.
- **Steam** [42] encompasses user reviews for video games on the Steam Store.
- **Beauty** [43] contains movie details and user reviews from Jun 1996 to Sep 2023.
- **Toys** [43] includes user reviews and metadata for toys and games from Jun 1996 to Jul 2014.
- **Sports** [43] comprises reviews and metadata for sports and outdoor products from 1996 to 2014.

Following prior works [1, 2], we adopt the user-splitting strategy, which has been shown to effectively prevent information leakage in test sets [44]. Specifically, we sort all sequences chronologically for each dataset and then split the data into training, validation, and test sets with an 8:1:1 ratio, while preserving the last 10 interactions as the historical sequence. The statistical characteristics of the processed dataset are shown in Table 2. As observed from the table, the recommendation datasets face a significant challenge of severe data sparsity.

Table 2: Statistics of datasets after preprocessing.

Dataset	# users	# items	# Interactions	sparsity
Movies	6040	3883	1001456	04.27%
Steam	39795	9265	2949605	00.80%
Beauty	22,363	12,101	198,502	00.07%
Toys	19,412	11,924	138,444	00.06%
Sports	35,598	18,357	256,598	00.04%

F.2 Baselines

We compare PreferGrow with both traditional discriminative recommenders using negative sampling and diffusion-based generative recommenders, including classical recommenders (SASRec [42], Caser [45], GRURec [46]), item-level diffusion-based recommenders (DreamRec [1], PreferDiff [2]), and preference score-level diffusion-based recommenders (DiffRec [3], DDSR [5]):

- **SASRec** [42] leverages the self-attention mechanism in Transformer to model user preference scores from interaction histories, addressing data sparsity through negative sampling.
- **Caser** [45] utilizes horizontal and vertical convolutional filters to capture sequential patterns at the point-level and union-level, allowing for skip behaviors, and models user preference scores while addressing data sparsity through negative sampling.
- **GRURec** [46] adopts RNNs to model user preference scores from interaction histories, mitigating data sparsity through negative sampling.
- **DreamRec** [1] reshapes sequential recommendation as oracle item generation, addressing data sparsity by adding Gaussian noise to dense item embeddings.
- **PreferDiff** [2] introduces an optimization objective specifically designed for item-level DM-based recommenders, which can integrate multiple negative samples, addressing data sparsity by adding noise to dense item embeddings and using negative sampling both.
- **DiffRec** [3] is a preference score-level diffusion-based generative recommender assuming a Gaussian prior, addressing data sparsity by adding Gaussian noise to preference scores, without considering the constraints of the probability simplex.
- **DDSR** [5] is a preference score-level diffusion-based generative recommender assuming a categorical prior, addressing data sparsity by adding discrete noise to preference scores while respecting the constraints of the probability simplex.

F.3 Implementation Details

Training settings: We implement all models using Python 3.7 and PyTorch 1.12.1 on an Nvidia GeForce RTX 3090. During training, all methods are trained with a fixed batch size of 256 using the Adam optimizer. Additionally, we apply early stopping based on the model's performance on the validation set.

To ensure reproducibility, we fix all random seeds to 100, a randomly chosen value. For the classic recommenders with negative sampling, we employ the binary cross-entropy (BCE) loss. PreferGrow uses SASRec as the encoding model for the user's historical sequence, and we adopt both Hybrid-Wise and Adaptive settings for the fading matrix. For all SASRec modules, we apply RoPE position encoding [47]. The search space of hyperparameters for the baselines is shown in Table 3, and the optimal parameters for our PreferGrow under both hybrid and adaptive settings are presented in Table 4.

Evaluation Protocols and Metrics: To ensure a comprehensive evaluation and mitigate potential biases, we adopt the all-rank protocol [48–50, 1, 2], which evaluates recommendations across all items. We employ two widely used ranking-based metrics: *Normalized Discounted Cumulative Gain* (**N**@K) and *Mean Reciprocal Rank* (**M**@K), to assess the effectiveness of the models.

Table 3: Hyperparameters Search Space for Baselines.

Method	Hyperparameter Search Space
Shared	lr $\sim$ {1e-2, 1e-3, 1e-4, 1e-5} with decay 0, embedding size $d \sim$ {128, 256, 512} the number of negative sampling (if using) $\sim$ {64, 128, 256}, bath size = 256
DreamRec	$w \sim$ {0, 1, 2, 5, 10}, $T \sim$ {500, 1000, 2000,3000}, $p \sim$ {0.05, 0.1, 0.15, 0.2, 0.25, 0.3}
PreferDiff	$\lambda \sim$ {0.2, 0.4, 0.6, 0.8}, $w \sim$ {0, 1, 2, 5, 10}, $T \sim$ {500, 1000, 2000,3000}
DiffRec	noise scale $\sim$ {1e-1, 1e-2, 1e-3, 1e-4, 1e-5}, $T \sim$ {2, 5, 20, 50, 100}
DDSR	$T \sim$ {500, 1000, 2000,3000}
PreferGrow	$T \sim$ {5, 10, 20, 30, 40}, $p \sim$ {0.05, 0.1, 0.15, 0.2, 0.25, 0.3} $w \sim$ {0, 1, 2, 5, 10}, $\lambda \sim$ {0.9, 0.99, 0.999, 0.9999, 0.99999}

Table 4: Best Hyperparameters for PreferGrow on five dataset.

Variant	Dataset	lr	d	p	T	w	λ
Hybrid	**MovieLens**	1e-4	256	0.1	20	10	0.9999
	Steam	1e-3	256	0.1	20	10	0.99999
	Beauty	1e-4	256	0.1	20	5	0.999
	Toys	1e-3	256	0.2	20	10	0.9999
	Sports	1e-3	256	0.2	20	1	0.9999
Variant	**Method**	lr	d	p	T	w	x_{-1}
Adaptive	**Dataset**	1e-4	256	0.2	20	10	True
	Steam	1e-3	256	0.05	20	10	True
	Beauty	1e-4	256	0.1	20	2	True
	Toys	1e-4	256	0.2	20	5	True
	Sports	1e-4	256	0.2	20	5	True

Table 5: Comparison of efficiency on Steam.

Models	# Trainable Parameters	# training epochs	Inference GFLOPs	Inference steps
SASRec	2.70M	61	0.85	1
Caser	2.49M	58	0.38	1
GRURec	2.77M	55	4.06	1
DreamRec	7.25M	52	0.85	20
PreferDiff	7.25M	55	0.85	20
DiffRec	18.55M	1000	/	20
DDSR	3.03M	142	1.77	20
PreferGrow	3.03M	480	0.93	20

F.4 Efficiency Analysis

As shown in Table 5, we present the trainable parameters of each model, the number of training epochs, the GFLOPS per model output (note that diffusion models require multiple outputs for denoising), and the number of model outputs.

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