Benchmarking Optimizers for Large Language Model Pretraining

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Abstract

The recent development of Large Language Models (LLMs) has been accompanied by an effervescence of novel ideas and methods to better optimize the loss of deep learning models. Claims from those methods are myriad: from faster convergence to removing reliance on certain hyperparameters. However, the diverse experimental protocols used to validate these claims make direct comparisons between methods challenging. This study presents a comprehensive evaluation of recent optimization techniques across standardized LLM pre-training scenarios, systematically varying model size, batch size, and training duration. Through careful tuning of each method, we provide guidance to practitioners on which optimizer is best suited for each scenario. For researchers, our work highlights promising directions for future optimization research. Finally, by releasing our code and making all experiments fully reproducible, we hope our efforts can help the development and rigorous benchmarking of future methods.

1 Introduction

Over the past five years, Large Language Models (LLMs) [15, 59, 22, 48] have shown growth in performance and size, demonstrating proficiency in various downstream tasks [80, 7, 85]. The success of LLM pretraining hinges on three key pillars: high-quality data [65, 44], architectural innovations [31, 15], and scalable optimization techniques.

Among these, the choice of optimizer has remained notably consistent in recent years, with `Adam(W)` [38, 50] dominating deep learning for nearly a decade. However, recent advances [33, 47, 84, 62, 66, 17] challenge this status quo, offering alternatives that surpass `AdamW` in speed, communication efficiency [1] or final downstream performance on various benchmarks [12, 37], particularly for autoregressive language modeling [70]. Despite these innovations, current benchmarks and ablation studies [96, 34] remain narrow in scope, often examining only isolated aspects of optimizer design. This lack of systematic comparison makes it difficult to obtain trustworthy insights for practitioners, or identify the next promising research directions.

In this work, our goal to revisit the problem of benchmarking optimizers for LLM pretraining. We do so through standardized experiments which vary important parameters such as batch size, model size, and the number of training iterations. This allows us to formulate an up-to-date list of best-performing methods for the community of researchers and practitioners. We demonstrate the efficiency of each considered method through careful tuning, and present insightful ablations along the way. Furthermore, we provide a set of best practices for LLM pretraining that are applicable regardless of the optimizer chosen.

We summarize our contributions as follows:

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(Contribution 1) We conduct the first large-scale, controlled benchmark of 11 different optimization methods across diverse LLM training scenarios. A fair comparison is ensured by precise accounting for compute costs, and extensive hyperparameter tuning. We identify optimal optimizer choices in several relevant training regimes, for both dense and MoE architectures.

(Contribution 2) We perform comprehensive ablations of critical training hyperparameters—including warmup duration, initialization schemes, gradient clipping, final learning rates, and learning rate scheduler choices—providing actionable insights for optimizing LLM training in practice.

(Contribution 3) We open-source our full benchmarking toolkit, including training scripts, evaluation pipelines, and hyperparameter configurations, to enable reproducible research and facilitate future optimizer development.

For practitioners, our work provides an evidence-based answer to the burning question: "*Is `Adam` still the most effective optimizer in the age of LLMs, or can we achieve better performance at scale with novel optimizers?*". **For researchers**, our work delivers a unified benchmarking framework for LLM pretraining, along with extensive ablation studies which systematically evaluate both popular and overlooked optimizer designs—revealing previously unexplored tradeoffs between efficiency, stability, and final model performance. Overall, our findings not only challenge long-held assumptions about optimizer selection but also establish a foundation for future advances in large-scale model training. By bridging the gap between theoretical innovation and practical deployment, this work aims to accelerate progress in both research and industry applications of LLM training.

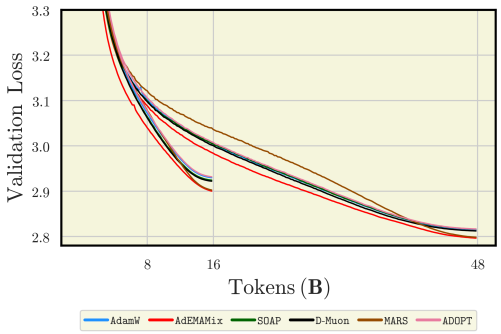


Figure 1: **A comparison of leading optimizers, for training a 720M parameter LLM.**

2 Background & Related Work

Optimizers. While computer vision models often show comparable performance between `SGD` [72] and `AdamW` [94], the landscape differs dramatically in LLM training. Recent work [95] demonstrates that adaptive methods like `AdamW` provide substantially better optimization characteristics for transformer-based language models. The question of why `AdamW` works so well has been a long-standing topic of research [2, 60, 93, 43, 41]. Modern methods often inherit `AdamW`'s core ideas in their structure, such as `ADOPT` [83] and `AdEMAMix` [62]. `ADOPT` has been motivated by solving long-standing convergence issues in `AdamW`. By normalizing the second-order moment prior to the momentum update, they eliminate the non-convergence issues of `AdamW` on smooth non-convex functions. Meanwhile `AdEMAMix` extends `AdamW` with an additional slower momentum buffer, i.e. a slower exponential moving average (EMA), which allows the use of much larger momentum values, accelerating convergence.

One interpretation of `AdamW`'s effectiveness lies in its sign-based update [42]: without the exponential moving average (EMA), `AdamW` resembles `signSGD` [6]. Recent work [96, 36] has shown that `Signum` (`signSGD` with momentum), can perform comparably to `AdamW`. Earlier, the community also discussed `Lion` [9], a method with a similar sign-based structure. `Signum` and `Lion` offer memory benefits due to the use of only a single instead of Adam's two buffers for optimizer states.

Another family of methods stems from `AdamW`'s approximate second-order structure, where the diagonal of the Fisher information matrix or other preconditioning approaches [52, 24] are used as the second moment estimate. This idea has given rise to `Sophia` [46], `SOAP` [84], and, to some extent, `Muon` [33].

The parameter-free concept [61] has led to the development of `Schedule-Free AdamW` (`SF-AdamW`) [17] and `Prodigy` [54]. These optimizers do not require a decreasing learning rate schedule, making them relevant for continual training. Last but not least, `MARS` [88], builds upon this line of research and incorporates a variance reduction mechanism in its update rule.

Benchmarks. To a large extent, the benchmarking setup determines the final conclusions. Some benchmarks are designed for short speedruns in terms of training or validation loss [32], while others focus on a downstream target metric after training [96, 12, 76]. Methods that perform well in short speedruns might not be optimal for longer training horizons as in real LLM training runs (see Figure 3). *"But what constitutes a sufficiently long horizon?" "What should be the compute budget for LLM training?"* These are questions explored by scaling laws [35]. Early benchmarks for optimizers and other ablation studies often rely on Chinchilla scaling laws [26] with a ratio of roughly 20 tokens per parameter (TPR) needed for pretraining. However, recent research [69, 74] argues that this is far from sufficient for production-ready models.

Another important issue is the choice of loss function. Recent setups have been using an auxiliary z-loss [86, 11] in addition to cross-entropy, which requires further investigation. We believe this choice is influenced by the use of the OLMo [58] codebase, which we also address in our work.

Additionally, we found that previous setups for comparing optimizers do not align with recent best practices regarding weight decay, learning rate decay, and overall hyperparameter tuning. All of these questions are revisited in our work.

3 Experimental Setup

Notations. We use the following notations. Let γ be the learning rate, λ the weight decay coefficient, and T the total number of iterations. Momentum-related parameters are represented by the symbol β.

Models & Data. For most experiments, we use a Llama-like transformer [48] architecture, including SwiGLU activations [77], RMSNorm [91], and RoPE embeddings [81]. We experiment with four sizes of models: $124\mathbf{M}$, $210\mathbf{M}$, $583\mathbf{M}$, $720\mathbf{M}$. We train on a $100B$ tokens[1] subset of FineWeb [64]. It consists of a cleaned and deduplicated corpus for LLM pretraining, which we tokenize using the GPT-2 tokenizer prior to splitting into train and validation sequences. MoE setup described in Appendix D.

Iterations & Batch size. Throughout our experiments, we use a sequence length of 512 tokens. For clarity, we often report the batch size in tokens by writing *Batch size×sequence length*. For the $124\mathbf{M}$ model, we use batch sizes of $32 \times 512 = 16\mathbf{k}$, $256 \times 512 = 131\mathbf{k}$, and $512 \times 512 = 262\mathbf{k}$ tokens; for the $210\mathbf{M}$ model, we use a batch size of $256 \times 512 = 131\mathbf{k}$; and for $583\mathbf{M}$ model, we leverage the batch sizes of $1024 \times 512 = 524\mathbf{k}$ and $3936 \times 512 = 2\mathbf{M}$ tokens. Depending on the model size, we vary the number of iterations — also measured in tokens for compatibility with scaling laws and to accommodate different batch size settings. We train $124\mathbf{M}$ and $210\mathbf{M}$ models for equal durations of $\{1, 2.1, 4.2, 6.3, 8.4, 16.8\}\mathbf{B}$ tokens. This corresponds to $T \in \{64, 128, 256, 384, 512, 1024\}\mathbf{k}$ iterations for a batch size of 32, and $T \in \{8, 16, 32, 48, 64, 128\}\mathbf{k}$ iterations for a batch size of 256. For $583\mathbf{M}$ models, we train on $\{13, 32\}\mathbf{B}$ tokens, corresponding to $T \in \{6.5, 16\}\mathbf{k}$ iterations, resp. $T \in \{25, 61.5\}\mathbf{k}$ iterations, for a batch size of 3936, resp. 1024. In the setup with $720\mathbf{M}$ model, we have $T \in \{8, 16, 48\}\mathbf{k}$ iterations for a batch size of $1\mathbf{M}$ tokens. Thus, for all model scales, we include both Chinchilla optimal lengths of training and beyond. More details are available in Appendix C.

Loss. We train using the classical cross-entropy next token prediction loss. Some prior works introducing optimizers use a z-loss in addition to cross-entropy [30, 11, 86, 84, 96]. We found that this has little impact and, therefore, do not use z-loss. An ablation showing results with and without z-loss is in the appendix.

Hyperparameter Tuning. Training LLMs is a computationally intensive task. As a guidance, practitioners often rely on insights gathered at lower scales, scaling laws, and other rules [87, 18]. It is also commonplace to run experiments for only a shorter duration of training, as a way to test certain hyperparameters prior to extending the training horizon to more iterations. Because a full grid search over every hyperparameter, for each setting and optimizer, would be too costly, we resort to a similar approach. More precisely, for each model size, batch size, and optimizer, we tune optimization hyperparameters extensively for a number of training tokens which is near-Chinchilla optimal. We then keep those hyperparameters when we increase the number of iterations. While we found that the sensitivity to several hyperparameters can change as we increase the training horizon, we found this approach simple and yet effective. The hyperparameters being considered depend on the

[1] https://huggingface.co/datasets/HuggingFaceFW/fineweb

optimizer. We proceeded from small to large model scale, and used insights gathered at smaller scales to guide the hyperparameter search at larger scales. Our hyperparameter sweeps are summarized in Appendix D. We present the clarifications regarding the connection between the number of iterations and tokens for different batch size settings as well as the Chinchilla optimal length of training for our models in Tables 3 and 5. As learning rate schedulers, we compare cosine [49], linear and warmup-stable-decay (WSD) [27, 90, 28]. Unless specified, we use a cosine scheduler. Results with WSD and linear schedulers are discussed in Section 4. Recent works also emphasize the importance of sufficiently decaying the learning rate [4, 75, 28]. As such, we take care to decay to $0.01 \times \gamma$ instead of the often used $0.1 \times \gamma$. To give an idea of how much effort was put into tuning each method, across all model sizes, batches and iterations, we trained a total of 2400 models, and have spent roughly 30000 GPU hours.

Optimizers. Here is a list of the optimizers we considered in our work. For each algorithm, we write in parentheses the optimizer-specific hyperparameters we tuned: `AdamW`(β_1, β_2), `SOAP`(β_1, β_2) and preconditioning frequency, `Lion`(β_1, β_2), `MARS`(η, β_1, β_2) and Newton-Schulz hyperparameters, `ADOPT`(β_1, β_2), `Signum`(β), `Prodigy`(β_1, β_2), `SF-AdamW`(β_1, β_2), `Muon`$(\gamma^{\texttt{M}}, \beta, \beta_1, \beta_2)$, `Sophia`$(\rho, \beta_1, \beta_2)$, `AdEMAMix`$(\beta_1, \beta_2, \beta_3, \alpha)$. When an optimizer has several momentum variants e.g. Nesterov [57] or Polyak [67], we try both. In addition, we tune the learning rate γ extensively for all methods. We also try different gradient clipping, warmup steps, and weight-decay values. A summary of the hyperparameters tested and selected for each model size is in Appendix D. All the optimizers are described in depth in Appendix A.

4 Results

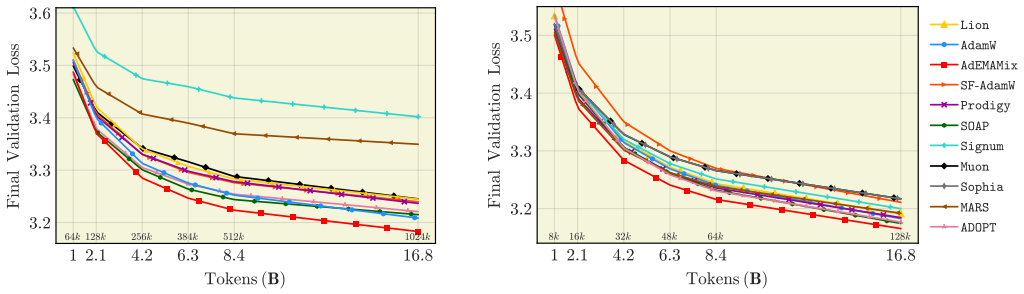
We structure our story starting with smaller models and batch sizes, and gradually scaling up to larger configurations. In some instances, we complement the core benchmarking results with additional ablations and possible best-practices.

4.1 Benchmarking at Small Scale: Training Models of 124M Parameters

Using "small" batches. We first report results when using batches of 32×512 tokens in Figure 3. We tune hyperparameters by training for $2.1\mathbf{B}$ tokens ($128\mathbf{k}$ iterations) and then keep those hyperparameters for all other training durations. The best hyperparameters are reported in Appendix D.1. We observe how, for the smallest number of iterations we considered ($1\mathbf{B}$ tokens $\equiv 64\mathbf{k}$), `SOAP`, `ADOPT` and `AdEMAMix` all outperform `AdamW`, with `SOAP` being the best. As we increase the number of iterations, `AdEMAMix` takes the lead while `AdamW` closes the gap with both `ADOPT` and `SOAP`. A sign-based methods such as `Lion` and `Signum` are expected to perform poorly when the batch size is small. Intuitively, this is due to the $\text{sign}(\cdot)$ operator being sensitive to gradient noise. As described in its original paper, `MARS` also performs poorly when the batch size is small. We found `Prodigy`, `Muon` and `SF-AdamW` to underperform in this setting compared to `AdamW`. On this scale, `Prodigy` suffers from the lack of bias correction of the learning rate, as well as being sensitive to (β_1, β_2) (see Figure 17.

Using "large" batches. We now report results when using batches of 256×512 tokens — $8\times$ larger than for our small batch setting. Results in Figure 2 show how `Signum`, `Mars`, `Lion`, `Prodigy` greatly benefit from the increased batch size. Remarkably, we observe that the `Prodigy` method scales similarly to `AdamW`. We emphasize the possible community interest in this algorithm as its EMA `Prodigy` adaptively emulates the learning rate behaviour. For a small number of iterations (e.g. $T \in \{8\mathbf{k}, 16\mathbf{k}\}$ corresponding to $1\mathbf{B}$ and $2\mathbf{B}$ tokens), all methods outperform `AdamW` except for `SF-AdamW` and `Sophia`. As we increase the number of iterations `ADOPT`, `SOAP`, and `AdEMAMix` take the lead. In particular, `AdEMAMix` has a consistent lead over other methods. While we anticipated—in accordance with Vyas et al.[84]—that `SOAP` would greatly benefit from the larger batch size, its behavior remains relatively consistent compared to our previous small batch setting.

Stability across training horizons. As mentioned in Section 3, we tune hyperparameters training on $2.1\mathbf{B}$ tokens and keep those hyperparameters when extending the training horizon. In Figure 3 we study whether it is possible to find better parameters for `AdamW`, `SOAP`, and `AdEMAMix`. When training on $16.8\mathbf{B}$ tokens, we see it is beneficial to increase the β_3 from 0.999 to 0.9999. Without this improvement, `SOAP` ends up matching the performances of `AdEMAMix` when extending the training horizon further to $33.6\mathbf{B}$ tokens ($\equiv 256\mathbf{k}$ iterations). In our experiments, $\beta_3 = 0.999$ is only better



(a) Batch size 32×512 tokens.

(b) Batch size 256×512 tokens.

Figure 2: **Ranking of optimizers for 124M models with small and large batch sizes.** In both **(a)** and **(b)**, we show the *final* validation loss for different training durations, corresponding to different numbers of tokens. Above each token number, we write the number of training iterations corresponding. In **(a)**, we use a "small" batch size of 32×512 tokens. In **(b)**, we use a larger batch size of 256×512 tokens.

than $\beta_3 = 0.9999$ when the number of training iterations is less than 32k. This matches observation from [62], which recommends reducing β_3 when training for fewer iterations. We also test whether the learning rate γ changes as we increase the number of tokens/iterations. In Figure 5, we run a sweep over γ when training for 16.8B tokens. While for most methods, the best γ obtained in the previous sweep remains optimal, this is not the case for `SOAP` and `SF-AdamW`, which can benefit from a larger $\gamma = 0.002$.

WSD vs. cosine & linear γ-schedulers. Learning rate schedulers received a lot of attention recently [79, 28]. We conducted a series of experiments comparing WSD [27, 90] and linear with cosine [49] learning rate schedulers. Surprisingly, the performance gap between these two schedulers observed in Figure 25 is often significant[2] for benchmarking optimizers. Consequently, we decided to adopt the cosine scheduler for all further experiments.

Decaying γ sufficiently. In Figure 8 we show the impact of decaying more or less the learning rate $\gamma^{(t)}$. From $\gamma = 10^{-3}$ we train models using cosine decay down to $\gamma_{\text{end}} \in \{10^{-4}, 10^{-5}, \dots, 10^{-9}\}$. We found that decaying the learning rate sufficiently matters. In particular, the often use rule consisting in decaying to $0.1 \times \gamma$ is suboptimal. This agrees with the recent works [28, 75, 4]. Building on this findings, we consistently use cosine decay down to $0.01 \times \gamma$.

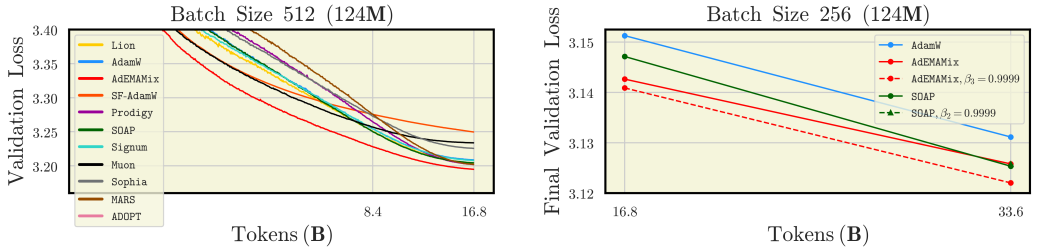
Takeaway 1. After the experiment in the small-batch setting, we conclude that: (i) `AdEMAMix` scales in the best manner with the number of iterations, `SOAP` underperforms `AdamW` when the length of training increases. `ADOPT` and `Prodigy` show almost equal performance across all training durations. Sign-based methods, predictably, underperform when the batch size is small, but what is interesting, is that `Sophia` diverges at all, even if trains with sufficiently small learning rate.

Increasing the batch size further. We also run an experiment with batches of $512 \times 512 = 262$k tokens, training for 64k iterations. Results in Figure 3 show mostly consistent results. Noticeably `MARS` becomes the second best performing method behind `AdEMAMix`, followed closely by `Prodigy`, `Lion`, and `SOAP`. Interestingly, `Signum` performs comparably to `AdamW`.

Takeaway 2. Taking into consideration large batch size setting, we found that many methods, once properly tuned, can show a remarkable performance compared to `AdamW` and also outperform it.

Weight decay ablation. As recent frameworks for LLM pretraining or ablation studies omit weight decay as a default non-zero hyperparameter, some setups even mislead by not incorporating weight decay in their experiments. In this work, we demonstrate the importance of weight decay and its

[2]We emphasize that the difference between the two schedulers is generally less than 5% of the total compute spent. However, this still represents a significant gap in our benchmarking setup, e.g., `SF-AdamW` may outperform `AdamW` in some settings (see Figure 25).

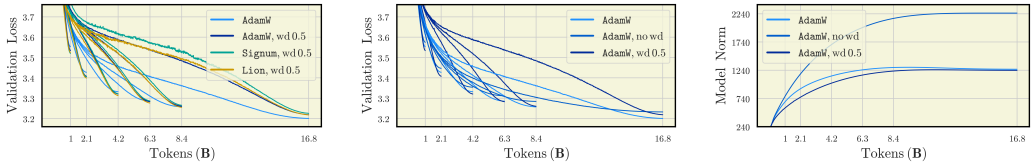


(a) Scaling batch size. (b) Scaling number of tokens.

Figure 3: **Our results demonstrate that (a)**: scaling the batch size significantly improves `MARS`, `Signum`, `Lion` and `Prodigy` making them as good as `AdamW` even for a long training for 16.8**B** tokens. Which was not the case in Figure 2 (b), where we still observed a significant gap in performance; and **(b)**: indeed, with scaling of the number of iterations, the gap between `SOAP` and `AdEMAMix` narrow and, finally, increases. But, on the other hand, with increase of the `AdEMAMix` β_3 parameter, the performance gap with `SOAP` reappears.

impact across different optimizers. Surprisingly, increasing weight decay while keeping the learning rate constant proves to be an effective technique for training on shorter horizons. This approach is so effective that methods like `Signum` and `Lion` with high weight decay significantly outperform `AdamW` without weight decay (see Figure 4). Implementation details also warrant attention. Coupled weight decay is still used in some settings, including the PyTorch [63] optimizer implementations. Notably, the popular implementation of `Signum` becomes ineffective when weight decay is applied. Highlighting this oversight for the community, we contribute by demonstrating our implementation of `Signum` (Algorithm 6) with decoupled weight decay. The influence of weight decay on model weights is intriguing. As is known, model weights typically grow during training, but weight decay, by modifying the optimized function, significantly reduces the growth of the model's parameter norm. Such ablations of weight decay are also of interest to the community [13, 40].

Regarding the ablation of weight decay for optimizers, we again select the best setup for each and conduct a sweep over weight decay values. Our results are presented in Figure 4 and in Figure 23.



(a) (b) (c)

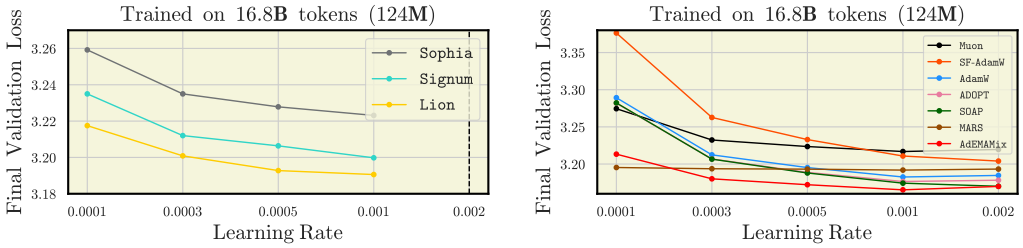
Figure 4: **Larger weight decay achieves significantly better results when training on fewer tokens.** In **(a)** we observe that runs of `AdamW`, `Signum`, and `Lion` with the large weight decay of 0.5 consistently outperform the baseline `AdamW` with weight decay of 0.1 for all training durations except for the last one. Notably, `Signum` and `Lion` with large weight decay perform even better than `AdamW` with the same learning rate. In **(b)**, we also consider a setting without weight decay. We observe that this is suboptimal not only for `AdamW`, but also for the majority of other optimizers (see Appendix E.2), while the typical weight decay of 0.1 remains the best for large training durations. Importantly, in **(c)**, we ablate the impact od weight decay on the model's ℓ_2 norm.

With our weight decay ablation, we are ready to provide one more insight.

> **Takeaway 3.** The use of weight decay, particularly a large decoupled weight decay term, can significantly impact the final loss value and optimizer behavior. However, for extended training horizons, a moderate, non-zero weight decay proves to be a robust option.

Learning rate sensitivity. Since we tune optimizers at a smaller scale and then extrapolate, we pose the question whether the best learning rate we have found so far transfers to the larger training duration. To verify this, we run 124**M** model on 16.8**B** tokens in 256×512 batch size setting,

sweeping the learning rate across five typical values: $\{1e^{-4}, 3e^{-4}, 5e^{-4}, 1e^{-3}, 2e^{-3}\}$. The best learning rate for each method at the moment of hyperparameter tuning on near Chinchilla-optimal $2.1\mathbf{B}$ training duration we report in Appendix D.1. A summary of our results for larger number of tokens is provided in Figure 5 and detailed results of the sweep are presented in Appendix E.2.



(a) (b)

Figure 5: **Optimal learning rate stability across optimizers.** The optimal learning rate determined during tuning on $2.1\mathbf{B}$ tokens remains consistent after a learning rate sweep on $16.8\mathbf{B}$ tokens for most optimizers. In **(a)**, we observe that sign-based methods and similar to them `Sophia` diverge with increasing learning rate. Interestingly, in **(b)**, `SF-AdamW` and `SOAP` demonstrate the best performance with a large learning rate of 0.002. In our work, we further show that it is possible to increase the learning rate even more for such methods.

Warmup ablation. Another important ingredient of the pretraining is learning-rate warmup in the initial phase of training. Recent studies have explored the necessity of warmup in modern deep learning, with some investigating its elimination [39] and others ablating it to improve model performance and stability [92]. We focus on the latter, examining how warmup affects optimizer setup and whether it can significantly enhance performance. For each optimizer's best configuration, we vary warmup across three values: $\{0.27, 1, 4.2\}\mathbf{B}$ tokens, which corresponds to $\{2, 8, 32\}\mathbf{k}$ iterations. Our choice of the largest warmup value is inspired by [92]. We describe this experiment in Appendix E.2. Mainly, we observe that `Signum` and `SF-AdamW` perform better with a larger warmup of $8\mathbf{k}$ steps when training on $16.8\mathbf{B}$ tokens. We also ablate the claim from [92] that a warmup of 25% of the Chinchilla optimal duration is the best. However, our findings contradict this assertion (see Figure 19). We show that a moderate values of the warmup, generally, is better, however, different optimizers could prefer different number of warmup steps. As such, `SF-AdamW`, `Sophia`, `Signum` prefer larger warmup, which is clearly depicted in Figure 6.

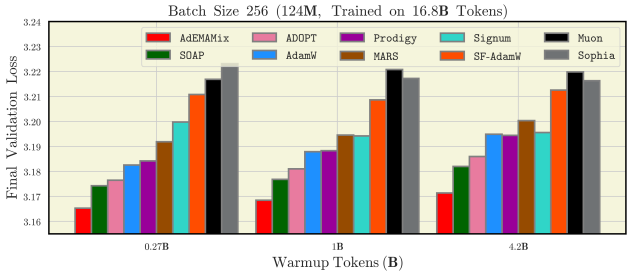
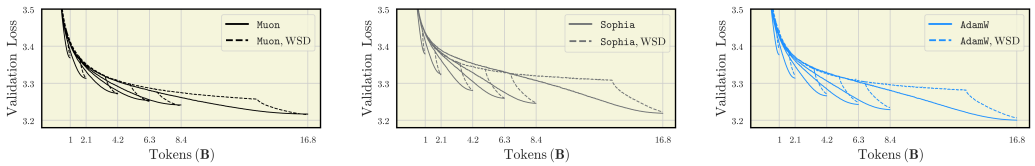


Figure 6: **Warmup ablation.** We report the final validation loss on the FineWeb dataset for $124\mathbf{M}$ model trained on the batch size of 256. We sweep over the batch sizes of $\{1.56\%, 6.25\%, 25\%\}$ of the length of training, which corresponds to $\{2000, 8000, 32000\}\mathbf{k}$ iterations, respectively.

Cosine vs WSD. At the outset of our study, we indicated a preference for the cosine scheduler over WSD. In this section, we provide a more detailed ablation of this choice. Having optimally tuned the cosine scheduler for each optimizer, we replicate the setup of [28], which allows us to avoid adjusting additional hyperparameters. Our findings, which demonstrate the superiority of the cosine scheduler across various optimization methods, are presented in Figure 7, and in the Appendix Figures 25 and 26. These results not only validate our initial preference but also provide insights into the interaction between learning rate schedules and different optimizers in large-scale language model training.



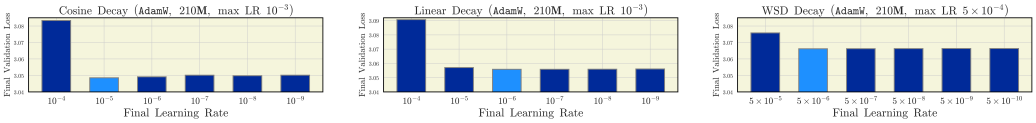
(a) (b) (c)

Figure 7: **Comparisons between WSD and cosine scheduler.** Notably, WSD and cosine scheduler behave differently with respect to optimizer. In **(a)**, the `Muon` optimizer shows a preference for WSD across most training durations. `Sophia` exhibits an almost perfect match between both schedulers. However, for `AdamW`, along with the majority of other optimizers studied (see Figure 26), we get a better performance with cosine. We also report a detailed comparison with linear scheduler in Appendix E.2.

4.2 Benchmarking at medium scale: Training Models of 210M Parameters

In this section, we verify if our selected hyperparameters from smaller 124M allow accurate transfer to a slightly larger model. We point out that the most important hyperparameters to be sweeped are learning rate and gradient clipping. Regarding the learning rate, we observe that it only becomes a sensitive choice for sign-based methods, while the optimal hyperparameters for `AdamW` remain the same.

Results with a batch size of 256×512. Results provided in the Appendix in Figure 21 are consistent with those obtained training 124M models with large batches.



(a) (b) (c)

Figure 8: **Decaying the learning rate down to $0.01 \times \gamma_{\max}$ and beyond, instead of only to 10%** We observe a common pattern for different schedulers that decreasing the learning rate to moderate 10^{-2} value is a better choice than decreasing it down to zero. Interestingly, the linear learning rate scheduler for models at a given scale, requires $0.001 \times \gamma_{\max}$.

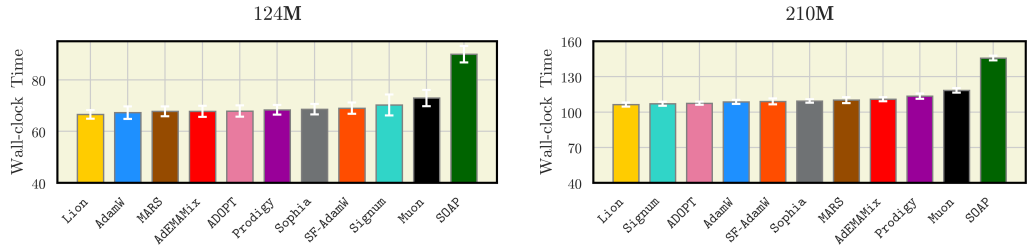


Figure 9: **Wall-clock time comparison.** After conducting experiments for 124M and 210M models, we are ready to present the wall-time comparison for each methods. For this purposes, we use a single GPU, and run each optimizer for 100 iterations on a small batch size without gradient accumulation and `torch.compile`. We report the wall-clock time per 100 iterations. We observe that all methods take the roughly the same time or very close time to complete 100 iterations, with the exception of `Muon` and `SOAP`. In addition, we point out that `SOAP`'s runtime exhibits a non-linear dependence on the model size, due to its preconditioner matrices operations which are fast only for certain matrices smaller than a predefined size.

4.3 Scaling Up: Benchmarking models of 583M and 720M Parameters

We pick three methods: AdamW, SOAP, and AdEMAMix, and run experiments with a larger model of 583M parameters, and a large batch size of 2M tokens. The goal being to get closer to one of the

settings described in [84]. We train for 6500 and 16000 iterations, corresponding to 13**B** and 32**B** tokens respectively.

Comparison between our setting and [84]. We found several key differences between our codebase and [84]: (i) we decay the learning rate to $0.01 \times \gamma$ instead of $0.1 \times \gamma$, with γ being the maximum learning rate, (ii) we use typical weight decay values of e.g. 0.1 instead of smaller values such as 0.01 or 0.0001, (iii) we do not use a z-loss in addition to ours. It has been shown recently that properly decaying the learning rate has an important effect on the optimization [4]. We run an ablation to compare both settings and conclude that removing the z-loss and increasing the weight decay to 0.1 improves the results. Results further improve when the learning rate is decayed more. This ablation is shown in Figure 8.

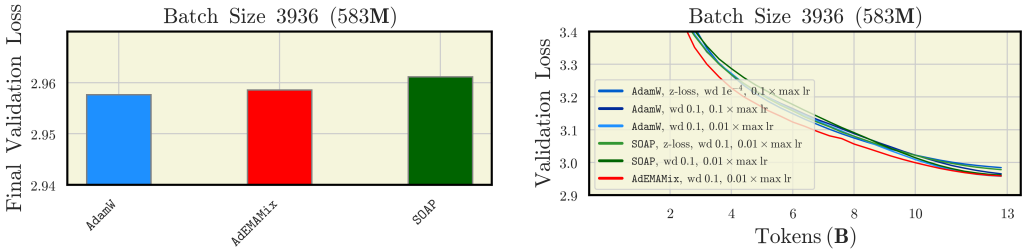


Figure 10: **Results for the** 583M **model.** On the **left**, we show our results when training for 6500 iterations. In this setting, AdamW gives best results, followed by AdEMAMix and then SOAP. This is surprising as it conflicts with findings from [84]. Those results are partly reconciled with the figure on the **right**. And we see that the difference in performance between models trained with and without the z-loss regularizer is quite minor.

5 Extension to MoEs

Setup & Comparison. Besides training dense Llama-like transformers, we also conver a comparison for MoE architectures [78]. Our variant of MoE is based on the Switch-Transformer implementation [20]. We use a classical linear gating with softmax and top-k routing ($k = 2$) and 8 experts. The activation functions remains the same as for the dense base model from Section 3. Such a configuration of the MoE model gives us approximately 520M parameters. We cover additional details in Appendix E.6. In this setting we train with a batch size of 256×512 for $T \in \{42, 336\}$**k** iterations. Again we cover both a Chinchilla-optimal horizon and the beyond. We summarize the results in the following Table 1.

Opt.	42k	336k
AdEMAMix	22.37	18.47
D-Muon	22.67	18.51
ADOPT	22.70	18.58
AdamW	22.85	18.69
Prodigy	22.82	18.78

Opt.	42k	336k
Lion	23.20	18.87
Signum	23.31	19.09
SF-AdamW	23.34	19.13
Sophia	23.41	19.22
MARS	22.73	19.33

Table 1: **Final validation perplexity for MoE training** (↓)**.**

6 Discussion

Our advices on tuning each method. Overall, we validate the already widely used hyperparameters of $\lambda = 0.1$ and $T_{\text{warmup}} \approx 2$**k**. For Lion—as mentioned in [9]—we find that the best value for β_1 is consistently 0.99. The mechanism for Lion seems similar to AdEMAMix, one can imagine that Lion could be better with larger β_1, which would require schedulers. We also pose an interesting observation toward Prodigy: while it may not be so efficient with a super small batch sizes, with scaling of the model size and the batch size it becomes almost as competitive as AdamW. Importantly, Muon and D-Muon performed poorly at a small scale with relatively small batch sizes (32, 256), however, as we see in Figure 1

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Contents

A Optimizers we study

In this section, we describe all considered algorithms, presenting them in a unified formalism. We start with notation and then discuss the algorithms according to their logical grouping:

1. Adam-like methods: AdamW (Algorithm 1), ADOPT (Algorithm 2) and AdEMAMix (Algorithm 3).

2. Sign-based methods: Lion (Algorithm 4), Signum (Algorithms 5 and 6).

3. Approximate second-order optimizers: Muon (Algorithm 8), SOAP (Algorithm 10) and Sophia (Algorithm 11).

4. Learning rate / scheduler-free learning algorithms: Schedule-Free AdamW (Algorithm 12), Prodigy (Algorithm 13).

5. MARS algorithms: (Algorithms 14 to 16).

Notation. In our work, we denote all vectors and matrices using bold symbols, while non-bold symbols represent scalars. Let $\mathcal{L} : \mathcal{D} \to \mathbb{R}$ be an empirical loss function parameterized by $\boldsymbol{x}$, and mapping batch of inputs $\boldsymbol{\xi} \subset \mathcal{D}$ to $\mathbb{R}$. As $\boldsymbol{g} = \nabla_{\boldsymbol{x}} \mathcal{L}(\boldsymbol{x}, \boldsymbol{\xi})$ we denote a stochastic gradient of the loss w.r.t. parameters $\boldsymbol{x}$. For simplicity, we omit $\boldsymbol{x}$ in ∇ and write $\nabla \mathcal{L}(\boldsymbol{x}, \boldsymbol{\xi})$. We use the following standardized notation for specific symbols in our work: batch size — $|\boldsymbol{\xi}|$, learning rate — γ, weight decay — λ, momentum — β, iteration counter t with the total number of iterations — T.

And basic notation for symbols in the algorithms: $\boldsymbol{m}, \boldsymbol{v}$ – are first and second moment estimates, respectively, with their bias corrected versions $\hat{\boldsymbol{m}}, \hat{\boldsymbol{v}}$, and beta parameters — β_1, β_2. We denote the dot product of two vectors $\boldsymbol{z}$, $\boldsymbol{y}$ as $\langle \boldsymbol{z}, \boldsymbol{y} \rangle$, while $\boldsymbol{z} \odot \boldsymbol{y}$ stands for their element-wise product. All division and addition operations in the described algorithms are element-wise.

A.1 AdamW, ADOPT, AdEMAMix

AdamW. Our baseline — Adam(W), has become a de facto optimizer for deep learning, demonstrating impressive performance across diverse domains: from tabular data to diffusion and language models.

The method originated from the ideas of Adagrad [19] and RMSProp [23], which utilize a second moment estimate $\boldsymbol{v}$ in their update rule. However, Adam(W) enhanced this prior scheme by incorporating momentum [55, 82], establishing itself as a state-of-the-art method for a wide range of tasks. All other algorithms we consider also employ a similar, if not identical, momentum scheme.

Another key aspect of AdamW is its decoupled weight decay λ [50], unlike Adam. We use the decoupled weight decay scheme for all methods to ensure consistency and emphasize its importance for optimizer comparison, hyperparameter tuning, and final performance. This is clearly observable, e.g., for Signum (Algorithm 5).

Algorithm 1 AdamW

Input: Initial parameters $\boldsymbol{x}_0$, number of iterations T, learning rate γ_t, weight decay λ, β_1, β_2, ε.
Initialize: $\boldsymbol{m}_0 \leftarrow \boldsymbol{0}$, $\boldsymbol{v}_0 \leftarrow \boldsymbol{0}$
for $t \in [T]$ **do**
  $\boldsymbol{g}_t \leftarrow \nabla \mathcal{L}(\boldsymbol{x}_t, \boldsymbol{\xi}_t)$
  $\boldsymbol{m}_t \leftarrow \beta_1 \boldsymbol{m}_{t-1} + (1 - \beta_1) \boldsymbol{g}_t$
  $\boldsymbol{v}_t \leftarrow \beta_2 \boldsymbol{v}_{t-1} + (1 - \beta_2) \boldsymbol{g}_t \odot \boldsymbol{g}_t$
  $\hat{\boldsymbol{m}}_t \leftarrow \boldsymbol{m}_t / (1 - \beta_1^t), \quad \hat{\boldsymbol{v}}_t \leftarrow \boldsymbol{v}_t / (1 - \beta_2^t)$
  $\boldsymbol{x}_{t+1} \leftarrow \boldsymbol{x}_t - \gamma_t \left(\frac{\hat{\boldsymbol{m}}_t}{\sqrt{\hat{\boldsymbol{v}}_t} + \varepsilon} + \lambda \boldsymbol{x}_t \right)$
end for
Return: $\boldsymbol{x}_T$

ADOPT. Recently, [83] proposed a modification of Adam, by removing the current gradient $\boldsymbol{g}_t$ from the second moment estimate $\boldsymbol{v}_t$ and changing the order of the momentum update $\boldsymbol{m}_t$ and the normalization by the second moment estimate. As shown in line 8 of Algorithm 2, the parameter update depends only on the previous value of the second moment estimate $\boldsymbol{v}_{t-1}$. The authors analyze the convergence of ADOPT with the following update rule:

$$\boldsymbol{m}_t \leftarrow \beta_1 \boldsymbol{m}_{t-1} + (1 - \beta_1) \frac{\boldsymbol{g}_t}{\max\{\sqrt{\boldsymbol{v}_{t-1}}, \varepsilon\}},$$
$$\boldsymbol{x}_{t+1} \leftarrow \boldsymbol{x}_t - \gamma_t \boldsymbol{m}_t.$$

However, the practical implementation differs in a few details. To tackle instabilities caused by near-zero gradients during the early stages of training, the authors propose using a clipping on $\boldsymbol{g}_t / \max\{\sqrt{\boldsymbol{v}_{t-1}}, \varepsilon\}$, which we formalize as the `clamp` operation. Given a vector $\boldsymbol{g}$ and a positive scalar c, it is defined as:

$$\texttt{clamp}\left(\boldsymbol{g}, c\right)^{(i)} = \min\left\{\max\left\{g^{(i)}, -c\right\}, c\right\}. \tag{1}$$

Thus, the element-wise `clamp` operation preserves $\boldsymbol{g}_t$ from the division by near-zero values.

The authors theoretically claim that `ADOPT` achieves the optimal convergence bound for non-convex objectives, regardless of the choice of the β_2 parameter. We empirically investigate this claim and observe that, contrary to the theoretical results, there is a significant performance gap for different choices of β_2 in practice.

Algorithm 2 `ADOPT`

Input: Initial parameters $\boldsymbol{x}_0$, number of iterations T, learning rate γ_t, weight decay λ, β_1, β_2, ε.
Initialize: $\boldsymbol{m}_0 \leftarrow \boldsymbol{0}$, $\boldsymbol{v}_0 \leftarrow \nabla\mathcal{L}(\boldsymbol{x}_0, \boldsymbol{\xi}_0) \odot \nabla\mathcal{L}(\boldsymbol{x}_0, \boldsymbol{\xi}_0)$
for $t \in [T]$ **do**
  $\boldsymbol{g}_t \leftarrow \nabla\mathcal{L}(\boldsymbol{x}_t, \boldsymbol{\xi}_t)$
  $c_t \leftarrow t^{1/4}$ ▷ Update clipping value schedule
  $\boldsymbol{m}_t \leftarrow \beta_1 \boldsymbol{m}_{t-1} + (1-\beta_1)\texttt{clamp}\left(\frac{\boldsymbol{g}_t}{\max\{\sqrt{\boldsymbol{v}_{t-1}}, \varepsilon\}}, c_t\right)$
  $\boldsymbol{v}_t \leftarrow \beta_2 \boldsymbol{v}_{t-1} + (1-\beta_2)\boldsymbol{g}_t \odot \boldsymbol{g}_t$
  $\boldsymbol{x}_{t+1} \leftarrow \boldsymbol{x}_t - \gamma_t\left(\boldsymbol{m}_t + \lambda \boldsymbol{x}_t\right)$ ▷ Update without $\boldsymbol{v}_t$
end for
Return: $\boldsymbol{x}_T$

`AdEMAMix`. Another `Adam`-like optimizer we study is `AdEMAMix` [62]. This work argues that using a single EMA to accumulate past gradients in the first moment estimate $\boldsymbol{m}$ can be sub-optimal, as it cannot simultaneously prioritize both immediate past and older gradients. In Algorithm 3, the authors incorporate two EMAs: one — `Adam`-like EMA for $\boldsymbol{m}$ (fast), and another — a slow EMA $\boldsymbol{m}^{\text{slow}}$ (see `line 7`) with an additional β_3 parameter. In its update rule, the algorithms balances fast and slow EMAs with the constant factor α (see `line 10` of Algorithm 3). This algorithmic design helps `AdEMAMix` benefit from older gradients and results in smoother loss curves during training.

However, to mitigate the effect of early instabilities, the authors use two additional schedulers for α and β_3 – `alpha_scheduler` and `beta_scheduler`, formalized in our work as follows:

$$\texttt{alpha\_scheduler}(t, \alpha, T_\alpha) = \min\left\{\frac{t\alpha}{T_\alpha}, \alpha\right\},$$

$$\texttt{beta\_scheduler}(t, \beta_3, \beta_{\text{start}}, T_{\beta_3}) = \min\left\{\exp\left(\frac{\log(\beta_{\text{start}})\log(\beta_3)}{\left(1-\frac{t}{T_{\beta_3}}\right)\log(\beta_3) + \frac{t}{T_{\beta_3}}\log(\beta_{\text{start}})}\right), \beta_3\right\}.$$

In all experiments, we set $\beta_{\text{start}} = \beta_1$, and the warmup parameters equal to the length of training: $T_\alpha = T_{\beta_3} = T$.

One thing that should be commented — α and β schedulers are seemingly contradict the idea of WSD scheduler. However, setting T_α, T_{β_3} to be longer than the first checkpoint of the WSD does not significantly impact the final performance. Thus, `AdEMAMix` can still be combined with recent findings regarding the WSD scheduler.

A.2 Sign-based methods: `Lion` and `Signum`

Another branch of methods includes sign-based `Lion` and `Signum`. To some extend, one can classify `Adam` as a sign-based method also, but we mention only `Lion` and `Signum` as they explicitly incorporate the `sign` operation in the update rule.

These methods, particularly `Signum`, have been unfairly overlooked in the context of LLM pretraining. However, our results demonstrate that, with sufficiently large batch sizes and at moderate model scales, these optimizers perform on par with `Adam`, and in some cases, even outperform it.

Algorithm 3 `AdEMAMix`

Input: Initial parameters $\boldsymbol{x}_0$, number of iterations T, learning rate γ_t, weight decay λ, β_1, β_2, β_3, β_{start}, α, `beta_scheduler`, `alpha_scheduler`, warmup parameters T_{β_3} and T_α, ε.
Initialize: $\boldsymbol{m}_0 \leftarrow \mathbf{0}$, $\boldsymbol{m}_0^{\text{slow}} \leftarrow \mathbf{0}$, $\boldsymbol{v}_0 \leftarrow \mathbf{0}$
for $t \in [T]$ **do**
  $\beta_3(t) \leftarrow \texttt{beta\_scheduler}(t, \beta_3, \beta_{\text{start}}, T_{\beta_3})$, $\alpha(t) \leftarrow \texttt{alpha\_scheduler}(t, \alpha, T_\alpha)$ ▷ Update β_3 and α schedulers
  $\boldsymbol{g}_t \leftarrow \nabla \mathcal{L}(\boldsymbol{x}_t, \boldsymbol{\xi}_t)$
  $\boldsymbol{m}_t \leftarrow \beta_1 \boldsymbol{m}_{t-1} + (1-\beta_1)\boldsymbol{g}_t$
  $\boldsymbol{m}_t^{\text{slow}} \leftarrow \beta_3(t)\boldsymbol{m}_{t-1}^{\text{slow}} + (1-\beta_3(t))\boldsymbol{g}_t$ ▷ Update slow EMA
  $\boldsymbol{v}_t \leftarrow \beta_2 \boldsymbol{v}_{t-1} + (1-\beta_2)\boldsymbol{g}_t \odot \boldsymbol{g}_t$
  $\hat{\boldsymbol{m}}_t \leftarrow \boldsymbol{m}_t / (1-\beta_1^t)$, $\hat{\boldsymbol{v}}_t \leftarrow \boldsymbol{v}_t / (1-\beta_2^t)$
  $\boldsymbol{x}_{t+1} \leftarrow \boldsymbol{x}_t - \gamma_t \left(\frac{\hat{\boldsymbol{m}}_t + \alpha(t)\boldsymbol{m}_t^{\text{slow}}}{\sqrt{\hat{\boldsymbol{v}}_t} + \varepsilon} + \lambda \boldsymbol{x}_t \right)$
end for
Return: $\boldsymbol{x}_T$

Lion. And the first method of this kind is `Lion` [9]. This optimizer is symbolically discovered in the program space of first-order optimization primitives. `Lion` updates its EMA of $\boldsymbol{m}$ after updating the parameters and has additional term $(1-\beta_1)\boldsymbol{g}$ whis adds to the momentum. This interpolation $\beta_1 \boldsymbol{m}_{t-1} + (1-\beta_1)\boldsymbol{g}_t$ (see `line 6` of Algorithm 4) makes the symbolic-discovered idea behind `Lion` similar to the idea of the `AdEMAMix` optimizer.

Algorithm 4 `Lion`

Input: Initial parameters $\boldsymbol{x}_0$, number of iterations T, learning rate γ_t, weight decay λ, β_1, β_2.
Initialize: $\boldsymbol{m}_0 \leftarrow \mathbf{0}$
for $t \in [T]$ **do**
  $\boldsymbol{g}_t \leftarrow \nabla \mathcal{L}(\boldsymbol{x}_t, \boldsymbol{\xi}_t)$
  $\boldsymbol{m}_t \leftarrow \beta_2 \boldsymbol{m}_{t-1} + (1-\beta_2)\boldsymbol{g}_t$ ▷ Update EMA of $\boldsymbol{g}_t$
  $\boldsymbol{x}_{t+1} \leftarrow \boldsymbol{x}_t - \gamma_t \left(\texttt{sign}\left(\beta_1 \boldsymbol{m}_{t-1} + (1-\beta_1)\boldsymbol{g}_t\right) + \lambda \boldsymbol{x}_t \right)$
end for
Return: $\boldsymbol{x}_T$

Signum. Another sign-based method, which is the adoptation of `signSGD` — `Signum` [6], or `signSGD` with momentum. This method differs from `Lion` in the interpolation term between EMA of momentum and current gradient, as well as in the `Signum`'s update rule a current EMA is used.

Importantly, while `Signum` is not as widespread for LLM pretraining and stands mostly as a theoretical artifact, recent practitioner's studies also start to use `Signum` for scalable training [96]. Mostly, due to the memory-efficiency of `Signum` compared to `AdamW`.

In this regard, we want to make an important point — many recent PyTorch [63] implementations of the `Signum` optimizer, unlikely, are not suitable for this method, impairing the potential performance from using it.

The main problem of many open-source implementations is a use of decoupled weight decay in the PyTorch implementation of `SGDM` (`SGD` with momentum) [82]. Indeed, with a decoupled weight decay, the update of Algorithm 5 transforms into:

$$\boldsymbol{x}_{t+1} \leftarrow \boldsymbol{x}_t - \gamma_t \texttt{sign}\left(\beta \boldsymbol{m}_{t-1} + (1-\beta)\boldsymbol{g}_t - \lambda(1-\beta)\boldsymbol{g}_t\right),$$

which affects the sign of the update, leading to potentially wrong optimization direction if the weight decay is sufficiently large.

Another popular failure while using `Signum` is incorrectly tractable PyTorch implementation of `SGDM`. It does not include such EMA as `line 5` in Algorithm 5, on the other hand, in PyTorch, the momentum update depends on the dampening parameter τ:

$$\boldsymbol{m}_t \leftarrow \beta \boldsymbol{m}_{t-1} + (1-\tau)\boldsymbol{g}_t,$$

which is zero by default. Therefore, the typical update rule, reflecting the actual `Signum` behavior in practice corresponds to the following update:

$$\boldsymbol{x}_{t+1} \leftarrow \boldsymbol{x}_t - \gamma_t \left(\texttt{sign} \left(\beta \boldsymbol{m}_{t-1} + (1-\tau) \boldsymbol{g}_t \right) + \lambda \boldsymbol{x}_t \right),$$

where the weight decay is decoupled and, consequently, does not affect the `sign`.

$$\boldsymbol{g}_t \leftarrow \boldsymbol{g}_t + \beta \boldsymbol{m}_t,$$

improves `Signum`. Since enabling Nesterov momentum requires zero dampening τ, we revisited the descrition of Algorithm 5, and propose more practical, PyTorch-compatible version of `Signum` in Algorithm 6.

Algorithm 5 `Signum` (basic)

Input: Initial parameters $\boldsymbol{x}_0$, number of iterations T, learning rate γ_t, weight decay λ, momentum β.
Initialize: $\boldsymbol{m}_0 \leftarrow \boldsymbol{0}$
for $t \in [T]$ **do**
  $\boldsymbol{g}_t \leftarrow \nabla \mathcal{L}(\boldsymbol{x}_t, \boldsymbol{\xi}_t)$
  $\boldsymbol{m}_t \leftarrow \beta \boldsymbol{m}_{t-1} + (1-\beta) \boldsymbol{g}_t$
  $\boldsymbol{x}_{t+1} \leftarrow \boldsymbol{x}_t - \gamma_t \left(\texttt{sign} \left(\boldsymbol{m}_t \right) + \lambda \boldsymbol{x}_t \right)$
end for
Return: $\boldsymbol{x}_T$

Algorithm 6 `Signum` (our PyTorch variant)

Input: Initial parameters $\boldsymbol{x}_0$, number of iterations T, learning rate γ_t, weight decay λ, momentum β.
Initialize: $\boldsymbol{m}_0 \leftarrow \boldsymbol{0}$
for $t \in [T]$ **do**
  $\boldsymbol{g}_t \leftarrow \mathcal{L}(\boldsymbol{x}_t, \boldsymbol{\xi}_t)$
  $\boldsymbol{m}_t \leftarrow \beta \boldsymbol{m}_{t-1} + \boldsymbol{g}_t$
  $\boldsymbol{x}_{t+1} \leftarrow \boldsymbol{x}_t - \gamma_t \left(\texttt{sign} \left(\beta \boldsymbol{m}_t + \boldsymbol{g}_t \right) + \lambda \boldsymbol{x}_t \right)$
end for
Return: $\boldsymbol{x}_T$

Moreover, to prevent other researchers and practitioners from the possible wrong use of `Signum` and for the reproducibility purposes, we provide our Python code.

Listing 1: `Signum` code skeleton using PyTorch

```
from typing import Dict

import torch


class Signum(torch.optim.Optimizer):
    def __init__(
        self,
        params,
        lr=1e-3,
        momentum=0,
        dampening=0,
        weight_decay=0,
        nesterov=False,
        sign_update=True,
    ):
        if lr < 0.0:
            raise ValueError(f"Invalid learning rate: {lr}")
        if momentum < 0.0:
            raise ValueError(f"Invalid momentum value: {momentum}"
                )
        if weight_decay < 0.0:
            raise ValueError(f"Invalid weight_decay value: {
                weight_decay}")

        defaults = dict(
            lr=lr,
```

```
            momentum=momentum,
            dampening=dampening,
            weight_decay=weight_decay,
            nesterov=nesterov,
            sign_update=sign_update,
        )
        if nesterov and (momentum <= 0 or dampening != 0):
            raise ValueError("Nesterov momentum requires a
                momentum and zero dampening")
        super().__init__(params, defaults)

    def __setstate__(self, state):
        super().__setstate__(state)
        for group in self.param_groups:
            group.setdefault("nesterov", False)

    @torch.no_grad()
    def _init_state(self, example, state=None):
        assert isinstance(example, torch.Tensor)
        assert isinstance(state, Dict) or state is None
        if state is None:
            state = {}
        state["step"] = 0
        state["momentum_buffer"] = torch.clone(example).detach()
        return state

    @torch.no_grad()
    def _compute_update(
        self, grad, state, lr, momentum, nesterov, dampening,
            sign_update, **kwargs
    ):
        if momentum != 0:  # Signum check
            buf = state["momentum_buffer"]
            buf.mul_(momentum).add_(grad, alpha=1 - dampening)

            if nesterov:
                grad = grad.add(buf, alpha=momentum)
            else:
                grad = buf

        if sign_update:
            grad = grad.sign()

        return grad * (-lr)

    @torch.no_grad()
    def step(self, closure=None):
        """Performs a single optimization step.

        Args:
            closure (Callable, optional): A closure that
                reevaluates the model
                and returns the loss.
        """
        loss = None
        if closure is not None:
            with torch.enable_grad():
                loss = closure()
```

```
        for group in self.param_groups:
            for p in group["params"]:
                if p.grad is None:
                    continue

                grad = p.grad
                state = self.state[p]

                if group["weight_decay"] != 0:
                    p.mul_(1 - group["lr"] * group["weight_decay"
                        ])

                if len(state) == 0:
                    self._init_state(example=p, state=state)
                    if not group["momentum"]:
                        state.pop("momentum_buffer", None)

                state["step"] += 1

                update = self._compute_update(
                    grad,
                    state,
                    group["lr"],
                    group["momentum"],
                    group["nesterov"],
                    group["dampening"],
                    group["sign_update"],
                )

                p.add_(update)

        return loss
```

A.3 `Muon`, `SOAP`, `Sophia`

Next page of the methods covers algorithms that rather aim to use more information about the problem's curvature (`SOAP` [84], `Sophia` [46]) or perform fast updates of matrix parameters involving higher order computations (`Muon` [33]).

Contrary to the chronological order, we discuss them starting from the recent one — `Muon` and end up with `Sophia`.

`Muon`. Specifically designed for the speedrun comparisons, this method surpasses `AdamW` baseline on nanoGPT pretraining benchmark [32]. Claims from the `Muon` extend to faster learning, lower memory usage and better sample-efficiency with a small wall-clock time overhead. However, there are not much to say about the final performance given a particular budget of tokens to train on.

The reason why the `Muon` is a good option for speedrun pretraining lies in its structure — `Muon` treats different layers in different way. One dimensional (1D) parameters, large embedding layers, scalar parameters (such as Layer Norm parameters) and the output layer of LLM (`lm_head`) are optimized by `AdamW`. And all parameters with two or more dimensions, e.g., Multi-Head Attention layers are optimized by Algorithm 7, which we call `MuonNon1D`.

Inspired by `Shampoo`'s preconditioners [24], the authors of `MuonNon1D` incorporated an orthogonalization step to compute the `SVD` transformation of gradient matrix. Before the orthogonalization step, `MuonNon1D` represents `SGD` with Nesterov momentum. To ensure fast orthogonalization procedure, the authors, insiped by [5], use Newton-Schulz procedure [25]. As the number of Newton-Schulz iterations increases, the closer resulting matrix becomes to $\boldsymbol{U}\boldsymbol{V}^\top$ from `SVD` transformation. The authors also mention that `Muon` can be thought of as a second way of smoothing spectral steepest descent [8], with a different set of memory and runtime trade-offs compared to `Shampoo`.

Algorithm 7 `MuonNon1D` (for non-1D parameters)

Input: Initial non-1D parameters $\boldsymbol{x}_0$, number of iterations T, learning rate γ_t, momentum β, number of Newton-Schulz iterations T_{NS}, a, b, c coefficients.
Initialize: $\boldsymbol{m}_0 \leftarrow \boldsymbol{0}$
for $t \in [T]$ **do**
  $\boldsymbol{g}_t \leftarrow \nabla \mathcal{L}(\boldsymbol{x}_t, \boldsymbol{\xi}_t)$
  $\boldsymbol{m}_t \leftarrow \beta \boldsymbol{m}_{t-1} + \boldsymbol{g}_t$
  $\boldsymbol{g}_t \leftarrow \beta \boldsymbol{m}_t + \boldsymbol{g}_t$ ▷ Practical implementation of Nesterov momentum
  Set: $\boldsymbol{w}_0 \leftarrow \boldsymbol{g}_t / \|\boldsymbol{g}_t\|_F$
  for $n \in [T_{\text{NS}}]$ **do**
    $\boldsymbol{w}_{n+1} \leftarrow a\boldsymbol{w}_n + b\boldsymbol{w}_n\boldsymbol{w}_n^\top + c\left(\boldsymbol{w}_n\boldsymbol{w}_n^\top\right)^2 \boldsymbol{w}_n$ ▷ Newton-Schulz iteration
  end for
  $\boldsymbol{x}_{t+1} \leftarrow \boldsymbol{x}_t - \gamma_t \boldsymbol{w}_{T_{\text{NS}}}$
end for
Return: $\boldsymbol{x}_T$

Algorithm 8 `Muon` (general scheme)

Input: Initial parameters $\boldsymbol{x}_0$, number of iterations T. `Muon`'s parameters: learning rate $\gamma_t^{\texttt{M}}$, momentum β, number of Newton-Schulz iterations T_{NS}, a, b, c coefficients. `AdamW`'s parameters: learning rate $\gamma_t^{\texttt{A}}$, weight decay λ, β_1, β_2, ε.
for $t \in [T]$ **do** $\boldsymbol{x}_t \in \{\texttt{embeds}, \texttt{scalar\_params}, \texttt{lm\_head}\}$
  $\boldsymbol{x}_t^{\texttt{A}} \leftarrow \boldsymbol{x}_t$
  $\boldsymbol{x}_{t+1}^{\texttt{A}} \leftarrow \texttt{AdamW}\left(\boldsymbol{x}_t^{\texttt{A}}, \gamma_t^{\texttt{A}}, \lambda, \beta_1, \beta_2, \varepsilon, T = 1\right)$ ▷ One iteration of `AdamW`
  $\boldsymbol{x}_t^{\texttt{M}} \leftarrow \boldsymbol{x}_t$
  $\boldsymbol{x}_{t+1}^{\texttt{M}} \leftarrow \texttt{MuonNon1D}\left(\boldsymbol{x}_t^{\texttt{M}}, \gamma_t^{\texttt{M}}, T_{\text{NS}}, \beta, a, b, c, T = 1\right)$ ▷ One iteration of `MuonNon1D`
end for
Return: $\boldsymbol{x}_T^{\texttt{A}}, \boldsymbol{x}_T^{\texttt{M}}$

Importantly, we noticed that the original algorithmic description of `Muon` optimizer, provided in the official repository[3], differs from the actual one, presented in Algorithm 7. In the original code, as well as in our benchmarking, weight decay do not applies to the matrix parameters in the optimizer state of `MuonNon1D`, which means that the only weight decay used during training is `AdamW`'s weight decay. From this perspective, we observe that the gap between the final loss values for runs with 0.1 and 0 weight decay values almost disappears, while the run with 0.5 weight decay becomes the worst, which is not the case for other optimizers. We describe this in our weight decay ablations.

`SOAP`. [84] proposed new, improved modification of `Shampoo` [24]. `SOAP` reduces the computational overhead optimizing only two dimensional layers (2D) via Algorithm 9, while running `AdamW` for 1D layers. At initialization, preconditioners are computed via eigenvector decomposition of the initial gradient matrices $\texttt{eigenbasis}\left(\nabla\mathcal{L}(\boldsymbol{x}_0, \boldsymbol{\xi}_0)\nabla\mathcal{L}(\boldsymbol{x}_0, \boldsymbol{\xi}_0)^\top\right)$: $\nabla\mathcal{L}(\boldsymbol{x}_0, \boldsymbol{\xi}_0)\nabla\mathcal{L}(\boldsymbol{x}_0, \boldsymbol{\xi}_0)^\top = \boldsymbol{q}\Lambda\boldsymbol{q}^{-1}$, where Λ stands for the diagonal matrix whose diagonal elements are the corresponding eigenvalues. For the rest of the iterations, `SOAPNon1D` performs the `QR` decomposition (see `lines 15, 16` of Algorithm 9) for all 2D layers, which is the main computational part of the method.

A key idea behind the `SOAP` optimizer is:

1. Given the slowly changing coordinate basis provided by eigenvectors $\boldsymbol{l}$ and $\boldsymbol{r}$, `SOAP` updates its second moment estimates in this basis, i.e., it runs `AdamW` in another, rotated space.

2. To update the eigenvectors of $\boldsymbol{l}$ and $\boldsymbol{r}$, `SOAP` runs `QR` decomposition with preconditioning frequency ϕ.

In Algorithm 9, if one would set both $\boldsymbol{q}_l$ and $\boldsymbol{q}_r$ to identity, then we would recover `AdamW`.

The overall `SOAP` algorithm can be formalized as follows:

[3] https://github.com/KellerJordan/modded-nanogpt

Algorithm 9 `SOAPNon1D` (for non-1D parameters)

Input: Initial parameters $\boldsymbol{x}_0$, number of iterations T, learning rate γ_t, weight decay λ, β_1, β_2, preconditioning frequency ϕ, ε.
Initialize: $\boldsymbol{m}_0 \leftarrow \mathbf{0}$, $\boldsymbol{v}_0 \leftarrow \mathbf{0}$
Initialize preconditioners: $\boldsymbol{q}_l, \boldsymbol{q}_r \leftarrow \texttt{eigenbasis}\left(\nabla\mathcal{L}(\boldsymbol{x}_0, \boldsymbol{\xi}_0)\nabla\mathcal{L}(\boldsymbol{x}_0, \boldsymbol{\xi}_0)^\top\right)$
for $t \in [T]$ **do**
  $\boldsymbol{g}_t \leftarrow \nabla\mathcal{L}(\boldsymbol{x}_t, \boldsymbol{\xi}_t)$
  $\boldsymbol{g}'_t \leftarrow \boldsymbol{q}_l^\top \boldsymbol{g}_t \boldsymbol{q}_r$ ▷ Rotate $\boldsymbol{g}_t$
  $\boldsymbol{m} \leftarrow \beta_1 \boldsymbol{m}_{t-1} + (1-\beta_1)\boldsymbol{g}_t$
  $\boldsymbol{m}'_t \leftarrow \boldsymbol{q}_l^\top \boldsymbol{m}_t \boldsymbol{q}_r$ ▷ Compute `Adam`'s statistics in rotational space
  $\boldsymbol{v}_t \leftarrow \beta_2 \boldsymbol{v}_{t-1} + (1-\beta_2)\boldsymbol{g}'_t \odot \boldsymbol{g}'_t$
  $\gamma_t \leftarrow \gamma_t \frac{\sqrt{1-\beta_2^t}}{1-\beta_1^t}$ ▷ Optional: use bias correction
  $\boldsymbol{x}_{t+1} \leftarrow \boldsymbol{x}_t - \gamma_t \left(\boldsymbol{q}_l \frac{\boldsymbol{m}'_t}{\sqrt{\boldsymbol{v}_t}+\varepsilon} \boldsymbol{q}_r^\top + \lambda \boldsymbol{x}_t\right)$ ▷ Perform update in original space
  $\boldsymbol{l}_t \leftarrow \beta_2 \boldsymbol{l}_{t-1} + (1-\beta_2)\boldsymbol{g}_t \boldsymbol{g}_t^\top$ ▷ Update preconditioners
  $\boldsymbol{r}_t \leftarrow \beta_2 \boldsymbol{r}_{t-1} + (1-\beta_2)\boldsymbol{g}_t^\top \boldsymbol{g}_t \; t \equiv 1 \pmod{\phi}$
  $\boldsymbol{q}_l \leftarrow \texttt{QR}\left(\boldsymbol{l}_t \boldsymbol{q}_l\right)$
  $\boldsymbol{q}_r \leftarrow \texttt{QR}\left(\boldsymbol{r}_t \boldsymbol{q}_r\right)$
end for
Return: $\boldsymbol{x}_T$

Algorithm 10 `SOAP` (general scheme)

Input: Initial parameters $\boldsymbol{x}_0$, number of iterations T, learning rate γ_t, weight decay λ, β_1, β_2, preconditioning frequency ϕ, ε.
for $t \in [T]$ **do** $\boldsymbol{x}_t \in \{\texttt{embeds}, \texttt{scalar\_params}, \texttt{lm\_head}\}$
  $\boldsymbol{x}_t^{\texttt{A}} \leftarrow \boldsymbol{x}_t$
  $\boldsymbol{x}_{t+1}^{\texttt{A}} \leftarrow \texttt{AdamW}\left(\boldsymbol{x}_t^{\texttt{A}}, \gamma_t, \lambda, \beta_1, \beta_2, \varepsilon, T=1\right)$ ▷ One iteration of `AdamW`
  $\boldsymbol{x}_t^{\texttt{S}} \leftarrow \boldsymbol{x}_t$
  $\boldsymbol{x}_{t+1}^{\texttt{S}} \leftarrow \texttt{SOAPNon1D}\left(\boldsymbol{x}_t^{\texttt{S}}, \gamma_t, \lambda, \beta_1, \beta_2, \varepsilon, T=1\right)$ ▷ One iteration of `SOAPNon1D`
end for
Return: $\boldsymbol{x}_T^{\texttt{A}}, \boldsymbol{x}_T^{\texttt{S}}$

`Sophia`**.** Despite being named as second-order optimizer, `Sophia` [46] performs an update, quite similar to `Adam`'s. It also leverages the diagonal preconditioner $\boldsymbol{h}$, but not the curvature information of the optimization problem, which depends on the non-diagonal terms of the Hessian. One should notice that `Sophia` were introduced with two types of preconditioners — Hutchinson [3] and Gauss-Newton-Bartlett [51]. Since the latter one shows more promising performance, we consider only this type of preconditioner for `Sophia`.

Every ϕ iterations, `Sophia` updates its second moment estimate by computing the gradient $\hat{\boldsymbol{g}}$ of the empirical loss $\mathcal{L}$ given `softmax` of the logits instead of the true logits. Multiplying by the batch size, we obtain $\hat{\boldsymbol{h}}$, after that, `Sophia` updates the EMA of $\hat{\boldsymbol{h}}$.

Importantly, we found out, that algorithmic description of `Sophia` in the original paper differs in minor details from the code implementation[4]. Indeed, the update rule in their work formulates as follows:

$$\boldsymbol{x}_{t+1} \leftarrow \boldsymbol{x}_t - \gamma_t \texttt{clamp}\left(\frac{\boldsymbol{m}_t}{\max\{\rho \boldsymbol{h}_t, \varepsilon\}}, 1\right),$$

where `clamp` is defined as in Equation (1). On the other hand, the code from the official repository suggests:

Listing 2: `Sophia` update skeleton using PyTorch

```
# update step
```

[4]https://github.com/Liuhong99/Sophia

```
step_t += 1

# Perform stepweight decay
param.mul_(1 - lr * weight_decay)

# Decay the first and second moment running average coefficient
exp_avg.mul_(beta1).add_(grad, alpha=1 - beta1)

else:
    step_size_neg = -lr

    ratio = (exp_avg.abs() / (rho * bs * hess + 1e-15)).clamp(None
        , 1)
    param.addcmul_(exp_avg.sign(), ratio, value=step_size_neg)
```

Therefore, the actual update of `Sophia` is wrongly tractated in the original paper and should be corrected and equal to the `line 16` of Algorithm 11.

Takeaway 4. *The actual update rule of* `Sophia` *differs from its description in the original paper.*

Algorithm 11 `Sophia`

Input: Initial parameters $\boldsymbol{x}_0$, number of iterations T, learning rate γ_t, weight decay λ, β_1, β_2, estimator frequency ϕ, scaling factor ρ, ε.
Initialize: $\boldsymbol{m}_0 \leftarrow \mathbf{0}$, $\boldsymbol{h}_0 \leftarrow \mathbf{0}$
for $t \in [T]$ **do**
  $\boldsymbol{g}_t \leftarrow \nabla \mathcal{L}(\boldsymbol{x}_t, \boldsymbol{\xi}_t)$
  $\boldsymbol{m}_t \leftarrow \beta_1 \boldsymbol{m}_{t-1} + (1-\beta_1)\boldsymbol{g}_t \; t \equiv 1 \pmod{\phi}$
  $p_t \leftarrow \boldsymbol{\xi}_t$ ▷ Obtain logits from batch
  $p_t \leftarrow \texttt{softmax}\,(p_t)$ ▷ Sample from logits
  $\hat{\mathcal{L}}(\boldsymbol{x}_t, \boldsymbol{\xi}_t) \leftarrow p_t$ ▷ Loss, where p_t are labels
  $\hat{\boldsymbol{g}}_t \leftarrow \nabla \hat{\mathcal{L}}(\boldsymbol{x}_t, \boldsymbol{\xi}_t)$
  $\hat{\boldsymbol{h}}_t \leftarrow |\boldsymbol{\xi}_t| \hat{\boldsymbol{g}}_t \odot \hat{\boldsymbol{g}}_t$
  $\boldsymbol{h}_t \leftarrow \beta_2 \boldsymbol{h}_{t-\phi} + (1-\beta_2)\hat{\boldsymbol{h}}_t$
  $\boldsymbol{h}_t \leftarrow \boldsymbol{h}_{t-1}$
  $\boldsymbol{x}_{t+1} \leftarrow \boldsymbol{x}_t - \gamma_t \left(\texttt{sign}(\boldsymbol{m}_t) \min\left\{ \frac{|\boldsymbol{m}_t|}{\rho \boldsymbol{h}_t + \varepsilon}, 1 \right\} + \lambda \boldsymbol{x}_t \right)$
end for
Return: $\boldsymbol{x}_T$

A.4 `Schedule-Free AdamW`, `Prodigy`

In this section, we outline two more players — `Schedule-Free AdamW` [17] and `Prodigy` [54]. Both of them have a promising advantages and require less hyperparameter tuning which paves the road to parameter-free optimizers.

`Schedule-Free AdamW`. [17] introduced a concept of schedule-free optimizers. Underlying idea behind his `Schedule-Free SGD` and `Schedule-Free AdamW` is to remove the scheduler with iterate averaging. Particularly, schedule-free method uses an interpolation between Polyak-Ruppert averaging [68, 73] and Primal averaging [56] for momentum update instead of usual EMA (`line 4` of Algorithm 12). To avoid undesirable behavior during scalable training the authors also propose internal warmup (see `line 7` of Algorithm 12) which uses the general number of warmup iterations parameter in the code, this gradually increases the learning rate and, at the same time, ensures `Adam`'s bias correction.

An interesting result we observe, `SF-AdamW` shows the best performance with larger number of warmup iterations compared to other methods.

Another key point — training with SF-AdamW is sensitive to the choice of beta parameters. Unlike in AdamW, these parameters serve different purposes in SF-AdamW: as β_1 acts as an interpolation parameter between two sequences, and β_2 controls the EMA of the second moment estimate, which relates to $\boldsymbol{y}$ sequence rather than the model parameters $\boldsymbol{x}$ (line 6 of Algorithm 12). For Adam it is common to analyze in theory the case, when $\beta_2 = 1 - 1/T$ [89, 10], i.e., the choice of the "optimal" beta parameters depends on the length of training. Which is also the case for SF-AdamW, making it not fully schedule-free. [28] observed this sensitivity to beta parameters, and we go beyond this ablation also.

Importantly, the authors mention that disabling the gradient norm clipping is crucial for schedule-free runs, however, we do not observe this in practice, demonstrating the contrary results.

Algorithm 12 SF-AdamW

Input: Initial parameters $\boldsymbol{x}_0$, number of iterations T, learning rate γ, weight decay λ, β_1, β_2, warmup iterations T_{warmup}, ε.
Initialize: $\boldsymbol{z}_0 \leftarrow \boldsymbol{x}_0$, $\boldsymbol{v}_0 \leftarrow \mathbf{0}$
for $t \in [T]$ **do**
  $\boldsymbol{y}_t \leftarrow (1-\beta_1)\boldsymbol{z}_t + \beta_1 \boldsymbol{x}_t$
  $\boldsymbol{g}_t \leftarrow \nabla \mathcal{L}(\boldsymbol{y}_t, \boldsymbol{\xi}_t)$
  $\boldsymbol{v}_t \leftarrow \beta_2 \boldsymbol{v}_{t-1} + (1-\beta_2)\boldsymbol{g}_t \odot \boldsymbol{g}_t$
  $\gamma_t \leftarrow \gamma \sqrt{1-\beta_2^t}\min\{1, t/T_{\text{warmup}}\}$
  $\boldsymbol{z}_{t+1} \leftarrow \boldsymbol{z}_t - \gamma_t \left(\boldsymbol{g}_t/(\sqrt{\boldsymbol{v}_t} + \varepsilon) + \lambda \boldsymbol{y}_t\right)$
  $c_{t+1} \leftarrow \frac{\gamma_t^2}{\sum_{i=0}^{t}\gamma_i^2}$
  $\boldsymbol{x}_{t+1} \leftarrow (1-c_{t+1})\boldsymbol{x}_t + c_{t+1}\boldsymbol{z}_{t+1}$
end for
Return: $\boldsymbol{x}_T$

Prodigy. Improving the D-Adaptation concept [16], [54] derived an Adam-like method, which use an EMA for the learning rate (see lines 8, 9 of Algorithm 13), approximating the Adam's update of the second moment estimate EMA. The derived update reflects an EMA of $d_t \boldsymbol{g}_t$ sequence rather than $\boldsymbol{g}_t$. Idea of such a method is to come up with a scheme that is able to remove the hand-tuned learning rate via sequence which adapts during training on the fly. In Algorithm 13, d_t is such a sequence that affects botth first and second moment estimates, and evolves according to line 10.

Crucially, Prodigy does not need the learning rate tuning (typically, we initialize $\gamma = 1$), however, it still can be compatible with learning rate schedules, which we verify experimentally at scale. We also show that d_t sequence indeed acts similarly to cosine learning rate scheduler, with usually smaller learning rate at initialization and a bit larger values of it at maximum Moreover, this method scales reliably similar to AdamW, making it a promising choice for future development of parameter-free methods.

A.5 MARS

Very recently, [88] introduce MARS — a series optimizers, which incorporate modern adaptive methods [50, 9] and approximate second-order methods [24] with variance reduction update update style.

This optimization framework gave a birth to: MARS-AdamW — our main baseline which we call simply MARS, MARS-Lion and MARS-Shampoo. We mainly include MARS-AdamW in our ablation studies, but report results for other two optimizers.

The authors modified a variance reduction update introducing a sclaing parameter η, which we call variance reduction scaling in the outlined algorithms and experiments. This parameter controls the scale of gradient correction – see line 5 of Algorithms 14 to 16.

Algorithm 13 `Prodigy`

Input: Initial parameters $\boldsymbol{x}_0$, number of iterations T, learning rate γ, weight decay λ, β_1, β_2, ε.
Initialize: $d_0 \leftarrow 10^{-6}$, $\gamma \leftarrow 1$, $\boldsymbol{m}_0 \leftarrow \mathbf{0}$, $\boldsymbol{v}_0 \leftarrow \mathbf{0}$, $r_0 \leftarrow 0$, $\boldsymbol{s}_0 \leftarrow \mathbf{0}$ ▷ Optional: use scheduler on γ
for $t \in [T]$ **do**
  $\boldsymbol{g}_t \leftarrow \nabla \mathcal{L}(\boldsymbol{x}_t, \boldsymbol{\xi}_t)$
  $\boldsymbol{m}_t \leftarrow \beta_1 \boldsymbol{m}_{t-1} + (1-\beta_1) d_t \boldsymbol{g}_t$
  $\boldsymbol{v}_t \leftarrow \beta_2 \boldsymbol{v}_{t-1} + (1-\beta_2) d_t^2 \boldsymbol{g}_t \odot \boldsymbol{g}_t$
  $\gamma_t \leftarrow \gamma \sqrt{1-\beta_2^t}/(1-\beta_1^t)$ ▷ Optional: use bias correction
  $r_t \leftarrow \sqrt{\beta_2} r_{t-1} + (1-\sqrt{\beta_2}) \gamma_t d_t^2 \langle \boldsymbol{g}_t, \boldsymbol{x}_0 - \boldsymbol{x}_t \rangle$
  $\boldsymbol{s}_t \leftarrow \sqrt{\beta_2} \boldsymbol{s}_{t-1} + (1-\sqrt{\beta_2}) \gamma_t d_t^2 \boldsymbol{g}_t$
  $d_{t+1} \leftarrow \max\{d_t, \frac{r_t}{\|\boldsymbol{s}_t\|_1}\}$
  $\boldsymbol{x}_{t+1} \leftarrow \boldsymbol{x}_t - \gamma_t d_t \left(\boldsymbol{m}_t / \left(\sqrt{\boldsymbol{v}_t} + d_t \varepsilon \right) + \lambda \boldsymbol{x}_t \right)$
end for
Return: $\boldsymbol{x}_T$

An important detail, we follow only the approximate scheme of `MARS`-like optimizers, i.e., we evaluate the gradient $\boldsymbol{g}_t$ in different stochasticity, meaning

$$\boldsymbol{g}_t = \nabla \mathcal{L}\left(\boldsymbol{x}_t, \boldsymbol{\xi}_t\right),$$
$$\boldsymbol{g}_{t-1} = \nabla \mathcal{L}\left(\boldsymbol{x}_{t-1}, \boldsymbol{\xi}_{t-1}\right).$$

Importantly, in the same spirit as for `SOAP` and `Muon`, the authors use `MARS`-like algorithms for layers with two and more dimensions, for 1D layers, embeds, scalar parameters and final the head layer of neural network, this method utilize `AdamW`. Such a choice allows use `MARS` in the more fast and still efficient way. Following the practices from their work, we also use `MARS` only for 2D layers.

`MARS` (`MARS-AdamW`). For `AdamW`-like algorithm, the difference occurs at the computation of $\boldsymbol{m}_t$ and $\boldsymbol{v}_t$, where instead of the gradient, the variance reduction update $\boldsymbol{c}_t$ is used.

Algorithm 14 `MARS` (`MARS-AdamW`)

Input: Initial parameters $\boldsymbol{x}_0$, number of iterations T, learning rate γ_t, weight decay λ, β_1, β_2, variance reduction scaling η, ε.
Initialize: $\boldsymbol{m}_0 \leftarrow \mathbf{0}$, $\boldsymbol{v}_0 \leftarrow \mathbf{0}$
for $t \in [T]$ **do**
  $\boldsymbol{g}_t \leftarrow \nabla \mathcal{L}(\boldsymbol{x}_t, \boldsymbol{\xi}_t)$
  $\boldsymbol{c}_t \leftarrow \boldsymbol{g}_t + \eta \frac{\beta_1}{1-\beta_1} \left(\boldsymbol{g}_t - \boldsymbol{g}_{t-1} \right) \|\boldsymbol{c}_t\|_2 > 1$
  $\boldsymbol{c}_t \leftarrow \boldsymbol{c}_t / \|\boldsymbol{c}_t\|_2$
  $\boldsymbol{m}_t \leftarrow \beta_1 \boldsymbol{m}_{t-1} + (1-\beta_1) \boldsymbol{c}_t$
  $\boldsymbol{v}_t \leftarrow \beta_2 \boldsymbol{v}_{t-1} + (1-\beta_2) \boldsymbol{c}_t \odot \boldsymbol{c}_t$
  $\hat{\boldsymbol{m}}_t \leftarrow \boldsymbol{m}_t / (1-\beta_1^t), \quad \hat{\boldsymbol{v}}_t \leftarrow \boldsymbol{v}_t / (1-\beta_2^t)$
  $\boldsymbol{x}_{t+1} = \boldsymbol{x}_t - \gamma_t \left(\frac{\hat{\boldsymbol{m}}_t}{\sqrt{\hat{\boldsymbol{v}}_t} + \varepsilon} + \lambda \boldsymbol{x}_t \right)$
end for
Return: $\boldsymbol{x}_T$

`MARS-Lion`. Similarly for `Lion`-like algorithm, the authors use scaled gradient correction with the current gradient — $\boldsymbol{c}_t$.

`MARS-Shampoo`. The same holds for `MARS-Shampoo`. One key comment here, is that to compute `SVD` of the first moment estimate, the authors also use Newton-Schulz iteration [5, 25]. In our experiments we use 10 iterations of this orthogonalization scheme for `MARS-Shampoo`.

Algorithm 15 `MARS-Lion`

Input: Initial parameters $\boldsymbol{x}_0$, number of iterations T, learning rate γ_t, weight decay λ, β_1, variance reduction scaling η, ε.
Initialize: $\boldsymbol{m}_0 \leftarrow \mathbf{0}$, $\boldsymbol{v}_0 \leftarrow \mathbf{0}$
for $t \in [T]$ **do**
  $\boldsymbol{g}_t \leftarrow \nabla \mathcal{L}(\boldsymbol{x}_t, \boldsymbol{\xi}_t)$
  $\boldsymbol{c}_t \leftarrow \boldsymbol{g}_t + \eta \frac{\beta_1}{1-\beta_1} \left(\boldsymbol{g}_t - \boldsymbol{g}_{t-1}\right) \|\boldsymbol{c}_t\|_2 > 1$
  $\boldsymbol{c}_t \leftarrow \boldsymbol{c}_t / \|\boldsymbol{c}_t\|_2$
  $\boldsymbol{m}_t \leftarrow \beta_1 \boldsymbol{m}_{t-1} + (1-\beta_1)\boldsymbol{c}_t$
  $\boldsymbol{x}_{t+1} = \boldsymbol{x}_t - \gamma_t \left(\texttt{sign}\left(\boldsymbol{m}_t\right) + \lambda \boldsymbol{x}_t\right)$
end for
Return: $\boldsymbol{x}_T$

Algorithm 16 `MARS-Shampoo`

Input: Initial parameters $\boldsymbol{x}_0$, number of iterations T, learning rate γ_t, weight decay λ, β_1, variance reduction scaling η, ε.
Initialize: $\boldsymbol{m}_0 \leftarrow \mathbf{0}$, $\boldsymbol{v}_0 \leftarrow \mathbf{0}$
for $t \in [T]$ **do**
  $\boldsymbol{g}_t \leftarrow \nabla \mathcal{L}(\boldsymbol{x}_t, \boldsymbol{\xi}_t)$
  $\boldsymbol{c}_t \leftarrow \boldsymbol{g}_t + \eta \frac{\beta_1}{1-\beta_1} \left(\boldsymbol{g}_t - \boldsymbol{g}_{t-1}\right)$
  $\boldsymbol{m}_t \leftarrow \beta_1 \boldsymbol{m}_{t-1} + (1-\beta_1)\boldsymbol{c}_t$
  $\boldsymbol{U}_t, \boldsymbol{\Sigma}_t, \boldsymbol{V}_t \leftarrow \texttt{SVD}(\boldsymbol{m}_t)$
  $\boldsymbol{x}_{t+1} = \boldsymbol{x}_t - \gamma_t \left(\boldsymbol{U}_t \boldsymbol{V}_t^\top + \lambda \boldsymbol{x}_t\right)$
end for
Return: $\boldsymbol{x}_T$

B Implementation

Our code is based on an extension of nanoGPT[5] and uses PyTorch [63] as well as FlashAttention [14]. We incorporate mixed-precision training [53], i.e., we train in `bfloat16` precision, except for normalization modules and softmax which we train in `float32`. The optimizer states are also stored in `float32`. The majority of the experiments were performed using a cluster of A100-SXM4-80GB GPUs as well as H100-HBM3-80GB. We trained both in a single GPU regime and in DDP [45] (from 2 to 8 GPUs per one run). We estimate that the full cost of all experiments for our project to roughly 30000 GPU hours.

C Model & Data

Architecture details. In our project, we use Llama-like family of models [48]. We implement the the popular in the community decoder-only transformer with SwiGLU activation functions [77], RoPE embeddings [81], RMSNorm [91]. The vocabulary is based on the GPT2 [71] tokenizer [6] and contains 50304 tokens.

The number of parameters in our models is fully configurable, and we present the exact configurations used in our experiment in Table 2.

Dataset. Our main findings are obtained on the subset of FineWeb [64] with $100\mathbf{B}$ tokens [7], cleaned and deduplicated corpus for LLM pretraining, which we split into train and validation sequences. During training, we evaluate the models with a fixed set of 32 batches of our chosen sequence length (512 for almost all experiments, the same context length as training) to establish the validation loss curves. At the end of training, we compute the full validation loss and perplexity (this loss is reported

[5] https://github.com/karpathy/nanoGPT
[6] https://github.com/openai/tiktoken
[7] https://huggingface.co/datasets/HuggingFaceFW/fineweb

Table 2: Hyperparameters for our Llama-like models.

# Parameters	124M	210M	600M
Hidden size	768	768	
# Attention heads	12	12	
# Layers	12	24	
Init std	0.02	0.02	0.02
Use bias	no	no	no
RMSNorm epsilon	0.00001	0.00001	0.00001
Positional encoding	RoPE	RoPE	RoPE

as Final Validation Loss in the figures). We also performed our initial results on the subset of OpenWebText2 dataset [21].

D Hyperparameter tuning

How do we tune hyperparameters? We perform systematic hyperparameter tuning for all algorithms, starting with smaller models (124M, 210M) and extrapolating to larger ones. Our tuning process focused on two primary settings: **Small batch setting** (32 batch size) and **Large batch setting** (256 batch size). For both settings, we use a sequence length of 512 tokens, resulting in 16k and 130k tokens per batch, respectively. If the batch cannot fit into memory, we use gradient accumulation steps, while maintaining the effective batch size.

We also include ablations on even larger batch size for 124M models, where we train on 512 batch size (260k tokens correspondingly). And larger, 583M models, we train on 3936 batch size, preserving the basic sequence length of 512, i.e., 4M tokens.

We first run multiple experiments, greed searching hyperparameters, on near Chinchilla optimal length of training using *cosine learning rate scheduler* (except for `SF-AdamW`):

- for 124M models we tune at 2.1B tokens for both small (32) and large (256) batch size setting,
- for 210M models we tune at 4.2B tokens for our large batch size setting,
- for 583M models we also consider a setting with and without z-loss.

We present the configurations for different training horizons in Tables 3 and 5.

Table 3: Lengths of training for **Small batch settings** (32×512).

# Parameters	Tokens (Iterations)						Ch. Tokens
124M	1B (64k)	2.1B (128k)	4.2B (256k)	6.3B (384k)	8.4B (512k)	16.8B (1024k)	2.5B
210M	1B (64k)	2.1B (128k)	4.2B (256k)	6.3B (384k)	8.4B (512k)	16.8B (1024k)	4.2B

Table 4: Lengths of training for **Large batch settings** (256×512).

# Parameters	Tokens (Iterations)						Chinchilla Tokens
124M	1B (8k)	2.1B (16k)	4.2B (32k)	6.3B (48k)	8.4B (64k)	16.8B (128k)	2.5B
210M	1B (8k)	2.1B (16k)	4.2B (32k)	6.3B (48k)	8.4B (64k)	16.8B (128k)	4.2B

Important to note, for larger models, we mostly kept the best hyperparameters found for the 124M model and re-tuned the learning rate and gradient clipping. We summarize this process in Appendices D.1 to D.3.

Additionally, when we report an of one particular hyperparameters, we mean that corresponding algorithm has already been tuned and, thus, we show only how one particular hyperparameter affects the overall performance.

Hyperparameters used in our WSD scheduler experiments. Once we found the best setting for each method using cosine learning rate scheduler, we are ready to obtain the optimal performance of our method with WSD scheduler [27]. Here we follow the rule of thumb from [28]:

Table 5: Lengths of training for **X-Large batch settings** (1984×512).

# Parameters	Tokens (Iterations)			Chinchilla Tokens
720M	8B (8k)	16B (16k)	48B (48k)	14.4B

- use half the optimal learning rate for the cosine scheduler,
- use 20% of iterations for cooldown phase,
- use $(1 - \sqrt{x})$ decay shape for the cooldown phase,

the only difference is that we do not employ stochastic weight averaging [29].

Therefore, we maintain most hyperparameters across optimizers, only re-tuning the learning rate. For methods like `Muon` and `MARS`, we reduce both `AdamW`'s learning rate and the learning rate for non-1D parameters. This approach ensures a fair comparison while accounting for the unique properties of each optimizer.

Indeed, this rule of thumb works better in our setting also, e.g., see the comparison between linear decay shape and $(1 - \sqrt{x})$ in

We report a series of comparisons between cosine learning rate scheduler and WSD in

Hyperparameters used in `z-loss` experiments.

D.1 124M parameters model

Table 6: `AdamW` hyperparameter tuning for our 124M parameter large language models. Bold hyperparameters are the best.

Hyperparameter	Small batch setting	Large batch setting
Learning rate	$0.0001, \mathbf{0.0005}, 0.0008, 0.001, 0.002$	$0.0001, 0.0003, 0.0005, \mathbf{0.001}, 0.002$
Batch size	32	256
Sequence length	512	512
Number of warmup steps	$\mathbf{3000}, 5000, 8000$	$500, 1000, \mathbf{2000}, 3000, 8000, 32000$
Weight decay	0.1	no, $0.1, \mathbf{0.5}, 0.7$
Learning rate decay scheduler	WSD, **cosine**	WSD, **cosine**
Gradient clipping	no, $\mathbf{0.5}, 1, 1.5$	no, $0.5, 1$
`AdamW` β_1	$0.5, \mathbf{0.8}, 0.9$	$\mathbf{0.8}, 0.9$
`AdamW` β_2	$0.95, \mathbf{0.999}$	$0.95, 0.99, \mathbf{0.999}, 0.9999$

Table 7: `ADOPT` hyperparameter tuning for our 124M parameter large language models. Bold hyperparameters are the best.

Hyperparameter	Small batch setting	Large batch setting
Learning rate	0.001	$0.0001, 0.0003, 0.0005, \mathbf{0.001}, 0.002$
Batch size	32	256
Sequence length	512	512
Number of warmup steps	$\mathbf{3000}, 8000$	$\mathbf{2000}, 8000, 32000$
Weight decay	0.1	no $\mathbf{0.1}, 0.5$
Learning rate decay scheduler	WSD, **cosine**	WSD, **cosine**
Gradient clipping	0.5	no, $0.5, 1$
`ADOPT` β_1	0.9	$0.8, \mathbf{0.9}$
`ADOPT` β_2	$\mathbf{0.999}, 0.9999$	$0.5, \mathbf{0.999}, 0.9999$

Table 8: AdEMAMix hyperparameter tuning for our 124M parameter large language models. Bold hyperparameters are the best.

Hyperparameter	Small batch setting	Large batch setting
Learning rate	$0.0001, \mathbf{0.0005}, 0.0008, 0.001, 0.002$	$0.0001, 0.0003, 0.0005, \mathbf{0.001}, 0.002$
Batch size	32	256
Sequence length	512	512
Number of warmup steps	$\mathbf{3000}, 8000$	$\mathbf{2000}, 8000, 32000$
Weight decay	0.1	no, $0.1, \mathbf{0.5}, 0.7$
Learning rate decay scheduler	WSD, **cosine**	WSD, **cosine**
Gradient clipping	no, $\mathbf{0.5}, 1, 1.5$	no, $\mathbf{0.5}, 1$
AdEMAMix β_1	$0.5, \mathbf{0.8}, 0.9$	$0.8, \mathbf{0.9}$
AdEMAMix β_2	0.999	$\mathbf{0.999}, 0.9999$
AdEMAMix β_3	$0.999, \mathbf{0.9999}, 0.99995$	$\mathbf{0.999}, 0.9999$
AdEMAMix α	$5, \mathbf{8}, 12$	8

Table 9: Lion hyperparameter tuning for our 124M parameter large language models. Bold hyperparameters are the best.

Hyperparameter	Small batch setting	Large batch setting
Learning rate	$0.00005, \mathbf{0.0001}, 0.0005, 0.001$	$0.0001, 0.0005, \mathbf{0.001}, 0.002$
Batch size	32	256
Sequence length	512	512
Number of warmup steps	3000	$\mathbf{2000}, 8000, 32000$
Weight decay	no, $0.1, 0.2, \mathbf{0.5}$	no, $0.1, \mathbf{0.5}, 0.7$
Learning rate decay scheduler	WSD, **cosine**	WSD, **cosine**
Gradient clipping	0.5	no, $\mathbf{0.5}, 1$
Lion β_1	$0.7, \mathbf{0.9}, 0.99$	$0.5, \mathbf{0.9}$
Lion β_2	$0.9, \mathbf{0.99}, 0.999$	$\mathbf{0.99}, 0.999$

D.2 210M parameters model

D.3 600M parameters model

E Additional results

E.1 Benchmarking: 124M

In this section, we provide complete results for the benchmarking part presented in Section 4.1. We cover both the **large batch setting** and the **small batch setting**, reporting the full curves with validation loss dynamics across different training durations.

Given the quite a lot number of methods under consideration, we divide them into two groups: those that outperform AdamW and those that underperform relative to AdamW. We use AdamW loss curves as the reference point in both figures. We summarize our findings for the small batch size of 32 in Figure 12. And for the large batch size of 256 in Figures 13 and 14.

E.2 Ablations for 124M model

Fail of Sophia.

Clipping & SF-AdamW.

Betas sensitivity.

Warmup ablation. In this section we detaily describe the main part in Section 4.1.

We study the impact of batch size on the final validation loss obtained. For all methods, we sweep over warmup lengths of $\{1.56\%, 6.25\%, 25\%\}$ of the total training duration to examine each method's

Table 10: `Signum` hyperparameter tuning for our $124\mathbf{M}$ parameter large language models. Bold hyperparameters are the best.

Hyperparameter	Small batch setting	Large batch setting
Learning rate	$0.0003, 0.0005, \mathbf{0.001}$	$0.0001, 0.00030.0005, 0.0003, \mathbf{0.001}, 0.002$
Batch size	32	256
Sequence length	512	512
Number of warmup steps	$2000, \mathbf{3000}$	$\mathbf{2000}, 8000, 32000$
Weight decay	no, $0, \mathbf{0.1}, 0.5$	no, $0, \mathbf{0.1}, 0.5, 0.7$
Learning rate decay scheduler	WSD, **cosine**	WSD, **cosine**
Gradient clipping	no, $\mathbf{0.5}, 1$	no, $\mathbf{0.5}, 1$
Momentum	no, $0.9, \mathbf{0.95}$	no, $0.9, \mathbf{0.95}, 0.99$
Nesterov momentum	no, **yes**	no, **yes**

Table 11: `Muon` hyperparameter tuning for our $124\mathbf{M}$ parameter large language models. Bold hyperparameters are the best.

Hyperparameter	Small batch setting	Large batch setting
Learning rate `AdamW`	$0.0001, 0.0003, 0.0005, \mathbf{0.001}, 0.002$	$0.0001, 0.0003, 0.0005, \mathbf{0.001}, 0.002$
Learning rate `Muon`	$0.001, \mathbf{0.01}, 0.02$	$0.001, \mathbf{0.01}, 0.02$
Batch size	32	256
Sequence length	512	512
Number of warmup steps	$\mathbf{3000}, 8000$	$\mathbf{2000}, 8000, 32000$
Weight decay	no, $\mathbf{0.1}, 0.5$	no, $\mathbf{0.1}, 0.5$
Learning rate decay scheduler	**WSD**, cosine	**WSD**, cosine
Gradient clipping	no, $\mathbf{0.5}$	no, $\mathbf{0.5}, 1.0$
Momentum `Muon`	$0.9, 0.95, \mathbf{0.99}$	$\mathbf{0.95}, 0.99$
Optimizer for 1D layers	`AdamW`	`AdamW`
Optimizer for 1D layers, β_1	$0.8, 0.9$	$\mathbf{0.8}, 0.9$
Optimizer for 1D layers, β_2	$0.99, \mathbf{0.999}, 0.9999$	$0.99, 0.999, 0.9999$
Newton-Schulz a	3.4445	3.4445
Newton-Schultz b	-4.7750	-4.7750
Newton-Schultz c	2.0315	2.0315
Nesterov momentum	no, **yes**	no, **yes**

sensitivity to warmup. For `AdamW`, we extend this sweep to $\{1.56\%, 5\%, 6.25\%, 10\%, 25\%\}$. We specifically consider the 1.56% and 6.25% percentages because the former represents a typical number of warmup steps (2000) for models of our scale, while the latter (6.25% of 128000 steps) aligns with the warmup strategy used in Llama [48].

Contrary to the insights from [92], we observe that 25% of the Cinchilla optimal duration is far from being the best batch size for pretraining. We emphasize that their results were obtained for $85\mathbf{M}$ models and then extrapolated to larger scales. However, in our setting, we found the basic 2000 steps a more suitable option for warmup. 25% of Chinchilla optimal length of training, for our $124\mathbf{M}$ model is $620\mathbf{M}$ tokens.

We provide the warmup sweep for `AdamW` in Figure 19

In addition to validating the results from [92], we report the sensitivity of different optimizers to the number of warmup steps by conducting a sweep over the aforementioned percentages. A summary of this experiment — Figure 6.

`Muon` **Newton-Schulz iterations.**

Weight decay ablation.

Learning rate sensitivity. In this part of the work, we meticulously replicate the learning rate sweep process and present comprehensive results. Consistent with our experimental setup, we aim to determine the true impact of the learning rate and its transferability to longer training horizons.

Table 12: SOAP hyperparameter tuning for our $124\mathbf{M}$ parameter large language models. Bold hyperparameters are the best.

Hyperparameter	Small batch setting	Large batch setting
Learning rate	$0.005, \mathbf{0.001}$	$0.0001, 0.0003, 0.0005, \mathbf{0.001}, 0.002$
Batch size	32	256
Sequence length	512	512
Number of warmup steps	$\mathbf{3000}, 8000$	$\mathbf{2000}, 4000, 8000, 12000, 16000, 32000$
Weight decay	0.1	no, $\mathbf{0.1}, 0.5$
Learning rate decay scheduler	WSD, **cosine**	WSD, **cosine**
Gradient clipping	0.5	no, $\mathbf{0.5}, 1$
Preconditioner dimension	10000	10000
Preconditioning frequency	$1, 5, \mathbf{10}$	$1, 5, \mathbf{10}$
SOAP β_1	$0.8, \mathbf{0.9}$	$0.8, \mathbf{0.9}, 0.95$
SOAP β_2	$0.95, 0.99, \mathbf{0.999}, 0.9999$	$0.95, 0.99, \mathbf{0.999}, 0.9999$

Table 13: Sophia hyperparameter tuning for our $124\mathbf{M}$ parameter large language models. Bold hyperparameters are the best.

Hyperparameter	Small batch setting	Large batch setting
Learning rate	$0.0001, \mathbf{0.0003}, 0.0005, 0.001, 0.002$	$0.0001, 0.0003, 0.0005, \mathbf{0.001}, 0.002, 0.01$
Batch size	32	256
Sequence length	512	512
Number of warmup steps	$\mathbf{2000}, 3000$	$\mathbf{2000}, 8000, 32000$
Weight decay	0.1	no, $\mathbf{0.1}, 0.5$
Learning rate decay scheduler	WSD, **cosine**	WSD, **cosine**
Gradient clipping	0.5	no, $\mathbf{0.5}, 1$
Estimator	Gauss-Newton-Bartlett	Gauss-Newton-Bartlett
Estimator frequency	10	10
Sophia β_1	0.9	$0.8, \mathbf{0.9}$
Sophia β_2	$0.95, \mathbf{0.999}, 0.9999, 0.99999$	$0.95, \mathbf{0.999}, 0.9999, 0.99999$
Sophia ρ	$0, 0.03, \mathbf{0.04}$	$0, 0.03, \mathbf{0.04}$

For each optimizer (except Prodigy), we vary only the learning rate while maintaining the best hyperparameter settings obtained during our initial tuning (see Appendices D and D.1) on $2.1\mathbf{B}$ tokens for the $124\mathbf{M}$ parameter model. We present the results of the learning rate sweep in Figure 24.

Cosine vs WSD. We present our results for two batch size settings: 32 and 256. At first, our initial results in small batch setting on the OpenWebText2 (OWT2) dataset, we present in Figure 25.

We report the final validation loss on the FineWeb dataset for $124\mathbf{M}$ model trained on the batch size of 256. We use our tuned with cosine scheduler methods. For WSD, we follow the rule of thumb from [28]: 20% of steps for the cooldown, $1 - \sqrt{x}$ decay shape, and the learning rate is half the optimal for cosine, i.e., 0.0005 if we have the best learning rate 0.001 for the method. Additionally, we point out that we do not include stochastic weight averaging in the comparison, which might potentially enhance the performance of optimizers with WSD.

Gradient norm patterns.

E.3 Benchmarking: $210\mathbf{M}$

E.4 Ablations for $210\mathbf{M}$ model

E.5 Wall-clock performance of optimizers across models of different scale

E.6 Extension to MoEs.

Table 14: `Schedule-Free AdamW` hyperparameter tuning for our 124M parameter large language models. Bold hyperparameters are the best.

Hyperparameter	Small batch setting	Large batch setting
Learning rate	$0.0001, 0.0003, 0.0005, \mathbf{0.001}, 0.005$	$0.0001, 0.0003, 0.0005, \mathbf{0.001}, 0.002, 0.005$
Batch size	32	256
Sequence length	512	512
Number of warmup steps	$\mathbf{3000}, 8000$	$2000, 4000, \mathbf{8000}, 12000, 16000, 32000$
Weight decay	no, $0.05, \mathbf{0.1}, 0.5$	no, $0.05, \mathbf{0.1}, 0.5$
Learning rate decay scheduler	no	no
Gradient clipping	no, $\mathbf{0.5}$	no, $\mathbf{0.5}, 1$
`Schedule-Free AdamW` β_1	$\mathbf{0.9}, 0.95, 0.98$	$\mathbf{0.9}, 0.95, 0.98$
`Schedule-Free AdamW` β_2	$0.95, 0.99, 0.999, \mathbf{0.9999}, 0.99999$	$0.95, 0.99, 0.999, \mathbf{0.9999}, 0.99999$

Table 15: `Prodigy` hyperparameter tuning for our 124M parameter large language models. Bold hyperparameters are the best.

Hyperparameter	Small batch setting	Large batch setting
Learning rate	$0.5, \mathbf{1}$	$0.5, \mathbf{1}, 2, 10, 100$
Batch size	32	256
Sequence length	512	512
Number of warmup steps	$\mathbf{3000}, 8000$	$\mathbf{2000}, 4000, 8000, 12000, 16000, 32000$
Weight decay	no, $\mathbf{0.1}, 0.5$	no, $\mathbf{0.1}, 0.5$
Learning rate decay scheduler	no, WSD, **cosine**	no, WSD, **cosine**
Gradient clipping	no, $\mathbf{0.5}, 1$	no, $\mathbf{0.5}, 1$
`Prodigy` β_1	0.9	$0.8, \mathbf{0.9}$
`Prodigy` β_2	$0.99, \mathbf{0.999}, 0.9999$	$\mathbf{0.999}, 0.9999$
`Prodigy` bias correction	no, **yes**	no, **yes**

Table 16: `MARS` (`MARS-AdamW`) hyperparameter tuning for our 124M parameter large language models. Bold hyperparameters are the best.

Hyperparameter	Small batch setting	Large batch setting
Learning rate `AdamW`	$0.0001, 0.0005, \mathbf{0.001}, 0.003$	$0.0001, 0.0005, \mathbf{0.001}, 0.003$
Learning rate `MARS`	$0.001, \mathbf{0.003}$	$0.001, \mathbf{0.003}$
Batch size	32	256
Sequence length	512	512
Number of warmup steps	$2000, \mathbf{3000}$	$\mathbf{2000}, 8000, 32000$
Weight decay `MARS`	no, $\mathbf{0.1}$	no, $\mathbf{0.1}, 0.5$
Weight decay for 1D layers	0.1	0.1
Learning rate decay scheduler	WSD, **cosine**	WSD, **cosine**
Gradient clipping	0.5	0.5
Optimizer for 1D layers	`AdamW`	`AdamW`
Optimizer for 1D layers β_1	$\mathbf{0.8}, 0.9$	$\mathbf{0.8}, 0.9, 0.95$
Optimizer for 1D layers β_2	$0.95, 0.99, \mathbf{0.999}$	$0.95, 0.99, \mathbf{0.999}$
`MARS` β_1	$0.9, \mathbf{0.95}$	$0.9, \mathbf{0.95}$
`MARS` β_2	$0.95, \mathbf{0.99}$	$0.95, \mathbf{0.99}$
VR scaling factor η	$0.024, \mathbf{0.025}$	$0.024, \mathbf{0.025}$

Table 17: MARS-Lion hyperparameter tuning for our 124M parameter large language models. Bold hyperparameters are the best.

Hyperparameter	Small batch setting	Large batch setting
Learning rate Lion	**0.0001**, 0.0005, 0.001, 0.003	**0.0001**, 0.0005, 0.001, 0.003
Learning rate MARS	**0.0001**, 0.001, 0.003	**0.0001**, 0.001, 0.003
Batch size	32	256
Sequence length	512	512
Number of warmup steps	2000, **3000**	**2000**, 8000, 32000
Weight decay MARS	no, **0.1**	no, **0.1**, 0.5
Weight decay for 1D layers	0.1	0.1
Learning rate decay scheduler	**WSD**, cosine	**WSD**, cosine
Gradient clipping	0.5	0.5
Optimizer for 1D layers	Lion	Lion
Optimizer for 1D layers β_1	0.8, **0.9**	0.8, **0.9**, 0.95
Optimizer for 1D layers β_2	0.95, 0.99, **0.999**	0.95, 0.99, **0.999**
MARS β_1	0.9, **0.95**	0.9, **0.95**
MARS β_2	0.95, **0.99**	0.95, **0.99**
VR scaling factor η	0.024, **0.025**	0.024, **0.025**

Table 18: MARS-Shampoo hyperparameter tuning for our 124M parameter large language models. Bold hyperparameters are the best.

Hyperparameter	Small batch setting	Large batch setting
Learning rate Shampoo	0.0001, 0.0005, **0.001**, 0.003	0.0001, 0.0005, **0.001**, 0.003
Learning rate MARS	0.001, **0.003**	0.001, **0.003**
Batch size	32	256
Sequence length	512	512
Number of warmup steps	2000, **3000**	**2000**, 8000, 32000
Weight decay MARS	no, **0.1**	no, **0.1**, 0.5
Weight decay for 1D layers	0.1	0.1
Learning rate decay scheduler	WSD, **cosine**	WSD, **cosine**
Gradient clipping	0.5	0.5
Optimizer for 1D layers	Shampoo	Shampoo
Optimizer for 1D layers β_1	0.8, **0.9**	0.8, **0.9**, 0.95
Optimizer for 1D layers β_2	0.95, 0.99, **0.999**	0.95, 0.99, **0.999**
MARS β_1	0.9, **0.95**	0.9, **0.95**
MARS β_2	0.95, **0.99**	0.95, **0.99**
VR scaling factor η	0.024, **0.025**	0.024, **0.025**

Table 19: Hyperparameters for our Llama-like models for the wall-clock experiments.

# Parameters	30M	52M	80M	124M	150M	210M	360M	720M	1B
Hidden size	384	512	768	768	768	768	1024	2048	1792
# Attention heads	6	8	6	12	12	12	16	16	14
# Layers	8	8	6	12	16	24	24	12	24
Init std	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Use bias	no	no	no	no	no	no	no	no	no
RMSNorm epsilon	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001	0.00001
Positional encoding	RoPE	RoPE	RoPE	RoPE	RoPE	RoPE	RoPE	RoPE	RoPE

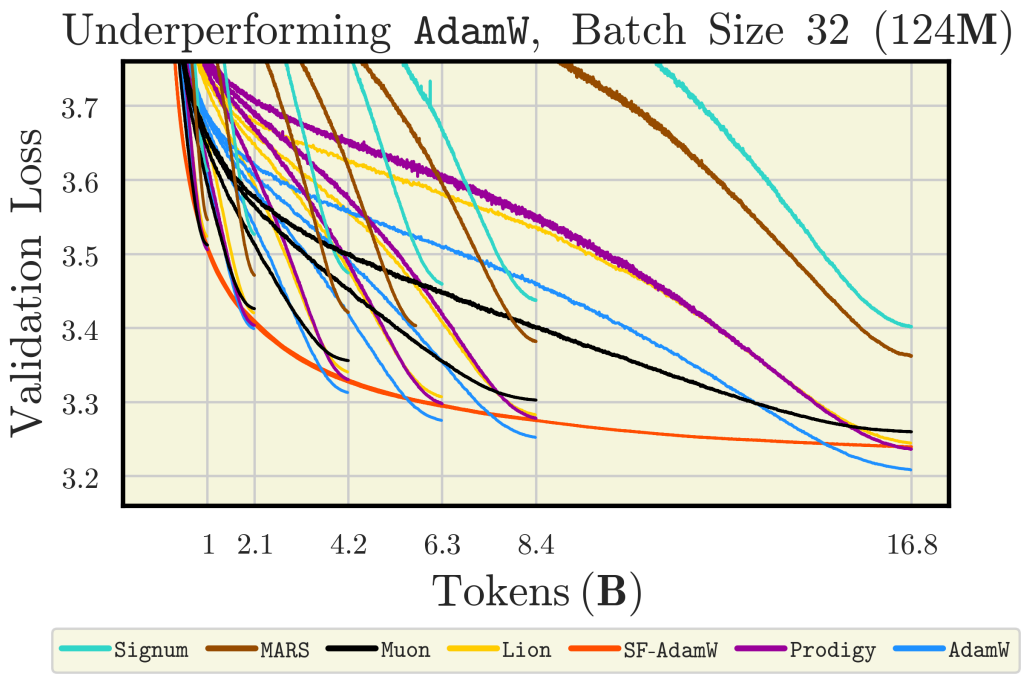


Figure 11: **Ranking of optimizers in the small-batch setting.** In the small-batch setting AdamW outperforms most of the optimizaers we study.

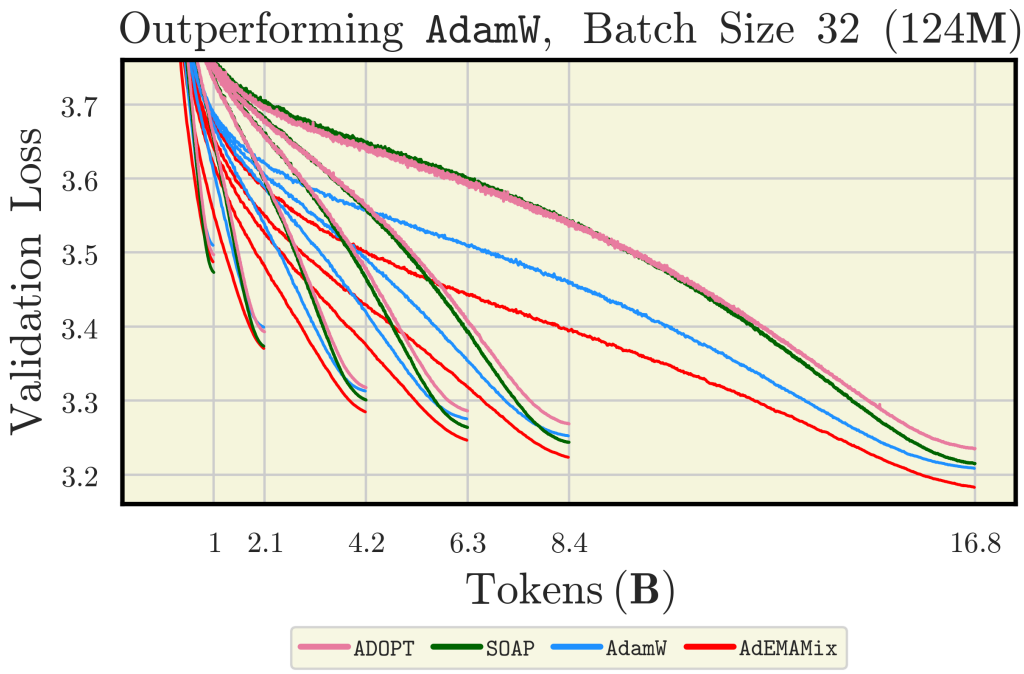


Figure 12: **Ranking of optimizers in the small-batch setting.** Here only AdEMAMix, SOAP and ADOPT show a remarkable performance compared to AdamW.

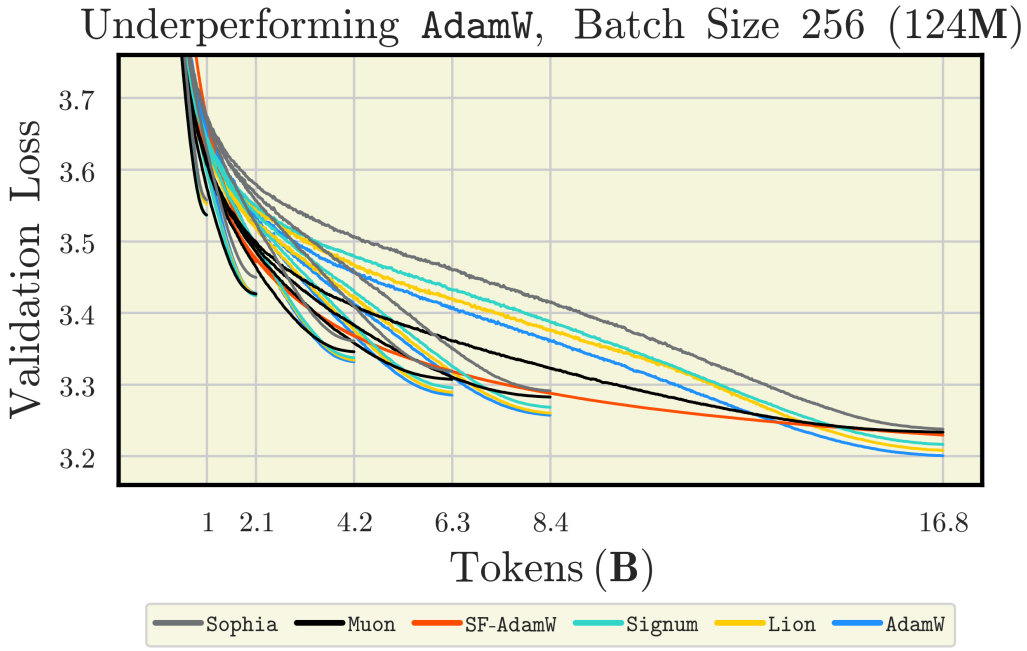


Figure 13: **Ranking of optimizers in the large-batch setting.**

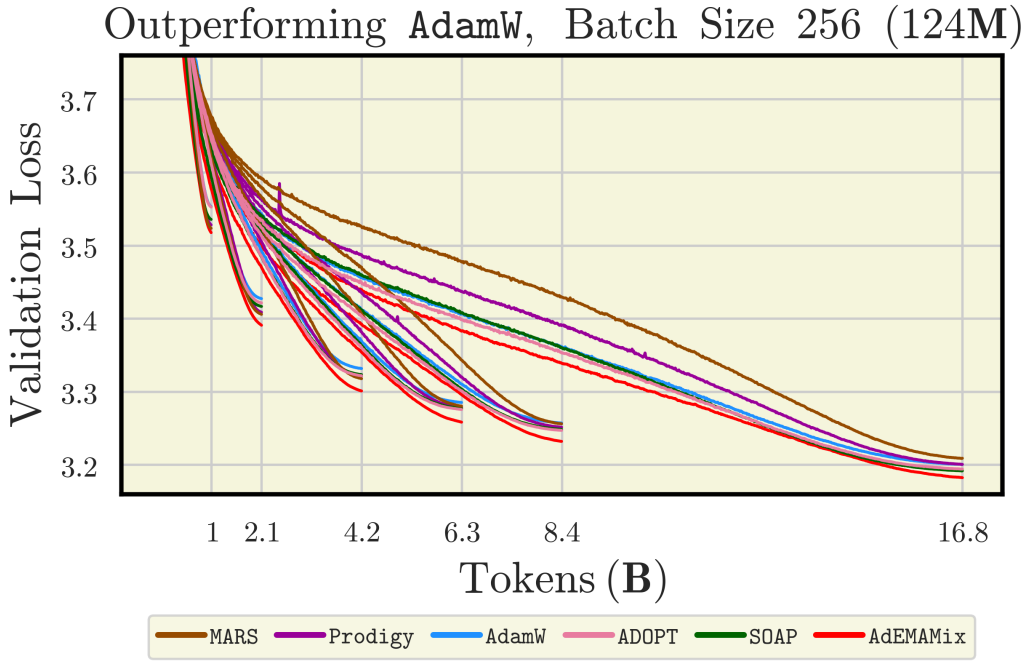
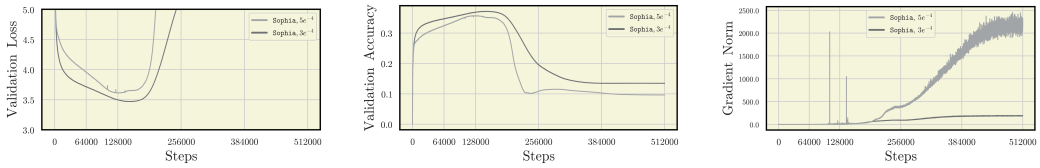
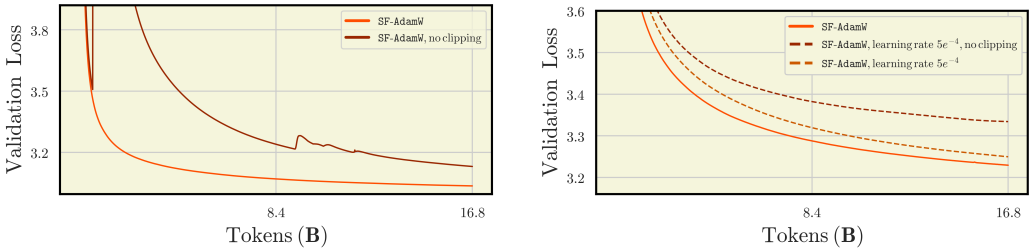


Figure 14: **Ranking of optimizers in the large-batch setting.**



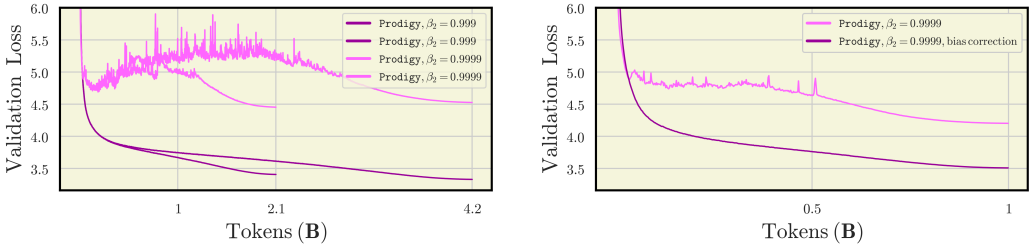
(a) (b) (c)

Figure 15: `Sophia` **diverges in the small-batch setting even with sufficiently small learning rate.**



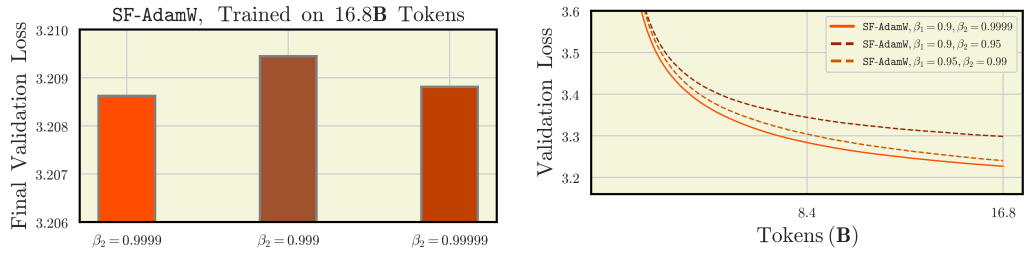
(a) (b)

Figure 16: **Clipping is significant for** `Schedule-Free`.



(a) (b)

Figure 17: `Prodigy` **is sensitive to beta parameters in the small-batch setting**.



(a) (b)

Figure 18: **Impact of beta parameters on** `Schedule-Free`.

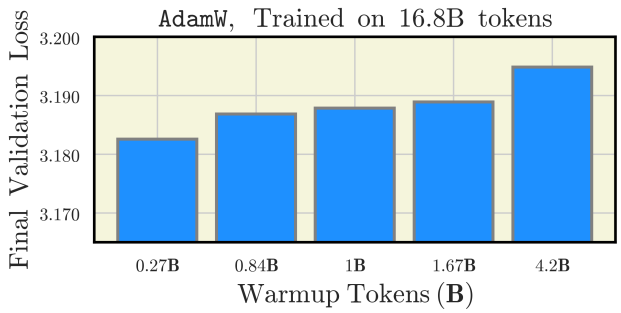


Figure 19: **Warmup sweep for `AdamW`.** We observe that the smaller yet reasonable warmup value is the best, however, this is not true for other methods like `Signum` and `SF-AdamW` (see **??**).

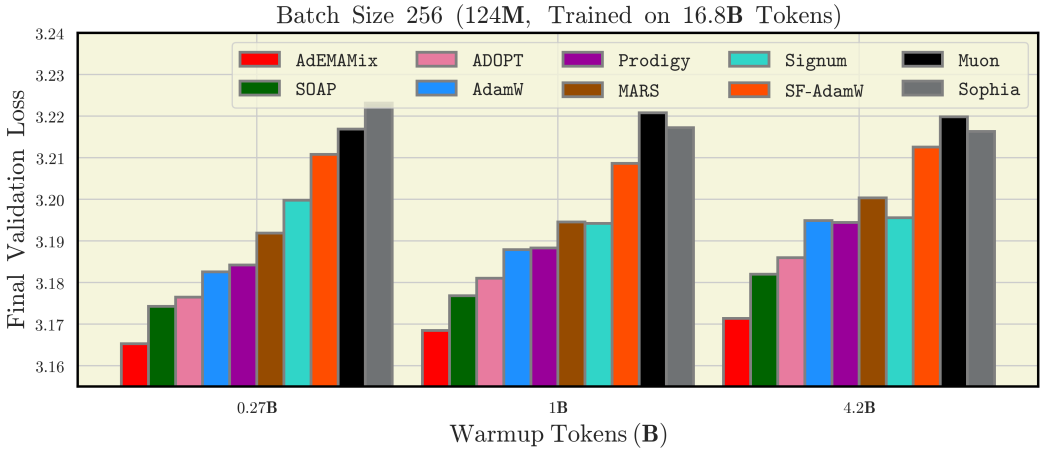


Figure 20: **Warmup ablation.** We report the final validation loss on the FineWeb dataset for 124M model trained on the batch size of 256. We sweep over the batch sizes of $\{1.56\%, 6.25\%, 25\%\}$ of the length of training, which corresponds to $\{2000, 8000, 32000\}$k iterations, respectively.

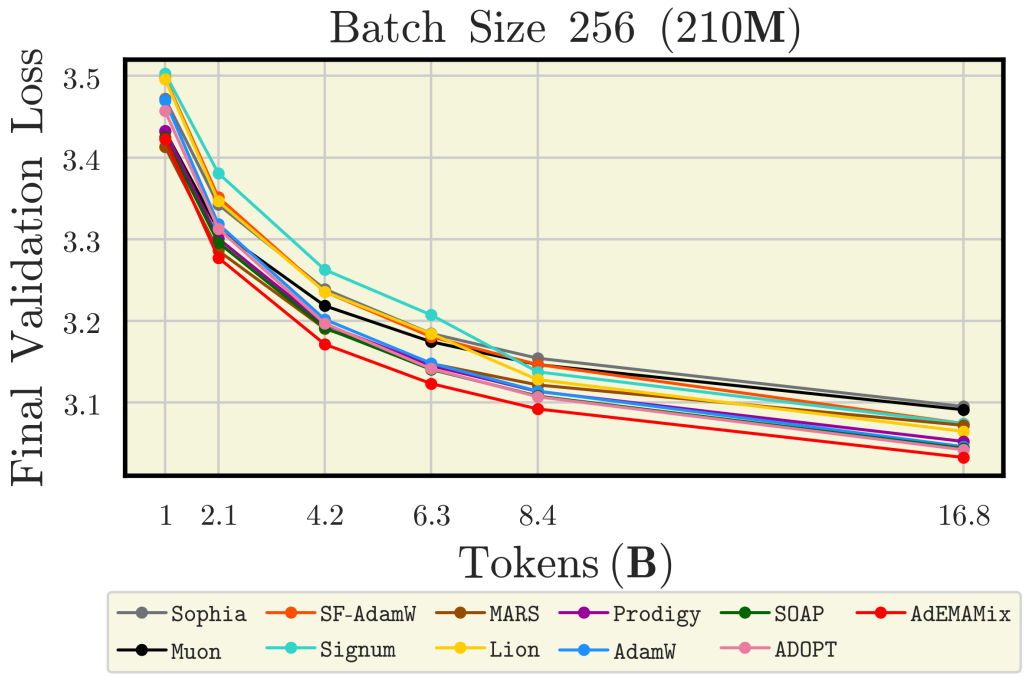


Figure 21: **Ranking of optimizers in the large-batch setting.**

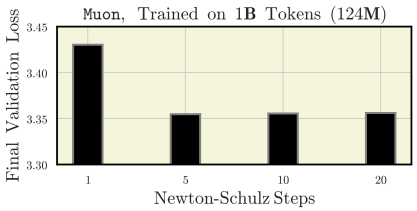


Figure 22: `Muon`**'s dependence on the number of Newton-Schulz iterations.**

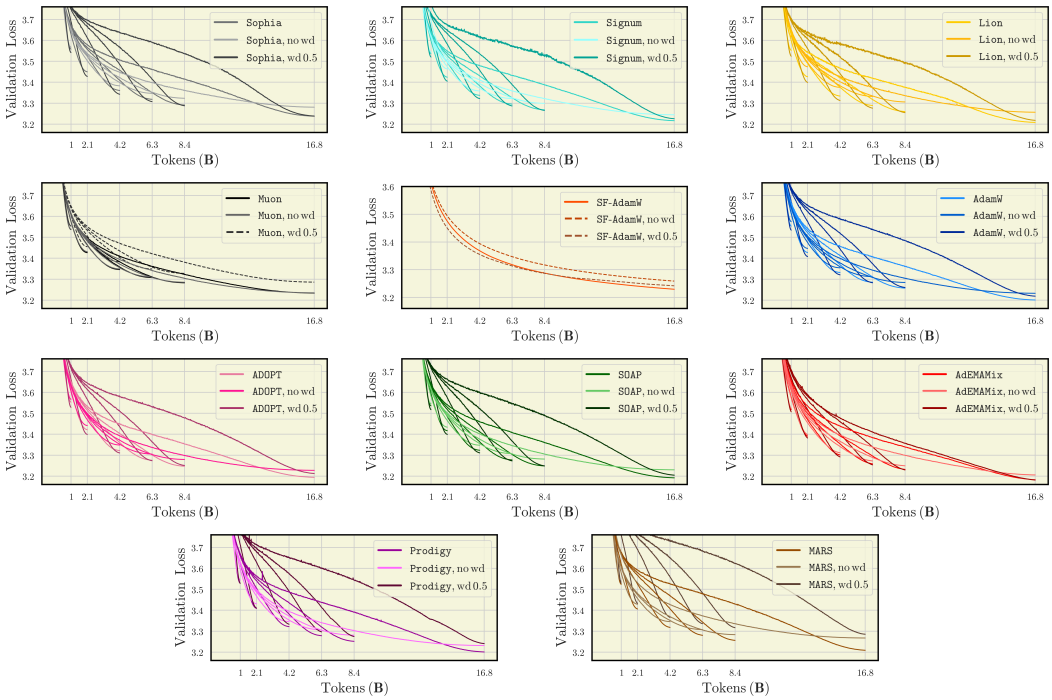


Figure 23: **Larger weight decay achieves significantly better results when training on fewer tokens.** We report the final validation loss on the FineWeb dataset for 124M model trained on the batch size of 256. We observe that the mmajority of runs with the large weight decay of 0.5 consistently outperform the same optimizer with weight decay of 0.1 for all training durations except for the last one. Notably, `Signum` and `Lion` with large weight decay perform even better than `AdamW` with the same learning rate. We also consider a setting without weight decay. We observe that this is suboptimal for most of other optimizers, while the typical weight decay of 0.1 remains the best for large training durations. An interesting thing we observe for optimizers that train one dimensional and two dimensional parameters in a different way — `Muon`, `MARS`. Indeed, the corresponding runs with the weight decay of 0.5 are always worse than then 0.1 baseline and, in some cases, even worse than runs without weight decay. For `Muon`, we connect this effect to its algorithmic design, where weight decay is not used to optimize matrix parameters (see Algorithm 7). For `MARS`, we only vary the weight decay that corresponds to matrix parameters, while keeping 0.1 for all scalar, one dimensional and final layer parameters. In this case, we conclude that the gap between large and small weight decay values narrows significantly faster.

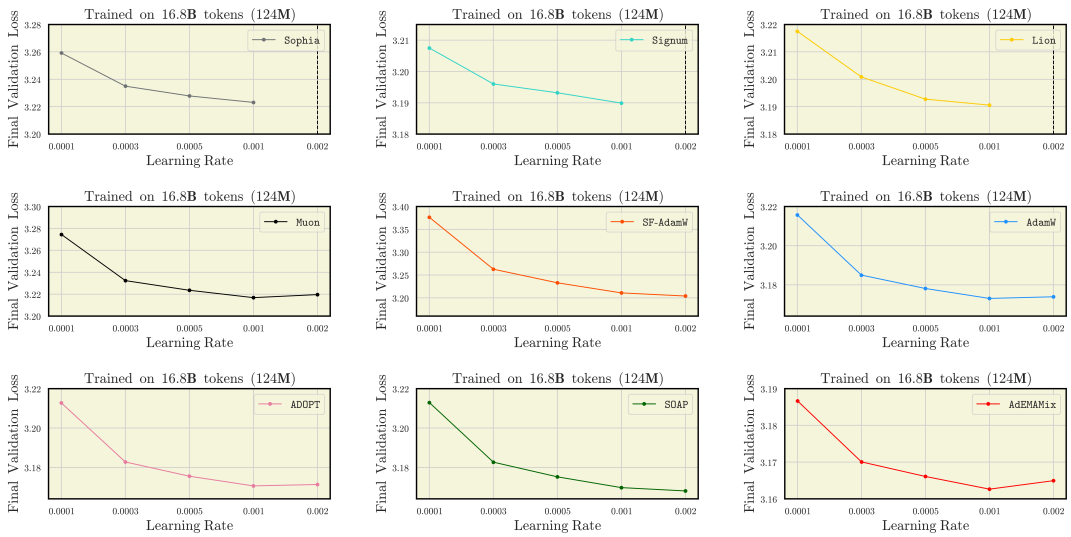


Figure 24: **Learning rate sensitivity.** We report the final validation loss on the FineWeb dataset for 124M model trained on the batch size of 256. In the current setting, only `SOAP` and `SF-AdamW` reach the better performance with the large leraning rate of 0.02. On the other hand, `Sophia` and all sign-based methods (`Signum` and `Lion`) diverge with this value of the learning rate.

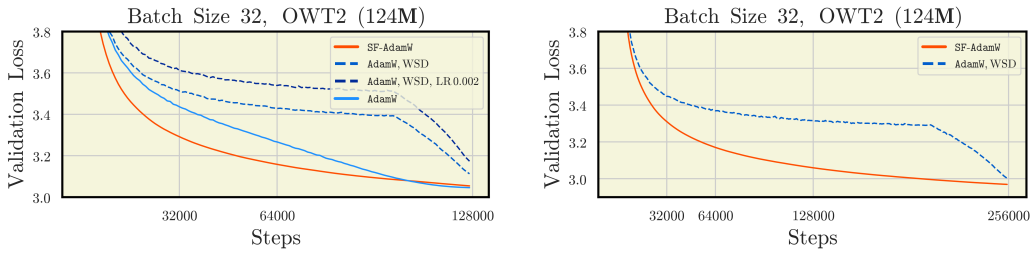


Figure 25: **WSD scheduler underperforms both `AdamW` with cosine scheduler and `SF-AdamW`.** Once the learning rate and beta parameters of `SF-AdamW` and `AdamW` are properly tuned, we observe a surprisingly large gap in performance between WSD scheduler and its competitors. Figure **(b)** suggests that this gap may potentially diminish with extended training. To investigate this further, we conduct a scalable comparison between tuned WSD and cosine baselines across longer training horizons.

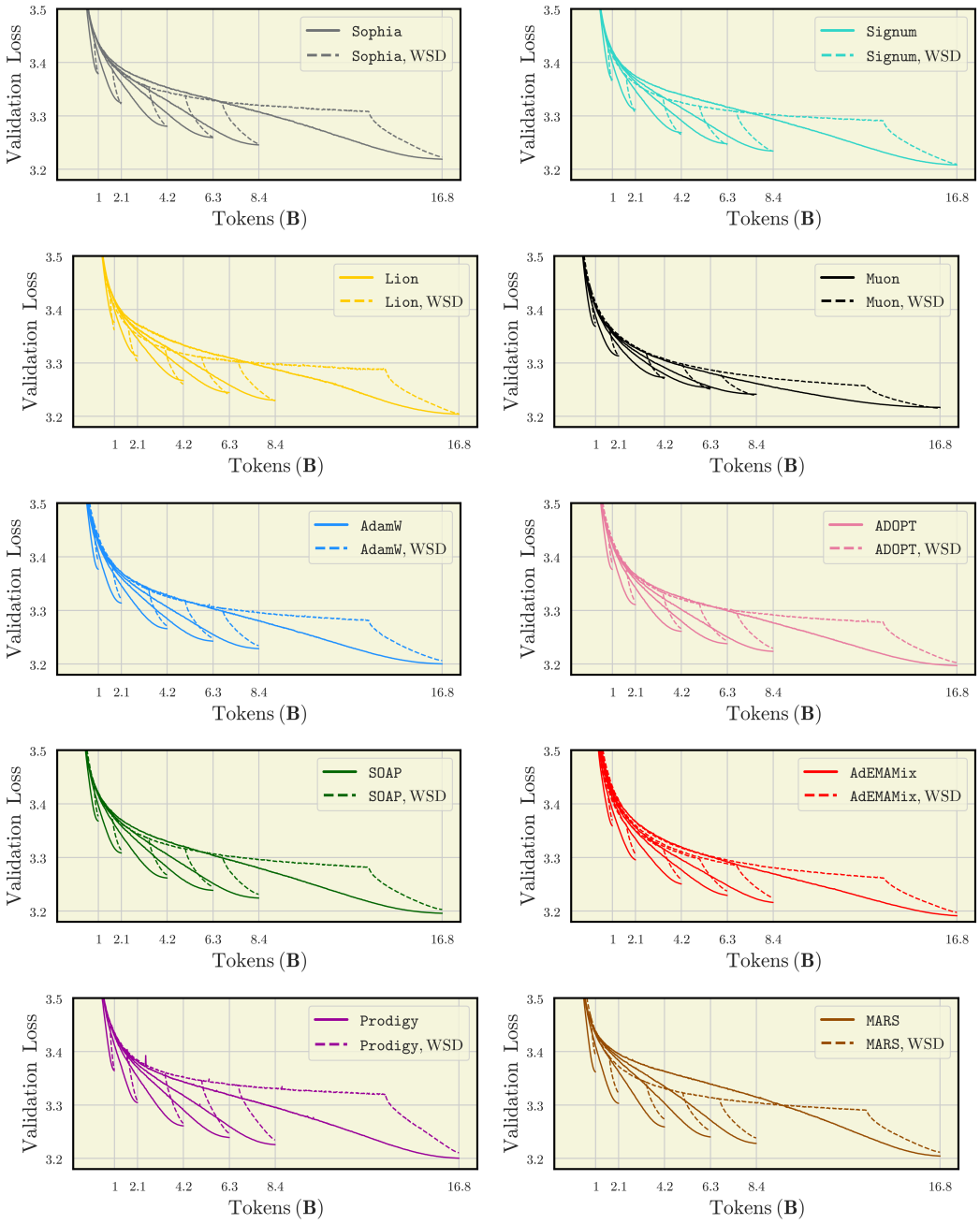


Figure 26: **WSD scheduler underperforms cosine.** We report the final validation loss on the FineWeb dataset for 124M model trained on the batch size of 256. We observe that WSD still can match the performance of cosine on `Sophia`, `Signum`, `Lion`, i.e., on the sign-based methods, and even outperform for `Muon`. Although the gap in performance is not particularly significant, but for benchmarking purposes, we decide to stick to the cosine scheduler because those gap still plays a substantial role in our setup.

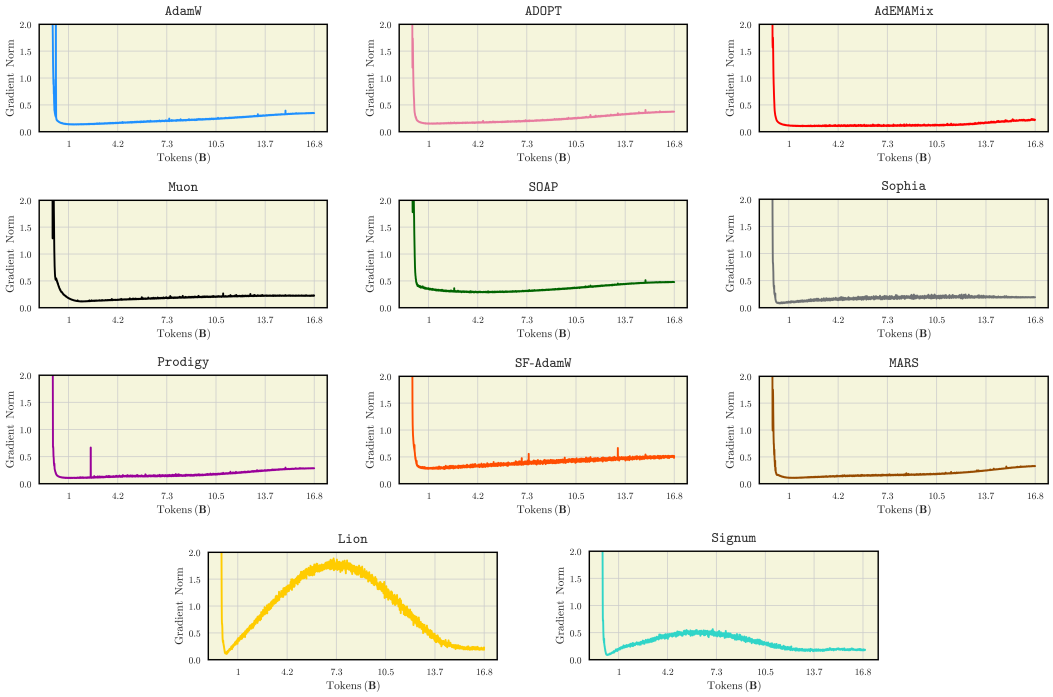


Figure 27: **Gradient Norm patterns for cosine scheduler.**

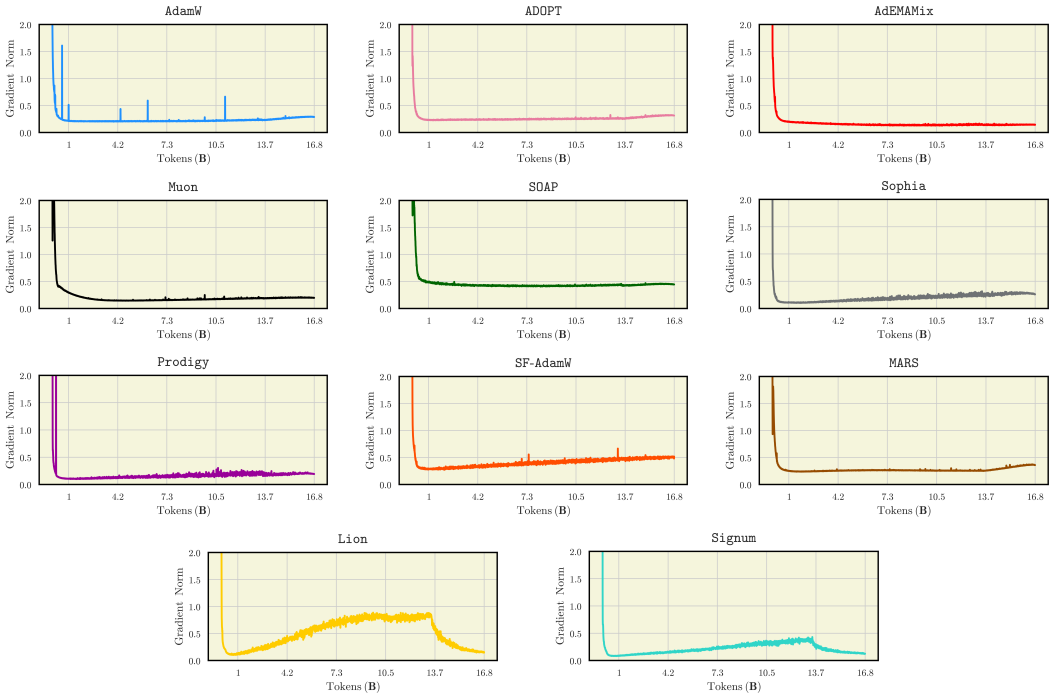


Figure 28: **Gradient Norm patterns for WSD scheduler.**

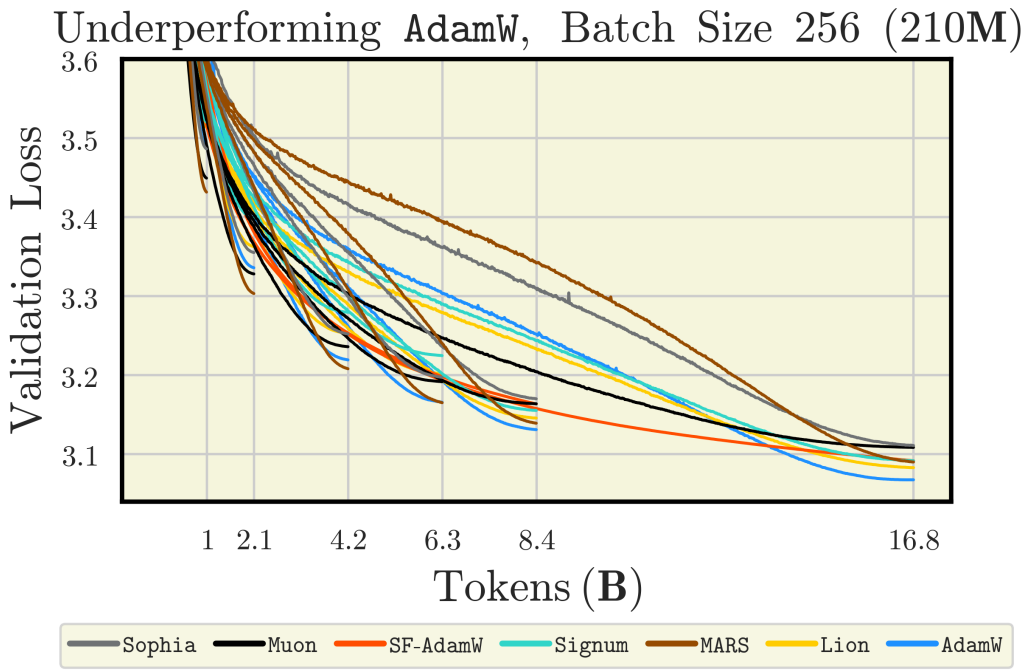


Figure 29: **Ranking of optimizers in the large-batch setting.**

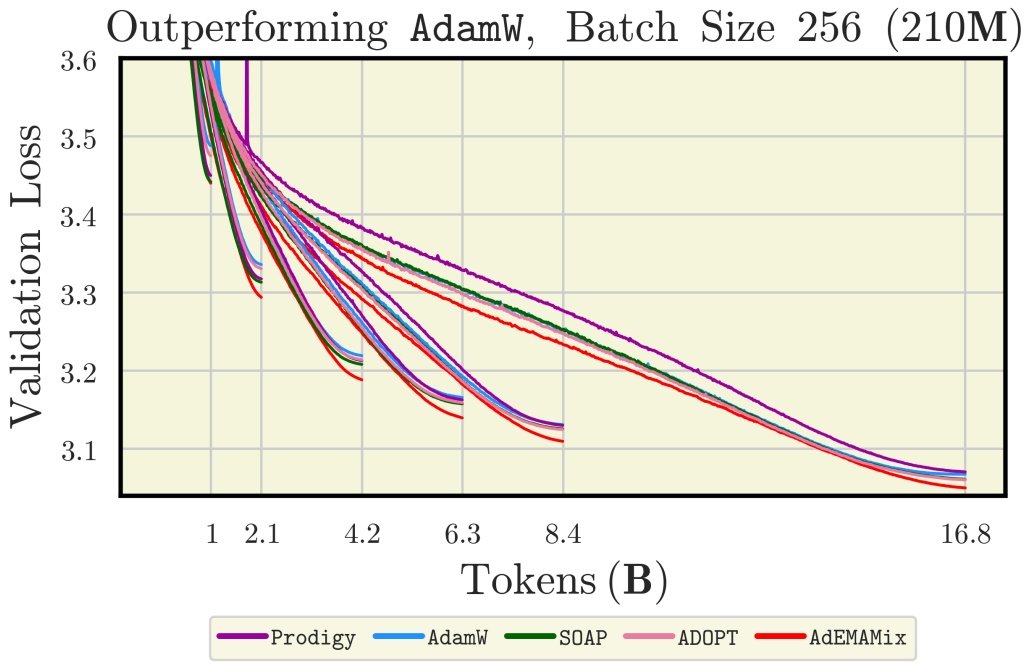


Figure 30: **Ranking of optimizers in the large-batch setting.**