Context-Dependent Manifold Learning in Dynamical Systems: A Neuromodulated Constrained Autoencoder Approach

Jérôme Adriaens Guillaume Drion Pierre Sacré

Neuroengineering Lab, Department of Electrical Engineering and Computer Science, University of Liège

Context-aware methods are required for ROM of systems with context

Objective

Reduce the order of dynamical systems *with* different physical parameters (or context).

Context-independent

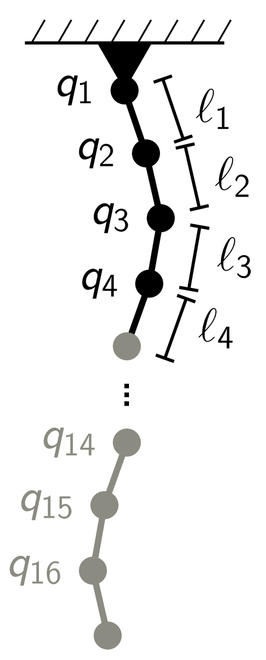
Coupling

$\vec{q}_{5:16} = \vec{f}(\vec{q}_{1:4})$

Examples

$q_9 = 1.5 \cdot \sin(q_2)$

$q_{15} = -2 \cdot q_3^2$



Context-dependent

Coupling:

$\vec{q}_{5:16} = \vec{f}(\vec{q}_{1:4}, \vec{\ell}_{1:4})$

Examples

$q_9 = 2\ell_2 1.5 \cdot \sin(q_2)$

$q_{15} = -2 \cdot q_3^{2 \cdot 2\ell_3}$

Our method combines

- **Constrained autoencoders** (cAE) from Otto et al. 2023 for the projection property when compressing the state, and
- **neuromodulation** mechanisms inspired by biology and used in Vecoven et al. 2020 to allow the adaptation to varying external parameters.

Constrained Autoencoder (cAE)

They comprise **biorthogonal layers** and **smooth activation function pairs**.

Smooth activation functions:

We extend standard activation function, defined as element-wise inverse pairs $\sigma_{\pm}(\cdot, \alpha)$, by changing their fixed parameter α with a context-modulated vector $\vec{\alpha}(\vec{c})$.

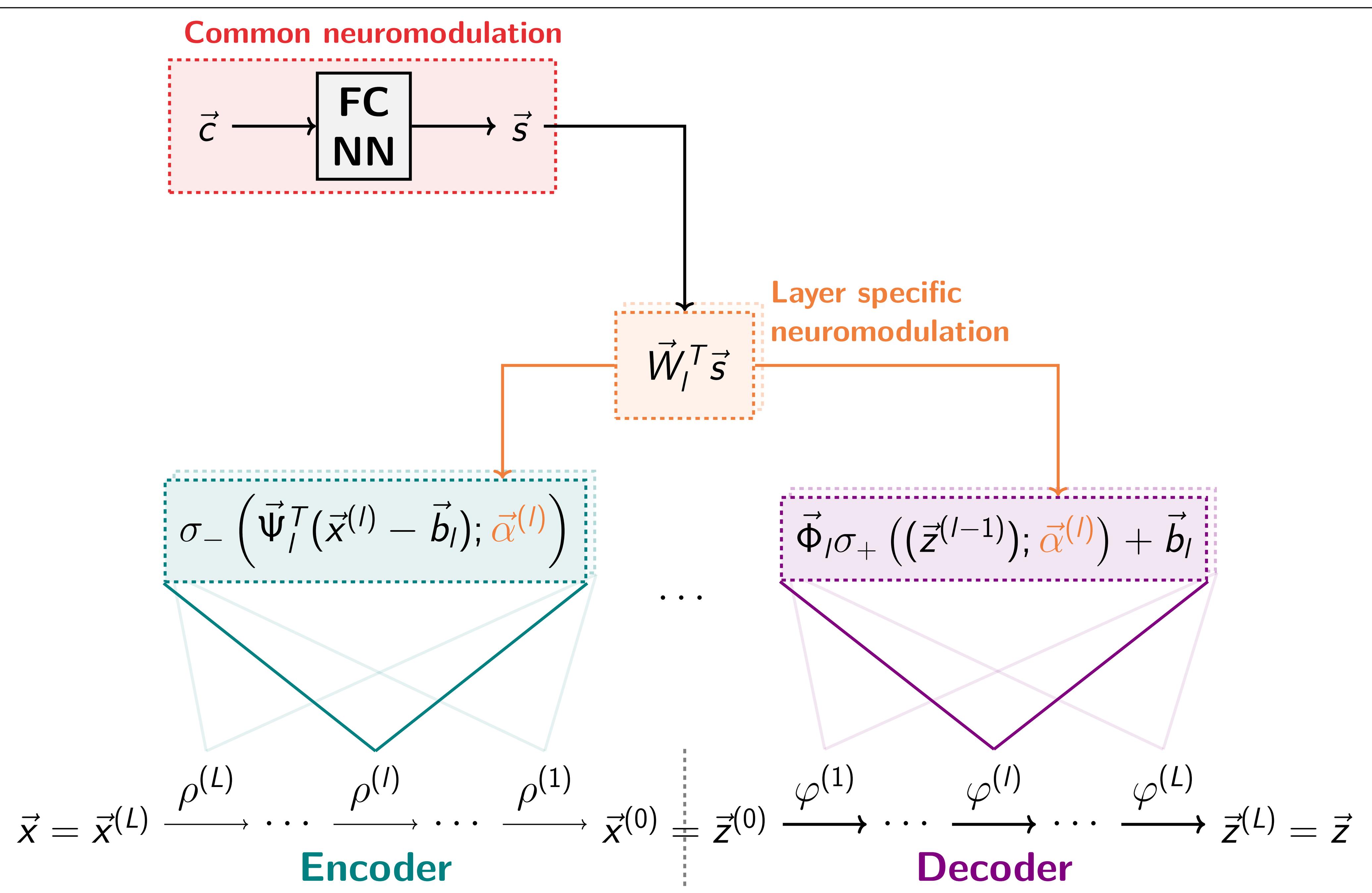
Biorthogonal layer pairs:

We denote the encoder ρ and decoder φ, and they satisfy $\rho \circ \varphi = Id$ such that $P = \varphi \circ \rho$ is a projection. The encoder ρ and decoder φ are composed of L layers :

$$\rho^{(l)}(\vec{x}^{(l)}) = \sigma_{-}\left(\vec{\Psi}_l^T(\vec{x}^{(l)} - \vec{b}_l)\right),$$

$$\varphi^{(l)}(\vec{z}^{(l-1)}) = \vec{\Phi}_l \sigma_{+}\left(\vec{z}^{(l-1)}\right) + \vec{b}_l.$$

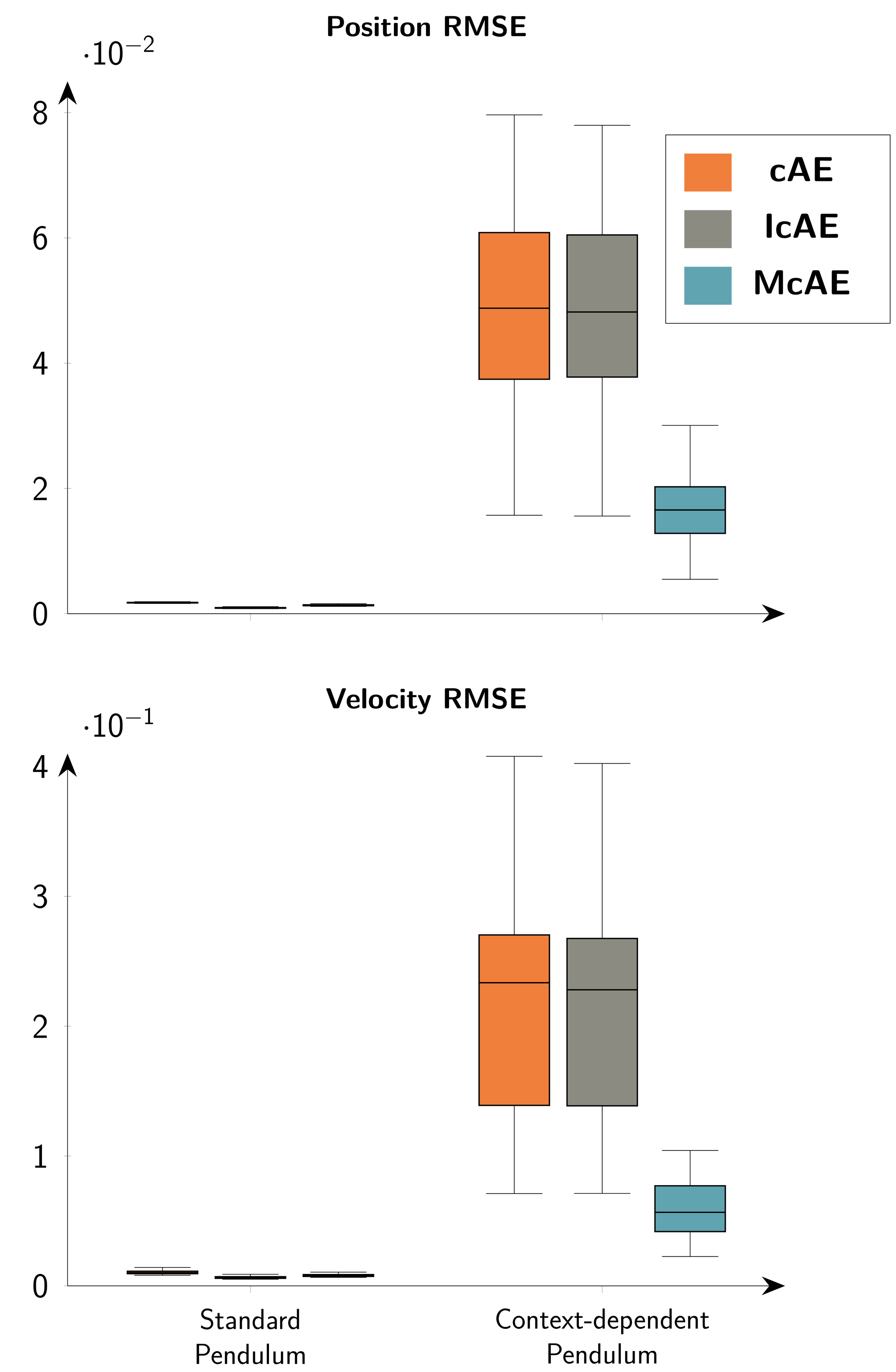
Neuromodulation can be incorporated in cAE



Our neuromodulation process

1. **Common neuromodulation**: A context vector $\vec{c}$ modulates the network by generating a common neuromodulation signal $\vec{s}$.
2. **Layer-specific neuromodulation**: Signal $\vec{s}$ is then multiplied by the layer-specific weight matrix $\vec{W}_l$ to produce the activation function parameters $\vec{\alpha}^{(l)}$.

Neuromod. has greater performance



Conclusions and perspectives

This work introduces a novel approach for context-dependent manifold learning using neuromodulated cAE. Preliminary results demonstrate the potential of our method, particularly in capturing context-dependent relationships in dynamical systems.

Future work

- Refine and rigorously validate our approach.
- Include a deeper analysis of the latent space.
- Extend experiments to other dynamical systems, for example, a 192 DoF rope, the Lorenz 96 model.

jadriaens@uliege.be