PhysDiff-VTON Supplementary Materials

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1 Theoretical Rationale for PRPO

The theoretical justification for the Potential-Regularized Path Optimization (PRPO) can be comprehensively analyzed through its foundational connections to stochastic optimal control, compatibility with probabilistic evolution equations, and consistency in discrete implementations. At its core, PRPO reinterprets the reverse process of the diffusion model through the lens of path integral control theory, where the action functional

$$\mathcal{S}[x(t)] = \mathbb{E}\left[\|s_\theta(x(t),t) - \nabla_x \log q_t(x(t))\|^2 + \lambda E(x) + \sigma\|\xi(t)\|^2\right]$$

encodes a trade-off between score-matching fidelity, domain-specific regularization, and controlled stochastic exploration. Minimizing this action corresponds to selecting the most probable paths under the Onsager-Machlup formalism [2], where the kinetic matching term ensures adherence to the data manifold, the potential term $E(x)$ imposes soft constraints like smoothness or physical consistency, and the noise energy term modulates exploration sensitivity. The derived modified stochastic differential equation (SDE)

$$dx = \left[f - g^2 s_\theta + \lambda g^2 \nabla_x E\right] dt + \sigma g d\bar{w}$$

maintains theoretical consistency through its Fokker-Planck equation [6]

$$\frac{\partial p_t}{\partial t} = -\nabla \cdot \left(\left[f - g^2 s_\theta + \lambda g^2 \nabla_x E\right] p_t\right) + \frac{\sigma^2 g^2}{2} \nabla^2 p_t,$$

where bounded regularization strength λ preserves the contraction properties of the primary drift term, and controlled noise intensity σ satisfies Novikov's condition [1] to maintain measure equivalence between forward and reverse processes. Discretization analysis reveals that the PRPO update step

$$x_{t-1} = \frac{1}{\sqrt{\alpha_t}}\left(x_t - \frac{\beta_t}{\sqrt{1-\bar{\alpha}_t}}\epsilon_\theta(x_t,t)\right) + \lambda\beta_t \nabla_x E(x_t) + \sigma\sqrt{\beta_t} z$$

achieves $O(\beta^{3/2})$ approximation error through Itō-Taylor expansion [4], matching conventional diffusion model discretization accuracy while introducing non-invasive regularization. The potential function $E(x)$ operates under weak interference principles ($\|\lambda\nabla E\| \ll \|s_\theta\|$) and manifold preservation constraints, ensuring adjustments remain proximal to the support of the data distribution. Dynamic noise annealing via $\sigma(t) = \sigma_{\max} e^{-kt}$ implements simulated annealing-inspired exploration, probabilistically converging to global minima while enabling early-stage diversity exploration. Crucially, PRPO's inference-time adaptation paradigm preserves pretrained score networks, unlike energy-based fine-tuning methods, achieving task-specific regularization through deterministic path optimization rather than retraining. This synthesis of variational action minimization with controlled stochastic dynamics provides a rigorous mathematical grounding while maintaining practical flexibility across domains.

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2 From Action Functional to Reverse-time SDE

The derivation of the modified reverse-time dynamics equation from the action functional can be systematically explained through variational optimization within the stochastic calculus framework. Starting with the action functional

$$\mathcal{S}[x(t)] = \int_0^T \|s_\theta(x,t) - \nabla_x \log q_t(x)\|^2 dt + \lambda \int_0^T E(x) dt + \sigma \int_0^T \|\xi(t)\|^2 dt,$$

we parameterize the reverse process dynamics by a stochastic differential equation (SDE) $dx = a(x,t)dt + b(x,t)dw$, where the drift term $a(x,t)$ and diffusion term $b(x,t)$ are optimized to minimize $\mathcal{S}[x(t)]$. The first term in $\mathcal{S}$, enforcing score matching, directly recovers the conventional reverse drift $a(x,t) = f(x,t) - g(t)^2 s_\theta(x,t)$ through the equivalence between score matching loss minimization and drift correction.

Variational analysis of the potential regularization term $\lambda \int E(x)dt$ introduces an additional gradient correction: perturbing the path $x(t) \to x(t) + \delta x(t)$ yields a variation

$$\delta\left(\lambda \int E(x)dt\right) = \lambda \int \nabla_x E(x) \cdot \delta x \, dt,$$

which corresponds to augmenting the drift with $\lambda g(t)^2 \nabla_x E(x)$, scaled by the noise coefficient $g(t)^2$ from the original diffusion process. Simultaneously, the stochastic control term $\sigma \int \|\xi(t)\|^2 dt$ regulates noise energy via optimal control theory, leading to a diffusion term adjustment $b(x,t) = \sigma g(t)$ that preserves the Wiener process structure while modulating exploration intensity.

Combining these contributions, the optimized SDE becomes,

$$dx = \left[f(x,t) - g(t)^2 s_\theta(x,t) + \lambda g(t)^2 \nabla_x E(x)\right] dt + \sigma g(t) d\bar{w}.$$

The compatibility of this modified dynamics with the target distribution $q_0(x)$ is verified through its Fokker-Planck equation [6],

$$\frac{\partial p_t}{\partial t} = -\nabla_x \cdot \left(\left[f - g^2 s_\theta + \lambda g^2 \nabla_x E\right] p_t\right) + \frac{\sigma^2 g^2}{2} \nabla_x^2 p_t,$$

where the original reverse process is recovered when $\lambda = 0$ and $\sigma = 1$. Theoretical consistency requires bounded regularization strength λ to avoid destabilizing the primary drift term and adherence to Girsanov's theorem [1] for noise intensity $\sigma g(t)$. This derivation rigorously unifies score matching, potential-guided regularization, and controlled stochasticity within a single variational framework, establishing the mathematical foundation for the path optimization mechanism of PRPO.

3 From Reverse-time SDE to PRPO Sampling

In a variance-preserving forward diffusion governed by the SDE

$$dx = -\tfrac{1}{2}\beta(t)\, x \, dt + \sqrt{\beta(t)}\, dw,$$

we discretize with $\Delta t = 1$ according to the DDPM [3] parametrization $\beta_t = \beta(t)$, $\alpha_t = 1 - \beta_t$, and $\bar{\alpha}_t = \prod_{s=1}^{t} \alpha_s$. The corresponding reverse-time SDE takes the form

$$dx = \left[-\tfrac{1}{2}\beta(t)x - \beta(t)s_\theta(x,t)\right] dt + \sqrt{\beta(t)}\, d\bar{w},$$

where $s_\theta(x,t) = -\epsilon_\theta(x,t)/\sqrt{1-\bar{\alpha}_t}$. Introducing an energy-gradient correction and a controllable noise term yields the modified dynamics

$$dx = \underbrace{\left[-\tfrac{1}{2}\beta(t)x - \beta(t)s_\theta(x,t)\right] dt}_{\text{standard reverse drift}} + \underbrace{\lambda\, \beta(t)\, \nabla_x E(x)\, dt}_{\text{energy correction}} + \underbrace{\sigma \sqrt{\beta(t)}\, d\bar{w}}_{\text{controllable noise}}.$$

Applying the Euler–Maruyama scheme [5] with $\Delta t = 1$ to each term gives for the step $t \to t-1$ the updates

$$-\tfrac{1}{2}\beta_t x_t - \beta_t s_\theta(x_t,t) = -\tfrac{\beta_t}{2}x_t + \tfrac{\beta_t}{\sqrt{1-\bar\alpha_t}}\epsilon_\theta(x_t,t) \quad \Longrightarrow \quad \frac{1}{\sqrt{\alpha_t}}\big(x_t - \tfrac{\beta_t}{\sqrt{1-\bar\alpha_t}}\epsilon_\theta(x_t,t)\big),$$
$$\lambda\, \beta_t\, \nabla_x E(x_t),$$

and

$$\sigma\,\sqrt{\beta_t}\, z, \quad z \sim \mathcal{N}(0, I).$$

Thus, one arrives at the PRPO sampling rule

$$x_{t-1} = \frac{1}{\sqrt{\alpha_t}}\Big(x_t - \frac{\beta_t}{\sqrt{1-\bar\alpha_t}}\epsilon_\theta(x_t,t)\Big) + \lambda\,\beta_t\,\nabla_x E(x_t) + \sigma\,\sqrt{\beta_t}\,z.$$

Matching coefficients confirms that the discrete energy-gradient term $\lambda\beta_t\nabla_x E$ and noise coefficient $\sigma\sqrt{\beta_t}$ exactly reflect their continuous-time origins, while the discretization error remains $O(\beta_t^{3/2})$, ensuring numerical stability provided $\lambda\beta_t\|\nabla E\|$ remains small relative to the deterministic update and $\sigma\sqrt{\beta_t}$ decays appropriately.

4 Numerical Stability

In order to ensure numerical stability in the PRPO discrete update

$$x_{t-1} \;=\; \frac{1}{\sqrt{\alpha_t}}\Big(x_t - \frac{\beta_t}{\sqrt{1-\bar\alpha_t}}\epsilon_\theta(x_t,t)\Big) \;+\; \lambda\,\beta_t\,\nabla_x E(x_t) \;+\; \sigma(t)\,\sqrt{\beta_t}\;z,$$

we require two conditions. First, the energy-correction term must remain bounded, which entails

$$\lambda\,\beta_t\,\|\nabla_x E(x_t)\| \;\ll\; \Big\|\frac{\beta_t}{\sqrt{\alpha_t(1-\bar\alpha_t)}}\;\epsilon_\theta(x_t,t)\Big\|.$$

Dividing both sides by β_t (with $\beta_t > 0$) gives

$$\lambda\,\|\nabla_x E(x_t)\| \;\ll\; \frac{\|\epsilon_\theta(x_t,t)\|}{\sqrt{\alpha_t(1-\bar\alpha_t)}}.$$

Under the common score-matching assumption $\|\epsilon_\theta(x_t,t)\| \propto \sqrt{1-\bar\alpha_t}$, this further simplifies to

$$\lambda\,\|\nabla_x E(x_t)\| \;\ll\; \frac{1}{\sqrt{\alpha_t}},$$

so that when $\alpha_t \to 0$ one must choose λ sufficiently small or design $E(x)$ so that $\|\nabla_x E(x_t)\|$ decays naturally.

Second, the noise amplitude must be controllable by designing $\sigma(t)$ to decay over t. If we set

$$\sigma(t) = \sigma_{\max} e^{-kt},$$

then the variance of the noise term $\sigma(t)\sqrt{\beta_t}\,z$ is

$$\mathrm{Var}\big(\sigma(t)\sqrt{\beta_t}\,z\big) = \sigma_{\max}^2 e^{-2kt}\,\beta_t\,I,$$

and requiring $\lim_{t\to 0}\sigma_{\max}^2 e^{-2kt}\,\beta_t = 0$ guarantees that as $t \to 0$ (late in generation) the stochastic perturbation vanishes. For example, if $\beta_t = (\beta_{\max}/T)\,t$ (a linear schedule), then one enforces

$$e^{-2kt}\,\frac{\beta_{\max}\,t}{T} \le \varepsilon$$

by tuning k and $\sigma_{\max}$.

Together, these two requirements,

$$\lambda\,\|\nabla_x E(x_t)\| \ll \frac{1}{\sqrt{\alpha_t}} \quad \text{and} \quad \sigma(t) = \sigma_{\max}e^{-kt}$$

, ensure that the PRPO algorithm remains numerically stable by balancing the deterministic score-matching update against energy-based correction and diminishing noise.

References

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