Geometry matters: Insights from Ollivier Ricci Curvature and Ricci Flow into representational alignment

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Overview

Do neural systems develop similar representations given the same computational problems? [1][2]

> Standard RSA can miss key geometric structure in **representations based on human judgements**.

> We introduce a **geometry-aware framework** for representational alignment.

Key insights ::

> Our framework detects representational geometries

> High behavioral alignment ≠ geometric alignment

Our approach

Geometry-aware framework

> **Ollivier Ricci Curvature** [3] :: discrete curvature

> **Ricci Flow** [4] :: curvature-driven density estimate :: communities ***(Fig.1)***

> Heat diffusion [5] :: weighted graph distance

Neural networks

> VGG-Face

> Human-aligned VGG-Face [6] ***(Fig.2)***

Human data & stimuli [6]

> Stimulus :: 3D heads (n_{IDs} = 100, $n_{wo|men}$ = 50)

> Task :: triplet odd-one-out task ***(Fig.3)***
:: similarity judgements in 2D & 3D

> Human online experiment ($n_{subjects}$ = 2,700)
:: > 300K trials

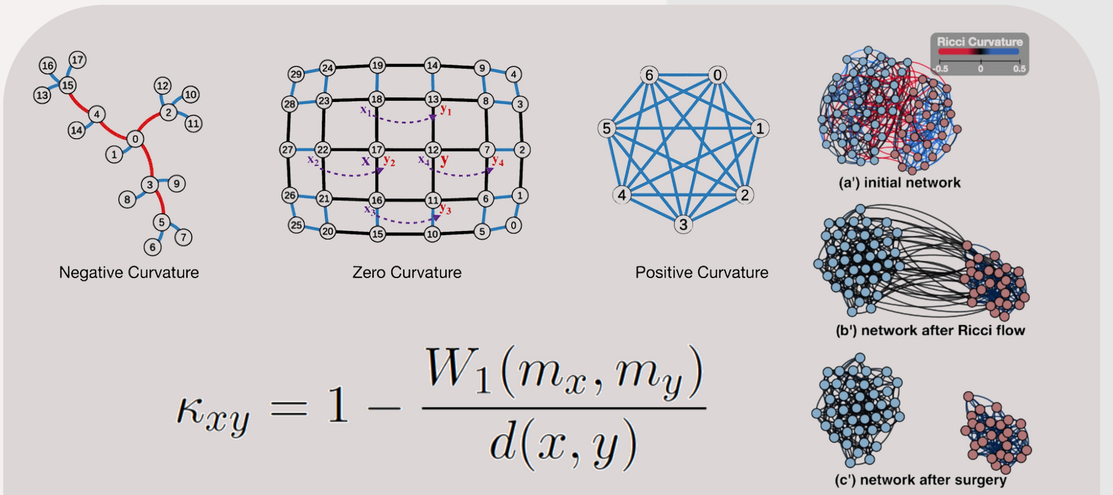

Fig.1 *Discrete case :: **Olliver Ricci curvature** and **Ricci Flow***

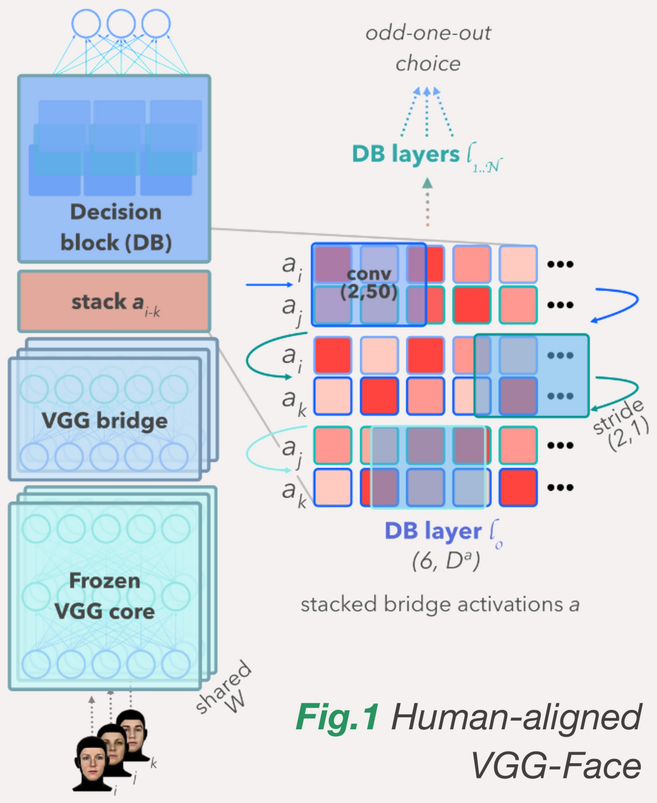

Fig.1 *Human-aligned VGG-Face*

Results

> Human-like geometry emerges in 2D but generalizes weakly to 3D conditions ***(Fig.4)***

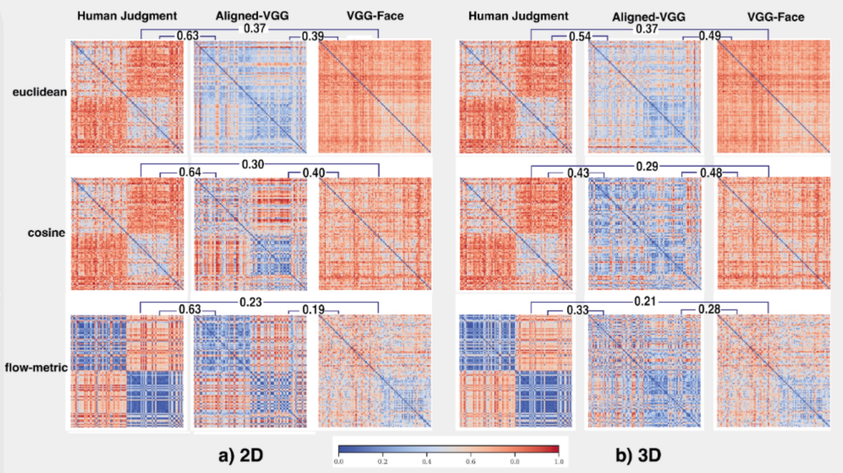

Fig.4 *Cross-metric correlations RDMs*

> Curvature distributions reveal geometric misalignment ***(Tab.1)***

Model	HJ vs. A-VGG	HJ vs. VGG-Face	A-VGG vs. VGG-Face
WS-distance 2D	0.04 ± 0.01	0.12 ± 0.01	0.12 ± 0.01
WS-distance 3D	0.07 ± 0.01	0.13 ± 0.01	0.07 ± 0.01
Heat-distance 2D	1.6 ± 0.04	2.05 ± 0.03	1.98 ± 0.03
Heat-distance 3D	1.69 ± 0.04	2.08 ± 0.04	1.79 ± 0.03

Tab.1 *Representational distances between model configs. (WS: Wasserstein; Heat: Heat diffusion)*

> Community structure reveals divergent alignment ***(Fig.5, Tab.2)***

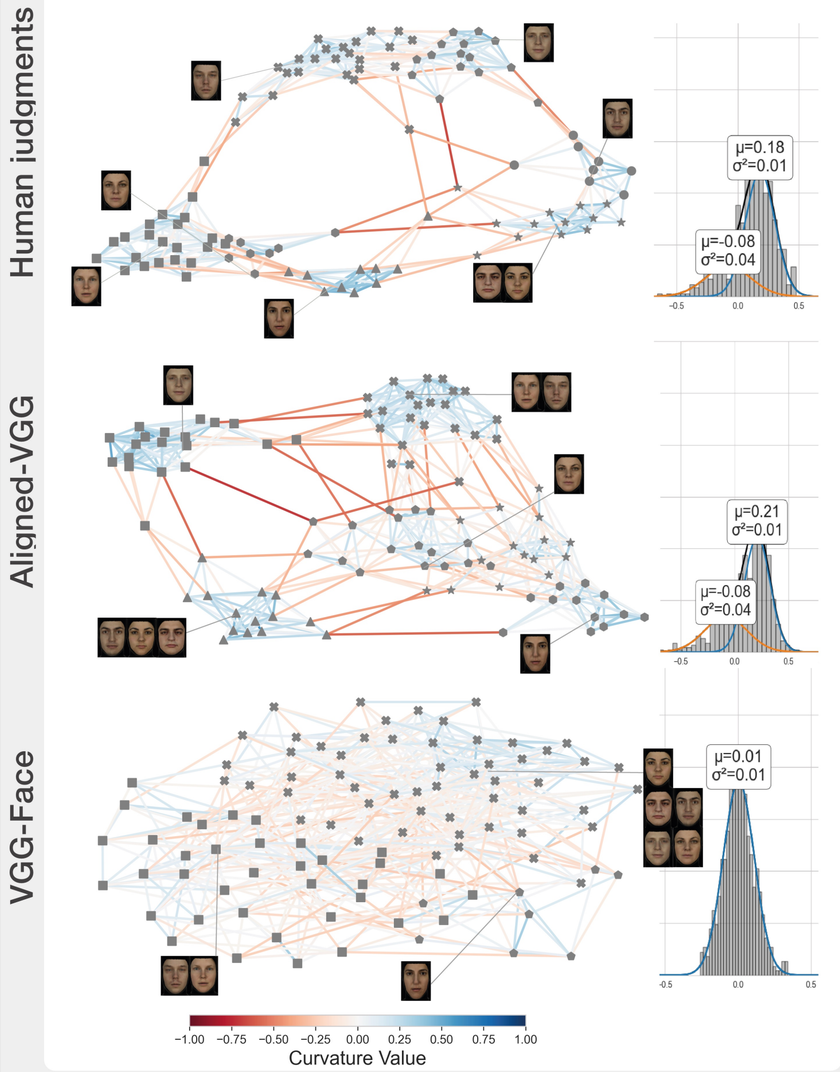

Fig.5 *Ricci flow-based communities (2D condition)*
Right*: Curvature distributions*

Community ::
Conductance *:: community seperation*
IED *:: internal edge density within community*
Modularity *:: community strength*
AE *:: average embeddedness, shared neighbours for pair of nodes within community*

Graph	Communities	Conductance	IED	Modularity	AE
Human Judgment 2D	7	0.23	0.60	34.73	0.78
Human Judgment 3D	8	0.31	0.68	31.54	0.69
Aligned-VGG 2D	6	0.20	0.52	35.98	0.80
Aligned-VGG 3D	6	0.34	0.57	21.57	0.68
VGG-Face	3	0.33	0.32	11.75	0.69

Tab.2 *Comparison of graph structures*

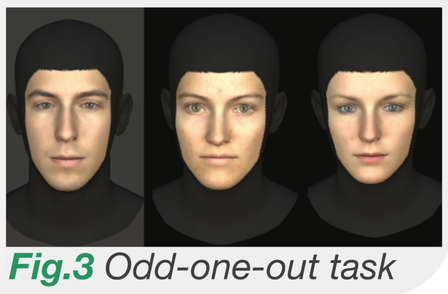

Fig.3 *Odd-one-out task*

References

[1] Sucholutsky et al., *TMLR* (2025)
[2] Muttenthaler et al., *Nat.* (2025)
[3] Ollivier, *C.R.Math.* (2007)
[4] Ni et al., *Sci.Rep.* (2025)
[5] Hammond et al., *IEEE GlobalSIP* (2025)
[6] Hofmann et al., *arXiv* (2025)

Our paper

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