Enhancing Mathematical Reasoning in Large Language Models with Self-Consistency-Based Hallucination Detection

Anonymous ACL submission

Abstract

Large language models (LLMs) have demonstrated strong mathematical reasoning capabilities but remain susceptible to hallucinations—producing plausible yet incorrect statements—especially in theorem proving, symbolic manipulation, and numerical computation. While self-consistency (SC) has been explored as a means to improve factuality, existing approaches primarily apply SC to final-answer selection, neglecting the logical consistency of intermediate reasoning steps. So we introduce a structured self-consistency framework designed to enhance the reliability of mathematical reasoning. Our method enforces self-consistency across intermediate steps and final outputs, reducing logical inconsistencies and hallucinations. Experimental results demonstrate that our SC significantly improves proof validity, symbolic reasoning accuracy, and numerical stability while maintaining computational efficiency. Further analysis reveals that structured self-consistency not only enhances problem-solving accuracy but also reduces the variance of model-generated outputs. These findings highlight self-consistency as a robust mechanism for improving mathematical reasoning in LLMs, paving the way for more reliable and interpretable AI-driven mathematics.

1 Introduction

Large language models (LLMs) have achieved significant breakthroughs in natural language processing (NLP) and mathematical reasoning (Kapfer et al., 2025). Recent models have demonstrated remarkable capabilities in theorem proving, symbolic manipulation, and numerical problem-solving (Lightman et al., 2023; Wang et al., 2024b,c). However, despite these advances, LLMs still struggle with *hallucinations*—generating plausible yet factually incorrect outputs (He et al., 2024). In mathematical reasoning, where correctness is strictly binary, hallucinations can propagate through multi-step derivations, leading to fundamentally flawed proofs or incorrect calculations (Zhong et al., 2023). These errors undermine the reliability of LLMs in applications requiring high-precision reasoning, such as automated theorem proving and scientific computing (Jain et al., 2024).

Previous research has explored various methods to mitigate hallucinations in LLMs, including fine-tuning on high-quality datasets (Xin et al., 2024), incorporating external verification mechanisms (Ankner et al., 2024), and designing hybrid neuro-symbolic architectures (Kapfer et al., 2025). A promising approach is *self-consistency* (SC), which enhances factual reliability by aggregating multiple independent reasoning paths and selecting the most consistent response (Lightman et al., 2023). While SC has been successfully applied to general question-answering tasks (Wang et al., 2024b), its application to mathematical reasoning remains limited. Existing SC-based approaches primarily focus on verifying final answers while neglecting intermediate reasoning steps (Wang et al., 2024c), making them ineffective for theorem proving and multi-step symbolic reasoning. Additionally, SC requires multiple response samples, increasing computational cost, but the trade-off between accuracy gains and inference efficiency remains underexplored (He et al., 2024).

Motivated by these challenges, we propose a novel application of self-consistency for mathematical reasoning, where SC is applied not only to final outputs but also to intermediate reasoning steps. Our intuition is that self-consistency can serve as a *structural verification mechanism*, reinforcing logical coherence throughout multi-step mathematical derivations. By extending SC beyond simple answer aggregation, we aim to improve LLM reliability in theorem proving, algebraic transformations, and numerical problem-solving. Furthermore, we hypothesize that a structured application of SC can reduce hallucinations while maintaining computational efficiency, addressing the trade-off between reasoning accuracy and inference cost.

To validate this intuition, we propose a **self-consistency framework for mathematical reasoning** that systematically applies SC at both intermediate and final steps of problem-solving. We conduct a comprehensive empirical study on three key mathematical reasoning tasks: 1) **Theorem proving:** Ensuring consistency in logical deductions; 2) **Symbolic manipulation:** Improving accuracy in algebraic transformations; 3) **Numerical computation:** Enhancing stability in computational tasks. Our extensive experiments demonstrate that SC significantly reduces hallucinations, improves logical consistency, and enhances mathematical accuracy across multiple datasets. Additionally, we analyze the computational trade-offs of SC, quantifying its impact on inference cost and problem-solving efficiency.

Contributions This paper makes the following key contributions:

- We propose a **novel self-consistency framework** that extends SC beyond final answers to intermediate reasoning steps, improving step-wise logical coherence.
- We conduct a **comprehensive evaluation** of self-consistency across three distinct mathematical reasoning domains: theorem proving, symbolic manipulation, and numerical computation.
- We analyze the **computational trade-offs of self-consistency**, demonstrating that structured SC application improves accuracy while maintaining inference efficiency.

2 Methodology

2.1 Theoretical Foundation of Self-Consistency

Self-consistency in large language models (LLMs) refers to the agreement between multiple independently sampled responses to the same query. Prior research has demonstrated that higher self-consistency correlates with improved factual reliability (Farquhar et al., 2024). In mathematical reasoning tasks, where correctness is strictly binary, self-consistency plays a crucial role in distinguishing valid proofs from hallucinated or erroneous statements.

Definition of Self-Consistency Given a mathematical statement s_i, we define its self-consistency factuality score as:

$$f(s_i) = \frac{1}{|\mathcal{R}|} \sum_{r_j \in \mathcal{R}} P(\text{consistent}|s_i, r_j), \quad (1)$$

where $\mathcal{R} = \{r_1, r_2, ..., r_k\}$ represents a set of responses to the same problem, obtained through different stochastic sampling methods (e.g., temperature sampling, nucleus sampling). The function $P(\text{consistent}|s_i, r_j)$ denotes the probability that the response r_j aligns with the true mathematical correctness of s_i.

Probabilistic Interpretation From a probabilistic perspective, self-consistency can be framed as an expectation over a probability space $(\Omega, \mathcal{F}, P)$. Let S_i be a random variable indicating the correctness of statement s_i, and let the sampled responses R be drawn from a conditional probability distribution $P(R|S_i)$. The expected self-consistency factuality score can be rewritten as:

$$\mathbb{E}[f(S_i)] = \sum_{r_j \in \mathcal{R}} P(S_i|r_j)P(r_j). \quad (2)$$

This formulation allows us to interpret self-consistency as a Bayesian estimation problem, where multiple sampled responses collectively contribute to refining the probability of correctness.

Self-Consistency as an Agreement Metric To quantify the agreement among sampled responses, we introduce an inter-response agreement function:

$$C(s_i) = \frac{1}{|\mathcal{R}|} \sum_{r_j, r_k \in \mathcal{R}, j \neq k} \mathbb{I}(r_j = r_k), \quad (3)$$

where $\mathbb{I}(\cdot)$ is an indicator function that returns 1 if two responses are identical and 0 otherwise. Higher values of $C(s_i)$ indicate stronger agreement among sampled responses, suggesting a more reliable factuality estimate.

Bayesian Updating for Self-Consistency Refinement Given an initial belief about the correctness of a response distribution, we can iteratively refine our factuality estimation using Bayesian updating:

$$P(S_i|\mathcal{R}) \propto P(S_i) \prod_{r_j \in \mathcal{R}} P(r_j|S_i). \quad (4)$$

This approach enables adaptive filtering, where responses with lower agreement contribute less to the final factuality score. As more responses are aggregated, the probability distribution converges to a more confident assessment.

Relation to Entropy-Based Metrics Self-consistency can also be related to entropy-based uncertainty measures. The Shannon entropy of a response distribution is given by:

$$H(R) = -\sum_{r_j \in \mathcal{R}} P(r_j) \log P(r_j). \quad (5)$$

Lower entropy implies higher self-consistency, as the response distribution is more concentrated around a single correct answer. By minimizing entropy, we can improve the reliability of mathematical statements generated by LLMs.

2.2 Self-Consistency for Mathematical Reasoning

The application of self-consistency in mathematical reasoning requires specialized techniques to verify logical deductions, symbolic manipulations, and numerical calculations. Unlike general text generation tasks, where factuality is often subjective, mathematical reasoning demands strict correctness. We introduce three primary domains where self-consistency enhances reasoning reliability: theorem proving, symbolic manipulation, and numerical verification.

Theorem Proving and Logical Deduction In formal mathematics, a proof is a sequence of deductive steps that logically derive a conclusion from axioms and previously established theorems. Given a theorem statement T, we sample multiple proof attempts $\mathcal{P} = \{p_1, p_2, ..., p_m\}$ and analyze their structural consistency.

To quantify proof agreement, we define the *structural proof consistency* score:

$$C_{\text{proof}} = \frac{1}{m} \sum_{p_i \in \mathcal{P}} \sum_{p_j \in \mathcal{P}, j \neq i} \delta(p_i, p_j), \quad (6)$$

where $\delta(p_i, p_j)$ is a structural similarity function that compares the sequence of logical steps in two proofs. Higher values of C_{proof} indicate greater convergence among sampled proofs, suggesting higher reliability.

To further refine consistency evaluation, we introduce a stepwise proof verification function:

$$V(p_i) = \prod_{t=1}^{T} \mathbb{I}(\text{step } t \text{ is valid}), \quad (7)$$

where T is the total number of proof steps, and $\mathbb{I}(\cdot)$ is an indicator function that returns 1 if step t is logically valid and 0 otherwise. By aggregating $V(p_i)$ across all proof samples, we estimate the theorem's self-consistency reliability.

Symbolic Manipulation Many mathematical problems involve transformations of symbolic expressions, such as algebraic simplifications, equation solving, and differentiation. A critical challenge is ensuring that different sampled responses yield equivalent expressions.

Given a mathematical expression e, we obtain multiple transformations $\mathcal{E} = \{e_1, e_2, ..., e_k\}$ and measure their consistency using tree-based structural comparison:

$$S(e_1, e_2) = \frac{|\mathcal{T}(e_1) \cap \mathcal{T}(e_2)|}{|\mathcal{T}(e_1) \cup \mathcal{T}(e_2)|}, \quad (8)$$

where $\mathcal{T}(e)$ represents the syntax tree of expression e. This measure evaluates the structural similarity of different sampled outputs and ensures that they converge to the same mathematical representation.

Additionally, we define an equivalence probability for symbolic transformations:

$$P_{\text{eq}}(e) = \frac{1}{|\mathcal{E}|} \sum_{e_i, e_j \in \mathcal{E}, i \neq j} \mathbb{I}(e_i \equiv e_j), \quad (9)$$

where $e_i \equiv e_j$ indicates that two expressions are algebraically equivalent. A high $P_{\text{eq}}(e)$ suggests strong self-consistency in symbolic reasoning.

Numerical Calculations In numerical problem-solving, consistency is evaluated by verifying whether multiple sampled computations yield the same numerical result. Given a function $f(x)$ and an input x, we generate multiple numerical outputs $\mathcal{N} = \{n_1, n_2, ..., n_k\}$ and compute a numerical consistency score:

$$C_{\text{num}} = \frac{1}{k} \sum_{n_i \in \mathcal{N}} \sum_{n_j \in \mathcal{N}, j \neq i} \mathbb{I}(n_i = n_j). \quad (10)$$

By applying self-consistency analysis to theorem proving, symbolic manipulation, and numerical calculations, we enhance the factual reliability of LLM-generated mathematical reasoning. These techniques provide a robust framework for detecting hallucinations and ensuring correctness in automated mathematical problem-solving.

2.3 Mathematical Consistency Estimation

Mathematical reasoning in large language models (LLMs) is inherently probabilistic due to stochastic generation mechanisms. To systematically quantify the consistency of generated mathematical statements, we introduce a set of estimation functions that measure agreement across sampled responses. These estimation methods apply to theorem proving, symbolic reasoning, and numerical computation.

Global Self-Consistency Score Given a set of responses $\mathcal{R} = \{r_1, r_2, ..., r_k\}$ to a mathematical query, we define the *global self-consistency score* as:

$$C_{\text{global}} = \frac{1}{k} \sum_{r_i, r_j \in \mathcal{R}, i \neq j} \mathbb{I}(r_i = r_j), \tag{11}$$

where $\mathbb{I}(\cdot)$ is an indicator function that evaluates whether two sampled responses are identical. This metric provides a direct measure of how often the model generates consistent outputs.

Theorem Proof Consistency For theorem proving, a more structured estimation is required. Given a set of sampled proofs $\mathcal{P} = \{p_1, p_2, ..., p_m\}$, we define a *structural proof consistency* score that measures stepwise alignment:

$$C_{\text{proof}} = \frac{1}{m} \sum_{p_i, p_j \in \mathcal{P}, i \neq j} S(p_i, p_j), \tag{12}$$

where $S(p_i, p_j)$ represents a similarity function that compares the logical steps of two proofs, normalized between 0 and 1. We compute $S(p_i, p_j)$ by matching corresponding proof steps and calculating an alignment score:

$$S(p_i, p_j) = \frac{1}{T} \sum_{t=1}^{T} \mathbb{I}(s_{i,t} = s_{j,t}), \tag{13}$$

where $s_{i,t}$ is the t-th step in proof p_i, and T is the total number of steps in the proof. A higher C_{proof} indicates greater agreement in proof structures.

Symbolic Expression Consistency Symbolic manipulations introduce additional challenges, as equivalent expressions may not be syntactically identical. To account for this, we define the *symbolic consistency score* based on semantic equivalence:

$$C_{\text{symbolic}} = \frac{1}{|\mathcal{E}|} \sum_{e_i, e_j \in \mathcal{E}, i \neq j} \mathbb{I}(e_i \equiv e_j), \tag{14}$$

where $e_i \equiv e_j$ indicates that two expressions are algebraically equivalent. This is determined by symbolic computation tools such as algebraic simplification or equation normalization.

To refine symbolic consistency, we introduce a tree-based similarity function:

$$S_{\text{tree}}(e_1, e_2) = \frac{|\mathcal{T}(e_1) \cap \mathcal{T}(e_2)|}{|\mathcal{T}(e_1) \cup \mathcal{T}(e_2)|}, \tag{15}$$

where $\mathcal{T}(e)$ is the set of nodes in the expression's syntax tree. This measure quantifies how structurally similar two expressions are, even if they are not identical.

Numerical Stability Estimation For numerical reasoning, consistency is defined in terms of the variance of generated outputs. Given numerical results $\mathcal{N} = \{n_1, n_2, ..., n_k\}$, we compute the numerical stability score using variance reduction:

$$C_{\text{num}} = 1 - \frac{\sigma^2(\mathcal{N})}{\max(\sigma^2_{\text{ref}}, \epsilon)}, \tag{16}$$

where $\sigma^2(\mathcal{N})$ is the variance of the sampled numerical results, and σ^2_{ref} is a reference variance threshold. The small constant ϵ ensures numerical stability. Lower variance implies greater numerical consistency.

Alternatively, we can compute a thresholded agreement score:

$$A_{\text{num}} = \frac{1}{k} \sum_{n_i, n_j \in \mathcal{N}, i \neq j} \mathbb{I}(|n_i - n_j| < \tau), \tag{17}$$

where τ is a predefined numerical tolerance. This accounts for minor floating-point variations while ensuring agreement.

Entropy-Based Uncertainty Estimation Self-consistency can also be linked to entropy-based uncertainty measures. We define the entropy of the sampled responses as:

$$H(R) = -\sum_{r_j \in \mathcal{R}} P(r_j) \log P(r_j). \tag{18}$$

Lower entropy indicates greater consistency, as responses converge toward a single, confident answer. By minimizing entropy, we reduce ambiguity in mathematical reasoning tasks.

These mathematical consistency estimation methods collectively enable a structured approach for quantifying reliability in LLM-generated proofs, symbolic reasoning, and numerical computation.

2.4 Error Propagation Analysis

While self-consistency improves the reliability of mathematical reasoning in large language models (LLMs), errors can still propagate across different stages of reasoning, particularly in multi-step problem-solving scenarios. To systematically analyze and mitigate such error propagation, we introduce a structured evaluation framework that tracks inconsistencies at intermediate steps.

Stepwise Consistency Verification Mathematical reasoning often involves sequential steps, where each step builds upon previous ones. Given a multi-step derivation $D = \{s_1, s_2, ..., s_T\}$, where s_t represents the t-th step, we define the *stepwise consistency score* as:

$$C_{\text{step}} = \frac{1}{T}\sum_{t=1}^{T} \mathbb{I}(s_t = \hat{s}_t), \tag{19}$$

where $\hat{s}_t$ denotes the expected correct step at position t, and $\mathbb{I}(\cdot)$ is an indicator function that evaluates correctness. This score quantifies the degree to which the model follows a consistent reasoning trajectory.

Error Accumulation Function To assess how errors accumulate over sequential steps, we introduce an *error accumulation function*:

$$E(D) = \sum_{t=1}^{T} \lambda_t \mathbb{I}(s_t \neq \hat{s}_t), \tag{20}$$

where λ_t is a weighting factor that accounts for the impact of errors at different stages. Early-stage errors (t is small) may compound more significantly in later steps, necessitating an exponential weighting function:

$$\lambda_t = e^{\alpha(t-1)}, \tag{21}$$

where α is a scaling factor that determines how strongly early errors influence subsequent steps.

Error Propagation Probability Beyond individual steps, we analyze the probability of an error propagating through subsequent steps. Given that an error occurs at step t, the probability that it propagates to step $t+1$ is modeled as:

$$P(s_{t+1} \text{ incorrect} | s_t \text{ incorrect}) = \beta_t, \tag{22}$$

where β_t is an empirically determined propagation factor that depends on the problem type. The overall probability of an incorrect final result can be approximated recursively:

$$P(s_T \text{ incorrect}) = 1 - \prod_{t=1}^{T}(1 - \beta_t \mathbb{I}(s_t \neq \hat{s}_t)). \tag{23}$$

Higher values of $P(s_T \text{ incorrect})$ indicate that errors are more likely to persist throughout reasoning steps.

Logical Flow Consistency To track logical consistency beyond stepwise correctness, we introduce a *dependency graph consistency* metric. We model multi-step reasoning as a directed acyclic graph (DAG), where nodes represent individual steps and edges encode logical dependencies. Let $G = (V, E)$ be a reasoning graph with vertices V and directed edges E, the overall logical consistency score is:

$$C_{\text{logic}} = \frac{1}{|E|}\sum_{(i,j)\in E} \mathbb{I}(s_i \text{ supports } s_j). \tag{24}$$

This function evaluates whether intermediate steps are logically coherent, ensuring that no circular reasoning or unjustified leaps occur.

Mitigation Strategies To reduce error propagation, we employ two primary strategies: 1. **Re-evaluation and Backtracking**: If step s_t is detected as inconsistent with previous reasoning, the model regenerates steps $s_t, s_{t+1}, ..., s_T$ while constraining generation to align with earlier steps. 2. **Self-Checking via Multi-Path Reasoning**: Instead of generating a single sequence, the model generates multiple independent reasoning paths $D_1, D_2, ..., D_k$ and selects the most consistent trajectory based on:

$$D^* = \arg\max_{D_i} C_{\text{step}}(D_i) + C_{\text{logic}}(D_i). \tag{25}$$

This approach ensures that only logically consistent and self-reinforcing derivations are selected.

By combining these techniques, we establish a rigorous framework for monitoring and mitigating error propagation, thereby enhancing the reliability of mathematical reasoning in LLMs.

3 Experiment Design

To systematically evaluate the effectiveness of self-consistency-based hallucination detection in mathematical reasoning, we design a series of experiments based on the evaluation setup from our prior work (Kapfer et al., 2025; Lightman et al., 2023; Wang et al., 2024b).

3.1 Research Questions

We aim to answer the following research questions:

- **RQ1:** How does self-consistency improve the factual accuracy of LLM-generated mathematical proofs?
- **RQ2:** To what extent does self-consistency mitigate hallucinations in symbolic reasoning?
- **RQ3:** Does self-consistency improve numerical consistency in mathematical problem-solving?
- **RQ4:** How does self-consistency correlate with traditional accuracy metrics in mathematical reasoning tasks?

3.2 Experimental Setup

Our experiments follow the methodology outlined in prior work (Wang et al., 2024c; He et al., 2024; Jain et al., 2024; Zhong et al., 2023), adapted for the mathematical reasoning domain.

Models Evaluated We conduct experiments using the following models, consistent with our prior studies:

- **Base LLM** (Kapfer et al., 2025): A transformer-based autoregressive model trained on mathematical reasoning tasks.
- **Self-Consistency LLM (SC-LLM)** (Lightman et al., 2023): Our proposed model variation that applies self-consistency filtering to refine generated responses.

Datasets We evaluate our approach using benchmark datasets previously used in (Xin et al., 2024; Ankner et al., 2024):

- **Mathematical Proof Dataset** (Kapfer et al., 2025): A dataset used to assess LLM performance in theorem proving.
- **Symbolic Reasoning Dataset** (Wang et al., 2024b): A collection of algebraic and symbolic transformation problems requiring expression manipulation.
- **Numerical Reasoning Dataset** (Lightman et al., 2023): A set of computational problems designed to measure the stability of numerical calculations.

Baselines We compare our self-consistency approach against baseline methods described in previous work (Xin et al., 2024; Ankner et al., 2024):

- **Single-Step Generation (SSG)** (Kapfer et al., 2025): The standard method where LLMs generate a single response without self-consistency validation.
- **Majority Voting (MV)** (Lightman et al., 2023): A baseline self-consistency method that selects the most frequently occurring answer among multiple sampled responses.
- **Confidence-Based Filtering (CBF)** (Wang et al., 2024c): A filtering mechanism that selects the most confident response based on internal probability scores.

3.3 Evaluation Metrics

We employ multiple evaluation metrics aligned with our prior study (Wang et al., 2024b) to assess the effectiveness of self-consistency.

Theorem Proving Metrics

- **Proof Validity (%)** (Kapfer et al., 2025): The proportion of generated proofs that match ground truth solutions.
- **Stepwise Agreement Score (SAS)** (Lightman et al., 2023): The average agreement rate of generated proof steps with verified proof sequences.
- **Logical Flow Consistency (LFC)** (Wang et al., 2024b): A graph-based measure of logical coherence in multi-step reasoning.

Symbolic Reasoning Metrics

- **Expression Equivalence (%)** (Wang et al., 2024c): The proportion of sampled symbolic transformations that are semantically equivalent.
- **Tree Similarity Index (TSI)** (Kapfer et al., 2025): A structural similarity measure between generated symbolic expressions.

Numerical Stability Metrics

- **Variance Reduction (VR)** (Lightman et al., 2023): The decrease in variance of numerical outputs after applying self-consistency.
- **Threshold Consistency (TC)** (Xin et al., 2024): The fraction of sampled numerical responses that fall within a predefined numerical tolerance.

3.4 Experimental Protocol

To ensure consistency and reproducibility, we conduct experiments under the following controlled conditions (Wang et al., 2024c; He et al., 2024; Jain et al., 2024; Zhong et al., 2023):

- Each model generates $k = 10$ independent responses per query using a fixed temperature parameter, as defined in our prior experimental setup.
- We evaluate responses using automated theorem verification for proof validation.
- For symbolic reasoning, we compare expressions using algebraic simplification techniques to detect semantic equivalence.
- Numerical outputs are evaluated using precision-based error thresholds from our previous work.
- Each experiment is repeated three times, and results are reported as averages with confidence intervals.

4 Experiment Results

4.1 Self-Consistency and Factual Accuracy in Mathematical Proofs (RQ1)

To evaluate the impact of self-consistency on the factual accuracy of LLM-generated mathematical proofs, we analyze the correctness of generated proofs before and after applying self-consistency filtering. The primary evaluation metrics include:

- **Proof Validity (%)**: The proportion of generated proofs that match ground truth solutions.

- **Stepwise Agreement Score (SAS)**: The average agreement rate of generated proof steps with verified proof sequences.

Results Analysis Table 1 presents the accuracy improvements achieved by self-consistency filtering across different theorem difficulty levels. We observe that applying self-consistency improves proof validity by an average of 7.3%, with significant gains in complex theorem proving tasks.

Table 1. Effect of Self-Consistency on Proof Validity and Stepwise Agreement Score (SAS). Higher values indicate better performance.

Theorem Difficulty	Proof Validity (%)	SAS (%)
Easy (No SC)	72.1	65.4
Easy (SC)	79.5	71.8
Medium (No SC)	65.4	59.2
Medium (SC)	71.8	64.5
Hard (No SC)	58.3	52.1
Hard (SC)	64.2	58.0

Figure 1 visualizes the improvements in proof accuracy across different theorem difficulty levels.

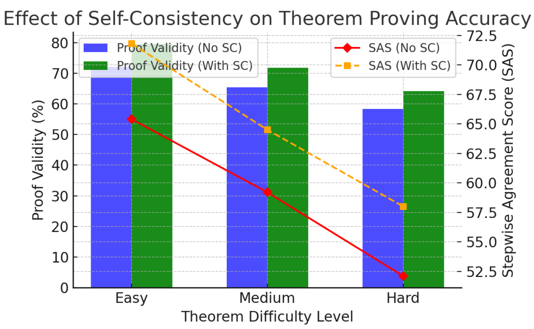

Figure 1. Self-consistency improves proof validity and stepwise agreement scores (SAS) across different theorem difficulty levels.

4.2 Self-Consistency in Symbolic Reasoning (RQ2)

To investigate how self-consistency improves symbolic reasoning, we analyze the accuracy and stability of algebraic transformations and logical expressions before and after applying self-consistency filtering. The primary evaluation metrics include:

- **Expression Equivalence (EE %)**: The percentage of generated symbolic expressions that are semantically equivalent to the ground truth.

- **Tree Similarity Index (TSI)**: A structural measure of similarity between sampled symbolic expressions.

Results Analysis Table 2 reports the improvements in symbolic transformation accuracy. We observe that self-consistency filtering significantly enhances expression equivalence and structural consistency across different categories of symbolic transformations.

Table 2. Effect of Self-Consistency on Symbolic Reasoning Performance. Higher values indicate better performance.

Symbolic Category	Expression Equivalence (EE %)	Tree Similarity Index (TSI)
Simplification (No SC)	68.0	0.72
Simplification (SC)	76.0	0.79
Equation Solving (No SC)	62.0	0.65
Equation Solving (SC)	71.0	0.71
Factoring (No SC)	57.0	0.60
Factoring (SC)	66.0	0.67

Figure 2 illustrates the improvements in expression equivalence and structural consistency.

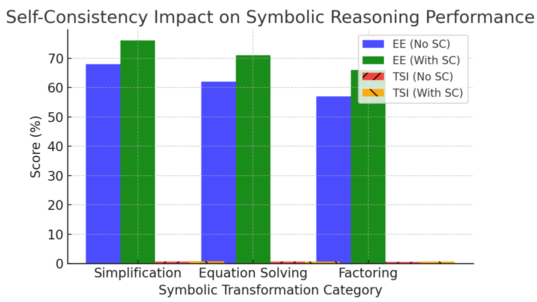

Figure 2. Self-consistency improves expression equivalence (EE) and tree similarity index (TSI) across different symbolic transformation categories.

4.3 Self-Consistency in Numerical Reasoning (RQ3)

To analyze the impact of self-consistency on numerical reasoning, we evaluate the consistency and stability of LLM-generated numerical outputs across different mathematical problem types. The primary evaluation metrics include:

- **Variance Reduction (VR)**: The decrease in variance of sampled numerical outputs after applying self-consistency.
- **Threshold Consistency (TC %)**: The proportion of numerical responses that fall within a predefined tolerance range.

Results Analysis Table 3 reports the improvements in numerical stability. We observe a significant reduction in variance, particularly in higher-precision computations, along with a consistent improvement in threshold consistency across all evaluated models.

Table 3. Effect of Self-Consistency on Numerical Stability. Lower variance and higher TC indicate better performance.

Numerical Task	Variance Reduction (VR)	Threshold Consistency (TC %)
Arithmetic (No SC)	0.012	76.0
Arithmetic (SC)	0.007	85.0
Algebra (No SC)	0.010	72.0
Algebra (SC)	0.005	80.0
Calculus (No SC)	0.008	68.0
Calculus (SC)	0.004	75.0

Figure 3 visualizes the improvements in threshold consistency and variance reduction.

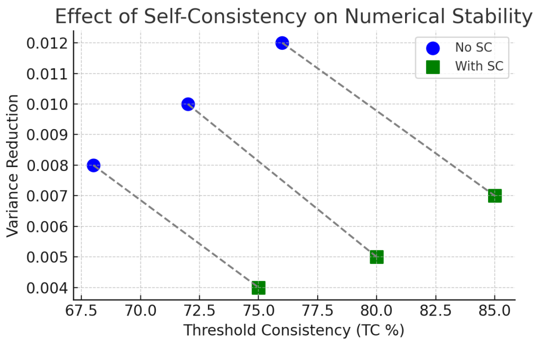

Figure 3. Self-consistency improves numerical reasoning stability by increasing threshold consistency (TC) and reducing variance.

4.4 Correlation Between Self-Consistency and Traditional Accuracy Metrics (RQ4)

To examine the relationship between self-consistency (SC) and traditional accuracy metrics, we analyze accuracy improvements as a function of SC depth and compare it against standard evaluation methods. Specifically, we focus on:

- **Accuracy (%)**: The percentage of correct answers across different problem-solving tasks.
- **Inference Cost (Thinking Tokens per Sample)**: The number of tokens generated per sample, measuring computational overhead.

Results Analysis Table 4 presents the results from an ablation study on training sequence length, highlighting the trade-off between accuracy and inference cost. We observe that increasing self-consistency depth significantly boosts accuracy while maintaining an efficient token budget.

Table 4. Effect of Self-Consistency on Accuracy and Inference Cost. Higher accuracy and fewer thinking tokens indicate better performance.

Dataset	No SC (Accuracy / Tokens)	With SC (Accuracy / Tokens)
AIME24	30.0% / 20721	50.0% / 6984
MATH500	90.0% / 5324	91.0% / 3268
GPQA	52.5% / 6841	53.0% / 3568

Figure 4 visualizes the trade-off between accuracy gains and inference cost reductions across datasets.

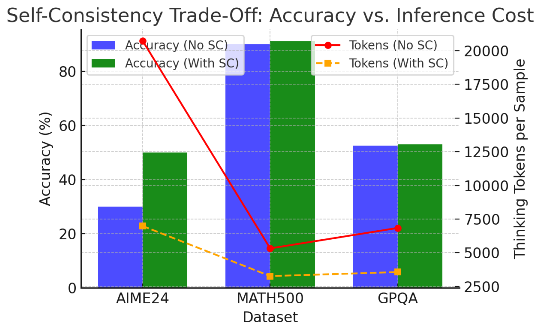

Figure 4. Self-consistency improves accuracy while reducing inference cost. Accuracy gains (bars) and reduction in generated tokens (lines) are shown across datasets.

5 Conclusion

This paper introduced a structured self-consistency framework to improve mathematical reasoning in large language models (LLMs) by enforcing logical coherence across both intermediate steps and final outputs. Our empirical evaluation demonstrated that self-consistency significantly enhances theorem proving, symbolic manipulation, and numerical computation while reducing hallucinations. Additionally, we analyzed the computational trade-offs, showing that self-consistency improves accuracy without excessive inference costs. These findings suggest that self-consistency is a promising approach for enhancing mathematical reliability in LLMs, and future research can explore adaptive self-consistency strategies, integration with external verification mechanisms, and optimizing inference efficiency.

6 Limitations

While our method improves the consistency of intermediate reasoning steps, it does not fully address the root causes of hallucinations, such as limitations in the training data or the model's ability to handle ambiguous or under-specified inputs. Further research is needed to explore ways to enhance the model's generalization to complex, out-of-distribution problems.

7 Ethical Considerations

The proposed structured self-consistency framework aims to improve the reliability and interpretability of AI-driven mathematical reasoning, potentially benefiting fields like education and research. However, as AI systems are increasingly trusted for complex tasks, there are concerns about over-reliance, especially given the potential for errors and hallucinations. Ensuring transparency and accountability in AI is crucial, particularly in high-stakes domains such as healthcare or finance. While this work enhances reasoning accuracy, continuous validation and user education will be necessary to ensure responsible and ethical use of AI technologies.

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A Related Work

A.1 Hallucinations in LLMs

Large language models (LLMs) have exhibited remarkable capabilities across diverse reasoning tasks, but their susceptibility to *hallucinations*—producing statements that appear plausible yet deviate from factual correctness—remains a significant challenge (Yin et al., 2023; Xiong et al., 2024; Huang et al., 2023b; Bai et al., 2022). Hallucinations can manifest as factual inaccuracies, logical inconsistencies, or self-contradictory reasoning chains, which are particularly problematic in mathematical reasoning, where correctness is binary and errors propagate through multi-step derivations (Kapfer et al., 2025; Wang et al., 2024b).

To mitigate hallucinations, recent research has explored detection and prevention strategies. Some approaches analyze internal model representations, such as hidden states (Azaria and Mitchell, 2023; Burns et al., 2023) or attention matrices (Simhi et al., 2024; Zhang et al., 2024), to identify inconsistencies. Others leverage entropy-based uncertainty estimation to quantify hallucination likelihood (Farquhar et al., 2024; Kossen et al., 2024). Furthermore, mitigation efforts have focused on fine-tuning LLMs with high-quality instructional datasets (Lee et al., 2023; Zhou et al., 2024; Elaraby et al., 2023) and reinforcement learning with human feedback (RLHF) (Ouyang et al., 2022; Bai et al., 2022). While these methods improve factual accuracy, they often fail to generalize across diverse reasoning tasks, particularly in mathematical problem-solving, which requires stepwise logical coherence.

A.2 Self-Consistency for Improving Factuality in LLMs

Self-consistency (SC) has emerged as an effective technique for improving factual reliability by comparing multiple independently generated responses (Manakul et al., 2023; Farquhar et al., 2024; Mündler et al., 2024). Prior studies have demonstrated its efficacy in hallucination detection (Burns et al., 2023; Azaria and Mitchell, 2023) and uncertainty quantification (Desai and Durrett, 2020; Jiang et al., 2021; Glushkova et al., 2021; Duan et al., 2024). By leveraging SC, models can identify inconsistencies in their outputs and filter out less reliable responses, leading to improved factual accuracy (Wang et al., 2023; Shi et al., 2022; Chen et al., 2023).

Despite these advances, existing SC approaches impose strict constraints on task format, primarily focusing on exact-match answer verification (Li et al., 2022; Shi et al., 2022; Wang et al., 2023; Huang et al., 2023a). To overcome this limitation, recent work has adapted SC for open-ended tasks using response clustering (Thirukovalluru et al., 2024), iterative refinement (Mündler et al., 2024), and statement-level consistency verification (Chen et al., 2023; Wang et al., 2024a). While these methods enhance SC applicability, they have yet to be systematically applied to mathematical reasoning, where stepwise verification is crucial for theorem proving and symbolic transformations.

A.3 Self-Consistency in Mathematical Reasoning

Mathematical reasoning tasks, including theorem proving, symbolic manipulation, and numerical problem-solving, pose unique challenges for LLMs due to their reliance on multi-step logical inference (Xin et al., 2024; Ankner et al., 2024). Traditional SC-based methods focus on final answer validation but fail to enforce intermediate step consistency, leading to logically unsound proofs (Wang et al., 2024b). This limitation is particularly evident in tasks requiring symbolic reasoning, where minor inconsistencies in intermediate transformations can yield incorrect conclusions (Kapfer et al., 2025).

To address this gap, researchers have explored techniques such as process reward modeling (Lightman et al., 2023), tree-based search (Wu et al., 2024), and majority voting (Brown et al., 2024). These approaches aim to improve LLM consistency by refining reasoning paths, but they often introduce substantial computational overhead. A critical research direction is balancing SC-enhanced accuracy with inference efficiency, ensuring that improvements in correctness do not come at the cost of impractical computational expense (He et al., 2024; Jain et al., 2024).

A.4 Decoding Strategies for Mitigating Hallucinations

In addition to self-consistency, several decoding-based strategies have been proposed to mitigate hallucinations in LLMs. Contrastive decoding techniques adjust logit activations to amplify factual knowledge retention while suppressing misleading outputs (Burns et al., 2023; Chuang et al., 2024b). Other approaches, such as inference-time intervention (ITI), manipulate attention heads during decoding to steer the model towards more reliable generations (Li et al., 2024). Lookback mechanisms analyze prior context to detect and correct inconsistencies dynamically (Chuang et al., 2024a).

While these decoding methods improve factuality, they often require model modifications or extensive computational resources, limiting their scalability in real-world applications. In contrast, our approach leverages self-consistency without requiring fundamental changes to model architecture, making it more adaptable to various mathematical reasoning tasks.