Robust Compressed Sensing MRI with Deep Generative Priors

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Multi-coil MRI

Robust to changes in scan and anatomy

Posterior Sampling via Annealed Langevin Dynamics

• Given measurements $y$, density $\mu$ over images, measurement likelihood $\pi(y|x)$, estimate is $\hat{x}$, such that:
  \[ \mu(\hat{x}|y) \propto \mu(x) \pi(y|x). \]
• Langevin dynamics:
  \[ x_{t+1} \leftarrow x_t + \beta_t \nabla \log \mu(x_t) + \beta_t \frac{d}{\sigma^2} \left( y - Ax_t \right) + N(0,2\beta_t). \]
• We use annealed Langevin dynamics and NCSNv2 generative model trained on brains [Song & Ermon].

Fine details like Meniscus tears are preserved

References:
2. Song and Ermon. Improved techniques for training score-based generative models. NeurIPS 2020

PSNR plots: Posterior sampling wins in most cases

Theory Results

Competitive for arbitrary measurements and performance metrics: For arbitrary metric $d$, if an oracle $x'$ achieves
\[ d(x,x') \leq \varepsilon \text{ with probability } 1-\delta, \]
then posterior sampling $\hat{x} \sim \mu_{x'y}$ achieves
\[ d(x,\hat{x}) \leq 2\varepsilon \text{ with probability } 1-2\delta. \]

Gaussian measurements are robust to distribution mismatch: If $x \sim \mu$, $A$ is a Gaussian matrix with $m \geq \log \left( \frac{1}{\delta} \right) + \log \text{Cov}_{\mathbb{R}^n}(\mu)$, and $\hat{x} \sim \nu_{x'y}$, then
\[ ||x - \hat{x}||_2 \leq C\varepsilon \text{ with probability } 1-\delta, \]
if some $1 - \delta, 1 - \varepsilon$ fraction of $\mu$ & $\nu$ are $\varepsilon$-close in $W_2$ distance.