Robust Compressed Sensing MRI with Deep Generative Priors

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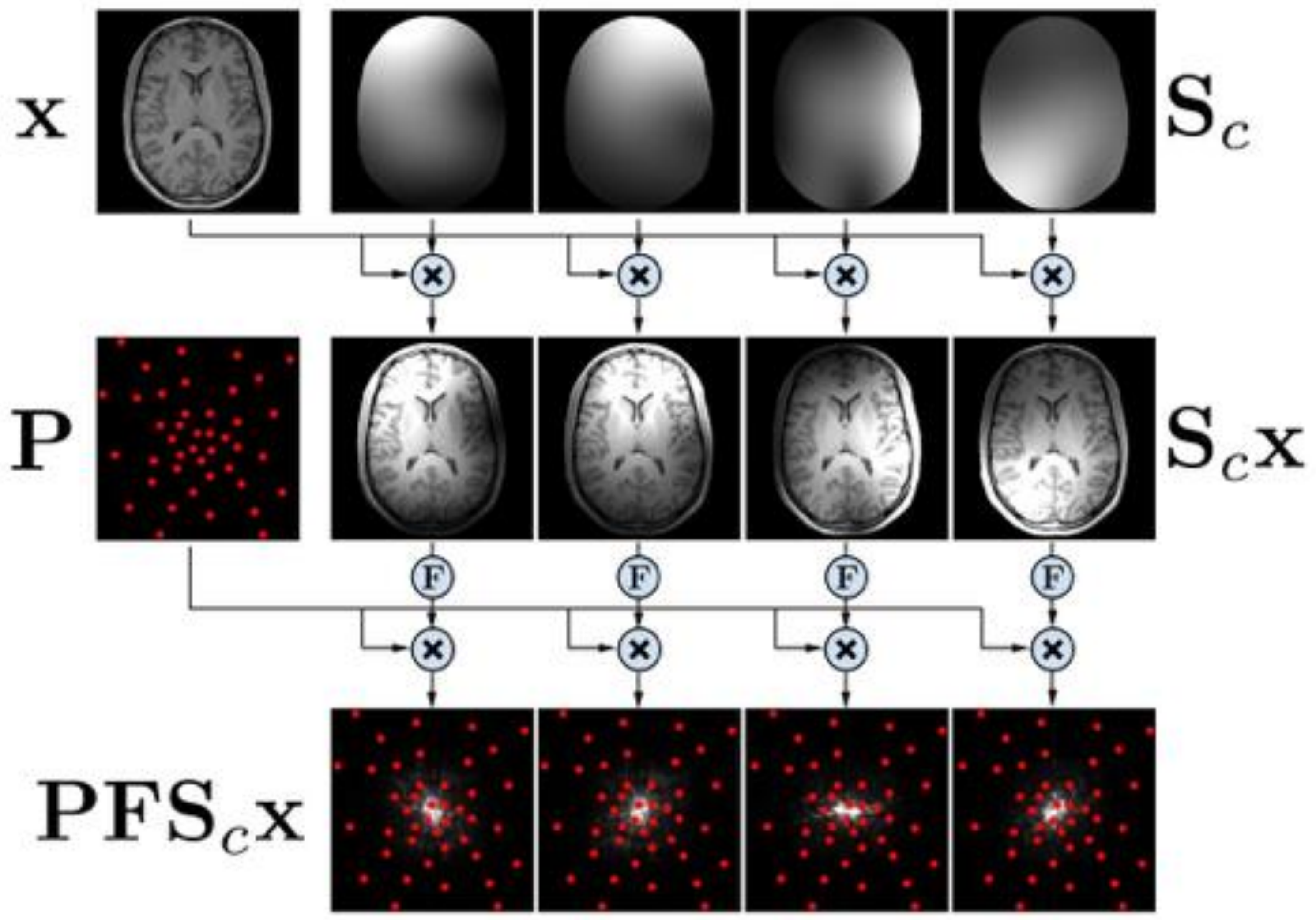
NEURAL INFORMATION PROCESSING SYSTEMS

Multi-coil MRI



Measurements from i^{th} coil :

$y_i = PFS_i x + \eta,\ \eta$ i.i.d. Gaussian

Posterior Sampling via Annealed Langevin Dynamics

- Given measurements y, density μ over images, measurement likelihood $\pi(y|x)$, estimate is $\hat{x}$, such that:

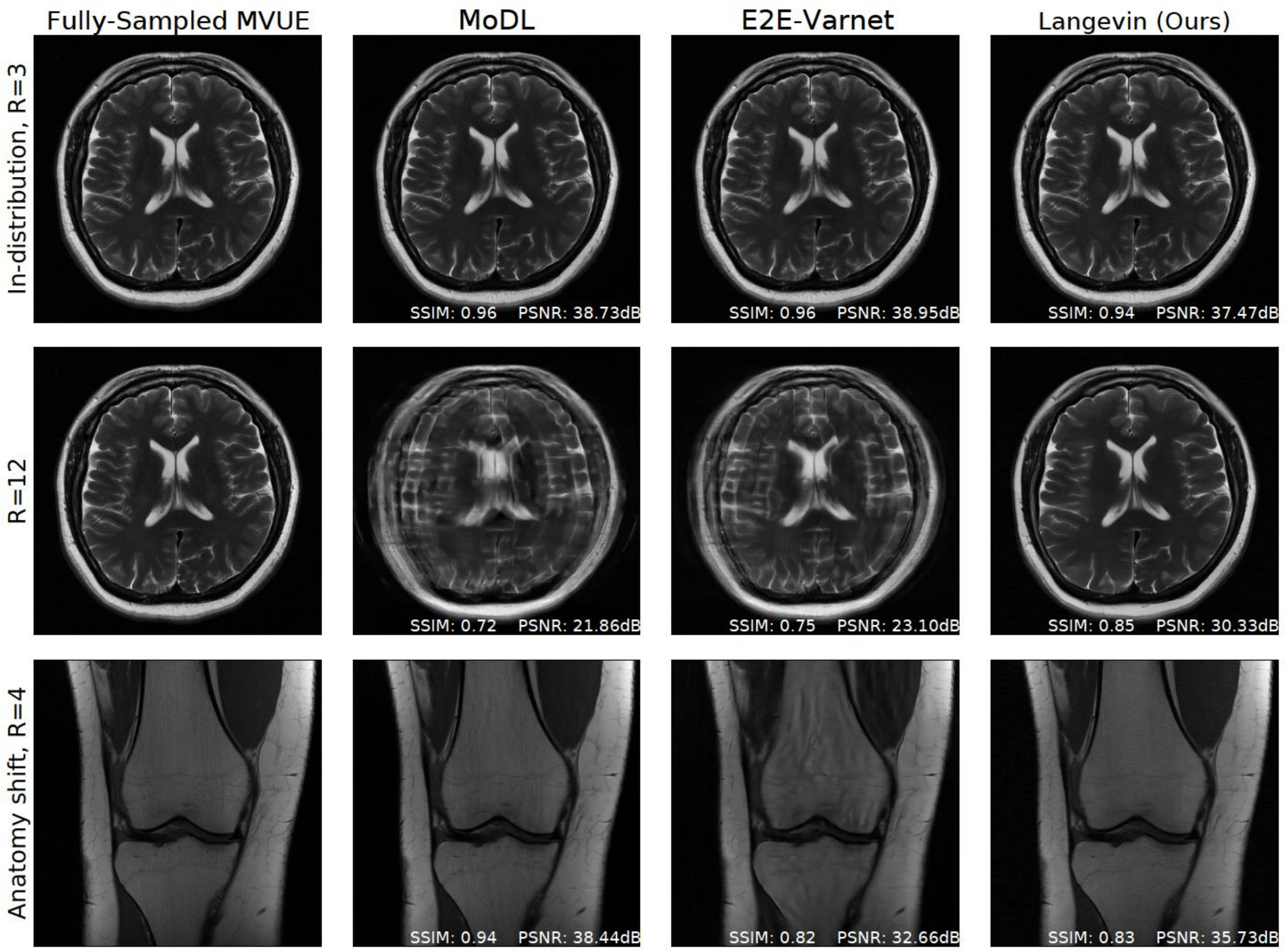
$$\mu(\hat{x}|y) \propto \mu(\hat{x})\pi(y|\hat{x}).$$

- Langevin dynamics:

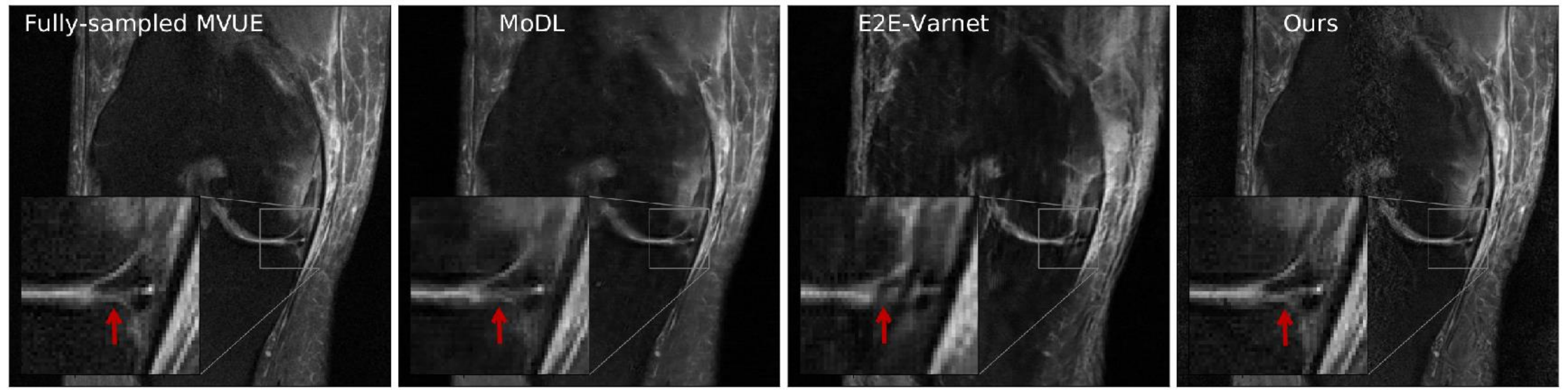
$$x_{t+1} \leftarrow x_t + \beta_t \nabla \log \mu(x_t) + \beta_t \frac{A^H(y - Ax_t)}{\sigma^2} + \mathcal{N}(0, 2\beta_t).$$

- We use annealed Langevin dynamics and NCSNv2 generative model trained on brains [Song & Ermon].

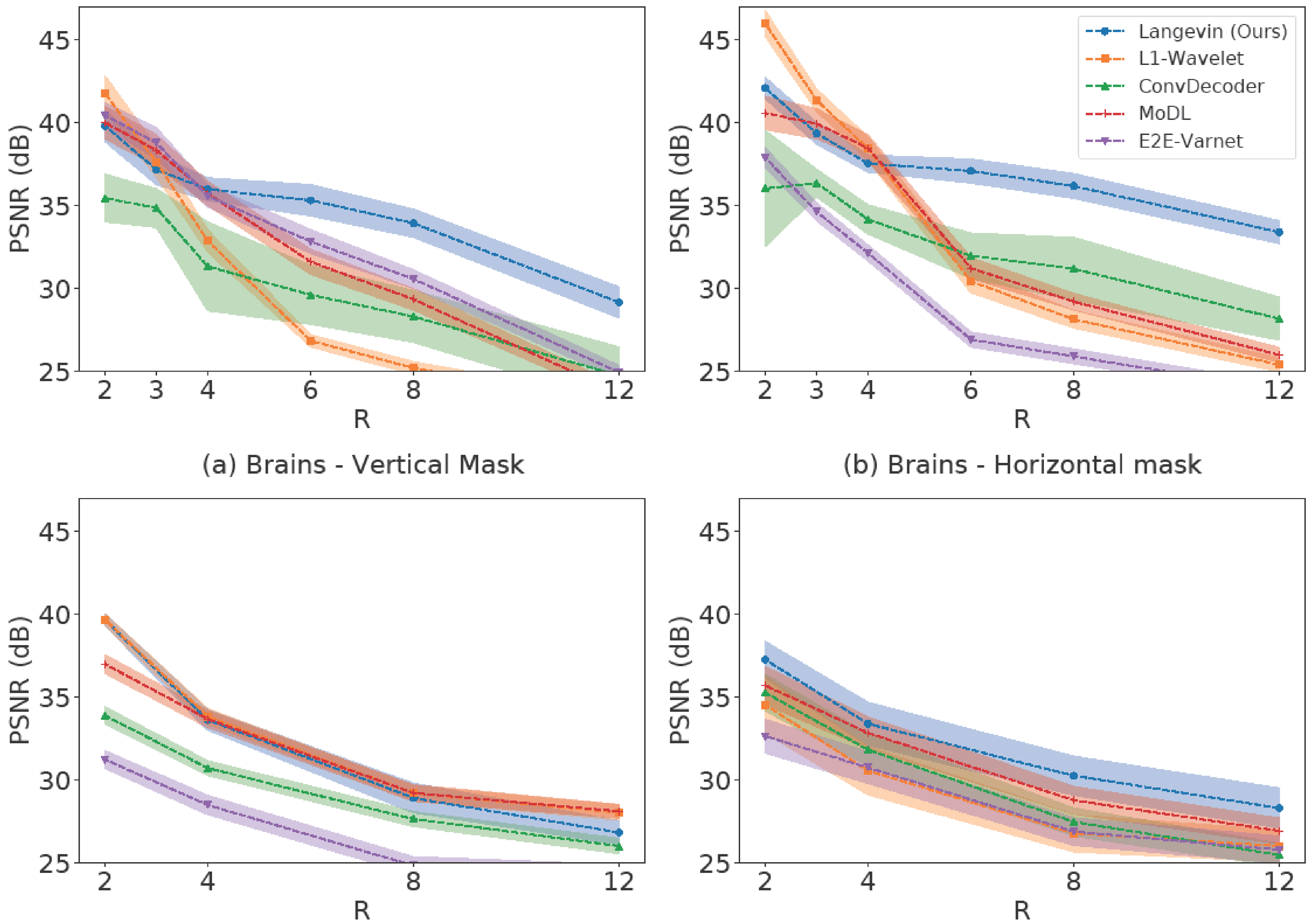
Robust to changes in scan and anatomy



Fine details like Meniscus tears are preserved



PSNR plots: Posterior sampling wins in most cases



(a) Brains - Vertical Mask

(b) Brains - Horizontal mask

(c) Abdomens

(d) Proton Density Knees

Theory Results

Competitive for arbitrary measurements and performance metrics: For arbitrary metric d, if an oracle x' achieves

$$d(x, x') \le \varepsilon \text{ with probability } 1\text{-}\delta,$$

then posterior sampling $\hat{x} \sim \mu_{x|y}$ achieves

$$d(x, \hat{x}) \le 2\varepsilon \text{ with probability } 1-2\delta.$$

Gaussian measurements are robust to distribution mismatch: If $x \sim \mu$, A is a Gaussian matrix with $m \ge \log\left(\frac{1}{1-\alpha}\right) + \log \mathrm{Cov}_{\sigma,\delta}(\mu)$, and $\hat{x} \sim \nu_{x|y}$, then

$$||x - \hat{x}||_2 \le C\sigma \text{ with probability } 1-\delta,$$

if some $1-\delta, 1-\alpha$ fraction of μ & ν are σ-close in $\mathcal{W}_\infty$ distance.

References:

1. Jalal, Karmalkar, Dimakis, Price. Instance-optimal compressed sensing via posterior sampling, ICML 2021
2. Song and Ermon. Improved techniques for training score-based generative models, NeurIPS 2020
3. Knoll et al., fastMRI: A publicly available raw k-space and dicom dataset of knee images for accelerated MRI reconstruction using machine learning. Radiology: Artificial Intelligence, 2020.