Motivation

Given limited measurements about some unknown ground truth \( x \), how can we select a measurement that would give us the most information about it?

**Proposed solution**

- GANs can model the posterior for inverse problems [1, 2].
- The variance of the posterior provides a natural criterion to guide sampling adaptively.
- The posterior quickly contracts to the correct mode.

Linear inverse problem

Recall a signal \( x \in \mathbb{C}^D \) from partial observations \( y_\omega \) obtained by subsampling a unitary transform matrix \( A \):

\[
y_\omega = P_\omega A x + \eta \in \mathbb{C}^N
\]

- \( \eta \): noise vector;
- \( \omega \subseteq [P] \): index set of sampling locations with \( |\omega| = N \);
- \( P_\omega \): diagonal matrix s.t. \( (P_\omega)_{ii} = 1_{i \in \omega} \).

**Posterior estimation**

Draw samples of the (approximate) posterior distribution:

\[
x_\omega = G_\theta(y_\omega, x).
\]

where \( z \) is a random vector drawn from a simple distribution; \( G_\theta(y_\omega, z) \) is a deep generative model mapping the input to a sample from \( p(x|y_\omega) \). Such models typically rely on conditional GANs [2] or VAEs [3].

**Sampling optimization**

**Ideal sampling optimization algorithm:** tailor the mask to each instance of \( x \sim p(x) \) solving

\[
\text{argmin}_{\omega \mid |\omega| \leq N} \mathbb{E}_{x \sim p(x)} \left[ \ell(x, \hat{x}_\theta(y_\omega = P_\omega A x)) \right].
\]

Solving this directly is impossible since this requires using the unknown ground truth signal \( x \) at testing time. Two main approaches circumvent this problem.

**Fixed (open-loop) sampling.** Design a mask at training time and keep it fixed at inference time.

**Uncertainty Estimate**

\[
\text{argmin}_{\omega \mid |\omega| \leq N} \mathbb{E}_{x \sim p(x)} \left[ \ell(x, \hat{x}_\theta(y_\omega = P_\omega A x)) \right],
\]

- constrained to a maximal sampling budget \( N \);
- want to find a mask that minimizes the loss \( \ell \);
- estimate the expected loss using the empirical estimate with the training samples: \( \hat{x}_\theta(y_\omega = P_\omega A x) \) denotes an estimate of the mean, obtained by averaging on \( z \) for equation 2, i.e., \( \hat{x}_\theta(y_\omega) = \mathbb{E}_z[G_\theta(y_\omega, z)] \).

**Adaptive (closed-loop) sampling.** Adapt at test time to a fixed, unknown data sample \( x \):

\[
v_t = \underset{v \in \{P\}}{\text{argmin}} \ell(x, \hat{x}_\theta(y_{v_t})) \text{ s.t. } \omega_t = \{\omega_{t-1}, v\}
\]

- \( v_t \): individual pixels to be sequentially observed from \( x \sim p(x) \);
- Use information of the previously obtained measurements \( y_{v_t-1} \) to determine what should be acquired at time \( t \) by conditioning on previous measurements.
- Optimize a heuristic \( \ell \) available at test time, or train a surrogate model to guide sampling [1, 7].

**GAS: Generative adaptive sampling**

Starting at \( t = 0 \) we iteratively sample the locations \( v_t \) with the largest variance in the domain \( A \):

\[
v_t = \argmax_{v \in \{P\}} \text{Var}[Ax|y_{v_t}] = \argmax_{v \in \{P\}} \text{Var}[Ax|y_{v_t}]
\]

- \( \text{Var}[Ax|y_{v_t}] \): element-wise variance in \( Ax \).
- No need for a surrogate model: the variance from the posterior model is sufficient to guide sampling adaptively.

**Results**

**Fixed (open-loop) sampling.** Design a mask at training time and keep it fixed at inference time.

- constrained to a maximal sampling budget \( N \);
- want to find a mask that minimizes the loss \( \ell \);
- estimate the expected loss using the empirical estimate with the training samples: \( \hat{x}_\theta(y_\omega = P_\omega A x) \) denotes an estimate of the mean, obtained by averaging on \( z \) for equation 2, i.e., \( \hat{x}_\theta(y_\omega) = \mathbb{E}_z[G_\theta(y_\omega, z)] \).

**Adaptive (closed-loop) sampling.** Adapt at test time to a fixed, unknown data sample \( x \):

\[
v_t = \underset{v \in \{P\}}{\text{argmin}} \ell(x, \hat{x}_\theta(y_{v_t})) \text{ s.t. } \omega_t = \{\omega_{t-1}, v\}
\]

- \( v_t \): individual pixels to be sequentially observed from \( x \sim p(x) \);
- Use information of the previously obtained measurements \( y_{v_t-1} \) to determine what should be acquired at time \( t \) by conditioning on previous measurements.
- Optimize a heuristic \( \ell \) available at test time, or train a surrogate model to guide sampling [1, 7].

**GAS: Generative adaptive sampling**

Starting at \( t = 0 \) we iteratively sample the locations \( v_t \) with the largest variance in the domain \( A \):

\[
v_t = \argmax_{v \in \{P\}} \text{Var}[Ax|y_{v_t}] = \argmax_{v \in \{P\}} \text{Var}[Ax|y_{v_t}]
\]

- \( \text{Var}[Ax|y_{v_t}] \): element-wise variance in \( Ax \).
- No need for a surrogate model: the variance from the posterior model is sufficient to guide sampling adaptively.

**Results**

**Fixed (open-loop) sampling.** Design a mask at training time and keep it fixed at inference time.

- constrained to a maximal sampling budget \( N \);
- want to find a mask that minimizes the loss \( \ell \);
- estimate the expected loss using the empirical estimate with the training samples: \( \hat{x}_\theta(y_\omega = P_\omega A x) \) denotes an estimate of the mean, obtained by averaging on \( z \) for equation 2, i.e., \( \hat{x}_\theta(y_\omega) = \mathbb{E}_z[G_\theta(y_\omega, z)] \).

**Adaptive (closed-loop) sampling.** Adapt at test time to a fixed, unknown data sample \( x \):

\[
v_t = \underset{v \in \{P\}}{\text{argmin}} \ell(x, \hat{x}_\theta(y_{v_t})) \text{ s.t. } \omega_t = \{\omega_{t-1}, v\}
\]

- \( v_t \): individual pixels to be sequentially observed from \( x \sim p(x) \);
- Use information of the previously obtained measurements \( y_{v_t-1} \) to determine what should be acquired at time \( t \) by conditioning on previous measurements.
- Optimize a heuristic \( \ell \) available at test time, or train a surrogate model to guide sampling [1, 7].

**GAS: Generative adaptive sampling**

Starting at \( t = 0 \) we iteratively sample the locations \( v_t \) with the largest variance in the domain \( A \):

\[
v_t = \argmax_{v \in \{P\}} \text{Var}[Ax|y_{v_t}] = \argmax_{v \in \{P\}} \text{Var}[Ax|y_{v_t}]
\]

- \( \text{Var}[Ax|y_{v_t}] \): element-wise variance in \( Ax \).
- No need for a surrogate model: the variance from the posterior model is sufficient to guide sampling adaptively.

**Results**

**Fixed (open-loop) sampling.** Design a mask at training time and keep it fixed at inference time.

- constrained to a maximal sampling budget \( N \);
- want to find a mask that minimizes the loss \( \ell \);
- estimate the expected loss using the empirical estimate with the training samples: \( \hat{x}_\theta(y_\omega = P_\omega A x) \) denotes an estimate of the mean, obtained by averaging on \( z \) for equation 2, i.e., \( \hat{x}_\theta(y_\omega) = \mathbb{E}_z[G_\theta(y_\omega, z)] \).

**Adaptive (closed-loop) sampling.** Adapt at test time to a fixed, unknown data sample \( x \):

\[
v_t = \underset{v \in \{P\}}{\text{argmin}} \ell(x, \hat{x}_\theta(y_{v_t})) \text{ s.t. } \omega_t = \{\omega_{t-1}, v\}
\]

- \( v_t \): individual pixels to be sequentially observed from \( x \sim p(x) \);
- Use information of the previously obtained measurements \( y_{v_t-1} \) to determine what should be acquired at time \( t \) by conditioning on previous measurements.
- Optimize a heuristic \( \ell \) available at test time, or train a surrogate model to guide sampling [1, 7].

**GAS: Generative adaptive sampling**

Starting at \( t = 0 \) we iteratively sample the locations \( v_t \) with the largest variance in the domain \( A \):

\[
v_t = \argmax_{v \in \{P\}} \text{Var}[Ax|y_{v_t}] = \argmax_{v \in \{P\}} \text{Var}[Ax|y_{v_t}]
\]

- \( \text{Var}[Ax|y_{v_t}] \): element-wise variance in \( Ax \).
- No need for a surrogate model: the variance from the posterior model is sufficient to guide sampling adaptively.