APPENDIX

A VISUAL PRESENTATION OF PREDICTION RESULTS

The visualization results for the other datasets are shown below: Figure1 demonstrates PastNet's proficiency in the KTH pedestrian motion dataset, accurately predicting future pedestrian trajectories; Finally, Figure2 illustrates PastNet's capability in predicting the systematic evolution of physical dynamics datasets such as RDS, EDPS, and FS.

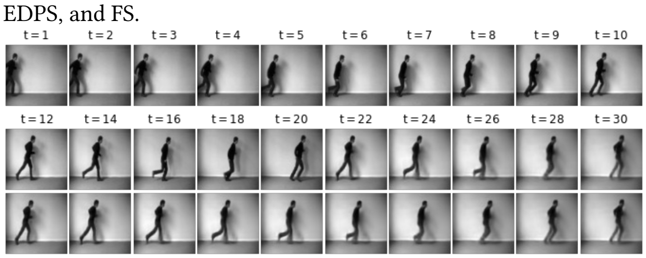

Figure 1: Example of prediction results on the KTH dataset. Top: input Pedestrian movement sequence; Middle: future real Pedestrian movement sequence; Bottom: PastNet predicted Pedestrian movement sequence.

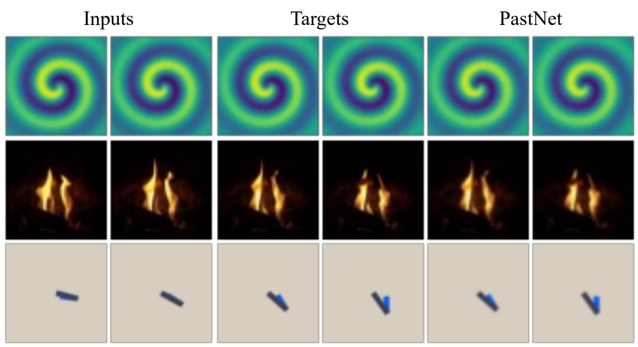

Figure 2: Examples of prediction results on the Reaction Diffusion System, Elastic Double Pendulum System and Fire System datasets; two input frames predict two output frames; from left to right, the input sequence, the target sequence and the PastNet prediction sequence.

B DESCRIPTION OF THE DETAILS OF THE PDE EQUATIONS

B.1 2D Navier-Stokes Equations

This paper considers the 2D Navier-Stokes equations for a viscous, incompressible fluid with vorticity in the form of a curl on the unit torus.

$$\begin{aligned} \partial_t w(x,t) + u(x,t)\cdot\nabla w(x,t) &= \nu\Delta w(x,t) + f(x) & x\in(0,1)^2, t\in(0,T] \\ \nabla\cdot u(x,t) &= 0, & x\in(0,1)^2, t\in[0,T] \\ w(x,0) &= w_0(x), & x\in(0,1)^2 \end{aligned} \tag{1}$$

In this context, $u\in C([0,T];H_r^{per}((0,1)^2;\mathbb{R}^2))$ for any $r>0$ is the velocity field, $w=\nabla\times u$ is the vorticity, $w_0\in L^2_{per}per((0,1)^2;\mathbb{R})$ is the initial vorticity, $\nu\in\mathbb{R}^+$ is the viscosity coefficient, and $f\in L^2_{per}((0,1)^2;\mathbb{R})$ is the forcing function. The time evolution of the equation is visualized in Figure 3.

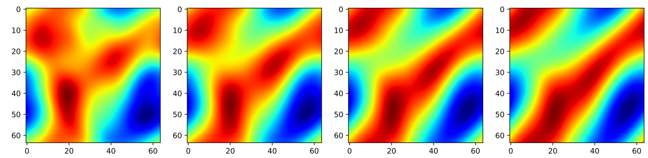

Figure 3: This visualization concerns the time evolution of data generated by the 2D Navier-Stokes equations.

B.2 2D Shallow-Water Equations

The Navier-Stokes equations are the fundamental equations that describe viscous flow in fluid mechanics. The shallow water equations can be derived from the Navier-Stokes equations and are used to model free-surface flow problems. In two dimensions, these equations can be expressed as a system of hyperbolic partial differential equations.

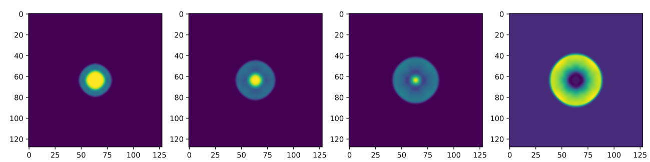

Figure 4: This visualization concerns the time evolution of data generated by the 2D shallow water equations.

$$\partial_t h + \partial_x hu + \partial_y hv = 0 \tag{2}$$

$$\partial_t hu + \partial_x\left(u^2h + \frac{1}{2}g_r h^2\right) = -g_r h\partial_x b \tag{3}$$

$$\partial_t hv + \partial_y\left(v^2h + \frac{1}{2}g_r h^2\right) = -g_r h\partial_y b \tag{4}$$

In the shallow water equations, which are used to model free-surface flow problems, u and v represent the horizontal and vertical velocities, h represents the water depth, and b describes the spatial variation of the depth. The terms hu and hv can be interpreted as the directional momentum components, and g represents the acceleration due to gravity.

The benchmark for the shallow water equation problem presented in Subsection ?? includes a specific simulation of a two-dimensional radial dam-breaking scheme. The simulation takes place on a square domain $\Omega=[-2.5,2.5]^2$, where the initial water height is represented by a circular bulge at the center of the domain.

$$h(t=0,x,y) = \begin{cases} 2.0, & \text{for } r<\sqrt{x^2+y^2} \\ 1.0, & \text{for } r\geq\sqrt{x^2+y^2} \end{cases} \tag{5}$$

The time evolution of the equation is visualized in Figure 4.