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A Further Related Works

Unpaired Multimodal Learning. Unpaired data has long been used for image-to-image [60, 30, 1, 47] and text-to-text translation [24]. More recently, several works have also proposed learning from unpaired data by inferring coarse- or fine-grained alignments through distribution matching or optimal transport objectives [55, 7, 46]. In contrast, we leverage unpaired data for learning representations without the need for explicit or inferred alignment. [53, 52] theoretically analyze the problem of identifying shared latent components and causal structures in unaligned multimodal mixtures. Most closely related to our work is [28], which leverages coarse-grained text data such as class names to improve image classification on CLIP using a shared linear head. Another related line of works [44, 42, 35, 12] leverage prompting templates and pretrained LLMs to generate descriptive class captions, showing improved image classification performance with CLIP. Nonetheless, these methods operate on CLIP with pre-aligned representation spaces, whereas our approach also learns from unpaired data without assuming prior alignment. Several works have also proposed learning large multitask multimodal models with joint encoders and unified embedding spaces [51, 50, 59, 14, 13], often using joint training over separate tasks and/or masked prediction objectives. In a similar vein, [5] uses a stage-wise training strategy with both unpaired and paired data, and [16] trains a single model across visual modalities. However, most of these methods rely on some amount of paired data for preliminary alignment and then leverage abundant modality-specific unpaired data for further improvement. In contrast, our approach demonstrates that a model can implicitly learn cross-modal correlations from purely unpaired data, without requiring explicit alignment as a prerequisite.

Multimodal Representation Alignment. Our method relies on the notion of shared information and structure between unaligned modalities. Closely related to this are works demonstrating that unimodal representations trained without multimodal data are nevertheless converging. [19] presents evidence that better-performing language models exhibit increased alignment to self-supervised vision models. Similarly, [34] shows a latent space alignment between vision and text encoders across backbones and training paradigms, and uses the CKA metric to connect unaligned encoders zero-shot. Earlier works also note alignment between models trained with different datasets and modalities [38, 40]. Several works have also shown that a linear projection or MLP is sufficient to stitch together the latent spaces of pretrained vision and language models [36, 29, 22]. [57] extends this to training a text encoder to align to a frozen pretrained image model; this method was in turn used to integrate DINOv2, a large self-supervised vision model, with a text encoder [21].

B Supplementary Experimental Details and Assets Disclosure

B.1 Assets

We do not introduce new data in the course of this work. Instead, we use publicly available widely used image datasets for the purposes of benchmarking and comparison.

B.2 Hardware and setup

Each experiment was conducted on 1 NVIDIA Tesla V100 GPUs, each with 32GB of accelerator RAM. The CPUs used were Intel Xeon E5-2698 v4 processors with 20 cores and 384GB of RAM. All experiments were implemented using the PyTorch deep learning framework.

B.3 Datasets

B.3.1 Image Classification Benchmarks

We evaluate on the following widely-used classification benchmarks: ImageNet [8], StanfordCars [23], UCF101 [49], Caltech101 [11], Oxford Flowers [39], SUN397 [56], DTD [6], FGVCAircraft [33], OxfordPets [41], and Food101 [4]. More details about the dataset and splits is provided in Table 2.

B.3.2 Constructing text templates

To construct conceptually related yet unpaired text data, we generate text templates that capture varying granularities of information about the dataset. Our first approach (*Vanilla*) uses the straight-

Table 2: Detailed statistics of the 10 datasets for image classification.

Dataset	Classes	Train	Val	Test
Caltech101 [11]	100	4,128	1,649	2,465
OxfordPets [41]	37	2,944	736	3,669
StanfordCars [23]	196	6,509	1,635	8,041
Oxford Flowers [39]	102	4,093	1,633	2,463
Food101 [4]	101	50,500	20,200	30,300
FGVCAircraft [33]	100	3,334	3,333	3,333
SUN397 [56]	397	15,880	3,970	19,850
DTD [6]	47	2,820	1,128	1,692
UCF101 [49]	101	7,639	1,898	3,783
ImageNet [8]	1,000	1.28M	N/A	50,000

forward template “a photo of a {}” with a natural language label for each category, resulting in a basic text description for each class. However, this simple textual corpus lacks fine-grained information necessary to distinguish between visually similar subcategories or to resolve contextually ambiguous terms. To address this, for the second template, we draw from the extensive literature on improving text prompts for zero-shot classification in CLIP [12, 35, 42, 44]. Specifically, for the second approach (*GPT-3 Descriptions*), we adopt the text prompt generation strategy developed by Pratt et al. [42], using large language models such as GPT-3 to generate diverse and contextually rich prompts for each image category. We use three generic hand-written sentences across the datasets:

Describe what a/the {} looks like:
Describe a/the {} :
What are the identifying characteristics of a/the {}?

The blank portion of each template is populated with the category name, along with the category type for specialized datasets (e.g., “pet” + {} for Oxford Pets or “aircraft” + {} for FGVC Aircraft). The type specification is important for disambiguating categories with multiple interpretations. Some examples of these descriptions are provided in Table 3 for the Oxford Pets dataset.

Table 3: Sample text descriptions per class for Oxford Pets dataset

Class	Examples
Wheaten Terrier	A wheaten terrier is a small, shaggy dog with a soft, silky coat. A wheaten terrier has a soft, wheat-colored coat that is low-shedding and hypoallergenic. The wheaten terrier is a medium-sized, hypoallergenic dog breed. A pet Wheaten Terrier usually has an intelligent expression and a soft, wheat-colored coat.
Great Pyrenees	A great pyrenees is a large, white, shaggy-coated dog. A Great Pyrenees is a large, fluffy dog with a calm, gentle disposition. The great pyrenees was originally bred to protect livestock from predators. Great Pyrenees are known for being very large, white dogs with thick fur.
Sphynx	A pet Sphynx typically has a small, wrinkled head and a hairless body. A Sphynx is a hairless cat breed known for its soft, warm skin. A Sphynx often displays large ears, pronounced cheekbones, and no fur. Sphynx are unique cats characterized by their lack of coat and wrinkled skin.
Birman	A Birman is a long-haired, color-pointed cat with a “mask” of darker fur on its face. A Birman has silky, pale cream to ivory fur with deep seal- or lilac-colored points. Birman cats possess striking blue eyes and contrasting white “gloves” on their paws. They are known for being gentle, affectionate, and smooth-coated companions.
Pomeranian	A Pomeranian is a small, fluffy dog with a thick double coat. Pomeranians are toy-sized, alert dogs with fox-like faces and plumed tails. A pet Pomeranian often comes in orange, black, white, or mixed coat colors. They are lively, outgoing, and known for their bold, friendly personalities.

B.3.3 ImageNet-ESC Dataset

Experimental Setup. We extend our results beyond vision and language to an audiovisual-language dataset: the ImageNet-ESC benchmark [28]. This benchmark combines ImageNet (1000 object categories) and ESC-50 (50 environmental sound classes) by matching classes that logically correspond. For example, the dog (barking) class from ESC-50 aligns with various dog breeds from ImageNet, while the clock-alarm sound maps to both analog clock and digital clock. This alignment captures the relationship between visual objects, their sounds, and their textual descriptions. The benchmark consists of two versions: 1) ImageNet-ESC-27: A broader set including loosely matched visual-audio pairs (e.g., drinking-sipping to water bottle); 2) ImageNet-ESC-19: A more precise subset containing only accurate visual-audio matches.

B.4 Training Protocol

B.4.1 Image Classification using Image and Unpaired Texts

For text, we use OpenLLaMA-3B as our default encoder and ablate against BERT-Large, RoBERTa-Large, GPT-2 Large, and the pre-aligned CLIP text encoder, keeping the text encoder frozen. For images, our main backbone is ViT-S/14 DINOv2, with ablations across other DINOv2 variants and the CLIP vision encoder. In the linear-probe setting, all encoder weights stay fixed and we train only a single linear classification head; in full fine-tuning, we jointly update the image backbone and that head, while still freezing the text encoder.

We optimize cross-entropy loss via AdamW [31] and perform an extensive grid search over learning rate, weight decay, cosine learning rate scheduling with linear warmup, dropout, and a learnable, modality-specific scaling on the logits. The results are reported for the best-performing model on the validation dataset. We report results for the model achieving highest validation accuracy; the full hyperparameter ranges are in Table 4.

For full fine-tuning, we jointly update the image backbone and classification head with a fixed learning rate of 5×10^{-5} , batch size 64, and omit learnable modality-specific scaling, since it showed no benefit in this setting.

Table 4: Hyperparameter grid for linear probing.

Hyperparameter	Values
Optimizer	adamw
Learning rate	{0.001, 1e-4}
Weight decay	{0.0, 0.01, 0.001}
LR scheduler	cosine
Batch size	{8, 32}
Max iterations	12,800
Warmup iterations	50
Warmup type	linear
Warmup min LR	1e-5
Dropout	{0.0}
Modality-specific learnable scaling	{False, True}
Early-stop patience	10

B.4.2 Evaluation on ImageNet-ESC

Similar to our vision-language experiments, we perform few-shot evaluation using the 5-fold splits defined in the benchmark. Each fold contains 8 samples per class, with one fold used for training and validation and the remaining four for testing. We repeat the process over 5 random splits and report the average performance. For audio encoding, we use AudioCLIP with an ES-ResNeXT backbone [17]. AudioCLIP is pretrained on AudioSet and generates audio embeddings in the same representation space as CLIP. Following the instructions in [17, 28], we use `train()` mode in Pytorch to extract the features since `eval()` mode yields suboptimal embeddings. We evaluate our models on two tasks—audio classification and image classification—comparing the unimodal baseline against

two multimodal variants in which the primary modality is each time augmented by one of the other modalities.

B.4.3 Transfer Learning from Language to Vision

To adapt a language model to image classification, we embed image patches using a linear projection and add positional encodings to capture spatial structure. We then use transformer layers initialized from pretrained BERT, and finally, a 2-layer MLP classification head. Specifically, we split each image of size 224×224 into patches of size 16×16 with 196 patch tokens. Each patch is then projected into the model’s embedding space of dimension d (e.g. $d=768$ for GPT-2, $d = 1024$ for BERT) via a learned linear layer. We then prepend a learnable “[CLS]” token, add learned positional embeddings of shape $(N + 1) \times d$, and apply dropout with probability $p = 0.1$. This $(N + 1) \times d$ sequence is passed into the pretrained transformer stack (either GPT-2 or BERT), using a full bidirectional attention mask over all patch tokens and the CLS token. We extract the final hidden state corresponding to the CLS token and feed it through a two-layer MLP classification head.

During training, we evaluate two scenarios: 1) one where the pretrained backbone is frozen and only the patch embedding and linear head are trained, and 2) another where the backbone is initially frozen to align the trainable layers (patch embedding and head) with the pretrained language backbone, and then unfrozen after 2000 steps for end-to-end training. This approach allows us to test whether the semantic richness captured by language models provides a strong initialization, leading to better convergence and performance compared to training ViT from scratch.

C Proofs of Theoretical Results

In this section, we present complete derivations and proofs of the main theoretical claims. [Appendix C.1](#) gathers all definitions and background required for our arguments. [Appendix C.2](#) formalizes the linear data-generating model, derives closed-form maximum-likelihood estimators for each modality and their joint estimator, and computes the corresponding block-wise Fisher information. Finally, [Appendix C.3](#) provides the detailed proofs of our variance-reduction claims, showing rigorously how unpaired multimodal estimation strictly lowers estimator variance.

C.1 Background and Definitions

In this section we revisit the mathematical definitions used in our theoretical analysis, including matrix-orderings, characterization of symmetric matrices and Fisher information.

Definition 1 (Positive Semidefinite Matrix). A real symmetric matrix $A \in \mathbb{R}^{d \times d}$ is *positive semidefinite* if for all vectors $v \in \mathbb{R}^d$, $v^\top A v \geq 0$. Equivalently, all eigenvalues of A are nonnegative. We denote the set of all $d \times d$ symmetric, positive-semidefinite matrices as $S_{\succeq 0}^d$.

Definition 2 (Positive Definite Matrix). A real symmetric matrix $A \in \mathbb{R}^{d \times d}$ is *positive definite* if for every nonzero $v \in \mathbb{R}^d$, $v^\top A v > 0$. Equivalently, all eigenvalues of A are strictly positive. We denote the set of all $d \times d$ symmetric, positive definite matrices as $S_{\succ 0}^d$.

Definition 3 (Loewner Order). For two real symmetric matrices $A, B \in \mathbb{R}^{d \times d}$, we write $A \preceq B \iff B - A$ is positive semidefinite and $A \prec B \iff B - A$ is positive definite. This defines a partial order on the cone of symmetric matrices.

Definition 4 (Fisher Information Matrix). Given a parametric family of densities $p(x; \theta)$ on data x , the *Fisher information matrix* at parameter θ is

$$I(\theta) = \mathbb{E}_{x \sim p(\cdot; \theta)} [\nabla_\theta \log p(x; \theta) \nabla_\theta \log p(x; \theta)^\top].$$

Equivalently, for regular models, $I(\theta) = -\mathbb{E} [\nabla_\theta^2 \log p(x; \theta)]$.

C.2 Maximum Likelihood Estimators and Fisher Contributions

In this section we revisit our linear data-generating model, introduce notations for the X -only, Y -only and joint likelihoods, derive the closed-form MLEs $\hat{\theta}_X$, $\hat{\theta}_Y$ and $\hat{\theta}_{X,Y}$, and formalize their information contributions towards estimating the ground truth parameters $\theta \equiv [\theta_c, \theta_x, \theta_y]^\top$.

Data Generating Process. Recall our linear data-generating process: Assume that all factors of variation in reality live in a single d -dimensional space $\mathcal{Z}^* \equiv \theta \in \mathbb{R}^d$ modeled using a linear data-generating pipeline. This parameter can further be decomposed as $\theta \equiv [\theta_c, \theta_x, \theta_y]^\top$ where $\theta_c \in \mathbb{R}^{d_c}, \theta_x \in \mathbb{R}^{d_x}, \theta_y \in \mathbb{R}^{d_y}$ and $d_c + d_x + d_y = d$. Here, θ_c captures the *common* (shared) parameters that affect both modalities, θ_x denotes the parameters that only affect modality X , and θ_y denotes the parameters that only affect modality Y . We observe two independent datasets, one from each modality $\{X_i\}_{i=1}^{N_x} \in \mathbb{R}^m$ and $\{Y_j\}_{j=1}^{N_y} \in \mathbb{R}^n$, each reflecting partial measurements of the ground truth latent space $\mathcal{Z}^*$:

$$X_i = A_{c,i} \theta_c + A_{x,i} \theta_x + \epsilon_{X,i}, \quad \epsilon_{X,i} \sim \mathcal{N}(0, \sigma_x^2 I_{m_i}) \quad (1)$$

$$Y_j = B_{c,j} \theta_c + B_{y,j} \theta_y + \epsilon_{Y,j}, \quad \epsilon_{Y,j} \sim \mathcal{N}(0, \sigma_y^2 I_{n_j}). \quad (2)$$

Here, $A_{c,i}, A_{x,i}, B_{c,j}, B_{y,j}$ are known design blocks capturing how each sample probes the latent factors and $\epsilon_{X,i}, \epsilon_{Y,j}$ represent the independent measurement noise.

In our linear setting, estimating the true latent state θ —and hence the underlying reality $\mathcal{Z}^*$ —is governed by the Fisher information matrix $I(\theta) = -\mathbb{E}[\nabla_\theta^2 \ell(\theta)]$, which measures how sharply the likelihood “curves” around the true θ . High curvature along a particular axis means the data tightly constrain that component, driving down estimator variance there.

Unimodal Estimators. We first estimate θ using only the X -dataset. Stacking $\{X_i\}_{i=1}^{N_x}$ yields a design matrix $\mathcal{A}$ with block rows $[A_{c,i}, A_{x,i}, 0]$. The least-squares solution

$$\hat{\theta}_X = \arg \min_{\theta} \sum_{i=1}^{N_x} \|X_i - A_{c,i} \theta_c - A_{x,i} \theta_x\|^2$$

omits θ_y entirely. Consequently, the Fisher information on θ_y vanishes, making it unidentifiable.

Analogously, stacking $\{Y_j\}_{j=1}^{N_y}$ defines $\mathcal{B}$ with block rows $[B_{c,j}, 0, B_{y,j}]$ and yields

$$\hat{\theta}_Y = \arg \min_{\theta} \sum_{j=1}^{N_y} \|Y_j - B_{c,j} \theta_c - B_{y,j} \theta_y\|^2.$$

This estimator doesn’t depend on θ_x , providing zero coverage for that component. Thus, each unimodal estimator entirely fails to recover the parameters exclusive to the omitted modality.

Multimodal Estimators. Despite the lack of one-to-one pairing, both $\{X_i\}$ and $\{Y_j\}$ share the common parameters θ_c . Since the two distributions are independent, the joint likelihood factorizes as

$$\prod_{i=1}^{N_x} p(X_i | \theta_c, \theta_x) \times \prod_{j=1}^{N_y} p(Y_j | \theta_c, \theta_y).$$

Maximizing this yields the combined estimator

$$\hat{\theta}_{X,Y} = \arg \min_{\theta_c, \theta_x, \theta_y} \left\{ \sum_{i=1}^{N_x} \|X_i - A_{c,i} \theta_c - A_{x,i} \theta_x\|^2 + \sum_{j=1}^{N_y} \|Y_j - B_{c,j} \theta_c - B_{y,j} \theta_y\|^2 \right\}.$$

Intuitively, there is no requirement to match up individual (X_i, Y_j) pairs. Instead, the estimate for θ_c is improved by both modalities while remaining unpaired.

Fisher Information. In our linear model, each dataset contributes block-structured Fisher information. For the X -dataset:

$$I_X = \sum_{i=1}^{N_x} \begin{pmatrix} A_{c,i}^\top A_{c,i} & A_{c,i}^\top A_{x,i} & 0 \\ A_{x,i}^\top A_{c,i} & A_{x,i}^\top A_{x,i} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

and for the Y -dataset:

$$I_Y = \sum_{j=1}^{N_y} \begin{pmatrix} B_{c,j}^\top B_{c,j} & 0 & B_{c,j}^\top B_{y,j} \\ 0 & 0 & 0 \\ B_{y,j}^\top B_{c,j} & 0 & B_{y,j}^\top B_{y,j} \end{pmatrix}.$$

Because X and Y samples are independent, their curvature contributions add pointwise, resulting in the joint Fisher information being simply the sum of the unimodal blocks.

$$I_{X,Y} = I_X + I_Y = \begin{pmatrix} \sum_i A_{c,i}^\top A_{c,i} + \sum_j B_{c,j}^\top B_{c,j} & * & * \\ * & \sum_i A_{x,i}^\top A_{x,i} & 0 \\ * & 0 & \sum_j B_{y,j}^\top B_{y,j} \end{pmatrix},$$

where “*” denotes the cross-modal blocks. In particular, we have the shared-parameter block as

$$(I_{X,Y})_{\theta_c, \theta_c} = \sum_{i=1}^{N_x} A_{c,i}^\top A_{c,i} + \sum_{j=1}^{N_y} B_{c,j}^\top B_{c,j},$$

C.3 Theorems and Proofs

The aim of this section is to detail the proofs of the theoretical results presented in the main manuscript. The key theoretical tools driving our analysis are already prepared in [Appendix C.1](#) and [Appendix C.2](#). Core to our theoretical analysis are a few lemmas around the Loewner-order monotonicity result for inverses that we prove below.

Lemma 1 (Loewner Order reversal for inverses). *Let $M, N \in \mathbb{S}_{>0}^d$ with $M \prec N$ (or $M \preceq N$). Then $N^{-1} \prec M^{-1}$ (or $N^{-1} \preceq M^{-1}$).*

Proof. Since $N \succ 0$, $N^{-1/2}$ exists and is nonsingular. Define $C := N^{-1/2} M N^{-1/2} \prec I$. Because a congruence with an invertible matrix preserves positive-definiteness, $C \succ 0$; hence C^{-1} is well defined and $C^{-1} \succ I$ (the scalar map $x \mapsto x^{-1}$ is strictly decreasing on $(0, \infty)$). Undoing the congruence gives

$$M^{-1} = N^{-1/2} C^{-1} N^{-1/2} \succ N^{-1/2} I N^{-1/2} = N^{-1}. \quad \square$$

Lemma 2 (Inverse-monotonicity of the Moore–Penrose pseudoinverse). *Let $M, N \in \mathbb{S}_{\geq 0}^d$ satisfy $M \prec N$ and $\ker M = \ker N =: K$. Then their pseudoinverses obey $N^\dagger \prec M^\dagger$.*

Proof. Set $S := K^\perp$ and let $P := P_S$ be the orthogonal projector onto S . Because M and N vanish on K , we have the decompositions $M = PMP$ and $N = PNP$. Restricted to S both matrices are positive-definite:

$$\tilde{M} := PMP, \quad \tilde{N} := PNP \in \mathbb{S}_{>0}^{\dim S}, \quad \tilde{M} \prec \tilde{N}.$$

Apply [Lemma 1](#) to $\tilde{M}, \tilde{N}$ to obtain $\tilde{N}^{-1} \prec \tilde{M}^{-1}$ on S . The Moore–Penrose pseudoinverse equals the ordinary inverse on S and is zero on K :

$$M^\dagger = P\tilde{M}^{-1}P, \quad N^\dagger = P\tilde{N}^{-1}P.$$

Therefore $N^\dagger = P\tilde{N}^{-1}P \prec P\tilde{M}^{-1}P = M^\dagger$. $\square$

Lemma 3 (Directional Loewner Order reversal). *Let $M, N \in \mathbb{S}_{>0}^d$ with $M \preceq N$. If a non-zero vector v satisfies $v^\top M v < v^\top N v$, then*

1. *For the vector v , it holds that $v^\top M^{-1} v \geq v^\top N^{-1} v$, with strict inequality $v^\top M^{-1} v > v^\top N^{-1} v$ if and only if $(N - M)M^{-1} v \neq 0$.*

2. *There exists a non-zero vector $u \in \mathbb{R}^d$ such that $u^\top M^{-1} u > u^\top N^{-1} u$.*

Proof. Denote the Loewner gap $\Delta := N - M \succeq 0$. Then, the assumption $v^\top N v > v^\top M v$ is equivalent to $v^\top \Delta v > 0$. Introduce the congruence-invariant normalisation $C := M^{-1/2} \Delta M^{-1/2} \succeq 0$. Now, using $\Delta = M^{1/2} C M^{1/2}$ and properties of inverse,

$$N = M^{1/2} (I + C) M^{1/2}, \quad N^{-1} = M^{-1/2} (I + C)^{-1} M^{-1/2},$$

since $I + C \succ 0$ (because $C \succeq 0$ and $I \succ 0$). Thus,

$$\begin{aligned} M^{-1} - N^{-1} &= M^{-1/2} \left[I - (I + C)^{-1} \right] M^{-1/2} \\ &= M^{-1/2} C (I + C)^{-1} M^{-1/2}, \end{aligned}$$

570 because $(I - (I + C)^{-1})(I + C) = C$. Finally, evaluating in the direction v , we have

$$\begin{aligned} v^\top (M^{-1} - N^{-1})v &= v^\top M^{-1/2}(I + C)^{-1}CM^{-1/2}v \\ &= u^\top (I + C)^{-1}Cu \quad (\text{where } u = M^{-1/2}v) \end{aligned}$$

571 Now, since $(I + C)^{-1} \in \mathbb{S}_{>0}$ and $C \in \mathbb{S}_{\geq 0}$ commute, the matrix $(I + C)^{-1}C$ is positive semidefinite
572 and it has exactly the same kernel as C . Thus, if $C = Q \text{diag}(\lambda_i)Q^\top$ ($\lambda_i \geq 0$), we have

$$u^\top C(I + C)^{-1}u = \sum_i \frac{\lambda_i}{1 + \lambda_i} (Q^\top u)_i^2 \geq 0.$$

573 This expression is strictly positive exactly when u has a component in any eigen-subspace with
574 $\lambda_i > 0$ i.e when $u \notin \ker(C)$. Since $M^{-1/2} \in \mathbb{S}_{>0}$, $Cu = 0 \implies M^{-1/2}\Delta M^{-1/2}u = 0 \implies$
575 $\Delta M^{-1}v = 0$. Thus, this expression is strictly positive if $\Delta M^{-1}v \neq 0$.

576 Now, from the premise $v^\top \Delta v > 0$, it follows that $\Delta \neq 0$. Since $M \succ 0$, $M^{-1/2}$ is invertible, C
577 is also not the zero matrix. Since $C \succeq 0$, this means that C must have at least one strictly positive
578 eigenvalue. Let $\lambda > 0$ be such an eigenvalue, and let $z \neq 0$ be a corresponding eigenvector. Define,
579 $x := M^{1/2}z \neq 0$. Thus, we have $x^\top (M^{-1} - N^{-1})x = z^\top C(I + C)^{-1}z = \frac{\lambda}{1 + \lambda} \|z\|^2 > 0$, showing
580 the existence of a non-zero vector x such that $x^\top M^{-1}x > x^\top N^{-1}x$.

581 □

582 **Theorem 1.** Let $\hat{\theta}_X, \hat{\theta}_Y$ be the least-squares estimators for θ using only $\{X_i\}$ and only $\{Y_j\}$ and
583 let $\hat{\theta}_{X,Y}$ be the joint estimator using both unpaired datasets. Then, under the assumption that
584 at least one $B_{c,j}$ where $j \in \{1, 2, \dots, N_y\}$ has full rank, the common-factor covariance satisfies
585 the strict Loewner ordering i.e. $\text{Var}(\hat{\theta}_{X,Y})_{\theta_c, \theta_c} \prec \text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c}$, or equivalently, the Fisher
586 information on θ_c strictly increases when combining both modalities, despite not having sample-wise
587 pairing: $(I_X + I_Y)_{\theta_c, \theta_c} \succ (I_X)_{\theta_c, \theta_c}$.

588 *Proof.* For any statistic $S(\theta) = \nabla_\theta \log p(x; \theta)$ and vector v ,

$$v^\top I(\theta) v = v^\top \mathbb{E}[S(\theta)S(\theta)^\top] v = \mathbb{E}[(v^\top S(\theta))^2] \geq 0.$$

589 Thus, a Fisher Information Matrix is a positive semidefinite matrix.

590 In our linear-Gaussian model, the X -dataset contributes $(I_X)_{\theta_c, \theta_c} = \sum_{i=1}^{N_x} A_{c,i}^\top A_{c,i}$ and the
591 Y -dataset gives $(I_Y)_{\theta_c, \theta_c} = \sum_{j=1}^{N_y} B_{c,j}^\top B_{c,j}$. Since at least one $B_{c,j}$ has full column rank, $(I_Y)_{\theta_c, \theta_c}$
592 is positive-definite on the θ_c subspace. Now, if at least one $B_{c,j} \in \mathbb{R}^{m \times d_c}$ has full column rank d_c ,
593 then for any $v \in \mathbb{R}^{d_c} \setminus \{0\}$,

$$v^\top B_{c,j}^\top B_{c,j} v = \|B_{c,j}v\|^2 > 0.$$

594 Hence, each summand in $(I_Y)_{\theta_c, \theta_c}$ is positive semidefinite and at least one is positive definite, so
595 their sum $\sum_j B_{c,j}^\top B_{c,j}$ is positive definite on the θ_c subspace. Thus,

$$(I_X)_{\theta_c, \theta_c} \prec (I_X)_{\theta_c, \theta_c} + (I_Y)_{\theta_c, \theta_c} = (I_X + I_Y)_{\theta_c, \theta_c}$$

Now, for regular exponential families (including Gaussian linear models), the covariance matrix of
the maximum likelihood estimator $\hat{\theta}$ near the true θ_0 is (asymptotically) the inverse of the Fisher
information matrix i.e. $\text{Var}(\hat{\theta}) \approx I(\theta_0)^{-1}$. Precisely, as the sample size $n \rightarrow \infty$, we have:

$$\sqrt{n}(\hat{\theta} - \theta_0) \xrightarrow{d} \mathcal{N}(0, I(\theta_0)^{-1}),$$

596 where θ_0 is the true parameter value, $I(\theta_0)$ is the Fisher Information Matrix evaluated at θ_0 and
597 $\mathcal{N}(0, I(\theta_0)^{-1})$ denotes a multivariate normal distribution with mean 0 and covariance matrix $I(\theta_0)^{-1}$.
598 Thus, we compare variances via the Moore-Penrose pseudoinverse of the information matrices.

599 Let $M_X = (I_X)_{\theta_c, \theta_c}$, $M_Y = (I_Y)_{\theta_c, \theta_c}$ and $M_{X,Y} = (I_X + I_Y)_{\theta_c, \theta_c}$. Since $M_Y \succ 0$, $M_{X,Y} =$
600 $M_X + M_Y$ is also positive definite (as $M_{X,Y} \succeq M_Y \succ 0$). Thus, $\text{Var}(\hat{\theta}_{X,Y}) = M_{X,Y}^{-1}$. We have

601 established $M_X \prec M_{X,Y}$. Assuming M_X is positive definite (to define the matrix $\text{Var}(\hat{\theta}_{X,Y})_{\theta_c, \theta_c}$),
 602 we apply Lemma 1 to get $M_{X,Y}^{-1} \prec M_X^{-1}$. Thus,

$$\text{Var}(\hat{\theta}_{X,Y})_{\theta_c, \theta_c} = M_{X,Y}^{-1} \prec M_X^{-1} = \text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c},$$

603 This proves the statement under the condition that M_X is positive definite. Note here that, on spaces
 604 unidentifiable by X -alone i.e. $v \in \ker(M_X)$, we have $\text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c} = \infty$. Since $M_{X,Y}$ is positive
 605 definite, it has finite variance along such v i.e. $\text{Var}(\hat{\theta}_{X,Y})_{\theta_c, \theta_c} < \infty$, thus strictly reducing the
 606 variance of the estimator. Thus, adding the unpaired Y -modality strictly reduces the variance (or,
 607 dually, increases the Fisher information) on the common factors θ_c .

608 □

609 **Theorem 2.** Let all notation be as in Theorem 1, and define $M_X := (I_X)_{\theta_c, \theta_c}$, $M_Y := (I_Y)_{\theta_c, \theta_c}$,
 610 and $M_{XY} := M_X + M_Y$. Let $v \in \mathbb{R}^{d_c} \setminus \{0\}$. If there exists at least one index $j \in \{1, 2, \dots, N_y\}$ such
 611 that $B_{c,j}v \neq 0$, then the following hold:

- 612 1. The Fisher information strictly increases in direction v i.e. $v^\top M_{XY} v > v^\top M_X v$.
- 613 2. The variance of the estimator in direction v is strictly reduced i.e. $v^\top \text{Var}(\hat{\theta}_{X,Y})_{\theta_c, \theta_c} v <$
 614 $v^\top \text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c} v$, if $v \notin \text{range}(M_X)$. For $v \in \text{range}(M_X)$, this strict inequality holds
 615 for v under an additional invertibility condition and is always guaranteed for some $u \in$
 616 $\text{range}(M_X)$ i.e. $\exists u$ s.t. $u^\top \text{Var}(\hat{\theta}_{X,Y})_{\theta_c, \theta_c} u < u^\top \text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c} u$.

617 *Proof.* Define $M_X := (I_X)_{\theta_c, \theta_c}$, $M_Y := (I_Y)_{\theta_c, \theta_c}$, and $M_{XY} := M_X + M_Y$. By assumption, $\exists j$
 618 such that $B_{c,j}v \neq 0$. Thus:

$$v^\top M_Y v = \sum_{j=1}^{N_y} \|B_{c,j}v\|^2 \geq \|B_{c,j}v\|^2 > 0.$$

619 Hence M_Y is positive-definite in direction v , implying $M_{X,Y} \succ M_X$ in this direction:

$$v^\top M_{XY} v = v^\top M_X v + v^\top M_Y v > v^\top M_X v,$$

620 thus proving the first part of the theorem.

621 **Case 1:** $v \notin \text{Range}(M_X)$. If $v \notin \text{Range}(M_X)$, then v has a non-zero component in $\ker(M_X)$. Let
 622 $v = v_S + v_K$, where $v_S \in \text{Range}(M_X)$ and $v_K \in \ker(M_X)$ with $v_K \neq 0$. The linear combination
 623 of parameters $v^\top \theta_c = v_S^\top \theta_c + v_K^\top \theta_c$. Since $v_K \in \ker(M_X)$, the component $v_K^\top \theta_c$ is not identifiable
 624 by the X -only model. Consequently, the asymptotic variance of an unbiased estimator for $v^\top \theta_c$ using
 625 only the X -dataset is infinite. We denote this as $v^\top \text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c} v = \infty$.

626 The strict inequality $v^\top M_{XY} v > 0$, ensures that $v \notin \ker(M_{XY})$, and thus $v \in \text{Range}(M_{XY})$.
 627 Since $v \in \text{Range}(M_{XY})$ and $v \neq 0$, $M_{XY}^\dagger v$ is well-defined. Furthermore, because M_{XY} is positive
 628 semidefinite, $M_{XY}^\dagger$ is also positive semidefinite and shares the same kernel as M_{XY} (since M_{XY} is
 629 symmetric). As $v \neq 0$ and $v \notin \ker(M_{XY})$, thus $v \notin \ker(M_{XY}^\dagger)$, which ensures $v^\top M_{XY}^\dagger v$ is a finite
 630 positive value. Thus,

$$v^\top \text{Var}(\hat{\theta}_{X,Y})_{\theta_c, \theta_c} v < \infty.$$

631 Comparing this to the variance from the X -only model in this case:

$$v^\top \text{Var}(\hat{\theta}_{X,Y})_{\theta_c, \theta_c} v < \infty = v^\top \text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c} v,$$

632 and the strict inequality holds.

633 **Case 2:** $v \in \text{Range}(M_X)$. Let $S := \text{Range}(M_X)$ and let P_S be the orthogonal projector onto S .
 634 Because $M_X = M_X P_S$ and $M_{XY} = M_X + M_Y$, the restrictions

$$\tilde{M}_X := P_S M_X P_S, \quad \tilde{M}_{XY} := P_S M_{XY} P_S = \tilde{M}_X + P_S M_Y P_S$$

are positive-definite on S ; To see this, take any non-zero $w \in S$. Since $w \in \text{range}(M_X)$, $P_S w = w$; hence

$$w^\top \tilde{M}_X w = w^\top M_X w > 0 \quad (P_S \text{ is identity when restricted to } S)$$

Thus $\tilde{M}_X \succ 0$ on S . Because $P_S M_Y P_S \succeq 0$, adding it preserves positive-definiteness, so

$$\tilde{M}_{XY} = \tilde{M}_X + P_S M_Y P_S \succeq \tilde{M}_X \succ 0 \quad \text{on } S.$$

Applying Lemma 3(1) to $\tilde{M}_X$ and $\tilde{M}_{XY}$ on S gives us $v^\top \tilde{M}_{XY}^{-1} v \leq v^\top \tilde{M}_X^{-1} v$. Strict inequality $v^\top \tilde{M}_{XY}^{-1} v < v^\top \tilde{M}_X^{-1} v$ holds if and only if the condition $C_v := ((\tilde{M}_{XY} - \tilde{M}_X) \tilde{M}_X^{-1} v \neq \mathbf{0})$ is met. Therefore, if condition C_v holds, the directional variance along this constrained space S is strictly reduced:

$$v^\top \text{Var}(\hat{\theta}_{X,Y})_{\theta_c, \theta_c} v = v^\top \tilde{M}_{XY}^{-1} v < v^\top \tilde{M}_X^{-1} v = v^\top \text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c} v.$$

Further, from Lemma 3(2), there exists some non-zero vector $u \in S$ such that $u^\top \tilde{M}_{XY}^{-1} u < u^\top \tilde{M}_X^{-1} u$. Thus we have,

$$u^\top \text{Var}(\hat{\theta}_{X,Y})_{\theta_c, \theta_c} u < u^\top \text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c} u.$$

Thus, completing the proof. $\square$

Corollary 1. Assume a direction $v \in \mathbb{R}^{d_c} \setminus \{0\}$ with $a = v^\top (I_X)_{\theta_c, \theta_c} v > 0$ and $b = v^\top (I_Y)_{\theta_c, \theta_c} v > 0$ where v is the common eigenvector of $(I_X)_{\theta_c, \theta_c}$ and $(I_Y)_{\theta_c, \theta_c}$. Then the variance in direction v contracts by the factor

$$\frac{v^\top \text{Var}(\hat{\theta}_{X,Y}) v}{v^\top \text{Var}(\hat{\theta}_X) v} = \frac{1/(a+b)}{1/a} = \frac{a}{a+b} < 1,$$

So the joint estimator achieves strictly lower error along v .

Proof. Let $M_X = (I_X)_{\theta_c, \theta_c}$ and $M_Y = (I_Y)_{\theta_c, \theta_c}$. By assumption, v is a common eigenvector of M_X and M_Y . Thus, $M_X v = \lambda_X v$ and $M_Y v = \lambda_Y v$ for some eigenvalues λ_X and λ_Y . From the assumptions, we have $\lambda_X = a/\|v\|^2 > 0$ and $\lambda_Y = b/\|v\|^2 > 0$. Since M_X is symmetric and $M_X v = \lambda_X v$ with $\lambda_X > 0$, the pseudoinverse acts as $M_X^\dagger v = \lambda_X^{-1} v$. Therefore, the variance in direction v for the X -only estimator is

$$v^\top \text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c} v = v^\top M_X^\dagger v = v^\top (\lambda_X^{-1} v) = \lambda_X^{-1} \|v\|^2 = a^{-1} \|v\|^4.$$

Since v is a common eigenvector, it is also an eigenvector of $M_{XY} = M_X + M_Y$:

$$(M_X + M_Y)v = M_X v + M_Y v = \lambda_X v + \lambda_Y v = (\lambda_X + \lambda_Y)v.$$

The corresponding eigenvalue is $\lambda_{XY} = \lambda_X + \lambda_Y$. Since $\lambda_X > 0$ and $\lambda_Y > 0$, $\lambda_{XY} > 0$. Thus, $(M_X + M_Y)^\dagger v = (\lambda_X + \lambda_Y)^{-1} v$. The variance in direction v for the joint estimator is

$$v^\top \text{Var}(\hat{\theta}_{X,Y})_{\theta_c, \theta_c} v = v^\top (M_X + M_Y)^\dagger v = (\lambda_X + \lambda_Y)^{-1} \|v\|^2 = (a+b)^{-1} \|v\|^4.$$

Now, we form the ratio of these variances:

$$\frac{v^\top \text{Var}(\hat{\theta}_{X,Y})_{\theta_c, \theta_c} v}{v^\top \text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c} v} = \frac{\lambda_X}{\lambda_X + \lambda_Y} = \frac{a}{a+b} < 1.$$

$\square$

Corollary 2. Assume a direction $v \in \mathbb{R}^{d_c} \setminus \{0\}$ with $v^\top (I_X)_{\theta_c, \theta_c} v = 0$ and $v^\top (I_Y)_{\theta_c, \theta_c} v > 0$. Then $v^\top \text{Var}(\hat{\theta}_X) v = \infty$ and $v^\top \text{Var}(\hat{\theta}_{X,Y}) v < \infty$ i.e. a direction unidentifiable from X alone becomes well-posed with even unpaired data from Y .

¹We note that true asymptotic variance defined as $v^\top \text{Var}(\hat{\theta}_{X,Y})_{\theta_c, \theta_c} v = v^\top M_{XY}^\dagger v$, $v^\top M_{XY}^\dagger v = v^\top \tilde{M}_{XY}^{-1} v$ if S is an invariant subspace of M_{XY} and M_{XY} is block-diagonal with respect to S and $S^\perp$ (i.e., $P_S M_{XY} P_{S^\perp} = \mathbf{0}$, which implies $P_S M_Y P_{S^\perp} = \mathbf{0}$).

663 *Proof.* This corollary follows directly from Case 1 of [Theorem 2](#). The condition $v^\top (I_X)_{\theta_c, \theta_c} v = 0$
 664 for $v \neq 0$ implies $v \in \ker((I_X)_{\theta_c, \theta_c})$, and thus $v \notin \text{range}((I_X)_{\theta_c, \theta_c})$. Given the additional condition
 665 $v^\top (I_Y)_{\theta_c, \theta_c} v > 0$, the conclusions of Case 1 of the theorem apply directly. $\square$

666 **Corollary 3** (Variance Reduction for Eigenvectors of M_X). *Let $v \in \mathbb{R}^{d_c} \setminus \{0\}$ be an eigenvector of*
 667 $M_X = (I_X)_{\theta_c, \theta_c}$ *with a corresponding eigenvalue $\lambda_X > 0$. If the Y -dataset provides information*
 668 *in this direction v (i.e., $v^\top M_Y v > 0$, where $M_Y = (I_Y)_{\theta_c, \theta_c}$), then the variance in direction v is*
 669 *strictly reduced by incorporating the Y -dataset:*

$$v^\top \text{Var}(\hat{\theta}_{X,Y})_{\theta_c, \theta_c} v < v^\top \text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c} v.$$

670 Specifically, $v^\top \text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c} v = \lambda_X^{-1} \|v\|^2$.

671 *Proof.* Let $M_X = (I_X)_{\theta_c, \theta_c}$ and $M_Y = (I_Y)_{\theta_c, \theta_c}$. Since v is an eigenvector of M_X with a positive
 672 eigenvalue $\lambda_X > 0$, it follows that $v \in \text{Range}(M_X)$. Let $S = \text{Range}(M_X)$. The variance using
 673 only the X -dataset in direction v is given by

$$v^\top \text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c} v = v^\top M_X^\dagger v.$$

674 Because v is an eigenvector of M_X with $\lambda_X > 0$, $M_X^\dagger v = \lambda_X^{-1} v$. Thus,

$$v^\top \text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c} v = v^\top (\lambda_X^{-1} v) = \lambda_X^{-1} \|v\|^2.$$

675 This scenario falls under Case 2 of [Theorem 2](#), specifically its conclusion regarding $v \in S$. According
 676 to that theorem, strict variance reduction $v^\top \text{Var}(\hat{\theta}_{X,Y})_{\theta_c, \theta_c} v < v^\top \text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c} v$ occurs if the
 677 condition $C_v = ((P_S M_Y P_S)(M_X|_S)^{-1} v \neq 0)$ holds. Here, P_S is the orthogonal projector onto S ,
 678 and $M_X|_S$ is the restriction of M_X to S , so $(M_X|_S)^{-1} v = \lambda_X^{-1} v$.

679 The condition C_v thus becomes $(P_S M_Y P_S)(\lambda_X^{-1} v) \neq 0$. Since $\lambda_X > 0$, this is equivalent to
 680 $P_S M_Y P_S v \neq 0$. We are given that $v^\top M_Y v > 0$. As $v \in S$, $P_S v = v$. Therefore, $v^\top M_Y v =$
 681 $v^\top P_S M_Y P_S v > 0$. Let $A_S = P_S M_Y P_S$ restricted to S . A_S is a positive semidefinite operator on
 682 S . The condition $v^\top A_S v > 0$ for $v \in S, v \neq 0$ implies that $A_S v \neq 0$ (because if $A_S v = 0$, then
 683 $v^\top A_S v = 0$, which contradicts $v^\top A_S v > 0$). Thus, $P_S M_Y P_S v \neq 0$, which means the condition C_v
 684 is satisfied.

685 Since $v \in S$ and the condition C_v for strict inequality is met, by [Theorem 2](#), it follows that
 686 $v^\top \text{Var}(\hat{\theta}_{X,Y})_{\theta_c, \theta_c} v < v^\top \text{Var}(\hat{\theta}_X)_{\theta_c, \theta_c} v$. $\square$

687 **Theorem 3.** Define for any m , $I_X^{(m)} = \sum_{i=1}^m A_{c,i}^\top A_{c,i}$ and $I_Y^{(m)} = \sum_{j=1}^m B_{c,j}^\top B_{c,j}$. If
 688 $\text{range}(I_Y^{(m)}) \not\subseteq \text{range}(I_X^{(m)})$, then there exists a nonzero $v \in \mathbb{R}^{d_c}$ such that $v^\top I_Y^{(m)} v > v^\top I_X^{(m)} v$.

689 *Proof.* Let $R_X := \text{range}(I_X^{(m)})$, $R_Y := \text{range}(I_Y^{(m)})$. By the assumption $R_Y \not\subseteq R_X$, choose a
 690 vector $w \in R_Y \setminus R_X$. Since $\mathbb{R}^{d_c}$ is a finite dimensional inner product space and R_X is its finite
 691 dimensional subspace, we can decompose $w = w_{||} + v$ with $w_{||} \in R_X$ and $v \in R_X^\perp$. Because
 692 $w \notin R_X$, the orthogonal component v is non-zero.

693 (i) *Term from $I_X^{(m)}$.* From the *Fundamental Theorem of Linear Algebra*, for any symmetric matrix S ,
 694 $\ker S = \text{range}(S)^\perp$; hence $R_X^\perp = \ker I_X^{(m)}$. Thus

$$v^\top I_X^{(m)} v = 0.$$

695 (ii) *Term from $I_Y^{(m)}$.* Because $w \in R_Y = \text{range}(I_Y^{(m)})$, there exists u with $w = I_Y^{(m)} u$. Suppose, for
 696 contradiction, that $I_Y^{(m)} v = 0$. Then $v \in \ker I_Y^{(m)} = R_Y^\perp$, so $v \perp w$. But $w \cdot v = (w_{||} + v) \cdot v =$
 697 $w_{||} \cdot v + \|v\|^2 = \|v\|^2 > 0$ because $v \perp w_{||}$ while $v \neq 0$. This contradicts $v \perp w$; therefore $I_Y^{(m)} v \neq 0$
 698 and, by positive semidefiniteness,

$$v^\top I_Y^{(m)} v > 0.$$

699 Combining the above inequalities yields $v^\top I_Y^{(m)} v > v^\top I_X^{(m)} v$, with $v \neq 0$, which is the desired
 700 inequality. $\square$

D UML Algorithm Pseudocode

In this section we present the full pseudocode for UML as shown in [Algorithm 1](#).

Algorithm 1 Pytorch Pseudocode

```
# f_img: image encoder (frozen or trainable)
# is_trainable: True if f_img is trainable else False
# f_text: frozen text encoder
# g: alignment network
# h: classification head

while not converged: # training loop
    x_img = fetch_next(image_loader) # image minibatch
    x_text = fetch_next(text_loader) # text minibatch (random/unaligned)

    z_img = f_img(x_img) # image embeddings
    z_text = f_text(x_text) # text embeddings

    logits_img = h(z_img) * alpha_image # predict image labels
    logits_text = h(z_text) * alpha_text # predict text labels

    loss_img = CE(logits_img, labels_img) # image classification loss
    loss_text = CE(logits_text, labels_text) # text classification loss
    loss = loss_img + lambda * loss_text # total loss

    loss.backward() # back-propagate
    update(h, f_img) if is_trainable else update(h) # SGD update

# Define Cross-Entropy loss
def CE(logits, labels):
    return -sum(labels * log_softmax(logits, dim=1)) / len(labels)
```

E Additional Experiments

E.1 Improving Image Classification using Unpaired Texts (Unaligned encoders)

In this section we report image-classification results on ten benchmarks (see [Appendix B.3](#)), covering three settings:

1. **Full-dataset fine-tuning**: train both the vision backbone and classification head ([Appendix E.1.1](#)).
2. **Full-dataset linear probe**: train only the classification head ([Appendix E.1.2](#)).
3. **Few-shot linear probe**: train only the classification head under few-shot conditions ([Appendix E.1.3](#)).

In each setting, we compare UML with baselines across all datasets and multiple DINO-initialized vision backbones.

E.1.1 Supervised Finetuning (across architectures)

In this section, we fine-tune both the vision backbone and the linear classifier on ten downstream tasks, comparing UML against strong image-only baselines. We evaluate four DINO-initialized backbones:

- ViT-B/16 in [Table 5](#)
- ViT-B/8 in [Table 6](#)
- DINOv2 ViT-S/14 in [Table 7](#)

721

• DINOv2 ViT-B/14 in Table 8

722

Results for DINOv2 ViT-L/14 are omitted due to computational constraints. Across all backbones, UML consistently improves over the image-only baseline by leveraging unpaired text embeddings. For some backbones such as DINOv2 ViT-B/16, our head-initialization variant (*Ours (init)*) outperforms training using unpaired multimodal data from scratch (*Ours*), while in others it does not.

724
725

Table 5: **Full finetuning on classification with ViT-B/16 DINO and OpenLLaMA-3B.** We compare our proposed approach with the image-only baseline when fine-tuning on the target dataset. All vision encoders are initialized from DINO weights, and our approach leverages unpaired text data using OpenLLaMA-3B embeddings.

Method	Dataset									
	Stanford Cars	SUN397	FGVC Aircraft	DTD	UCF101	Food101	Oxford Pets	Oxford Flowers	Caltech101	Average
Unimodal	78.41	63.99	62.12	74.17	81.43	82.38	92.00	98.24	96.31	81.01
Ours	82.56	67.04	67.38	76.42	84.06	81.79	93.20	98.98	97.04	83.16
Ours (init)	81.95	67.12	68.29	73.84	84.31	81.12	92.60	98.73	96.84	82.76

Table 6: **Full finetuning on classification with ViT-B/8 DINO and OpenLLaMA-3B.** We compare our proposed approach with the image-only baseline when fine-tuning on the target dataset. All vision encoders are initialized from DINO weights, and our approach leverages unpaired text data using OpenLLaMA-3B embeddings.

Method	Dataset									
	Stanford Cars	SUN397	FGVC Aircraft	DTD	UCF101	Food101	Oxford Pets	Oxford Flowers	Caltech101	Average
Unimodal	85.67	68.04	72.60	76.65	83.94	85.32	93.06	99.22	96.82	84.59
Ours	87.95	70.28	75.31	77.19	85.59	84.83	93.05	99.43	97.12	85.64
Ours (init)	87.44	70.03	76.09	76.24	86.49	84.71	93.81	99.27	97.16	85.69

Table 7: **Full finetuning on classification with ViT-S/14 DINOv2 and OpenLLaMA-3B.** We compare our proposed approach with the image-only baseline when fine-tuning on the target dataset. All vision encoders are initialized from DINOv2 weights, and our approach leverages unpaired text data using OpenLLaMA-3B embeddings.

Method	Dataset									
	Stanford Cars	SUN397	FGVC Aircraft	DTD	UCF101	Food101	Oxford Pets	Oxford Flowers	Caltech101	Average
Unimodal	79.45	66.20	66.99	72.16	83.18	80.65	90.67	99.18	95.45	81.54
Ours	84.87	66.72	71.54	74.14	84.77	81.16	91.87	99.55	97.03	83.52
Ours (init)	86.39	66.03	73.44	74.27	84.69	81.97	91.72	99.82	97.60	83.99

Table 8: **Full finetuning on classification with ViT-B/14 DINOv2 and OpenLLaMA-3B.** We compare our proposed approach with the image-only baseline when fine-tuning on the target dataset. All vision encoders are initialized from DINOv2 weights, and our approach leverages unpaired text data using OpenLLaMA-3B embeddings.

Method	Dataset									
	Stanford Cars	SUN397	FGVC Aircraft	DTD	UCF101	Food101	Oxford Pets	Oxford Flowers	Caltech101	Average
Unimodal	89.62	71.45	77.29	73.88	88.00	82.94	94.55	99.88	97.69	86.14
Ours	90.93	70.97	80.02	75.83	87.52	86.25	94.74	99.88	97.57	87.08
Ours (init)	90.73	70.92	80.23	75.87	87.60	83.43	94.47	99.80	97.93	86.77

E.1.2 Linear Probing (across architectures)

In this section, we train only the linear classifier, on top of the frozen vision and language backbone, on ten downstream tasks, comparing UML against strong image-only baselines. We evaluate five DINO-initialized backbones:

- ViT-B/16 in Table 9
- ViT-B/8 in Table 10
- DINOv2 ViT-S/14 in Table 11
- DINOv2 ViT-B/14 in Table 12
- DINOv2 ViT-L/14 in Table 13

Across all backbones, UML consistently improves over the image-only baseline by leveraging unpaired text embeddings. For all backbones, our head-initialization variant (*Ours (init)*) outperforms training using unpaired multimodal data from scratch (*Ours*).

Table 9: **Full linear probing on classification with ViT-B/16 DINO and OpenLLaMA-3B.** We compare our proposed approach with the image-only baseline when training a linear probe on the target dataset. All vision encoders are initialized from DINO weights, and our approach leverages unpaired text data using OpenLLaMA-3B embeddings.

Method	Dataset									
	Stanford Cars	SUN397	FGVC Aircraft	DTD	UCF101	Food101	Oxford Pets	Oxford Flowers	Caltech101	Average
Unimodal	67.10	64.63	56.02	72.42	81.27	74.96	93.07	98.32	95.01	78.08
Ours	68.71	65.14	57.42	72.95	82.06	75.30	93.18	98.46	96.19	78.82
Ours (init)	68.60	65.59	57.98	73.11	82.40	75.73	93.62	98.42	96.35	79.09

Table 10: **Full linear probing on classification with ViT-B/8 DINO and OpenLLaMA-3B.** We compare our proposed approach with the image-only baseline when training a linear probe on the target dataset. All vision encoders are initialized from DINO weights, and our approach leverages unpaired text data using OpenLLaMA-3B embeddings.

Method	Dataset									
	Stanford Cars	SUN397	FGVC Aircraft	DTD	UCF101	Food101	Oxford Pets	Oxford Flowers	Caltech101	Average
Unimodal	72.01	67.19	62.02	76.18	82.95	78.57	91.99	98.78	96.23	80.66
Ours	72.93	68.17	63.49	77.13	83.16	79.87	92.59	98.50	96.47	81.37
Ours (init)	72.81	68.36	64.09	76.48	83.72	80.01	92.50	98.74	96.43	81.46

Table 11: **Full linear probing on classification with ViT-S/14 DINOv2 and OpenLLaMA-3B.** We compare our proposed approach with the image-only baseline when training a linear probe on the target dataset. All vision encoders are initialized from DINOv2 weights, and our approach leverages unpaired text data using OpenLLaMA-3B embeddings.

Method	Dataset									
	Stanford Cars	SUN397	FGVC Aircraft	DTD	UCF101	Food101	Oxford Pets	Oxford Flowers	Caltech101	Average
Unimodal	77.48	70.72	66.28	78.25	82.64	84.39	94.29	99.62	97.00	83.40
Ours	78.45	71.53	67.33	78.70	83.51	84.67	94.70	99.82	97.11	83.98
Ours (init)	78.58	72.24	67.50	79.51	83.57	84.74	94.78	99.89	97.15	84.22

Table 12: **Full linear probing on classification with ViT-B/14 DINOv2 and OpenLLaMA-3B.** We compare our proposed approach with the image-only baseline when training a linear probe on the target dataset. All vision encoders are initialized from DINOv2 weights, and our approach leverages unpaired text data using OpenLLaMA-3B embeddings.

Method	Dataset									
	Stanford Cars	SUN397	FGVC Aircraft	DTD	UCF101	Food101	Oxford Pets	Oxford Flowers	Caltech101	Average
Unimodal	85.46	75.42	72.34	79.73	87.26	88.70	95.56	99.76	97.81	86.89
Ours	85.40	75.22	75.22	80.73	87.21	89.02	95.83	99.88	97.85	87.37
Ours (init)	85.74	75.70	74.17	81.32	87.26	88.78	95.78	99.88	97.93	87.40

Table 13: **Full linear probing on classification with ViT-L/14 DINOv2 and OpenLLaMA-3B.** We compare our proposed approach with the image-only baseline when training a linear probe on the target dataset. All vision encoders are initialized from DINOv2 weights, and our approach leverages unpaired text data using OpenLLaMA-3B embeddings.

Method	Dataset									
	Stanford Cars	SUN397	FGVC Aircraft	DTD	UCF101	Food101	Oxford Pets	Oxford Flowers	Caltech101	Average
Unimodal	88.16	77.26	74.32	81.56	89.82	90.95	96.27	99.84	97.97	88.46
Ours	88.45	77.20	76.93	82.39	90.19	91.09	96.51	99.92	98.01	88.97
Ours (init)	87.99	77.75	77.20	82.51	90.17	91.29	96.32	99.92	97.93	89.01

738 E.1.3 Few-shot Linear Probing (across architectures)

739 In this section, we train only the linear classifier, on top of the frozen vision and language backbone,
740 for few-shot classification on ten downstream tasks, comparing UML against strong image-only
741 baselines. We evaluate five DINO-initialized backbones: ViT-B/16 in Table 15, ViT-B/8 in Table 14,
742 DINOv2 ViT-S/14 in Table 16, DINOv2 ViT-B/14 in Table 18, DINOv2 ViT-L/14 in Table 18. Across
743 all backbones, UML consistently improves over the image-only baseline by leveraging unpaired text
744 embeddings. For all backbones, our head-initialization variant (*Ours (init)*) outperforms training
745 using unpaired multimodal data from scratch (*Ours*).

Table 14: **Linear evaluation of frozen features on 11 fine-grained benchmarks for few-shot learning.** We compare our proposed approach with the image-only baseline by training a linear classifier on top of frozen ViT-B/8 DINO features. Our method leverages unpaired text data using OpenLLaMA-3B

Train Shot	Method	Dataset										
		Stanford Cars	Sun397	Fgvc Aircraft	Did	Ucf101	Food101	Imagenet	Oxford Pets	Oxford Flowers	Caltech101	Average
1	Unimodal	7.40	26.37	12.16	28.62	39.75	19.23	42.81	54.97	58.22	74.13	36.37
	Ours	7.71	28.01	13.56	33.22	42.08	21.13	43.27	55.85	58.61	77.51	38.10
	Ours (init)	9.24	34.23	14.49	36.27	47.55	24.81	46.75	60.09	61.59	80.23	41.52
2	Unimodal	14.43	37.96	20.28	39.80	53.03	30.62	54.75	68.12	77.59	81.91	47.85
	Ours	15.71	40.74	21.04	43.74	55.86	33.52	54.49	69.86	77.18	84.52	49.67
	Ours (init)	16.94	45.16	22.17	45.43	59.02	35.89	56.78	71.57	77.94	86.06	51.70
4	Unimodal	25.67	49.23	29.39	52.52	64.27	43.82	61.64	75.85	87.41	90.36	58.02
	Ours	27.30	51.23	31.43	54.31	66.72	45.58	61.51	77.51	87.96	91.36	59.49
	Ours (init)	28.54	53.68	31.31	56.13	67.47	47.40	62.84	79.10	88.29	91.98	60.67
8	Unimodal	41.04	56.86	40.03	61.15	72.39	54.47	66.10	82.30	93.95	92.28	66.06
	Ours	43.76	58.14	42.56	63.12	73.13	56.30	66.36	84.27	94.25	92.71	67.46
	Ours (init)	44.16	59.80	42.30	64.46	74.30	57.07	67.18	84.85	94.00	93.24	68.14
16	Unimodal	57.72	61.74	52.63	67.69	76.18	62.63	68.87	87.31	96.41	94.27	72.54
	Ours	60.11	63.21	54.53	69.33	78.13	63.74	69.44	87.73	96.89	94.54	73.76
	Ours (init)	60.36	64.26	54.81	70.27	78.76	64.13	70.05	88.23	96.63	94.73	74.22

Table 15: **Linear evaluation of frozen features on 11 fine-grained benchmarks for few-shot learning.** We compare our proposed approach with the image-only baseline by training a linear classifier on top of frozen ViT-B/16 DINO features. Our method leverages unpaired text data using OpenLLaMA-3B

		Dataset										
Train Shot	Method	Stanford Cars										
		Sun397	Fgvc Aircraft	Did	Ucf101	Food101	Imagenet	Oxford Pets	Oxford Flowers	Caltech101	Average	
1	Unimodal	6.28	22.43	9.72	29.22	37.85	15.40	38.67	60.12	54.62	73.25	34.76
	Ours	7.89	26.08	10.41	32.45	40.27	18.14	39.28	60.88	58.32	75.66	36.94
	Ours (init)	8.96	31.34	12.12	34.22	44.32	21.46	42.68	66.39	60.37	79.74	40.16
2	Unimodal	12.64	35.64	14.98	38.93	51.14	26.05	50.34	70.84	75.61	83.16	45.93
	Ours	14.38	38.62	17.00	40.37	54.28	29.24	50.83	72.88	77.14	85.95	48.07
	Ours (init)	15.99	42.31	17.65	42.89	56.46	32.15	52.90	74.82	77.32	87.34	49.98
4	Unimodal	22.60	45.95	24.27	50.30	63.00	38.51	57.99	80.14	85.60	89.67	55.80
	Ours	24.83	48.62	25.76	52.64	64.39	40.74	57.96	80.92	87.20	91.17	57.42
	Ours (init)	25.83	51.01	26.35	55.06	65.86	42.69	59.32	82.23	87.83	91.99	58.82
8	Unimodal	37.68	52.94	33.67	59.18	70.62	49.48	62.97	85.26	92.83	93.17	63.78
	Ours	39.31	55.31	35.56	60.48	71.88	50.46	63.08	86.25	93.23	93.47	64.90
	Ours (init)	40.50	57.03	35.64	62.27	73.18	51.50	64.09	86.93	93.59	93.71	65.84
16	Unimodal	52.48	58.27	45.34	64.81	75.72	56.24	66.36	88.57	95.90	94.27	69.80
	Ours	55.84	60.57	47.70	66.21	76.81	58.26	66.47	89.60	96.55	95.12	71.31
	Ours (init)	55.82	61.73	48.14	67.02	77.39	58.76	67.08	90.53	96.62	94.98	71.81

Table 16: **Linear evaluation of frozen features on 11 fine-grained benchmarks for few-shot learning.** We compare our proposed approach with the image-only baseline by training a linear classifier on top of frozen ViT-S/14 DINOv2 features. Our method leverages unpaired text data using OpenLLaMA-3B

Train Shot	Method	Dataset										
		Stanford Cars	Sun397	Fgvc Aircraft	Dtd	Ucf101	Food101	Imagenet	Oxford Pets	Oxford Flowers	Caltech101	Average
1	Unimodal	13.18	34.15	14.09	36.60	46.74	35.18	36.48	63.51	89.62	76.66	44.62
	Ours	14.95	37.25	14.88	38.93	49.18	37.91	38.35	68.92	91.42	84.04	47.58
	Ours (init)	16.49	41.79	15.63	42.04	52.33	42.27	42.69	73.59	93.64	84.52	50.50
2	Unimodal	24.68	47.88	23.09	47.75	56.81	48.54	50.41	75.32	96.02	86.90	55.73
	Ours	26.93	49.65	24.29	50.99	61.67	51.77	51.31	79.44	96.90	89.80	58.28
	Ours (init)	28.65	53.15	24.78	53.25	63.86	54.44	54.21	81.41	97.63	90.55	60.19
4	Unimodal	38.76	57.51	32.10	59.69	67.75	60.79	58.73	83.89	98.59	93.48	65.12
	Ours	41.69	58.87	33.38	61.58	69.60	62.69	59.69	86.27	98.84	94.56	66.71
	Ours (init)	43.17	60.89	33.86	62.43	71.13	63.88	61.38	87.36	99.17	94.96	67.82
8	Unimodal	54.56	63.00	45.05	64.78	74.19	68.06	64.53	88.68	99.27	94.35	71.65
	Ours	56.27	64.57	45.98	66.31	75.19	69.22	65.14	89.78	99.27	95.42	72.71
	Ours (init)	57.91	65.82	47.40	67.81	75.99	69.71	66.40	90.29	99.54	95.84	73.67
16	Unimodal	67.96	67.35	55.89	71.36	77.92	73.24	68.14	90.73	99.63	96.43	76.22
	Ours	69.42	68.50	58.54	72.24	78.69	73.80	68.70	91.87	99.72	96.63	77.80
	Ours (init)	70.32	69.19	58.74	73.17	79.58	74.51	69.44	92.47	99.82	96.80	78.81

Table 17: **Linear evaluation of frozen features on 10 fine-grained benchmarks for few-shot learning with DINOv2 ViT-B/14.** We compare our proposed approach with the image-only baseline by training a linear classifier on top of frozen ViT-B/14 DINOv2 features. Our method leverages unpaired text data using OpenLLaMA-3B

Train Shot	Method	Dataset										
		Stanford Cars	Sun397	Fgvc Aircraft	Dtd	Ucf101	Food101	Imagenet	Oxford Pets	Oxford Flowers	Caltech101	Average
1	Unimodal	22.42	43.03	15.79	38.85	58.57	48.71	52.26	76.47	97.12	83.64	53.69
	Ours	23.10	45.12	16.22	42.69	61.05	51.30	52.45	78.14	98.08	87.68	55.58
	Ours (init)	25.47	48.56	16.83	45.31	63.53	54.16	55.56	81.08	97.94	88.13	57.66
2	Unimodal	35.17	55.41	25.54	51.16	69.49	62.13	62.35	84.31	99.58	89.55	63.47
	Ours	37.38	56.98	25.88	54.65	70.61	63.89	63.21	85.50	99.70	92.02	64.98
	Ours (init)	38.78	59.81	26.00	55.61	71.38	66.54	65.06	86.49	99.62	92.79	66.21
4	Unimodal	51.40	63.68	34.25	61.25	76.32	71.60	68.86	89.05	99.76	94.51	71.07
	Ours	54.26	64.65	35.52	62.63	76.87	72.33	69.14	90.00	99.70	95.51	72.06
	Ours (init)	55.01	66.55	35.14	63.97	77.57	73.25	70.30	90.31	99.57	95.65	72.73
8	Unimodal	66.01	68.88	48.17	66.67	79.92	76.26	72.48	90.97	99.80	95.54	76.47
	Ours	68.53	69.75	50.88	68.46	81.44	77.34	73.12	92.39	99.70	96.20	77.78
	Ours (init)	67.91	70.66	51.26	69.56	81.85	77.95	73.75	92.50	99.68	96.51	78.16
16	Unimodal	77.31	72.17	62.38	73.76	83.80	80.74	75.15	93.34	99.81	97.40	81.59
	Ours	78.92	72.80	64.51	75.16	84.62	81.00	75.46	92.92	99.59	97.38	82.24
	Ours (init)	78.52	73.18	65.81	75.65	84.77	81.18	75.82	93.28	99.78	97.57	82.56

Table 18: **Linear evaluation of frozen features on 10 fine-grained benchmarks for few-shot learning with DINOv2 ViT-L/14.** We compare our proposed approach with the image-only baseline by training a linear classifier on top of frozen ViT-L/14 DINOv2 features. Our method leverages unpaired text data using OpenLLaMA-3B

		Dataset										
Train Shot	Method	Stanford Cars		Fgvc Aircraft								
		Sun397			Dtd	Ucf101	Food101	Imagenet	Oxford Pets	Oxford Flowers	Caltech101	Average
1	Unimodal	24.89	48.36	17.69	38.77	66.46	59.27	57.50	79.83	98.13	82.96	57.39
	Ours	25.88	49.63	18.08	42.93	69.18	60.12	58.37	83.51	98.42	86.23	59.24
	Ours (init)	27.90	52.86	18.95	43.18	70.98	63.17	60.80	83.86	98.59	88.17	60.85
2	Unimodal	39.95	58.95	26.87	50.18	75.79	70.74	67.14	84.71	99.74	89.82	66.39
	Ours	41.22	60.82	27.15	53.01	76.61	72.07	67.90	86.07	99.72	91.95	67.65
	Ours (init)	42.93	63.36	28.14	54.96	77.72	73.87	69.20	87.13	99.81	91.71	68.88
4	Unimodal	56.49	66.37	38.59	59.08	80.84	77.39	72.41	89.90	99.73	94.44	73.52
	Ours	58.19	67.36	39.57	61.78	81.36	78.19	72.82	90.99	99.76	95.27	74.53
	Ours (init)	58.60	68.84	39.19	62.77	81.50	78.99	73.63	90.74	99.88	96.02	75.02
8	Unimodal	70.00	70.71	51.57	66.47	83.84	81.69	76.02	93.53	99.89	95.55	78.93
	Ours	71.63	71.59	55.13	67.91	84.47	82.12	76.43	93.62	99.88	96.36	79.91
	Ours (init)	72.02	72.51	55.49	69.03	84.57	82.52	76.78	93.80	99.89	96.73	80.33
16	Unimodal	80.84	73.83	64.13	73.96	87.43	84.58	77.78	94.69	99.91	97.36	83.45
	Ours	81.85	74.39	69.45	74.70	87.35	84.58	78.35	94.59	99.89	97.61	84.28
	Ours (init)	82.76	74.80	69.42	74.88	87.65	84.96	78.58	94.42	99.81	97.62	84.49

E.2 Improving Image Classification using Unpaired Texts (Aligned encoders)

E.2.1 Supervised Finetuning

In this section, we fine-tune both the vision backbone and the linear classifier on nine downstream tasks, comparing UML against strong image-only baselines. We evaluate two different backbones: ResNet-50 and ViT-B/16.

As shown in Table 19, across all backbones, UML consistently improves over the image-only baseline by leveraging unpaired text embeddings. Further, our head-initialization variant (*Ours (init)*) outperforms training using unpaired multimodal data from scratch (*Ours*).

Table 19: **Supervised finetuning on 9 fine-grained classification benchmarks with CLIP.** We compare our proposed approach with the image-only baseline when fine-tuning on the target dataset. All vision encoders are initialized from CLIP ResNet50 weights, and our approach leverages unpaired text data using the corresponding CLIP text encoder.

Method	Dataset									
	Stanford Cars	Sun397	Fgvc Aircraft	Dtd	Ucf101	Food101	Oxford Pets	Oxford Flowers	Caltech101	Average
Unimodal	36.12	25.93	37.70	51.06	52.49	69.24	63.17	88.42	83.61	56.42
Ours	37.00	24.05	41.34	55.67	60.48	69.77	74.49	92.57	84.79	60.02
Ours (init)	72.75	62.33	66.58	56.50	67.54	76.95	86.97	94.80	87.95	74.71

Table 20: **Full linear probing on classification with CLIP ResNet-50 Image Encoder and Text encoder.** We compare our proposed approach with the image-only baseline when training a linear probe on the target dataset. All vision encoders are initialized from ResNet-50 weights, and our approach leverages unpaired text data using the corresponding CLIP text embeddings.

Method	Dataset									
	Stanford Cars	SUN397	FGVC Aircraft	DTD	UCF101	Food101	Oxford Pets	Oxford Flowers	Caltech101	Average
Unimodal	76.36	70.97	41.88	72.81	81.23	81.60	88.39	97.89	92.78	78.21
Ours	77.23	71.18	42.66	71.81	81.81	81.51	87.84	97.65	93.01	78.30
Ours (init)	79.14	73.83	42.81	73.76	82.13	82.44	90.90	97.69	94.19	79.65

E.2.2 Linear Probing

In this section, we train only the linear classifier, on top of the frozen vision and language backbone from CLIP, on ten downstream tasks, comparing UML against strong image-only baselines. We evaluate two different backbones: ResNet-50 and ViT-B/16.

As shown in Table 20, across both backbones, UML consistently improves over the image-only baseline by leveraging unpaired text embeddings. Further, our head-initialization variant (*Ours (init)*) outperforms training using unpaired multimodal data from scratch (*Ours*).

E.2.3 Few-shot linear Probing (across architectures)

In this section, we train only the linear classifier, on top of the frozen vision and language backbone from CLIP, for few-shot classification on ten downstream tasks, comparing UML against strong image-only baselines. We evaluate two different backbones: ResNet-50 and ViT-B/16.

As shown in Table 21 and Table 22, across both backbones, UML consistently improves over the image-only baseline by leveraging unpaired text embeddings. Further, our head-initialization variant (*Ours (init)*) outperforms training using unpaired multimodal data from scratch (*Ours*).

E.3 Improving Visual Robustness Using Unpaired Texts

In this section, we evaluate the robustness of models trained with UML to test-time distribution shifts. We train a k -shot linear probe (where $k \in \{1, 2, 4, 8\}$) with DINOv2 on ImageNet and evaluate across four distribution-shifted target datasets: ImageNet-V2, ImageNet-Sketch, ImageNet-A, and ImageNet-R. Our method consistently improves robustness over the unimodal baseline (Figure 7, Figure 8, Figure 9 and Figure 10) across different training shots, indicating that language priors help capture more transferable features.

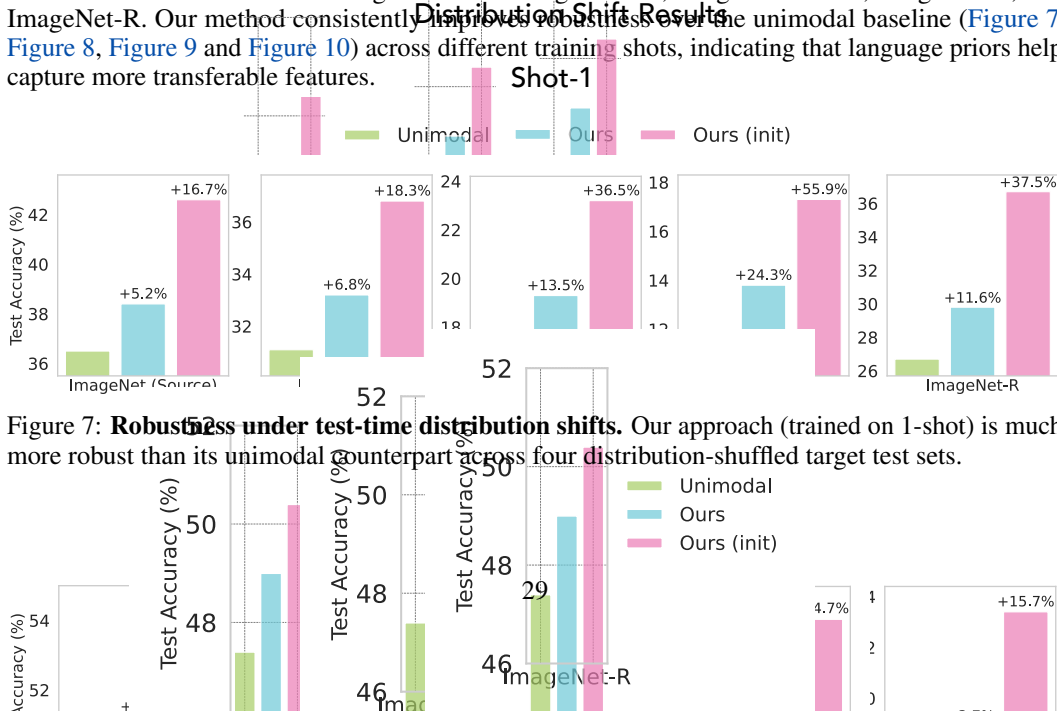
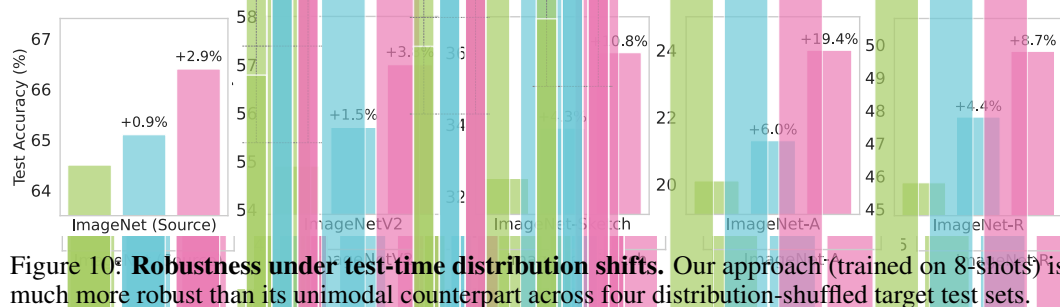
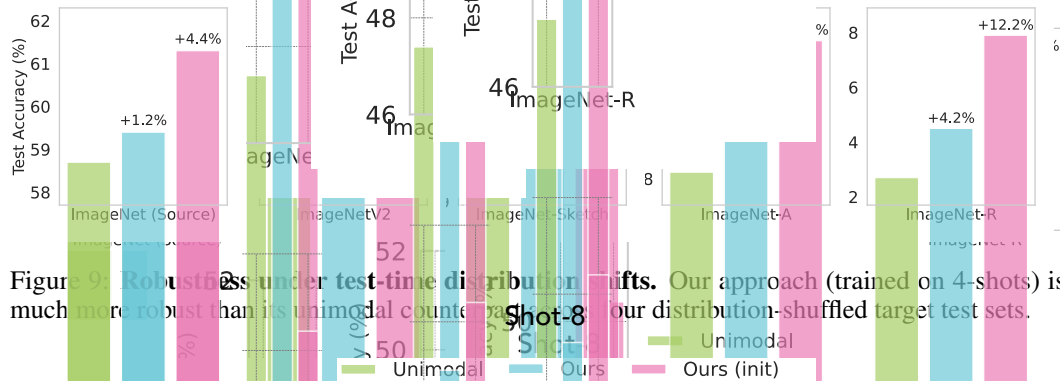
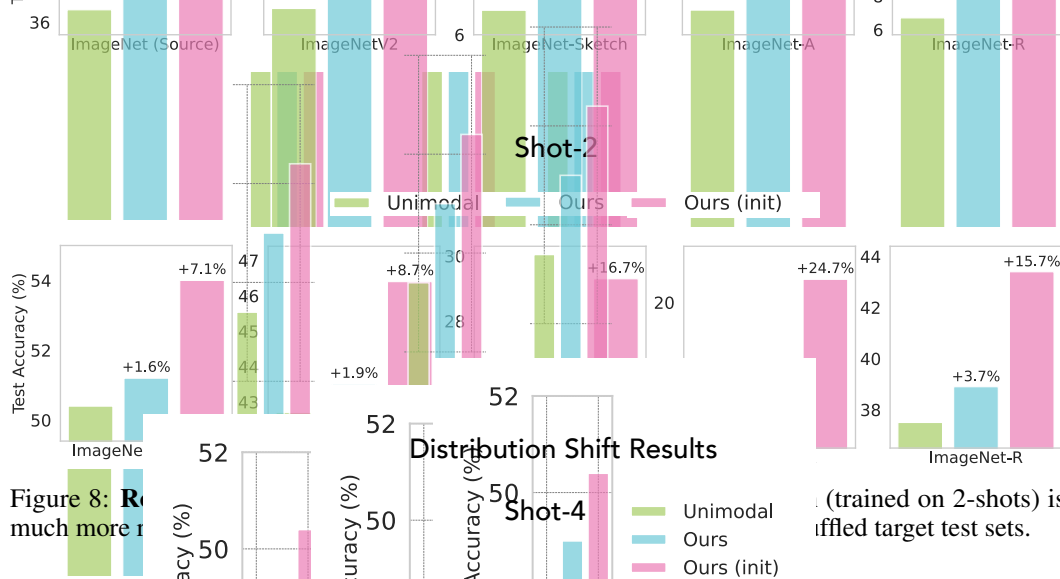


Table 21: **Linear evaluation of frozen features on 10 fine-grained benchmarks for few-shot learning.** We compare our proposed approach with the image-only baseline by training a linear classifier on top of frozen CLIP ResNet50 features. Our method leverages unpaired text data using the corresponding CLIP text encoder

Train Shot	Method	Dataset										
		Stanford Cars	Sun397	Fgvc Aircraft	Dtd	Ucf101	Food101	Oxford Pets	Imagenet	Oxford Flowers	Caltech101	Average
1	Unimodal	23.24	29.14	12.38	30.24	37.55	27.26	34.61	21.36	59.07	66.52	34.14
	Ours	36.32	45.40	16.84	40.92	53.19	49.76	53.03	36.48	68.56	76.80	47.73
	Ours (init)	57.88	64.59	22.23	50.85	65.99	76.73	86.59	60.92	81.08	83.79	65.06
2	Unimodal	38.37	43.83	18.63	40.33	53.25	44.60	47.75	32.62	75.03	78.90	47.33
	Ours	46.64	53.53	20.81	48.35	62.01	56.67	60.64	42.21	77.97	84.58	55.34
	Ours (init)	61.86	65.90	24.19	55.30	70.39	77.07	87.40	61.40	86.20	85.94	67.57
4	Unimodal	51.34	54.38	23.08	52.07	64.06	57.29	61.32	41.72	86.16	85.41	57.68
	Ours	55.21	59.48	24.77	56.78	67.65	62.68	67.31	47.04	86.46	87.23	61.46
	Ours (init)	65.80	68.11	27.49	60.13	73.62	77.79	86.54	62.37	91.60	87.57	70.10
8	Unimodal	61.74	61.47	30.22	60.15	70.16	64.63	68.94	49.48	92.20	89.14	64.81
	Ours	62.75	63.70	30.69	61.84	70.74	67.73	73.62	52.14	92.31	89.89	66.54
	Ours (init)	69.78	69.61	31.62	64.13	77.24	78.58	89.07	63.34	94.21	91.58	72.92
16	Unimodal	70.94	65.53	35.91	64.30	75.13	70.67	78.49	55.07	95.21	91.26	70.25
	Ours	71.58	67.08	36.23	65.62	76.09	71.63	79.52	56.92	95.44	91.94	71.20
	Ours (init)	74.56	71.33	37.13	68.09	78.66	79.06	89.71	64.31	96.17	93.31	75.23

Table 22: **Linear evaluation of frozen features on 10 fine-grained benchmarks for few-shot learning.** We compare our proposed approach with the image-only baseline by training a linear classifier on top of frozen CLIP ViT-B/16 features. Our method leverages unpaired text data using the corresponding CLIP text encoder

Train Shot	Method	Dataset										
		Stanford Cars	Sun397	Fgvc Aircraft	Dtd	Ucf101	Food101	Oxford Pets	Imagenet	Oxford Flowers	Caltech101	Average
1	Unimodal	31.53	33.51	17.76	31.72	43.64	39.40	37.43	27.65	67.95	71.68	40.23
	Ours	48.28	53.44	22.06	47.04	63.40	63.92	60.95	47.35	77.82	83.14	56.74
	Ours (init)	67.76	70.13	32.26	55.16	75.02	84.25	90.91	69.50	87.58	88.87	72.14
2	Unimodal	48.45	48.70	23.38	42.04	60.08	58.30	53.56	41.68	82.01	83.20	54.14
	Ours	57.89	59.95	27.19	52.27	69.60	71.18	66.78	54.24	87.43	90.20	63.67
	Ours (init)	70.75	71.52	33.99	60.17	78.37	85.39	90.67	70.19	92.18	90.09	74.33
4	Unimodal	61.64	60.66	31.01	54.37	70.49	71.91	69.35	52.15	90.99	91.08	65.36
	Ours	66.24	65.56	32.98	59.95	74.16	76.19	75.92	58.50	91.32	93.23	69.40
	Ours (init)	74.58	73.54	37.38	64.30	81.10	86.05	91.64	70.89	94.80	93.70	76.80
8	Unimodal	71.76	66.67	38.47	61.96	77.11	78.16	78.25	59.90	95.20	92.98	72.05
	Ours	72.77	69.50	39.09	64.89	79.01	80.07	80.85	62.63	94.98	94.36	73.82
	Ours (init)	78.43	75.07	41.77	68.50	83.41	86.87	92.55	71.97	96.94	95.27	79.08
16	Unimodal	78.76	71.49	44.74	68.79	80.43	82.08	85.16	63.87	96.97	94.54	76.68
	Ours	79.40	72.19	45.06	69.41	81.97	82.12	85.92	64.93	96.49	95.28	77.28
	Ours (init)	82.38	76.51	47.14	72.13	84.66	86.60	92.68	72.79	97.70	96.08	80.87



E.4 Marginal Rate-of-Substitution Between Modalities

How many words is an image worth? In this section, we extend our results to evaluate image-text conversion ratios using test accuracy isolines on the remaining eight datasets. We measure these global equivalence ratios by fitting a plane to the accuracy values given the number of image and text shots. [Figures 11 to 18](#) demonstrate the conversion ratios for DINOv2 ViT-S/14 as the vision backbone and OpenLLaMa-3B as the text backbone (unaligned encoders). Analogously, [Figures 19 to 26](#) show the same ratios for CLIP ResNet-50 as the vision and text encoders (aligned encoders). As expected, with the fully aligned CLIP backbone, each image equates to far fewer text prompts than under the unaligned DINO setting, showing the higher efficiency of aligned embeddings.

E.4.1 Unaligned Encoders

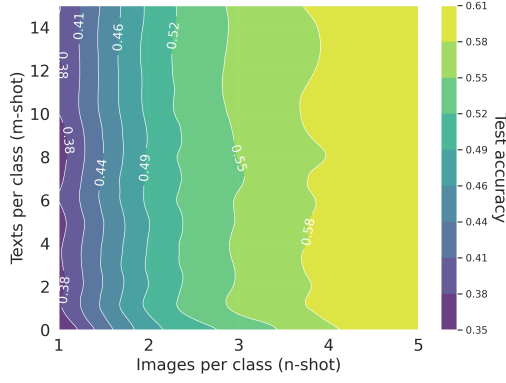


Figure 11: **SUN397**. 1 img $\approx$ 1568 words

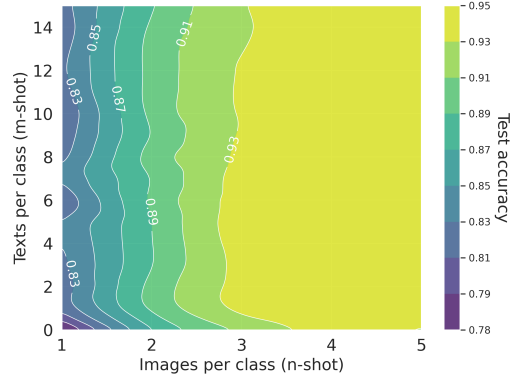


Figure 12: **Caltech101**. 1 img $\approx$ 1248 words

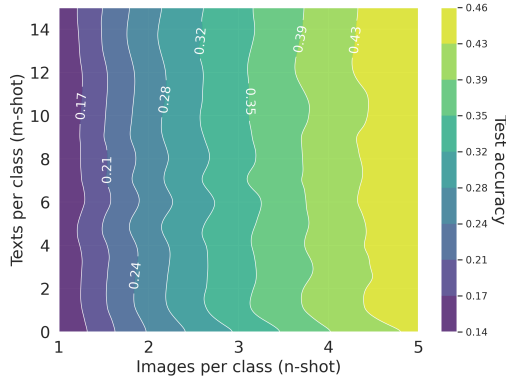


Figure 13: **Stanford Cars**. 1 img $\approx$ 1799 words

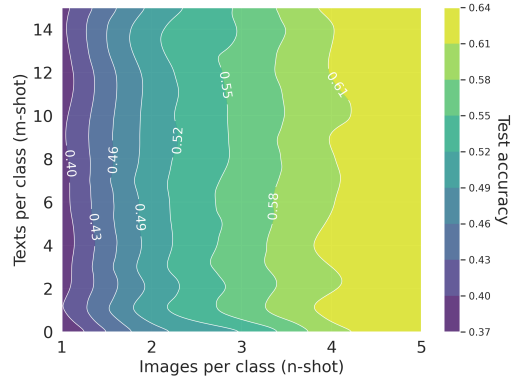


Figure 14: **DTD**. 1 img $\approx$ 2309 words

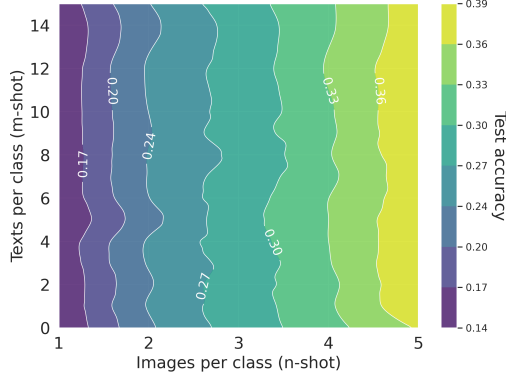


Figure 15: **FGVC Aircraft**. 1 img $\approx$ 3220 words

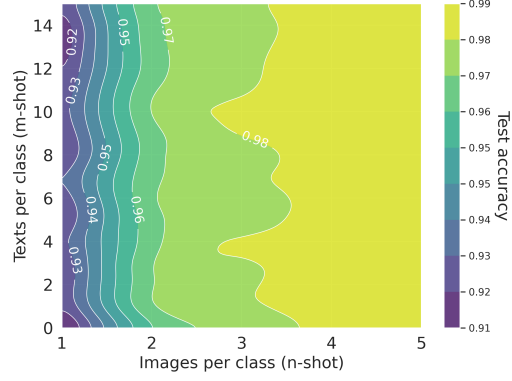


Figure 16: **Oxford Flowers**. 1 img $\approx$ 1895 words

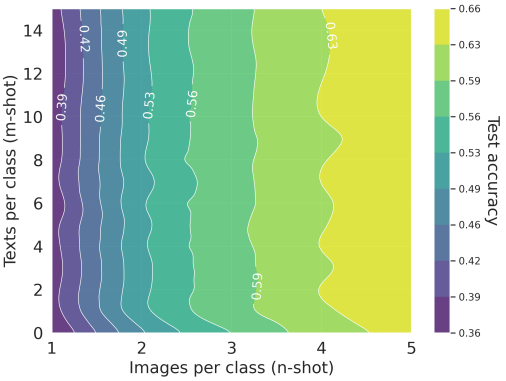


Figure 17: **Food101**. 1 img $\approx$ 2608 words

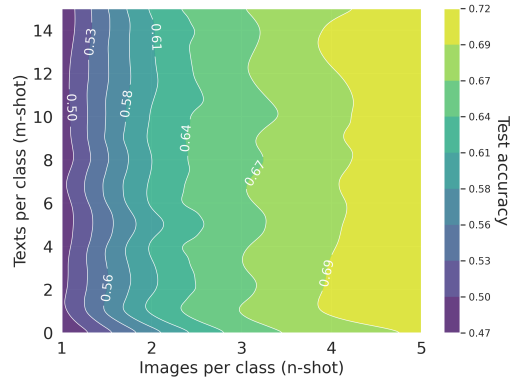


Figure 18: **UCF101**. 1 img $\approx$ 2617 words

785 **E.4.2 Aligned Encoders (CLIP)**

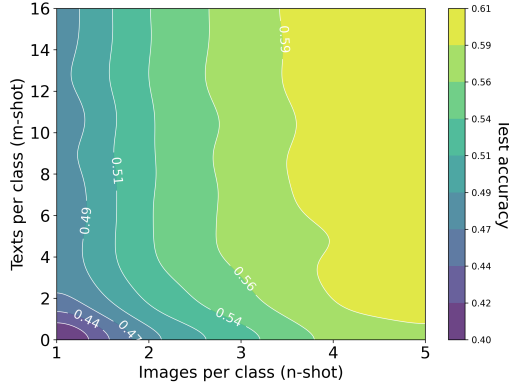


Figure 19: **SUN397**. 1 img $\approx$ 221 words

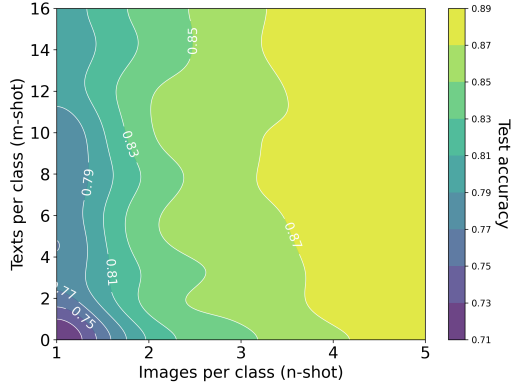


Figure 20: **Caltech101**. 1 img $\approx$ 256 words

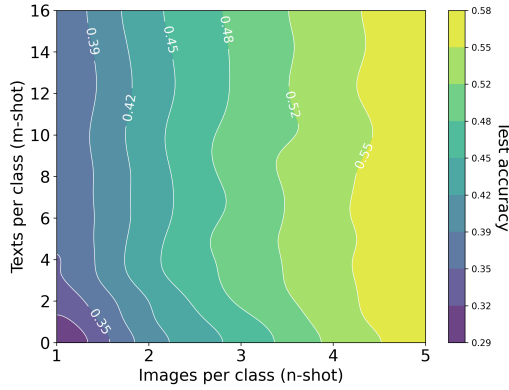


Figure 21: **Stanford Cars**. 1 img $\approx$ 649 words

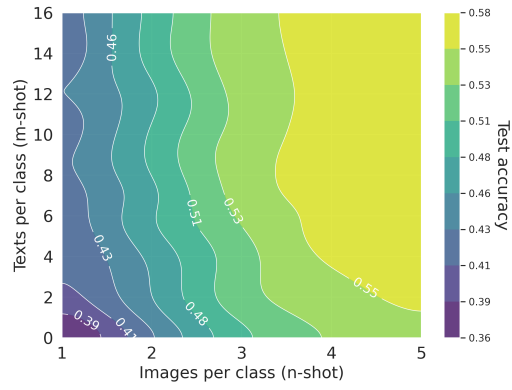


Figure 22: **DTD**. 1 img $\approx$ 228 words

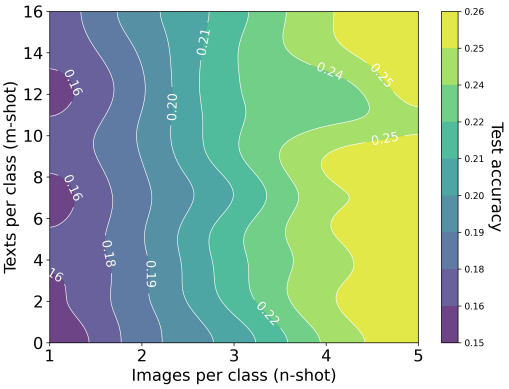


Figure 23: **FGVC Aircraft**. 1 img $\approx$ 691 words

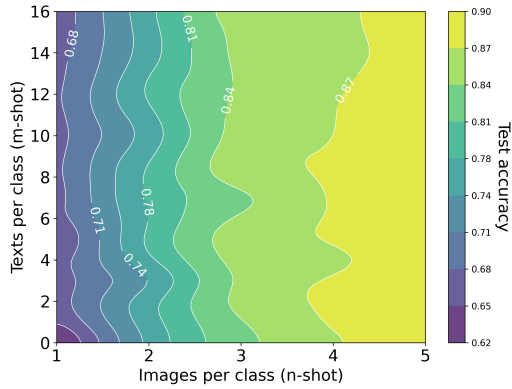


Figure 24: **Oxford Flowers**. 1 img $\approx$ 851 words

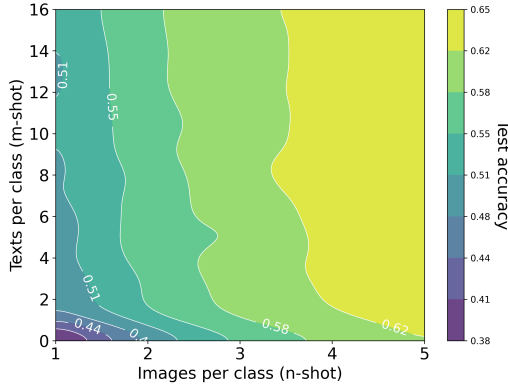


Figure 25: **Food101**. 1 img $\approx$ 202 words

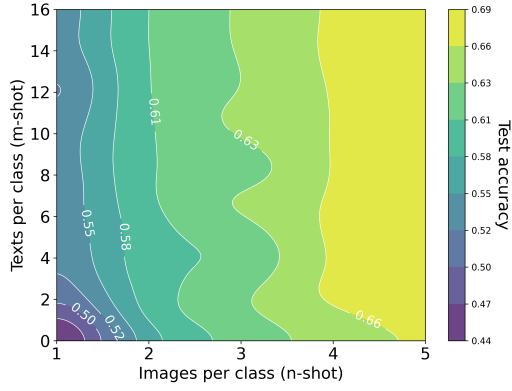


Figure 26: **UCF101**. 1 img $\approx$ 393 words



Figure 27: **Oxford Pets**. 1 img $\approx$ 228 words

E.5 Impact of Scaling Vision Backbone

In this section, we study how our method’s performance scales with the size and architecture of the vision backbone. In addition to ViT-S/14 DINOv2, we extend our analysis to a range of ViT-based architectures, including ViT-B/14 and ViT-L/14 DINOv2 and ViT-B/16 and ViT-B/8 DINO models. To ensure a fair comparison, we follow the same training protocol as in previous experiments. Our method consistently outperforms the unimodal baselines in every setting. In few-shot linear probing across ViT-B/8, ViT-B/16, DINOv2-ViTs and ViT-L/14 backbones (Tables 14 to 18), we see clear gains. The same holds for full-dataset end-to-end fine-tuning of both encoder and head (Tables 5, 6 and 8 and ??), and even when only the linear classifier is trained on the full splits (Tables 9 to 13).

E.6 Impact of Varying Text Encoders

In this section, we study how our method’s performance varies with different language models used for generating text embeddings. Through this experiment, we aim to understand how differences in embedding quality and model capacity affect the integration of textual information in our multimodal setup. Specifically, we cover LLMs with diverse architectures and scales, including BERT-Large, RoBERTa-Large and GPT-2 Large. As shown in Figure 28, adding unpaired text embeddings shows a significant boost in 1-shot accuracy and still decent gains at 16 shots on SUN397 dataset. Overall, OpenLLaMA-3B outperforms all other language models.

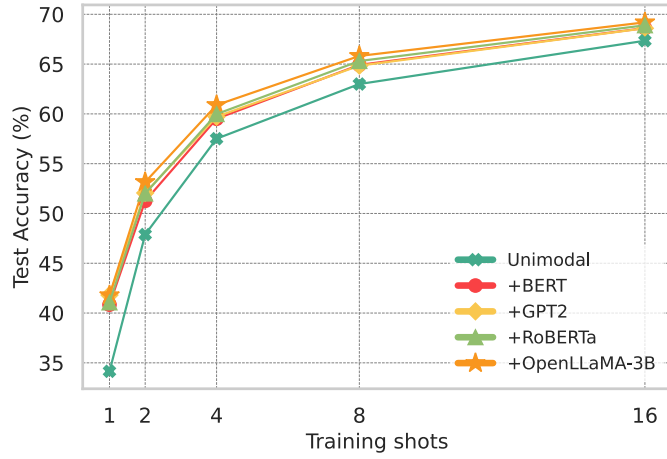


Figure 28: Few-shot classification accuracy on SUN397 using UML with unpaired, frozen embeddings from various pretrained language models.

803 E.7 Learning with Coarse-Grained vs. Fine-Grained Textual Cues

804 Understanding the type of information extracted from textual cues is crucial to assessing the effective-
 805 ness of our multimodal approach. A key question is whether the model merely utilizes class names or
 806 goes beyond to capture richer, more descriptive features. To investigate this, we compare the perfor-
 807 mance of our method using two types of text templates: a vanilla template that consists solely of the
 808 class name (e.g., "a photo of a [class]") and descriptive templates generated from GPT-3, as detailed
 809 in Section ?? and Figure 30, both multimodal approaches consistently outperform
 810 the unimodal baseline, with descriptions from GPT-3 offering a more substantial performance gain.
 811 This shows that leveraging richer, contextually diverse text cues can significantly enhance model
 812 performance, even in low-shot learning scenarios.

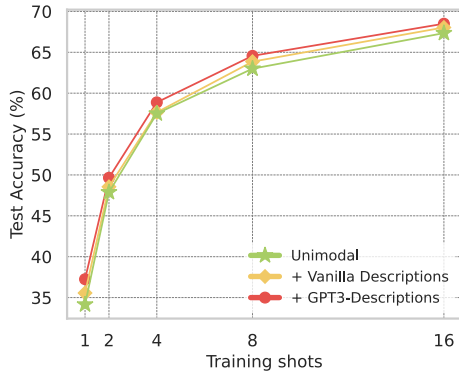


Figure 29: Few-shot SUN397 accuracy with UML using two levels of textual granularity: (a) vanilla class descriptions and (b) GPT-3-generated fine-grained descriptions.

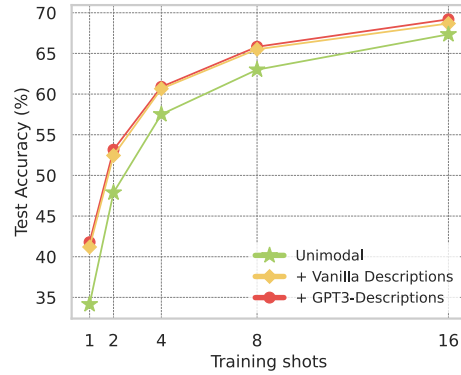


Figure 30: Few-shot SUN397 accuracy with UML (init) using two levels of textual granularity: (a) vanilla class descriptions and (b) GPT-3-generated fine-grained descriptions.

813 E.8 Impact on Performance with Increasing Unpaired Text Prompts

814 Here, we investigate how classification accuracy evolves as we augment each image with an increasing
 815 number of unpaired text prompts . Figure 31 shows these accuracy curves as we vary the number of

unpaired text prompts per image shot across five image-shot budgets. In every regime, our multimodal initialization (“Ours (init)”) outperforms training the head from scratch, with most of the gain coming from the first few prompts and gains tapering off thereafter. Note that we do not enforce diversity or novelty in the unpaired text prompts—simply adding more sentences does not guarantee additional information.

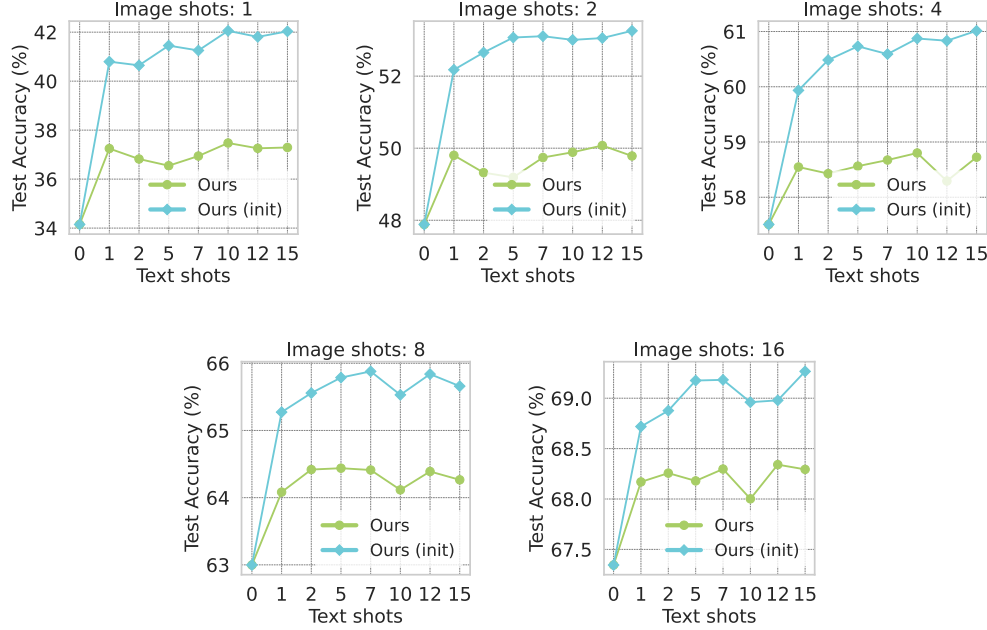


Figure 31: Classification accuracy as a function of the number of text prompts per image shot for the SUN397 Dataset.

E.9 Additional Experiments for Audio-Visual Setting

In this section, we extend our unpaired multimodal framework to the tri-modal ImageNet-ESC benchmark, examining how unpaired audio and text signals can enhance image classification under both aligned (Appendix E.9.2) and unaligned encoders (Appendix E.9.1). We then reverse the setting, showing that unpaired visual image benchmarks (Appendix E.9.3) improves audio classification (Appendix E.9.3).

E.9.1 Improving Image Classification with Unpaired Audio and Text (Unaligned encoders)

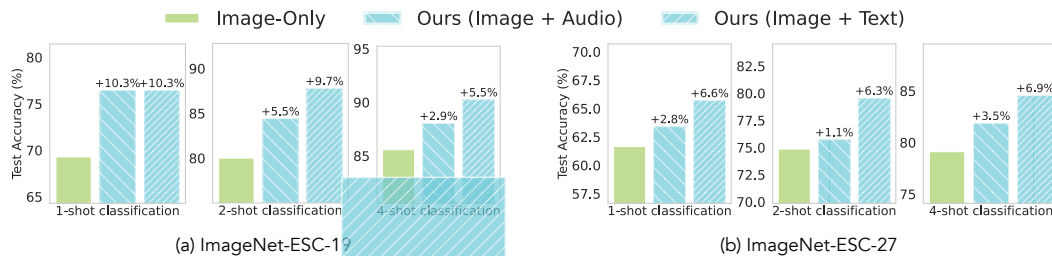


Figure 32: UML improves image classification using unpaired audio and text samples on both ImageNet-ESC-19 and ImageNet-ESC-27 benchmarks when trained on top of DINOv2 ViT-S/14 and OpenLLaMa-3B.

E.9.2 Improving Image Classification with Unpaired Audio and Text (Aligned encoders)

Image (RN50 CLIP)

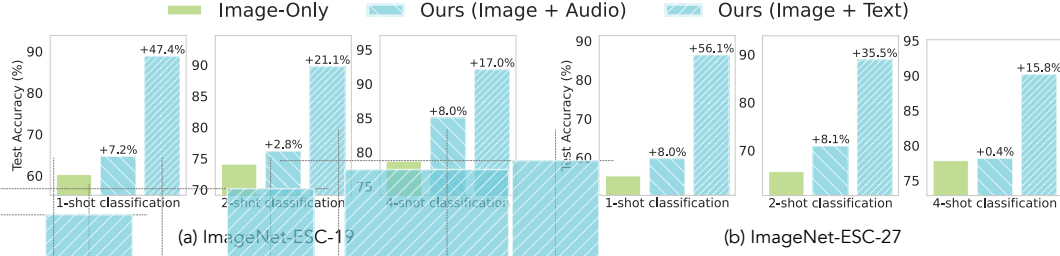


Figure 33: UML improves image classification using unpaired audio and text samples on both ImageNet-ESC-19 and ImageNet-ESC-27 benchmarks when trained on top of CLIP ResNet-50 image and text encoders.

Audio Benchmarks (Appendix)

E.9.3 Improving Audio Classification with Unpaired Image and Text (Aligned encoders)

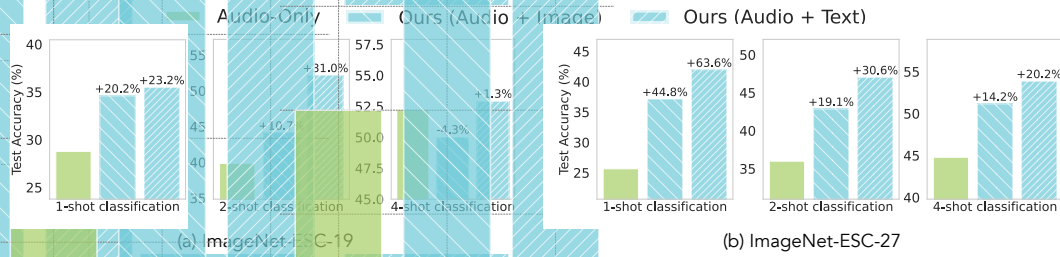


Figure 34: UML improves audio classification using unpaired image and text samples on both ImageNet-ESC-19 and ImageNet-ESC-27 benchmarks when trained on top of CLIP ResNet-50 image and text encoders.

E.10 Gaussian Experiments

Here, we shift our attention to a more nuanced and intriguing question: can incorporating unpaired multimodal data actually improve the *reconstruction* quality of a single modality? At first glance, this seems unlikely—why would adding data from a different modality make X reconstruction better than training with X ? Moreover, we push this question further: can incorporating data from a different modality, *while keeping the total dataset size fixed*, still improve the reconstruction of X compared to using the same number of samples X dataset alone? This setup isolates the importance of multimodal information from mere data scaling, and surprisingly, our experiments show that this improvement is indeed possible.

To investigate this, we design a synthetic experiment inspired by our theoretical framework in ?? . We generate data from two partially overlapping modalities, X and Y , derived from a shared latent space θ_c , while also containing unique components (θ_x and θ_y). The observations follow the same linear structure as in our theory:

$$\begin{aligned} X_i &= A_{c,i}\theta_c + A_{x,i}\theta_x + \epsilon_{X,i} \\ Y_j &= B_{c,j}\theta_c + B_{y,j}\theta_y + \epsilon_{Y,j} \end{aligned}$$

The overlap ratio, denoted as p , controls how much of the shared latent dimensions are jointly captured by both X and Y . We set $p = 0.2$, meaning that only 20% of the shared latent dimensions are observed by both modalities, while the remaining 40% are exclusively captured by X and Y respectively. This structured overlap ensures that neither modality alone can fully reconstruct the shared latent space, forcing the model to integrate complementary information from both.

Our architecture consists of a shared autoencoder with separate input projections for X and Y . Each modality is first encoded through a modality-specific linear projection layer, followed by a shared latent encoder composed of two layers with ReLU nonlinearity. The encoded representation is then passed through a decoder, also consisting of two linear layers, to reconstruct the input. We use separate heads for the final reconstruction, while keeping the latent space shared to promote cross-modal alignment.

As shown in Figure 35, the surprising outcome is that training on both modalities, even when they are unpaired, consistently improves the reconstruction of X compared to training solely on X . More strikingly, this improvement holds even when the total number of training samples is fixed, with half the data coming from X and half from Y ; showing that the model is not just benefiting from increased data quantity but from the diversity and complementary information provided by the second modality.

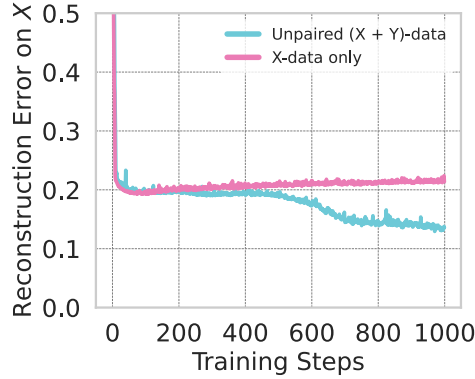


Figure 35: Training on $N/2$ samples from X and $N/2$ unpaired samples from Y improves test reconstruction on X , more than training on N samples from X .

F Analysis of the Learned Classifier

F.1 Change in Decision Boundaries with Unpaired Data from Another Modality

Our decision boundary visualizations are constructed by projecting the high-dimensional embedding space of a given classifier to a 2D plane. Axis 1 is computed as the normalized difference between the classifier weights of the two selected classes, representing the primary decision direction. Axis 2 is chosen to be orthogonal to Axis 1, constructed from the difference between the class mean embeddings after removing the component parallel to Axis 1. This orthogonalization ensures that the two axes capture complementary aspects: Axis 1 reflects the primary model decision boundary, while Axis 2 captures the variation orthogonal to that decision. The final 2D projection matrix combines these two vectors as columns, and embedding vectors are then mapped to this plane using a simple dot product. Figure 36 and Figure 37 show the change in decision boundary when adding unpaired textual information for 2-shot classification on top of frozen CLIP ResNet-50 features for DTD and Oxford Flowers datasets.

F.2 What do models learn from unpaired data?

To understand what the model is truly learning and how its weights evolve, we develop and analyze three key metrics: functional margin, silhouette score, and class-prototype vectors. These metrics inform on how well the model distinguishes between classes and how text information influences the structure of feature-space

Functional margin. This quantifies how confidently a model separates a given sample from the decision boundary. For a sample i belonging to class y , we calculate the margin relative to the next highest competing class. Specifically, we identify the second-highest logit among the incorrect classes, denoted as class j^* , and compute the functional margin as

$$\gamma_i = \frac{w_y^T x_i - w_{j^*}^T x_i}{\|w_y - w_{j^*}\|_2} \quad (3)$$

where $w_y^T x_i$ represents the logit for the true class, while $w_{j^*}^T x_i$ represents the highest logit among the competing classes. Larger margins indicate more confident and robust classification, while smaller

Additional Plots: DTD

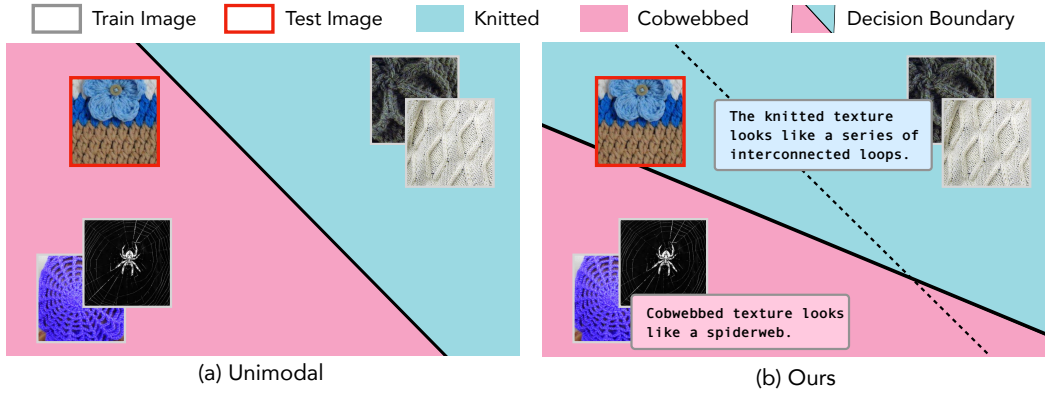


Figure 36: **Impact of unpaired text on decision boundaries (CLIP ResNet50).** (Left) Visual features alone learn ambiguous class boundaries between knitted and cobwebbed. (Right) Adding unpaired text sharpens the boundary, leveraging semantic cues to better distinguish similar categories

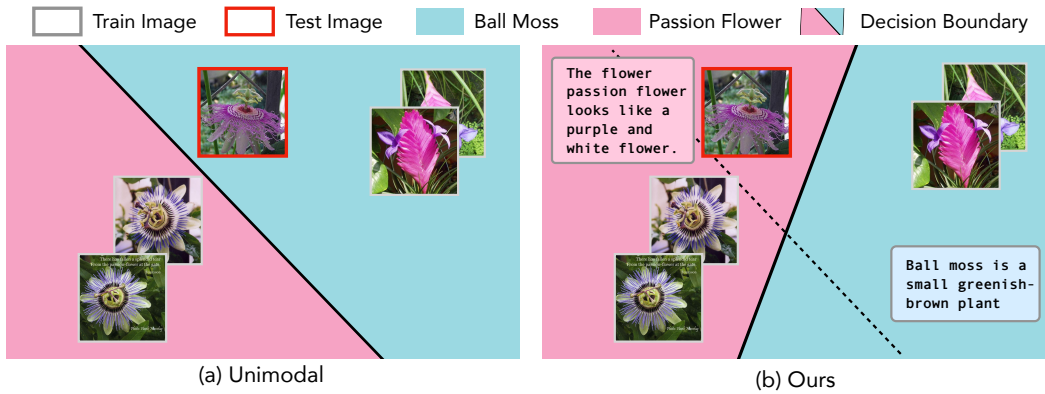


Figure 37: **Impact of unpaired text on decision boundaries (CLIP ResNet50).** (Left) Visual features alone learn ambiguous class boundaries between ball moss and passion flower. (Right) Adding unpaired text sharpens the boundary, leveraging semantic cues to better distinguish similar categories

884 margins imply that the sample lies closer to a misclassification boundary. As shown in Figure 38, both
 885 *Ours* and *Ours (init)* exhibit substantially larger classification margins than the unimodal baseline,
 886 demonstrating that augmenting primary-modality training with unpaired multimodal data improves
 887 confidence in predictions over the primary modality.

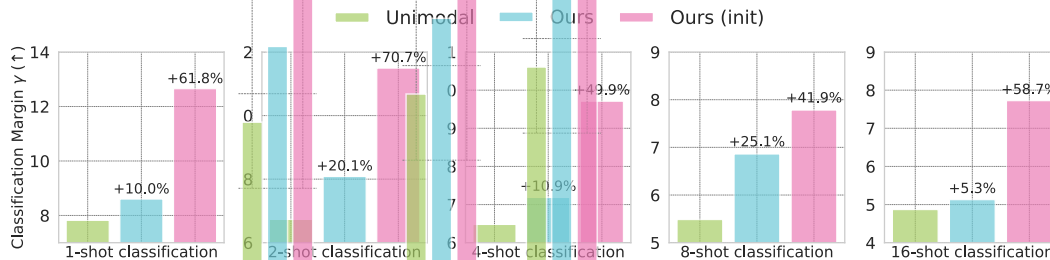
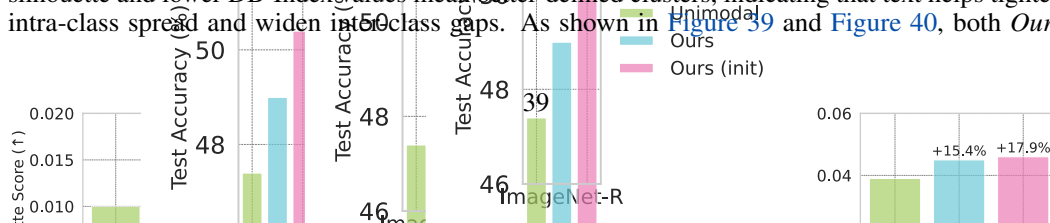


Figure 38: Functional margin of the linear head trained on SUN397 dataset for few-shot classification significantly increases when training with both UML and UML with linear head initialization.

888 **Silhouette Score and DB-Index.** The Silhouette Score indicates how well-separated the clusters
 889 are, while the DB-Index measures intra-class compactness versus inter-class separation. Higher
 890 silhouette and lower DB-Index values mean better-defined clusters, indicating that text helps tighten
 891 intra-class spread and widen inter-class gaps. As shown in Figure 39 and Figure 40, both *Ours*



and *Ours (init)* exhibit reduced intra-class distances and increased inter-class separations, further confirming improved class separability.

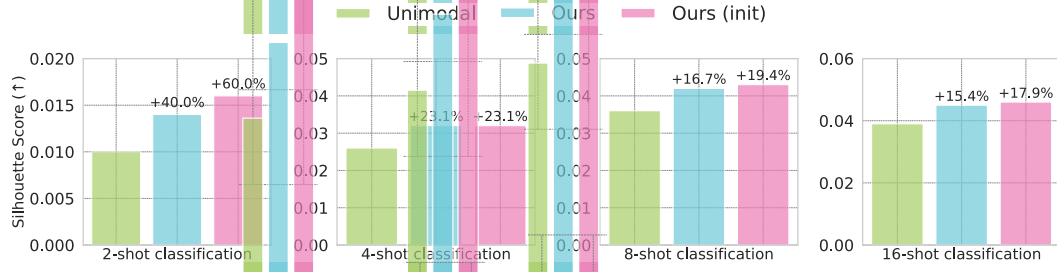


Figure 39: Silhouette Score of the linear head trained on SUN397 dataset for few-shot classification significantly increases when training with both UML and UML with linear head initialization.

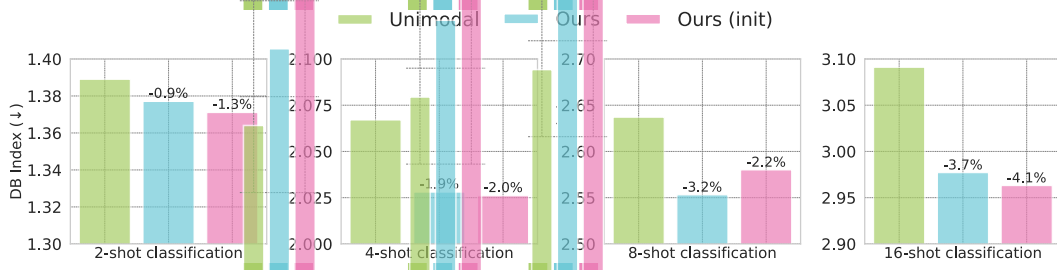


Figure 40: DB-Index of the linear head trained on SUN397 dataset for few-shot classification significantly improves when training with both UML and UML with linear head initialization.

Class-Prototype Vectors. These vectors are the rows of the final linear layer’s weight matrix, representing the class centroids in the shared embedding space. We compute a heatmap of inner products between class prototypes and average text embeddings of the corresponding class to assess how well text features align with class centers. This helps reveal how the model organizes multimodal information. Figure 41 shows a pronounced diagonal structure, indicating that each class’s text embedding aligns closely with the learned weights of the model.

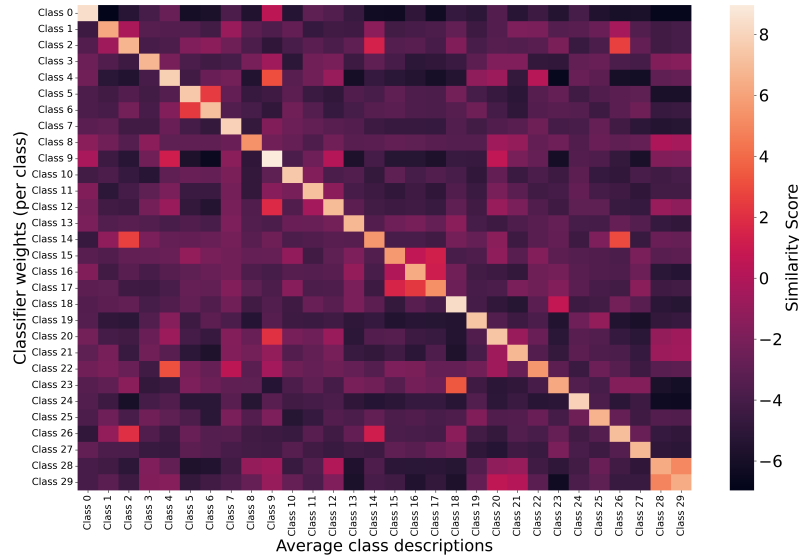


Figure 41: Inner products between each linear-head weight vector and its class’s mean text embedding, demonstrating that text features align well with class prototypes.

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