$2k_F$ instability and chiral spin density wave at the 1/9 magnetization plateau in the kagome antiferromagnets

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1. Introduction

Advanced machine learning (ML) techniques and in particular neural-network quantum states (NQS) based variational methods have recently been recognized as powerful computational methodology to study quantum many-body systems. Such techniques have proven to compete with, and in some cases surpass, well-established traditional formalisms like tensor networks and density matrix renormalization group (DMRG) methods. The main aim of various machine learning approaches is to allow unbiased high accuracy calculations that can outperform traditional approaches which are often biased towards particular kinds of states or face other limitations. For example, among the most successful traditional approaches are quantum Monte Carlo (QMC) methods and well-established DMRG algorithm. Whilst unbiased QMC methods face still unsolved sign problem, matrix product states within DMRG algorithm are biased towards gapped low entangled states and can fail to recognize highly entangled gapless phases. Development of unbiased high accuracy machine learning algorithms is therefore of utmost importance and presents crucial step in clarifying properties of various unconventional phases in numerous strongly correlated and frustrated many-body systems.

In this work we present novel ML study of the 1/9 magnetization plateau in the kagome antiferromagnets placed in an external magnetic field. Our ML approach combines symmetry enhanced NQS [1,2], variational Monte Carlo (VMC) and flux insertion method providing significant improvement of accuracy in comparison with previous studies with different methods. In addition to significantly improved results accuracy our study reveals novel $2k_F$ instability of the underlying spinon Fermi surface and a chiral spin density wave (CSDW) ground state at the 1/9 plateau [3] not previously found with other methods. The spin wave chirality results from the correlated spin order that reflects its nontrivial topology and properties of the found CSDW can readily explain recent experimental findings based on torque magnetometry measurements on YCOB single-crystal samples. The results in this work therefore provide an important starting point for further study of topologically nontrivial magnetization plateaus in the real kagome quantum antiferromagnets with material inspired additional interactions. Our findings could also contribute to our understanding of the density wave formation in high temperature superconducting materials which is one of the most intriguing problems in condensed matter physics. Additionally our study confirms advantage of the advanced ML approaches in discovering new materials properties and importance of further development of similar algorithms with improved efficiency and scalability.

Detailed description of our ML approach and results for the 1/9 magnetization plateau properies in the kagome antiferromagnets are presented in recent preprint: arXiv:2512.11670.

2. Symmetry enhanced NQS, free energy optimization and flux insertion method

Our numerical approach combines symmetry enhanced NQS ansätze, in particular group equivariant convolutional neural networks (GCNNs), that describe eigenstates of the system, VMC and flux insertion method [3]. GCNNs are deep neural network architectures that are in general composed of an arbitrary number of layers and that take into account all space group symmetries (kagome lattice translations and point group symmetries such as rotations and reflections). Within our VMC approach free energy loss function is minimized with natural gradient descent optimization algorithm (equivalent to the stochastic reconfiguration algorithm) used for neural network training. Our calculations are additionally supplemented by flux insertion method that improves stability and convergence of the results. The flux insertion method is illustrated in Fig. 1.

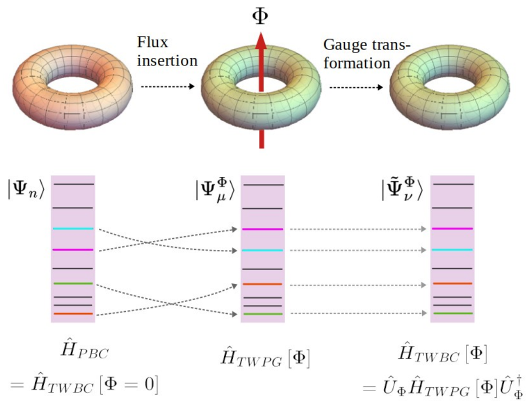

Fig. 1: Flux insertion method: Insertion of unit flux quantum through a torus cycle corresponds to a unitary transformation of the original Hamiltonian and therefore does not change many-body energy spectrum [3].

3. Ground state properties: Chiral spin density wave

Numerous studies show that presence of geometric frustration in quantum magnets results in a wide variety of exotic phases of matter. Kagome lattice antiferromagnets are paradigmatic examples of frustrated quantum magnets that can host various complex quantum states exhibiting unique properties such as long-range quantum entanglement, topological order and excitations with fractional quantum numbers. In the presence of an applied magnetic field, the magnetization exhibits a sequence of field induced plateaus.

Particularly intriguing state appears at the 1/9 magnetization plateau. The nature and exotic physical properties of the plateau however remain controversial since various studies using different methods lead to different conclusions about the properties of the ground state at the plateau due to an exceptional complexity of the state generated by geometrical frustration. While DMRG calculations [4] and recent VMC studies [5], based on fermionic parton representation for spin operators and Gutzwiller projection, indicate that the plateau ground state is a Z_3 quantum spin liquid, infinite projected entangled pair states (iPEPS) calculations find a valence bond crystal ground state exhibiting an hourglass pattern [6]. In addition, a separate variational calculation based on more general resonating valence bond (RVB) ansatz finds a VBC state with a 3×3 periodicity and a windmill-shaped motif [7]. Recent experimental findings based on torque magnetometry measurements on YCOB single-crystal samples, however, indicate presence of Dirac-like spinons at the 1/9 magnetization plateau that manifest as unconventional magnetic oscillations in magnetic torque [8]. Contrary to previous studies our ML study reveals that the ground state at the 1/9 plateau is a gapless 1×1 CSDW caused by $2k_F$ instability of the underlying composite Fermi liquid [3]. The spin wave chirality results from the correlated spin order that reflects its nontrivial topology.

Appearance of the spin density wave and $2k_F$ instability can in general be seen from the spin structure factor calculations. The spin-1/2 operators in a kagome antiferromagnet can be represented in terms of fermionic spinons. The spin structure factor can clearly identify $2k_F$ scattering features caused by the scattering close to the spinon Fermi surface and $2k_F$ instability that causes appearance of sharp peaks in the structure factor reflecting spin density wave ordering with the wave vector corresponding to the position of the peaks. In addition our calculations of the spin components parallel to the applied field reveal 1×1 density wave ordering. Our main results for the spin structure factors and spin components are shown in Fig. 2. A linear temperature dependence of the magnetic susceptibility found in the recent experiments on YCOB single-crystal samples appears as a consequence of the SDW antiferromagnetic correlations. Our results are therefore consistent with recent experimental findings.

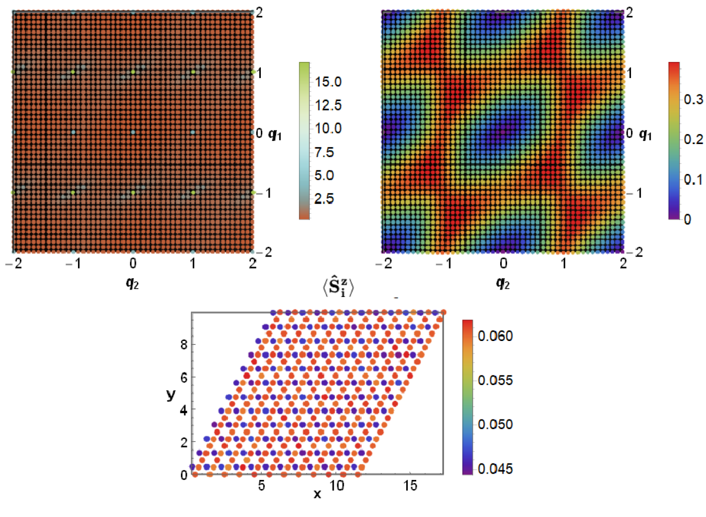

Fig. 2: *Spin structure factors reveal presence of the underlying spinon Fermi surface and an instability of the Fermi surface towards formation of a spin density wave [3].*

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