We want to highlight that **MTO in these results is obtained by training only ϕ while keeping θ frozen**, which supports our novelty and contribution in parameterizing and optimizing the *Adaptive Multidimensional Coefficient* and *Multidimensional Trajectory Optimization*.

SI (Stochastic Interpolants), FM (Flow Matching), and DDPM (Denoising Diffusion Probabilistic Model) are the flow and diffusion models used to validate the adversarial approach to MTO. X_{LPF} denotes the hypothesis space γ_ϕ with low-pass filtering applied, while $X_{\text{non-LPF}}$ represents the hypothesis space without low-pass filtering. The parameter s indicates the scale value used in these configurations.

Experiments for pre-training stage :

Table 1: Comparison of FIDs ↓ between unidimensional coefficient and multidimensional coefficient (non-LPF and LPF) for unoptimized paths using the Euler solver.

	CIFAR-10				ImageNet-32			
Method \ NFE	10	100	150	200	10	100	150	200
$\text{SI}_{\text{unidimensional}}$	**14.43**	4.75	4.51	4.30	17.72	8.08	7.79	7.63
$\text{SI}_{\text{non-LPF}_{s=0.005}}$	14.59	3.98	3.74	**3.63**	**17.41**	**6.33**	**6.21**	**6.20**
$\text{SI}_{\text{LPF}_{s=0.1}}$	15.44	**3.77**	**3.68**	3.75	17.86	6.63	6.47	6.44
$\text{FM}_{\text{unidimensional}}$	**13.70**	4.52	4.23	4.07	16.92	7.78	7.53	7.38
$\text{FM}_{\text{non-LPF}_{0.005}}$	13.81	**3.59**	**3.42**	**3.42**	**16.85**	**6.18**	**6.03**	**6.01**
$\text{FM}_{\text{LPF}_{0.1}}$	15.13	3.64	3.57	3.64	17.52	6.40	6.27	6.31
$\text{DDPM}_{\text{unidimensional}}$	98.47	6.64	4.84	4.10	**111.54**	8.13	7.40	7.14
$\text{DDPM}_{\text{non-LPF}_{0.005}}$	74.44	**3.77**	5.96	7.84	139.69	7.67	12.37	11.70
$\text{DDPM}_{\text{LPF}_{0.005}}$	72.23	4.73	**4.11**	**3.83**	135.48	6.84	**6.51**	**6.42**
$\text{DDPM}_{\text{LPF}_{0.1}}$	**71.80**	4.46	6.32	12.60	142.99	**6.70**	8.69	10.91

Table 2: FIDs for different σ for the kernel in low-pass filter in $\text{SI}_{\text{LPF}_{0.1}}$ using an unoptimized path on CIFAR-10.

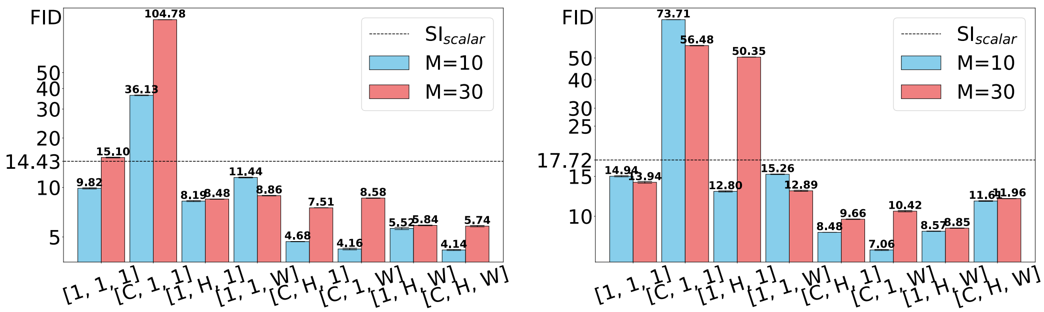
σ \ NFE	10	20	30	40	50	100	150	200
0.1	**14.89**	**8.05**	6.49	5.45	6.53	9.59	10.67	11.20
1.0	14.92	8.47	**5.32**	**4.45**	**4.68**	6.06	7.01	7.50
2.0	16.25	9.56	7.56	6.06	4.72	**3.77**	3.95	4.17
4.0	15.44	9.13	7.39	6.36	5.59	3.77	**3.68**	**3.75**

By the experiments in Table 1 and Table 2, we can identify the appropriate choice of hypothesis space for pre-training.

Experiments for adversarial training stage :

Table 3: FIDs for path optimizations with 10 NFE Euler solver on CIFAR-10.

Method \ M	5	10	15	20	25	30
$\mathrm{SI}_{\mathrm{LPF}_{0.1}}$	6.89	**4.14**	4.42	5.32	6.11	5.74
$\mathrm{FM}_{\mathrm{LPF}_{0.1}}$	**5.93**	6.13	6.70	6.18	5.97	6.42
$\mathrm{DDPM}_{\mathrm{LPF}_{0.1}}$	10.15	10.04	9.60	9.04	**8.94**	9.19



(a) CIFAR-10

(b) ImageNet-32

Figure 1: Comparison of FIDs for path optimization using $\mathrm{SI}_{\mathrm{LPF}_{0.1}}$ and 10 NFE with different constraints on the output dimension of γ_ϕ for $M = 10$ and $M = 30$.

In Figure 1, [X, X, X] represents the on/off values for the freedom of each axis in MTO. $\mathrm{X}_{\mathrm{scalar}}$ indicates a model trained with unidimensional coefficient.

Table 4: FIDs for path optimizations using $\mathrm{SI}_{\mathrm{LPF}_{0.1}}$ with different NFE on CIFAR-10.

Method \ NFE	4	6	8	10
$\mathrm{SI}_{\mathrm{LPF}_{0.1}}$	20.59	6.62	4.85	**4.14**
$\mathrm{FM}_{\mathrm{LPF}_{0.1}}$	16.42	8.17	6.56	**6.13**
$\mathrm{DDPM}_{\mathrm{LPF}_{0.1}}$	72.64	20.13	13.72	**10.04**

Table 5: FIDs for path optimizations with 10 NFE and different inputs to γ_ϕ on CIFAR-10.

Method \ Input	1	z	x_T
$\mathrm{SI}_{\mathrm{LPF}_{0.1}}$	7.84	6.48	**4.14**
$\mathrm{FM}_{\mathrm{LPF}_{0.1}}$	9.20	9.06	**6.13**
$\mathrm{DDPM}_{\mathrm{LPF}_{0.1}}$	26.09	23.31	**10.04**

As in Table 4, MTO can be applied to different sampling configurations. In Table 5, we validate the use of the "Adaptive Multidimensional Coefficient," which is conditioned on the starting point of the differential equation, x_T.

Table 6: FIDs for path optimizations with 10 NFE and SI trained using various hypothesis space γ_ϕ on CIFAR-10.

Method \ M	5	10	15	20
t	10.20	9.75	11.30	11.53
non-$\mathrm{LPF}_{0.005}$	6.60	4.79	4.45	5.28
$\mathrm{LPF}_{0.005}$	7.37	4.42	4.26	5.31
$\mathrm{LPF}_{0.1}$	7.21	**4.14**	5.59	5.32

In Table 6, we can see appropriate choices for the hypothesis space for MTO.