Approximate Message Passing (AMP) Algorithms

AMP is a family of PnP algorithms that have remarkable properties for large random A:

- The denoiser input-error is white and Gaussian with predictable variance.
- When used with an MMSE denoiser, AMP algorithms converge to the MMSE estimate of \( x_0 \) from \( y \).

Challenge: In most image recovery problems, \( A \) does not satisfy AMP’s randomness assumptions.

AMP for Fourier-Structured Matrix \( A = MF \)

Idea: Recover the wavelet coefficients \( c_0 \), not pixels \( x_0 \).

Why? The resulting model becomes \( y = Bc_0 + w \), where the masked Fourier-wavelet \( B = MF\Psi^T \) is approximately block-diagonal with sufficiently randomizing blocks.

With appropriate algorithm design, the denoiser input-error will be white and Gaussian in each wavelet subband.


Note: These algorithms provide well-characterized errors, but a non-standard denoiser is required to exploit them!