Filter Equivariant Functions

A symmetric account of length-general extrapolation on lists

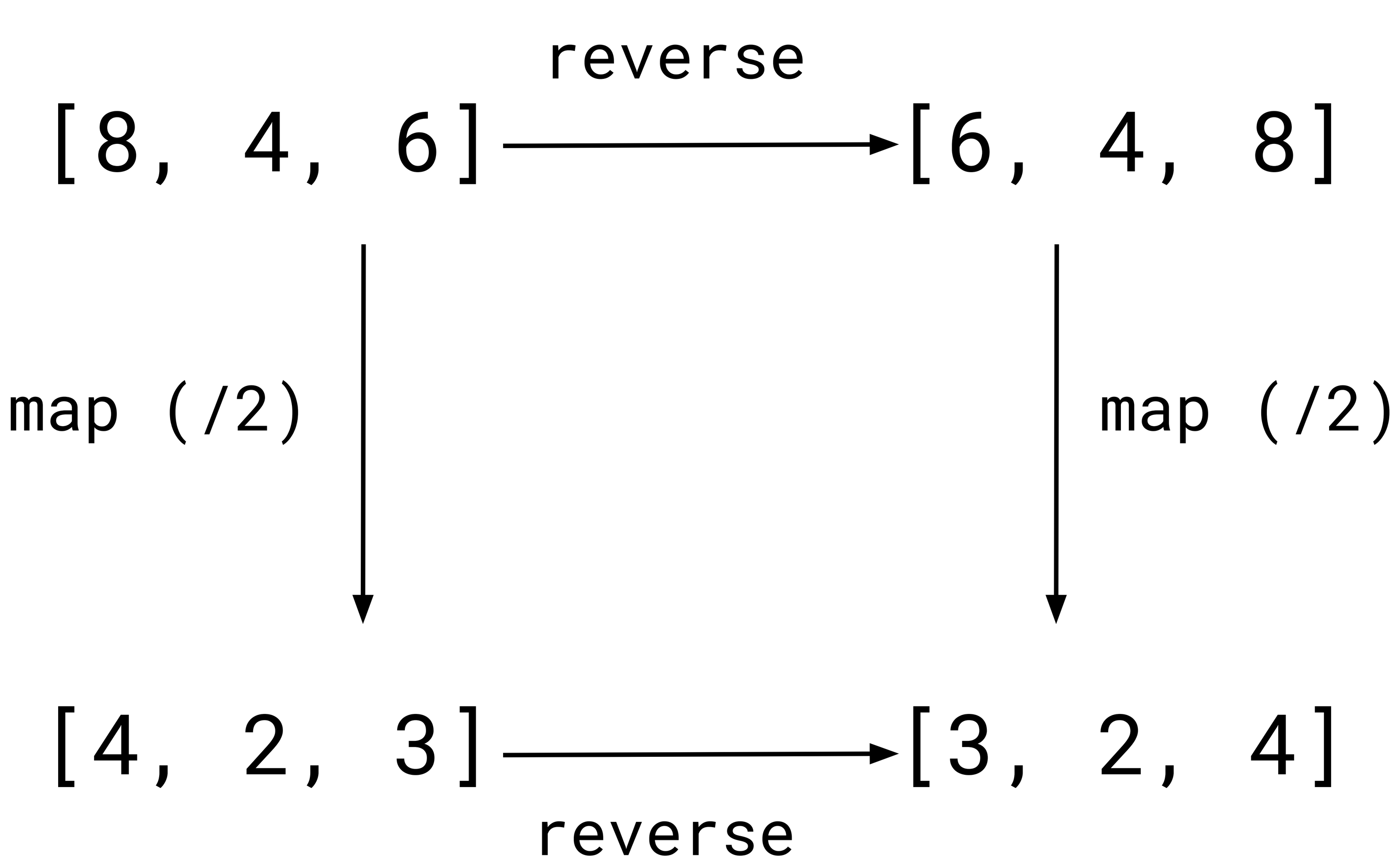
Owen Lewis, Neil Ghani, Andrew Dudzik, Christos Perivolaropoulos, Razvan Pascanu, Petar Veličković
Goodfire AI   Kodamai   Google DeepMind

Quantifying length generalization

```
[8, 4, 3]       → [3, 4, 8]
[8, 4, 3, 9, 1] → ?
```

Position generalization ⇔ translation symmetry
Length generalization ⇔ ?? symmetry

Background: element generalization via map symmetry

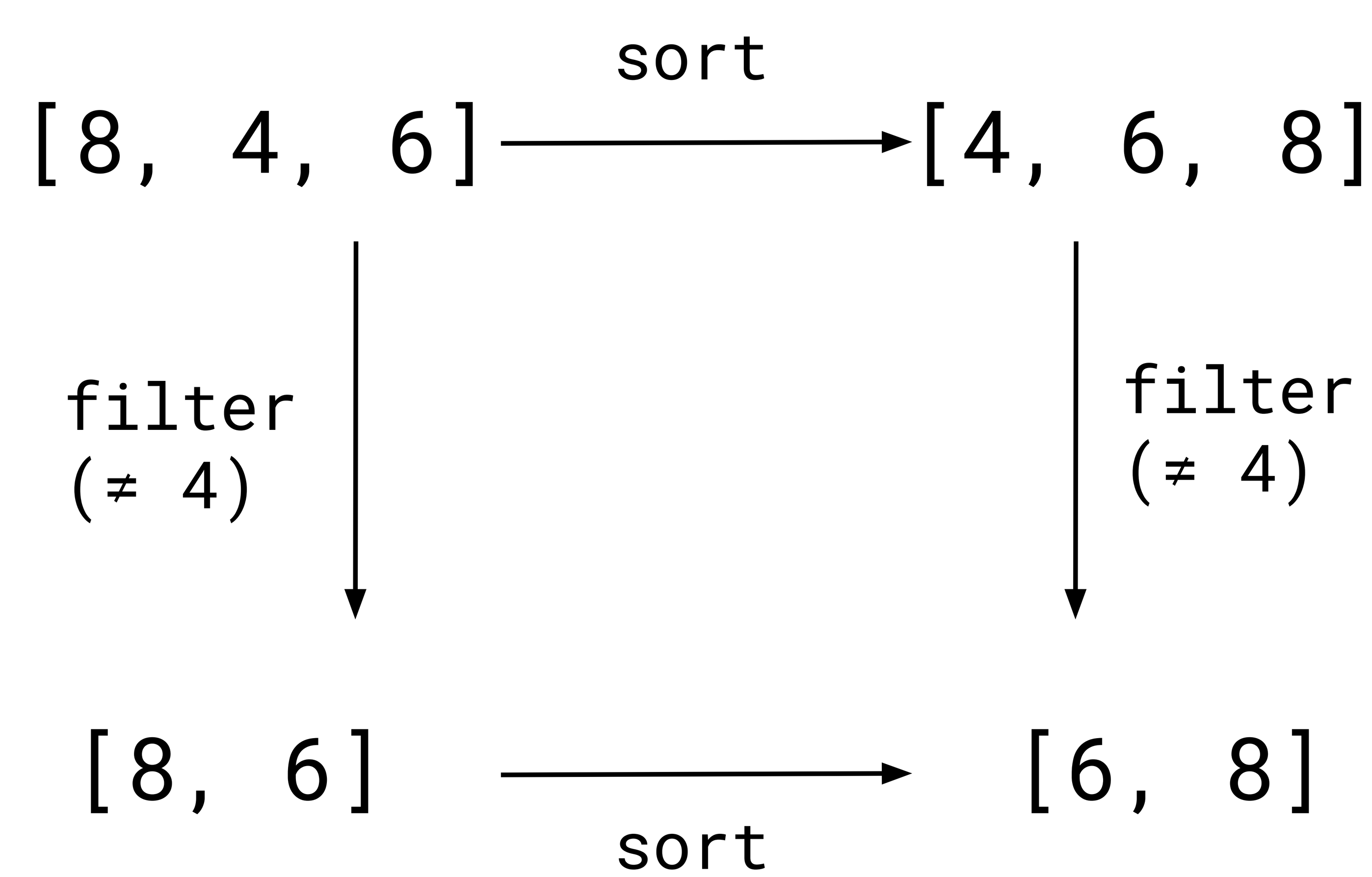



```
f : [a] → [a]
g : a → b
f ∘ (map g) = (map g) ∘ f
```

- E.g.: `reverse, double, inflate, etc.`
- Natural transformation from list functor to itself.

Length generalization via filter symmetry

```
filter :: (a -> Bool) -> [a] -> [a]
filter even [1, 2, 3, 4] = [2, 4]
```



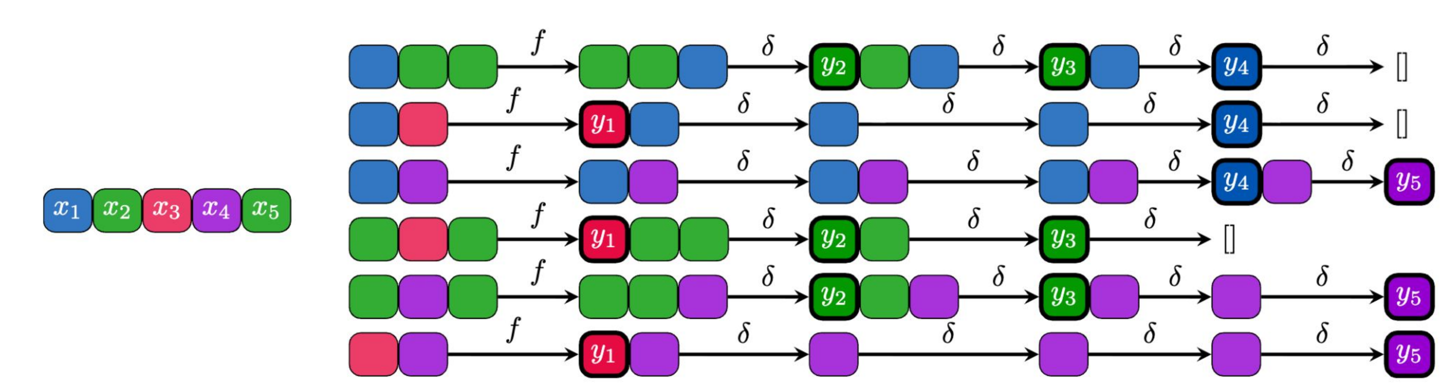
- Natural filter equivariants (NFEs) commute with `map` and `filter p`, for all p.
  - `reverse, double, inflate, …`
- Filter equivariants (FEs) (`filter` but not `map`)
  - `sort, filter, uniquify, …`
- `map` but not `filter`
  - `[8, 4, 6] ⟼ [8, 4, 4, 6, 6, 6]`

Short-to-long generalization

- NFEs are determined by length-two lists
- FEs are determined by lists with two unique elements

Intuition: outputs on filtered lists give order constraints

```
[8, 4, 3] → {[8, 4], [8, 3], [4, 3]}
             (apply f)
          → {[4, 8], [3, 8], [3, 4]}
          → {4 < 8, 3 < 8, 3 < 4}
          → [3, 4, 8]
```



Other results

- NFEs have categorical model in (semi-)simplicial sets
- NFEs have an an inductive type
- NFEs can be characterized by the length, k, to which the map a singleton list.
  - The number of k-NFEs is $2 \times 3^{k-1}$ when $k \geq 1$
- FEs obey certain closure laws under composition, concatenation, higher-order functions