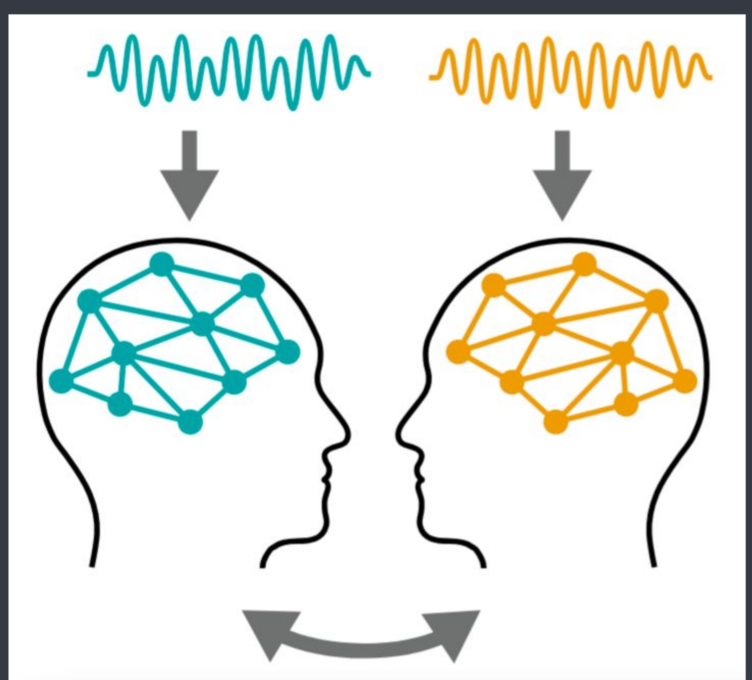
We propose leveraging **discrete curvatures** to examine the **dynamic reconfigurations** in **neural interactions** during **social exchanges**. Changes in **curvature distributions** across **interbrain networks** can reveal **phase transitions** and **information-routing strategies**, yielding more **mechanistic insight** than correlation-based synchrony metrics.

Hyperscanning generates interbrain networks...



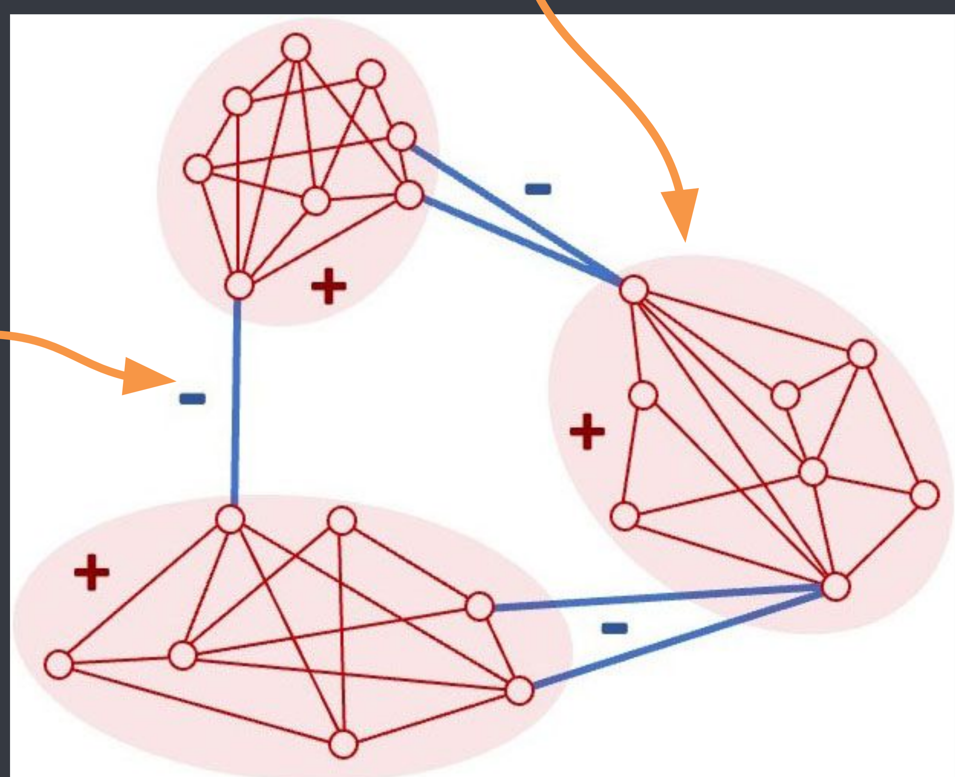
...which may be characterized by the curvatures of their edges:

Positive Curvature (Contraction):

The neighborhoods of these nodes have significant overlap, so information will "bounce around" this part of the network instead of flowing away

Negative Curvature (Expansion):

The neighborhoods of the two nodes connected by this edge do not overlap, so information spreads out away from these nodes



On a Geometry of Interbrain Networks

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Introduction

Interbrain networks represent the joint neural connectivity of two or more individuals as interconnected nodes within weighted graphs, constructed via hyperscanning (the simultaneous recording of neural signals from interacting individuals), with each node corresponding to a neural region and the edge weights derived by computing synchrony metrics from the neural activity in these regions.

Inspired by the successful integration of geometric insights in network science, we propose leveraging discrete geometry to examine the dynamic reconfigurations in neural interactions during social exchanges. Unlike conventional synchrony approaches, our method interprets inter-brain connectivity changes through the evolving geometric structures of neural networks.

Methods

Central to our proposal are discrete curvatures, one example of which is the Forman-Ricci curvature (FRC). Specifically, the FRC of an edge *e* connecting nodes *i* and *j* in a weighted network is defined as

$$F(e) = w_e \left(\frac{z_i}{w_e} + \frac{z_j}{w_e} - \sum_{e_i \sim i, e_i \neq e} \frac{z_i}{\sqrt{w_e w_{e_i}}} - \sum_{e_j \sim j, e_j \neq e} \frac{z_j}{\sqrt{w_e w_{e_j}}} \right)$$

where z_i and z_j represent node weights and w_e denotes edge weights corresponding to neural connectivity strength.

Ollivier-Ricci curvature (ORC) represents an alternative notion of discrete Ricci curvature which provides a comparable characterization of network geometry. Its definition via Markov chains lends itself toward mechanistic interpretations of inter-brain connectivity in terms of information routing strategies: regions with a high density of edges with low (negative) curvature promote shortest-path traversal, while regions with higher (positive) curvature promote diffusion.

To capture significant dynamic shifts in network configurations, we examine divergences over time in the differential entropy of graph curvature distributions of interbrain networks. These phase transitions can then be correlated with various measures of social interaction.

Results

As a proof-of-concept, we simulate time-varying brain networks modeled as weighted small-world networks with varying rewiring probability *p*. **A–D:** Four examples with nodes $N = 100$, mean degree $K = 5$, and different *p*. **E:** Entropy of the FRC distribution as *p* evolves from 0 to 1 for $N = 1000$, $K = 50$; note phase transition around $p = 10^{-2}$ due to increased neighborhood overlap and shortcut formation, marking a transition from a segregated, lattice-like topology to a more integrated small-world/random regime. **F:** Corresponding quantiles of the FRC distribution. Solid curves show the median over 200 replications; shaded areas mark 0.05 and 0.95 quantiles.