Can neural networks learn persistent homology features?

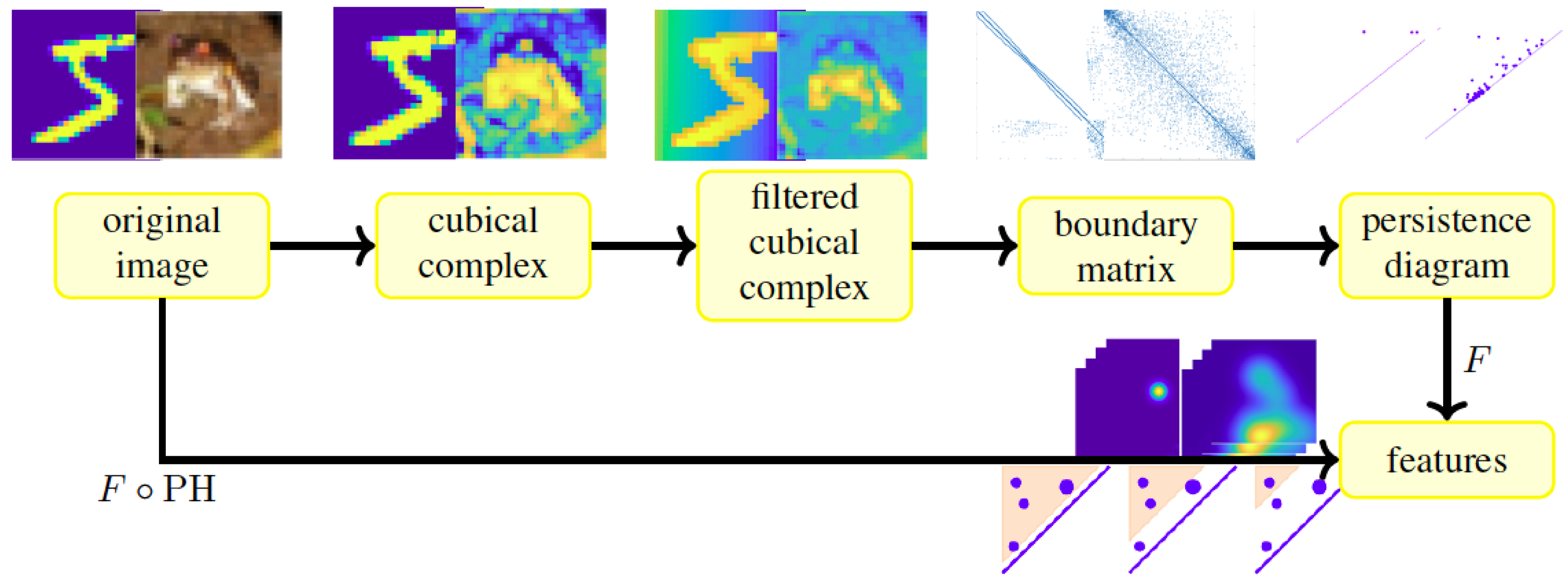
Guido Montúfar[⋆,⋄], Nina Otter[⋆], Yuguang Wang[⋄]

⋆ UCLA, Department of Mathematics, Los Angeles
⋄ Max Planck Institute for Mathematics in the Sciences, Leipzig

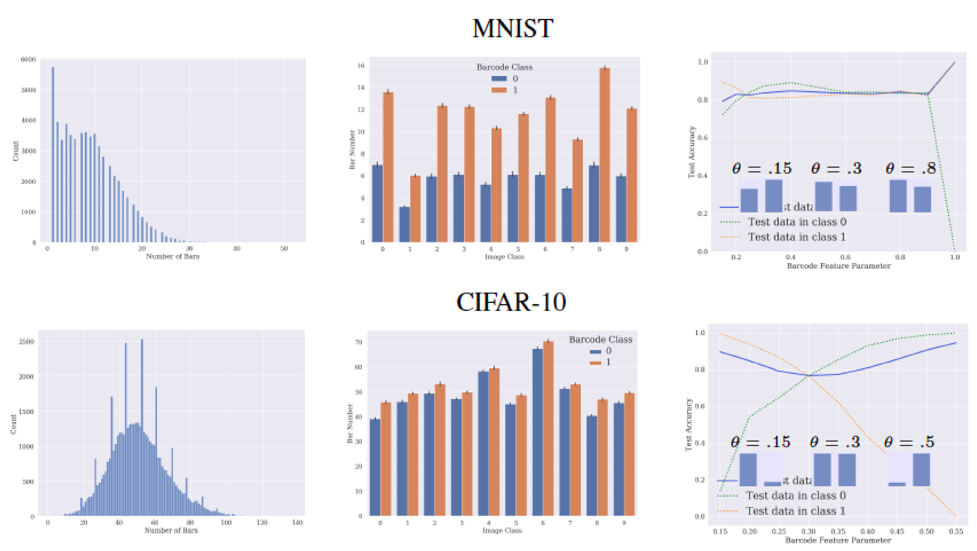
Summary

Topological data analysis uses tools from topology to create representations of data. In particular, in persistent homology, one studies one-parameter families of spaces associated with data, and persistence diagrams describe the lifetime of topological invariants, such as connected components or holes, across the one-parameter family. In many applications, one is interested in working with features associated with persistence diagrams rather than the diagrams themselves. In our work, we explore the possibility of learning several types of features extracted from persistence diagrams using neural networks.

Learning pipeline



Methods and experiments

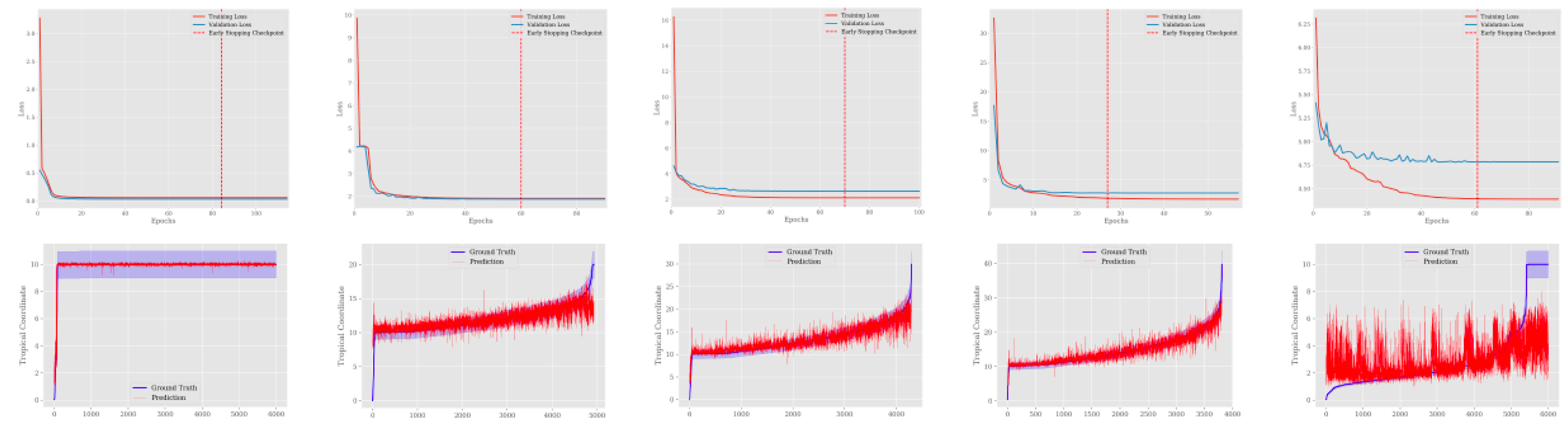


We use the data sets MNIST and CIFAR-10 and train neural networks on three different types of tasks, which consist in mapping:

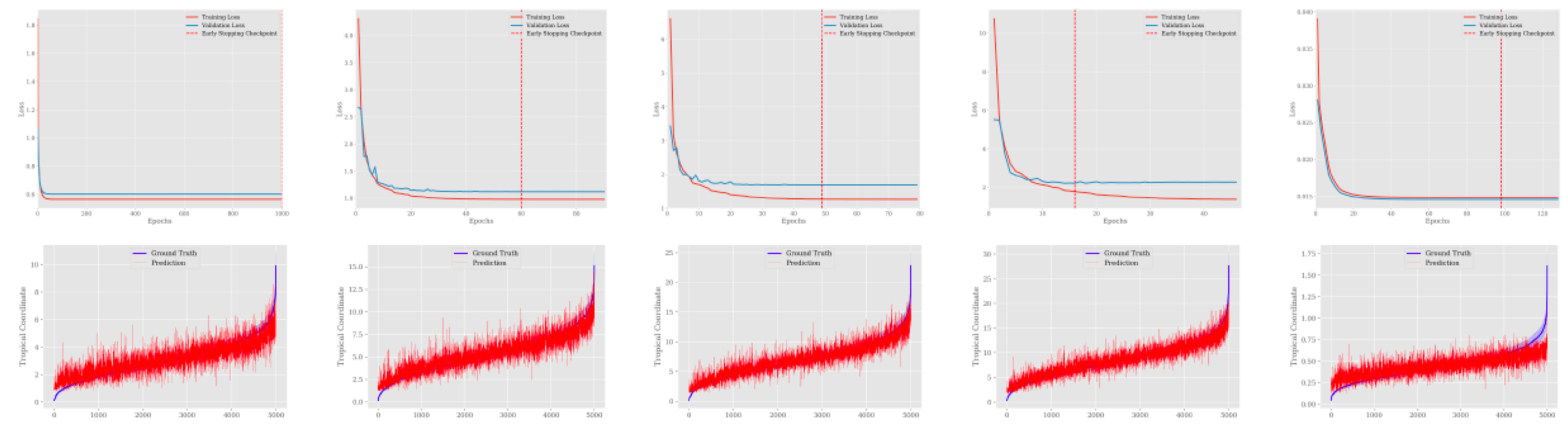
1. Images to binary features of their persistence diagrams.
2. Cubical complexes to binary features of the persistence diagrams.
3. Images to tropical coordinates of their persistence diagrams.

Experiments

MNIST - regress tropical coordinates of barcodes



CIFAR-10 - regress tropical coordinates of barcodes



Conclusion

- Using the MNIST and CIFAR-10 data sets, we show that we can train neural networks to compute several types of features of persistence diagrams. In our experiments a trained CNN can produce approximate values of persistence diagram features in 0.005s, which take 3s to compute with traditional methods.
- Two typical applications of TDA are homological inference and classification. In ongoing work, we are working on substituting the exact features with the approximations obtained by neural networks and evaluating how competitive they are. We suggest that neural networks (and GPUs) can be exploited to compute topological features for data sets that are traditionally computationally expensive, such as large 3D point clouds and 3D images.

References

1. G. Montúfar, N. Otter, Y. Wang. Can neural networks learn persistent homology features? *Topological Data Analysis and Beyond Workshop at the 34th Conference on Neural Information Processing Systems (NeurIPS 2020)*, Vancouver, Canada.
2. S. Kališnik. Tropical coordinates on the space of persistence barcodes. *Foundations of Computational Mathematics*, **19**(1), 2019.
3. K. N. Ramamurthy M. P. Buman P. Turaga A. Som, H. Choi. PI-Net: A deep learning approach to extract topological persistence images. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition Workshops*, 2020.