Unifying and Certifying Top-Quality Planning Supplementary Material

Primary Keywords: *(4) Theory;*

Definition 1 *Let Π be some planning task, $\mathcal{P}_\Pi$ be the set of its plans, and R be some relation over $\mathcal{P}_\Pi$. The dominance top-quality planning problem is defined as follows. Given a natural number q, find a set of plans $P \subseteq \mathcal{P}_\Pi$ such that*

1. *$\forall \pi \in P$, $cost(\pi) \leq q$,*
2. *$\forall \pi' \in \mathcal{P}_\Pi \setminus P$ with $cost(\pi') \leq q$, $\exists \pi \in P$ such that $(\pi, \pi') \in R$, and*
3. *P is minimal under $\subseteq$ among all $P' \subseteq \mathcal{P}_\Pi$ for which conditions 1 and 2 hold.*

Theorem 1 *The dominance top-quality planning problem for $R_{\ell\ell}$ is the loopless top-quality planning problem.*

Proof: Recall that $(\pi, \pi') \in R_{\ell\ell}$ if and only if (a) π is a loopless plan and (b) if S' are the states traversed by π, then π' traverses some $s \in S'$ more than once. Let P be a solution to the dominance top-quality planning problem for $R_{\ell\ell}$. Let $\pi' \in \mathcal{P}_\Pi \setminus \mathcal{P}_\Pi^{\ell\ell}$ be a plan with a loop such that $cost(\pi') \leq q$ and let π be some plan such that $cost(\pi) \leq q$ and $(\pi, \pi') \in R_{\ell\ell}$. Such a plan π always exists and can be obtained from π' by removing loops. Condition 2 of Definition 1 allows π' not to be in P and therefore condition 3 will ensure that $\pi' \notin P$. Together with condition 1 this gives us that P does not include plans with loops.

For a plan $\pi' \in \mathcal{P}_\Pi^{\ell\ell}$ with $cost(\pi') \leq q$, assume to the contrary that $\pi' \notin P$. Then, from condition 2 we have that there exists $\pi \in P$ such that $(\pi, \pi') \in R_{\ell\ell}$. That implies that there exists a state s that the plan π' traverses more than once, contradicting $\pi' \in \mathcal{P}_\Pi^{\ell\ell}$. □

Theorem 3 *If P is a solution to the dominance top-quality planning problem, then $\overline{P}$ is a solution to the top-quality planning problem.*

Proof: To show that $\overline{P}$ is a solution to the top-quality planning problem, we need to show that (i) $\forall \pi \in \overline{P}$, we have $cost(\pi) \leq q$, and (ii) $\forall \pi \in \mathcal{P}_\Pi \setminus \overline{P}$, we have $cost(\pi) > q$.

For (i), since $\overline{P} := P \cup \bigcup_{\pi \in P} \{\pi' \in \mathcal{P}_\Pi \mid cost(\pi') \leq q, (\pi, \pi') \in R\}$, we have the condition holds due to the first condition of Definition 1 for P.

For (ii), if $\pi' \in \mathcal{P}_\Pi \setminus \overline{P}$, then $\pi' \in \mathcal{P}_\Pi \setminus P$. Assume to the contrary that $cost(\pi') \leq q$. Then, from the second condition of Definition 1 for P, there exists a plan $\pi \in P$ such that $(\pi, \pi') \in R$. But then, by the definition of $\overline{P}$ we have $\pi' \in \overline{P}$, giving us a contradiction. □

Theorem 5 *Let Π be a planning task, π be its plan and $\Pi_\pi^{\ell\ell}$ be the loopless transformation under π. Then $\Pi_\pi^{\ell\ell}$ forbids exactly π and all plans dominated by π.*

Proof: The proof is similar to the proof of Theorem 2 of Katz et al. (2018). Let $r : \mathcal{O}' \mapsto \mathcal{O}$ be the mapping of action copies back to the original action: $r(o_i^f) = r(\overline{o_i}) = r(o_i^e) = r(\overline{o'}) = r(\overline{o}) = r(\overline{o_i}^e) = r(\overline{\overline{o}}) = r(\overline{\overline{o'}}) = r(\overline{\overline{o_i}}) = o$. Note that $\Pi_\pi^{\ell\ell}$ restricted to the variables $\mathcal{V}$ equals to the task Π, modulo the copies of the actions split into the cases of reaching one of the states s_i or none of them. Thus, for each plan π' for $\Pi_\pi^{\ell\ell}$, $r(\pi')$ is a plan for Π.

For the other direction, for each $o \in \pi$ at most one of the action copies is applicable in each state. So, every applicable in s_0 sequence of actions ρ in Π can be uniquely mapped into a sequence $r^{-1}(\rho)$ of $\Pi_\pi^{\ell\ell}$. Observe that $\pi = \langle o_1 \ldots o_n \rangle$ is mapped to $\langle o_1^f \ldots o_n^f \rangle$, which leads to a state where $\langle \overline{v}^d, F \rangle$ and therefore not a goal state. Looking now at a plan π' such that $(\pi, \pi') \in R_{\ell\ell}$, reaching some state s_i on π at least twice. When the action that reaches s_i for the second time (either o_i^e or $\overline{o_i}^e$) is applied in $\Pi_\pi^{\ell\ell}$, the value $\langle \overline{v}^e, T \rangle$ is reached and cannot be changed by any action. Therefore the corresponding sequence of actions is not a plan for $\Pi_\pi^{\ell\ell}$.

Let $\pi' \neq \pi$ some plan for Π such that $(\pi, \pi') \notin R_{\ell\ell}$ and let ρ and ρ' be the corresponding applicable sequences of actions in $\Pi_\pi^{\ell\ell}$. Let o be the first action where π' diverges from π. Then, the corresponding to o action in ρ' is one of $\{\overline{o_i}, \overline{o}, \overline{o'}, \overline{\overline{o_i}}, \overline{\overline{o}}, \overline{\overline{o'}}\}$ and achieves $\langle \overline{v}^d, T \rangle$. The value of $\overline{v}^d$ is never changed anymore, since there are no actions that achieve $\langle \overline{v}^d, F \rangle$. Since π' does not reach any of the states $s_0, \ldots, s_n$ more than once, none of the copies of the actions in ρ' are o_i^e or $\overline{o_i}^e$, which are the only copies that achieve $\langle \overline{v}^e, T \rangle$. Therefore, at the end of the execution of ρ' we have $\langle \overline{v}^e, F \rangle$. Since all the effects on the original variables are preserved precisely, we get that ρ' achieves a goal state and therefore a plan. □

References

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