Embedding Theoretical Baselines For Satellite Force Estimations

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1. Introduction

Over the years, the benefits and challenges surrounding satellite operation at very low Earth orbit (VLEO) are becoming better understood, and technological progress has gradually enabled its feasible implementation [1]. At such low altitudes (< 500km), the influence of atmospheric density is no longer negligible. As satellites are progressively deployed to operate in VLEO, the assumption of a constant drag will not hold up well.

While accurate force estimations can be realized through numerical simulations, the costs can be steep. In satellite mission planning and design, numerous sequential evaluations are required to accomplish orbit propagation over an operational lifetime. Conducting iterative shape and structural design over a single operating condition, or even optimized over the entire trajectory, compounds the computational demands. Various approaches have been explored to circumvent this, such as simplifying the geometry [2], optimizing in two-dimensions [3], selectively omitting physics [4], or developing general shaping strategies [5], with each presenting some form of trade-off in accuracy.

Although surrogate modelling can relieve these costs, sufficient data is still required to construct accurate and generalizable prediction models. As such, a pre-training strategy is explored to improve model accuracy under such data-scarce conditions. The use of machine-learning in enhancing physical analysis has been demonstrated to be achievable through careful integration [6, 7, 8]. In particular, pre-training strategies that tap on transfer-learning between related distributions have shown to be particularly effective in scientific tasks [9].

In this work, a baseline model is pre-trained on theoretical approximations to support the predictions of the ground truth satellite aerodynamic forces. The pre-training procedure involves residual-learning on the theory of free-molecular flat plates, in which the aerodynamic forces are approximated in a low-fidelity context and without any simulation data. The embedding of this theoretical baseline enables a simple corrector to be learned from sparse simulation samples. As such, the full model is inductively-biased towards physically reasonable predictions. The proposed approach is visualized in Figure 1.

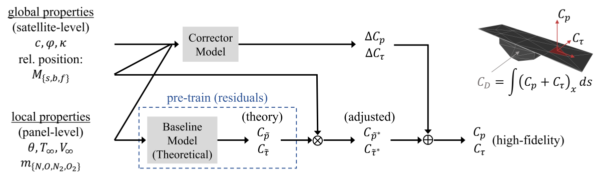

Fig. 1: Schematic of the proposed embedding of a theoretical baseline to estimate surface forces of a satellite.

2. Methods

2.1 Theory of free-molecular flat plates

The forces acting on the satellite can be approximated using the theory of free-molecular flow. The atmosphere at high orbital altitudes can be highly rarefied with mean free path of the gas particles being significantly greater than the characteristic length of the satellite, thereby justifying a collisionless environment (negligible inter-particle collisions). The forces imparted on the satellite are effectively determined by the interaction of the undisturbed flow with the surface. From these simplifications, the rarefied aerodynamics of flat surfaces can be described theoretically [10], which includes an α-parameterization of its gas-surface interactions [11]. Thus, any satellite geometry can be discretized into a compilation of flat panels, each oriented at angles θ towards the flow (of temperature T_∞ and speed V_∞) with individual force contributions of

$$C_{\tilde{l}_i} = G_i Z_i \sin\theta + \frac{V_{(\mathrm{re}/\infty)} \sin\theta}{2} \left(\sqrt{\pi} Z_i \cos\theta + P_i\right) \tag{1}$$

$$C_{\tilde{d}_i} = \frac{P_i}{\sqrt{\pi}} + Q_i Z_i \cos\theta + \frac{V_{(\mathrm{re}/\infty)} \cos\theta}{2} \left(\sqrt{\pi} Z_i \cos\theta + P_i\right) \tag{2}$$

where $G_i = \frac{1}{2S_i^2}$, $P_i = \frac{1}{S_i} \exp\left(-S_i^2 \cos^2\theta\right)$, $Q_i = 1 + G_i$, and $Z_i = 1 + \mathrm{erf}\left(S_i \cos\theta\right)$, for a speed ratio between the bulk velocity to the most-probable thermal velocity as $S_i = \frac{V_\infty}{\sqrt{2kT_\infty/m_i}}$. The error function is defined as $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp\left(-y^2\right) dy$, and the velocity ratio follows the wall temperature-corrected derivation as $V_{(\mathrm{re}/\infty)} = \sqrt{\frac{1}{2}\left[1 + \alpha \left(\frac{4RT_w}{V_\infty^2} - 1\right)\right]}$ [12]. The Boltzmann constant is defined as k and the molar gas constant defined as R. The contributions to the surface-normal pressure and surface-tangential shear stress coefficients are reconstructed from the local panel's theoretical lift and drag properties as $C_{\tilde{p}_i} = C_{\tilde{d}_i} \cos\theta + C_{\tilde{l}_i} \sin\theta$ and $C_{\tilde{\tau}_i} = C_{\tilde{d}_i} \sin\theta - C_{\tilde{l}_i} \cos\theta$.

2.2 Pre-training the Baseline Model on theory

The Baseline Model (**Base**) is pre-trained with this theoretical approximation through residual-learning of $f_{\tilde{p}} := \hat{C}_{\tilde{p}} - \sum_i^4 m_i C_{\tilde{p}_i}$ and $f_{\tilde{\tau}} := \hat{C}_{\tilde{\tau}} - \sum_i^4 m_i C_{\tilde{\tau}_i}$, where $\hat{C}_{\tilde{p}}$ and $\hat{C}_{\tilde{\tau}}$ are the output predictions by **Base** for each theoretical panel, while m_i is the mass fraction of each constituent gas species. The loss function defined as $\mathcal{L} = \mathbb{E}_{x_{\textbf{Base}} \sim \mathcal{P}} \left[{f_{\tilde{p}}}^2 + {f_{\tilde{\tau}}}^2 \right]$ is minimized, in which model inputs are $x_{\textbf{Base}} = \{\theta, T_\infty, V_\infty, m_{\mathrm{N}}, m_{\mathrm{O}}, m_{\mathrm{N}_2}, m_{\mathrm{O}_2}\}$. From the position of each panel relative to the global satellite geometry, masking ($M_{\{s,b,f\}}$) is conducted [13, 14] to provide an improved estimate of surface pressure $\hat{C}_{\tilde{p}^*}$ and shear stress $\hat{C}_{\tilde{\tau}^*}$.

2.3 Limitations of theory

In the absence of simulation data, a theoretical free-molecular flow can provide a physically-grounded approximation for the aerodynamic forces. While useful, there are apparent shortcomings arising from these simplifications. For instance, the influence of surface concavity is absent as every particle is reflected only once; local Knudsen numbers can be small at VLEO, hence the frequency of inter-particle collisions can be considerable; the flow may not be hyperthermal depending on the environment; and the lack of flow-shielding modelling.

2.4 Training the Corrector Model from sparse datasets

The ground truth satellite forces are defined using Direct Simulation Monte Carlo (DSMC) [15]. The prediction of the ground truth is expressed as a sum of the outputs from **Base** and a Corrector Model (**Corr**), i.e., $C_p = C_{\tilde{p}^*} + \Delta C_p$ and $C_\tau = C_{\tilde{\tau}^*} + \Delta C_\tau$. By randomly sampling from the full satellite flight path, a sparse representation of the ground truth can be established. These sparse samples are used to learn the corrections ΔC_p and ΔC_τ as a non-linear mapping from the inputs $x = \{\theta, T_\infty, V_\infty, m_{\mathrm{N}}, m_{\mathrm{O}}, m_{\mathrm{N}_2}, m_{\mathrm{O}_2}, M_{\{s,b,f\}}, c, \varphi, \kappa\}$, in which the panel baricenter c serves as a global identifier across various pitch φ and yaw κ satellite rotations. The full prediction model, **Base+Corr**, is formed as shown in Figure 1.

3. Results and Discussion

3.1 Set-up

As a toy problem, the satellite is projected on a circular, sun-synchronous orbit with a mean orbital altitude of 200 km, approximately 16 orbits per day, and inclination of $96.3°$ estimated using the J2 algorithm [16]. Only 24 hours of the flight path is used in the present analysis, and discretized into $N_t = 1440$ minutes of simulation data. Of which, only 3% and 10% of the full flight path is randomly sampled to form the training and test set, respectively. As visualized in Figure 2, four arbitrary maneuvers are prescribed onto the flight path consisting of pitch φ and yaw κ rotations towards $\pm 40°$ with each lasting around 35 minutes. No simulation data is required for the theoretical pre-training, and has a compute cost equivalent to ~2.5 converged simulation instances, which would correspond to 0.17% of the full flight path dataset. Fully data-driven models, **Direct**, are developed without pre-training using the same training sets.

3.2 Performance Comparison

In Figure 2, the local force coefficients of C_p and C_τ are integrated to obtain the satellite drag coefficients C_D across the entire 24-hour flight path. **Base** tends to over-predict the drag while **Direct** under-predicts, with both exhibiting substantial deviations. Notably, the latter performs much worse than the former in modelling the maneuvers, under data-scarce conditions. The **Base+Corr** reconciles the two models to best reflect the ground truth. In Table 1, the mean absolute percentage error is computed as $\mathbb{E}\left[|(C_D - \hat{C}_D) \,/\, C_D| \times 100\% \right]$ over ten randomly-initialized states, of which the flight path on and off maneuvers (MVR.) are also considered separately. Based on the full flight path, the proposed approach achieved error reductions by approximate factors of $0.5\times$ and $0.4\times$ from the models trained using only data and only theory, respectively. However, the extension of **Corr** reveals a trade-off in accuracy when on and off maneuvers.

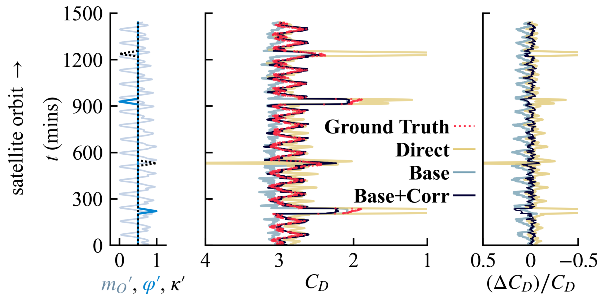

Fig. 2: Evaluation of satellite C_D predictions over the full 24-hour flight path. In general, the proposed approach (**Base+Corr**) better reflects the ground truth than the theoretical baseline (**Base**) and the model without pre-training (**Direct**).

Table 1: Absolute percentage error in C_D predictions (%)

	Satellite Path		
Model	Full	On MVR.	Off MVR.
Direct	7.64 ± 2.40	35.66 ± 31.08	4.57 ± 3.25
Base	5.75 ± 0.26	5.47 ± 0.07	5.78 ± 0.29
Base+Corr	2.93 ± 0.09	8.85 ± 3.29	2.29 ± 0.11

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