

SRTD: A SYMMETRIC DIVERGENCE FOR INTERPRETABLE COMPARISON OF REPRESENTATION TOPOLOGY

Yan Wang¹, Yue Zhu¹, Tianyang Hu²

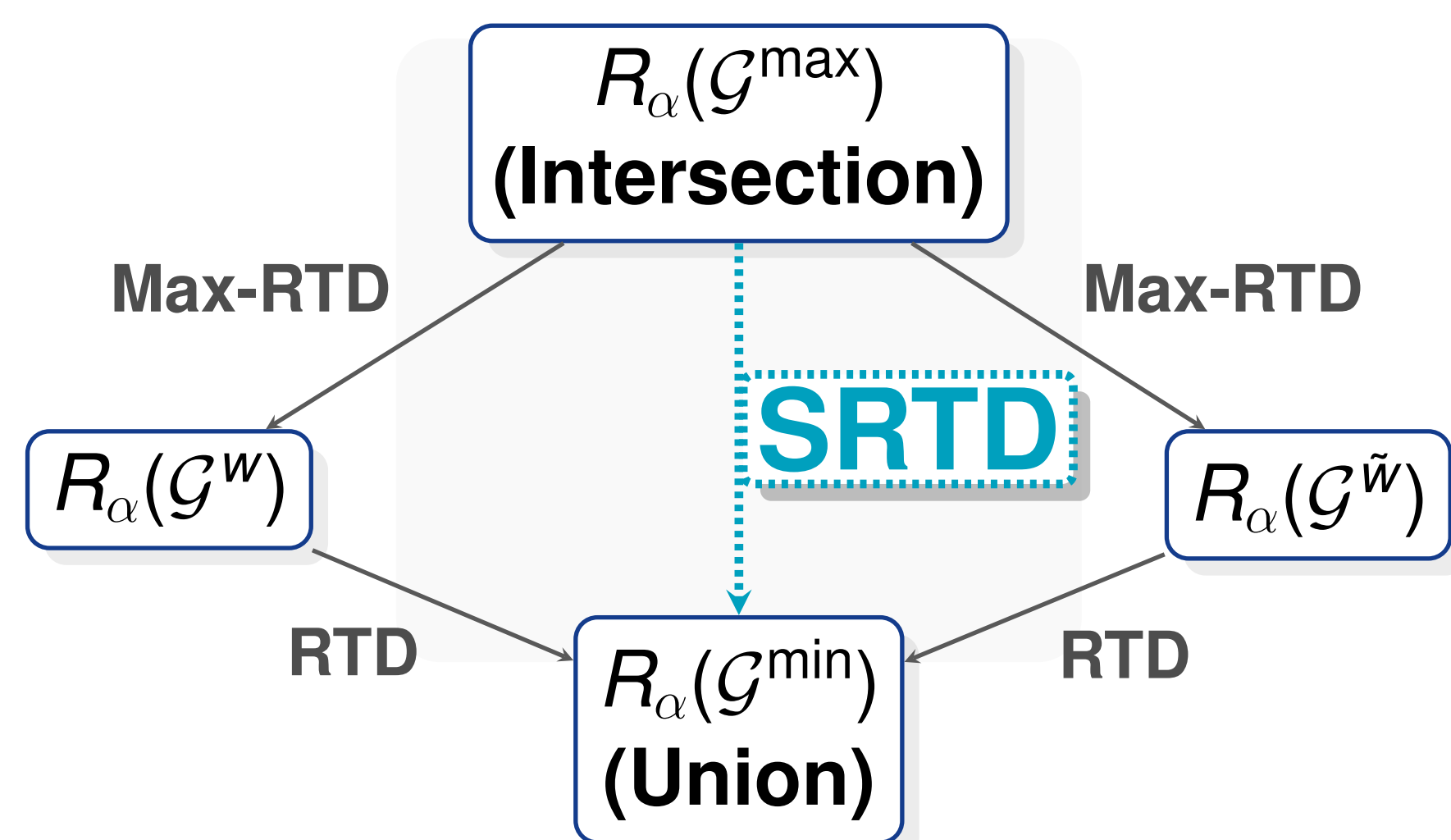
¹Tongji University

²The Chinese University of Hong Kong, Shenzhen

Abstract & Methodology

Motivation: Standard RTD is asymmetric ($RTD(P, Q) \neq RTD(Q, P)$). Averaging them lacks geometric interpretation.

Solution: We define **SRTD** on the auxiliary graph M_{sym} . It measures the topological gap between **Union** and **Intersection**.



Mathematical Properties

Our framework fits into a **Long Exact Sequence** in homology:

$$\cdots \rightarrow H_n(R_\alpha(\mathcal{G}^w), R_\alpha(\mathcal{G}^{\max})) \xrightarrow{\gamma_n} H_n(R_\alpha(\mathcal{G}^{\min}), R_\alpha(\mathcal{G}^{\max})) \xrightarrow{\beta_n} H_n(R_\alpha(\mathcal{G}^{\min}), R_\alpha(\mathcal{G}^w)) \xrightarrow{\delta_n} H_{n-1}(R_\alpha(\mathcal{G}^w), R_\alpha(\mathcal{G}^{\max})) \xrightarrow{\gamma_{n-1}} \cdots$$

Decomposition Theorems:

► **SRTD:** Decomposes into the two directional components.

$$SRTD \lesssim RTD + \text{Max-RTD}$$

► **SRTD-lite:** An elegant equality and inequality properties:

$$SRTD\text{-lite} = \text{Max-RTD-lite} + \text{RTD-lite}$$

$$\text{Max-RTD-lite} \geq SRTD\text{-lite} \geq \text{RTD-lite}$$

Why Intersection part Matters?

Incorporating information from $\max(w, \tilde{w})$ benefits RTD significantly.

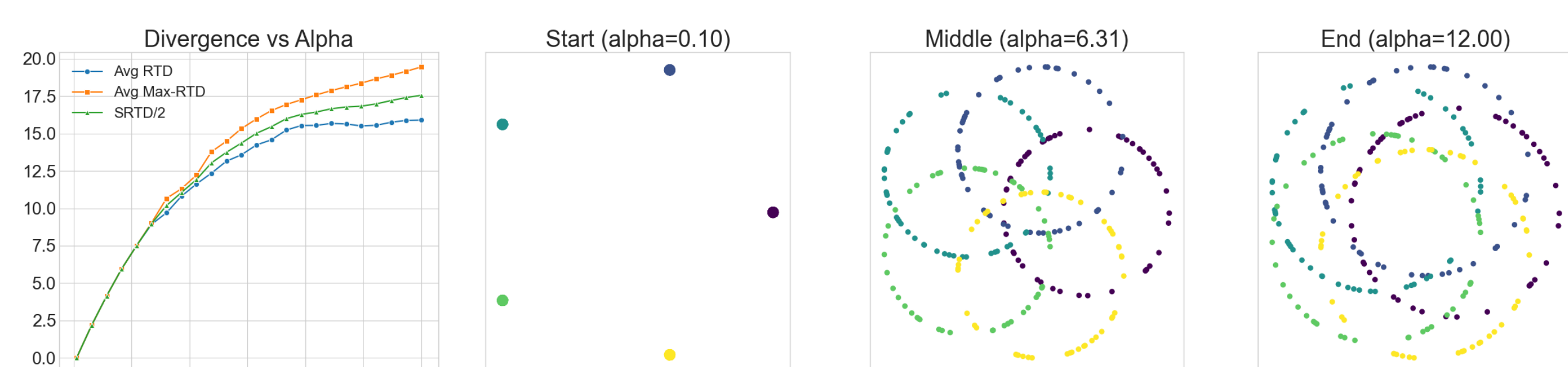


Fig. 1: Sensitivity Analysis

RTD becomes insensitive at large α , while SRTD maintains sensitivity.

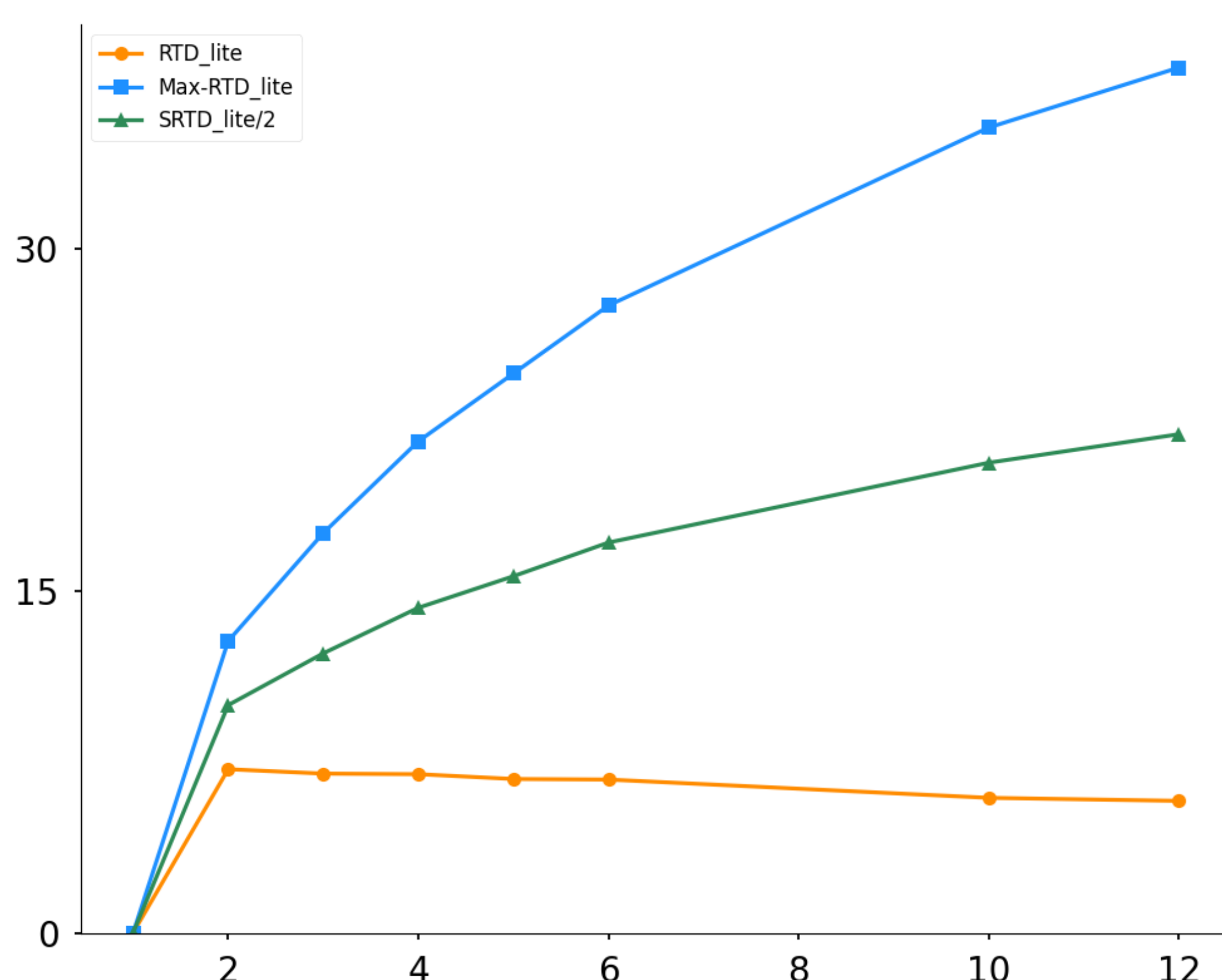
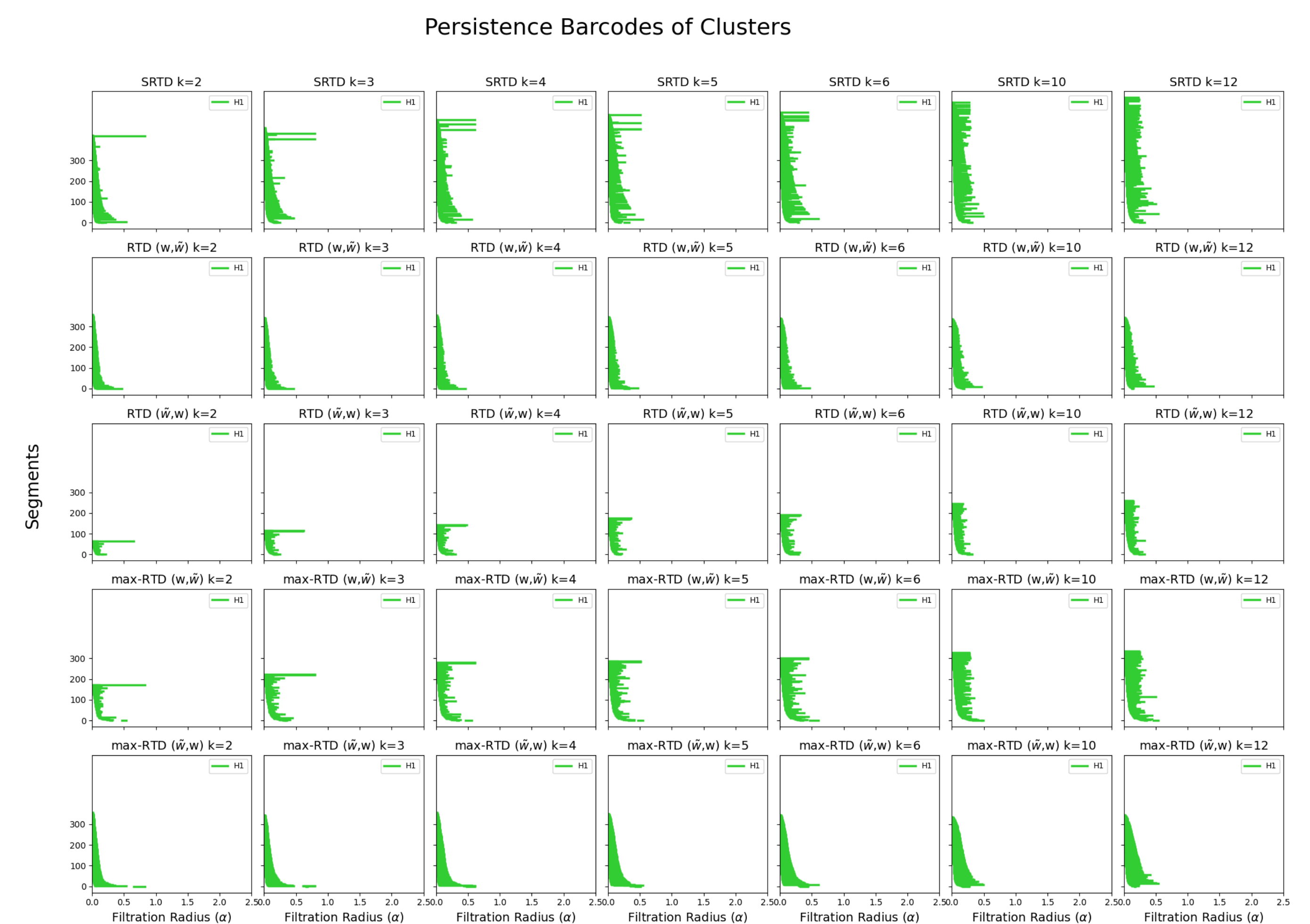


Fig. 2: Cluster Trend Correction

RTD-lite incorrectly suggests similarity increases with more clusters.

Richness of Features

SRTD captures a superset of features from both directions (Union + Intersection).



App I: Dimensionality Reduction

We projected COIL-20 and F-MNIST to 16D. SRTD achieves **state-of-the-art** topology preservation.

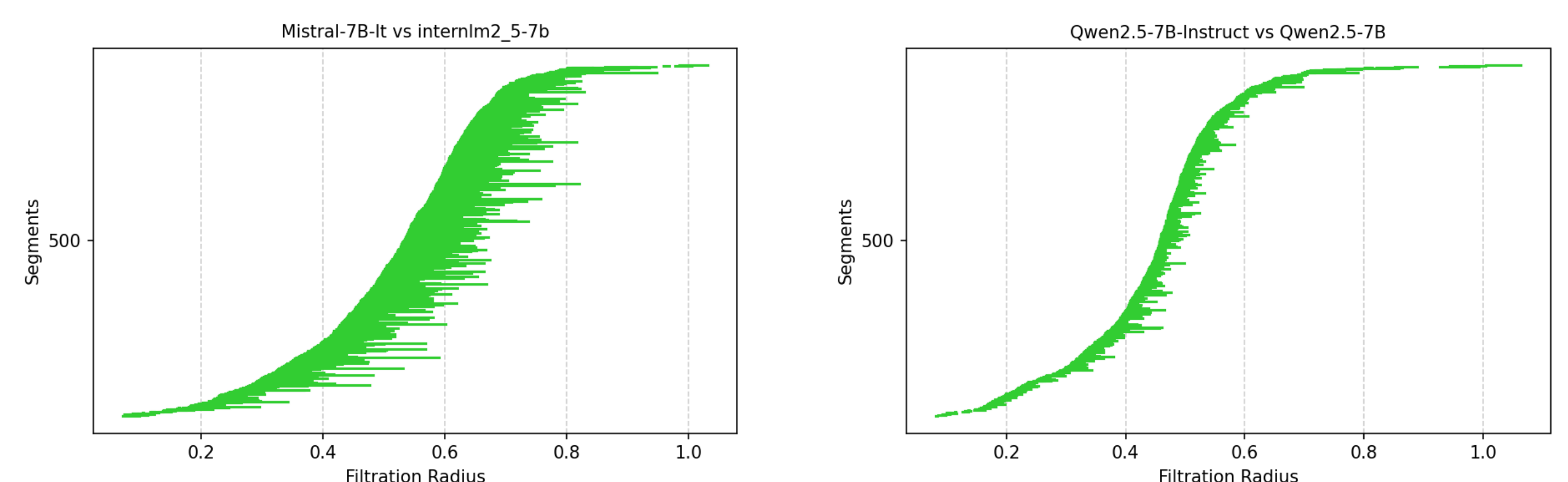
F-MNIST Results

Method	Triplet Acc	RTD Dist	Corr
RTD	0.907	1.28	0.954
Max-RTD	0.895	1.51	0.937
SRTD	0.910	1.29	0.957

~2x Faster
(vs RTD)

SRTD-lite, SRTD provides a lightweight alternative that maintains high quality while drastically reducing computational overhead.

App II: LLM Fingerprinting



Non-Homologous (long barcode)

Homologous (short barcode)

Through the barcode, SRTD can precisely determine the specific deviation of the input's representation across different models.

Conclusion

- **Theoretical Rigor:** Resolved the asymmetry of RTD by bridging the Union and Intersection via exact sequences.
- **Efficiency:** SRTD-lite offers a robust, high-speed alternative for massive datasets.
- **New Frontiers:** Demonstrated potential in **Model Phylogeny**.