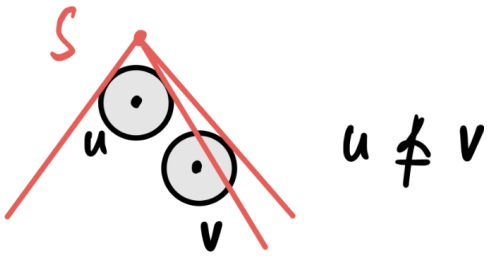

Figure 1: An example illustrating the necessity for v's ball to be entirely immersed in u's umbral shadow, in order that $u \preceq v$. In the marginal case shown here, $u \not\preceq v$ even though the point v is in u's umbral shadow. This is because part of v's ball and corresponding umbral shadow is outside of u's umbral shadow.

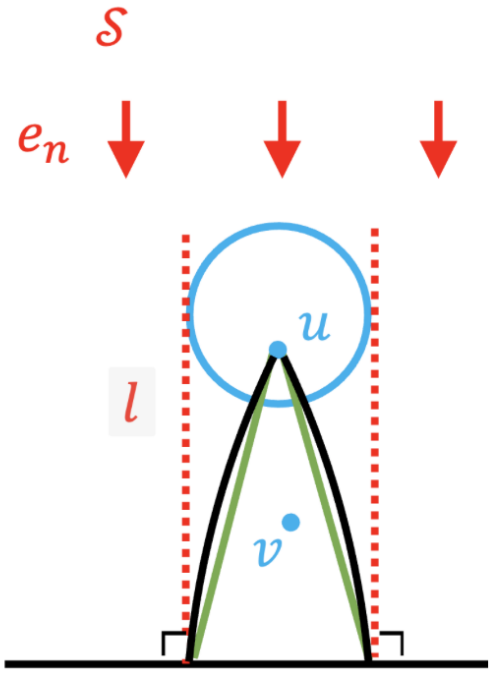

Figure 2: Umbral shadow cone (green) of u in half-space model with light source $\mathcal{S}$ at infinity, where e_n is the propagation direction of lights. l (dotted red) denotes the boundary of umbral shadow casted by u and its ball. Solid green lines denote the boundary of the umbral cone of u, while solid black lines delineate the geodesically convex hull of u's umbral cone. This figure shows that umbral cones are not geodesically convex, because they are strict subsets of their geodesically convex hulls.

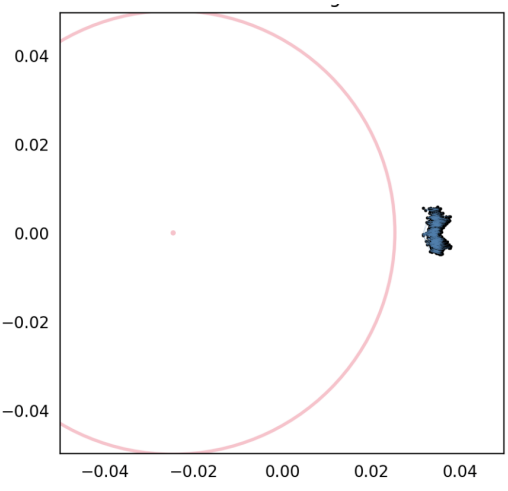

Figure 3: A global view of learned Mammal subgraph embeddings with a trainable light source radius, using Penumbral-Poincaré-ball.

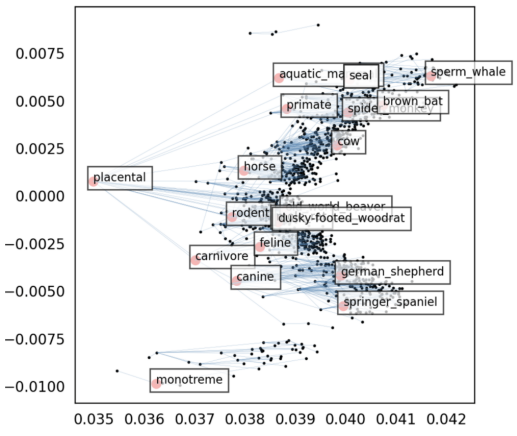

Figure 4: A zoomed-in view of learned Mammal subgraph embeddings with a trainable light source radius, using Penumbral-Poincaré-ball.