A Quantitative Law for the Organization of Verbal Valency

Abstract

A substantial body of work has provided evidence that the lexicons of natural languages are organized to support efficient communication. However, existing work has largely focused on word-internal properties, such as Zipf's classical observation that more frequent words are optimized in form to minimize communicative cost. Here, we investigate the hypothesis that efficient lexicon organization also affects valency, or the combinations and orders of additional words and phrases a verb selects for in a sentence. We introduce two measures of valency diversity for verbs: *valency frame count* (VFC), the number of distinct frames associated with a verb, and *valency frame entropy* (VFE), the average information content of frame selection associated with a verb. Using data from 79 languages, we provide evidence for a new quantitative law of language: more frequent verbs are associated with a larger diversity of valency frames, suggesting that the organization of valency helps optimize communicative efficiency. We discuss our findings in relation to classical findings such as Zipf's law of least effort, as well as implications for theories of valency and efficient language design.[1]

1 Introduction

The idea that functional pressures shape the organization of natural language lexicons has a long tradition, going back at least to Zipf's law of abbreviation: the length of the word is inversely related to the frequency of its use, which was motivated by the "principle of least effort" (Zipf, 1935, 1949). Further evidence has been provided by a functionalist line of research studying the emergence of the lexicon from the frequency of use (e.g. Bybee, 1998; Hopper and Bybee, 2001) as well as more recent work that formalizes such ideas using information theory (e.g. Regier et al., 2015; Wu et al.,

2019; Mahowald et al., 2022; Pimentel et al., 2021). Common across this literature is the idea that the lexicon reflects a pressure for efficient language use under the constraints posed by human cognition. Evidence has been adduced from domains such as word lengths (Piantadosi et al., 2011; Pimentel et al., 2023), morphological complexity (Wu et al., 2019), orthographic forms (Mahowald et al., 2018), and the partitioning of semantic space into word meanings (e.g. Regier et al., 2015; Kemp et al., 2018; Zaslavsky et al., 2018).

However, a key aspect of the lexicon has gone unaddressed in this literature: words are not used in isolation but systematically integrated within the broader structure of a sentence. Verbs in particular play a key role in determining the syntactic and semantic structure of a sentence, and the study of *valency*, or the number and type of arguments a verb takes, features prominently in theoretical linguistics (e.g. Chomsky et al., 1970; Levin and Rappaport Hovav, 2005).

In broad strokes, the *valency* of a verb indicates they type and number of dependents it takes. For example, a majority of verbs in many languages minimally requires a subject (Chomsky, 1982; McCloskey, 1994). The transitivity of a verb is a property that determines whether a verb takes an object and how many, considered a more narrow measure than valency (Bowers, 2002). A substantial literature has documented the many ways in which valency differs across verbs within a language, and across languages (e.g. Hartmann et al., 2013).

Fig. 1 shows how the sentences equivalent to English *The teacher helped the children with the homework* are rendered differently in several languages, with dependency relations of verb annotated according to Universal Dependencies (UD, v.2 Nivre et al., 2020) guidelines. We observe both cross-lingual similarities and difference: for example, it has a subject (UD relation `nsubj`), an object (`obj`) and an oblique dependent (`obl`) in

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[1] Code for reproducing our results is available under GNU GPL Version 2 at `https://anonymous.4open.science/r/verbal-valency-ud-8B7E`

both English and Finnish, but all three dependents are marked for case in Finnish (nominative, partitive and inessive, resp.). Other languages use other combinations of relation types to express the same meaning; for instance, the verb has only two dependents in Japanese and Chinese: in Japanese the verb takes takes *homework* as `obj` and in Chinese the sentence must be formulated with another verb as a clausal complement (`ccomp`).

For variation within a language, we focus on *valency frame*: a specific configuration of morphosyntactic features associated with a verb. The range of possible configurations varies between verbs of the same language; a substantial body of work has classified the different frames that verbs occur with within a language (e.g. Levin, 1993). For example, Fig. 2 shows a few more possible valency frames observed with the English verb "help" (also known as its *diathesis alternations*) .

Here, we investigate whether the valency properties of verbs show signatures of efficient organization. We argue that considerations of communicative efficiency predict that verbs will systematically differ in the diversity of valency frames they are associated with, so that high-frequency verbs will be associated with a greater diversity of frames. From a comprehension perspective, this kind of pattern ensures frames are predictable when verbs are not, reducing spikes in information density. We argue that similar considerations apply from the perspective of language production and learning.

We operationalize valency at a lexeme-level, and propose two novel information-theoretic metrics of the diversity of valency frames: (i) *valency frame count* (VFC) that measures the number of distinct frames associated with a verb; and (ii) *valency frame entropy* (VFE) that measures the average surprisal of valency frames as conditioned by the verb, determined by the diversity of frames associated with that verb. This allows us to posit and test hypotheses regarding the correlation between these metrics and the diversity of valency frames in experiments and show that effects predicted by learnability and efficiency considerations shape valency systems of languages cross-linguistically.

2 Theoretical Background and Predictions

The Notion of Valency   The notion of valency, and closely relate notions such as *argument structure*, *subcategorization* and *diathesis alternation*, have received a variety of formalizations across formal linguistic theories (e.g. Tesnière, 1959; Chomsky, 1965; Fillmore, 1982; Levin, 1993; Goldberg, 1995). While many theories make a formal distinction between arguments and adjunct (Chomsky et al., 1970; Ackema, 2015), others treat them in a similar fashion (van Noord and Bouma, 1994). For this study, we abstract away from such theoretical questions, and adopt a broad definition of verb valency within UD, a dependency grammar framework. In doing so, we aim for a broad descriptive basis for cross-linguistic comparison, as UD classifies dependency relations into cross-linguistically meaningful types (cf. Figures 1–2); pragmatically, this also allows applicable operationalization on large-scale cross-linguistic usage data available as UD treebanks. We formalize as *valency* the set of such relation types of dependents a verb co-occurs with, and a **valency frame** as a set of UD relations corresponding to dependents of a verb (see § 4.1).

Theories of Communicative and Processing Efficiency   One justification for efficient organization of the lexicon comes from *learnability* considerations. For instance, it has been suggested that infrequent irregular verbs are poorly acquired, leading to morphological irregularity being focused on high-frequency words (e.g. Bybee, 1998; Wu et al., 2019). A similar consideration applies to valency: When learning from a finite sample, a learner should be able to acquire a larger variety of frames for frequent verbs, as more data is available for them. Thus, if learnability affects the organization of valency, one expects that infrequent verbs should be associated with smaller diversity of frames. Another key justification for efficient organization of the lexicon (and language in general) comes from online processing, in both language production and comprehension. On the *production* side, accessibility, i.e. the ease of retrieval, has been shown to play a key role in shaping language form and structure (Kathryn Bock and Warren, 1985; MacDonald, 2013). To the extent that verbs may be planned in advance of their arguments in sentence production (e.g. Momma and Ferreira, 2019), reducing the diversity of frames associated with a verb should increase the accessibility of any of these frames. Such a pressure should be stronger for the long tail of low-frequency verbs, as producers will be less attuned to their production. On the *comprehension* side, we expect consequences for the distribution of information density. Specifically,

Figure 1: Example sentences in 6 languages showing relevant dependency relations. In each language, the verb expressing "helping" is associated with a different set of UD relation types linking it to its dependents.

Figure 2: Further examples of the verb "help" in English, showing its diathesis alternations .

the Uniform Information Density (UID) hypothesis (Fenk and Fenk-Oczlon, 1980; Levy and Jaeger, 2006) posits that language speakers prefer a more even distribution of surprisal values across utterances in order to maximize but not overload the capacity of the communication channel. Less frequent verbs will, on average, have high surprisal; high entropy in the valency frames associated with it would lead to an undesirable spike in overall surprisal. We believe that some other key processing theories, such as surprisal (Hale et al., 2002; Levy, 2008) and dependency locality (Gibson et al., 2000) do not make relevant predictions.

3 Data and Methodology

3.1 Data

We draw on Universal Dependencies (UD) as the source of data. It provids as a cross-linguistically consistent system for annotating morphosyntactic information within a dependency grammar framework (de Marneffe et al., 2021). We use the UD v2.11 release, and include all languages with at least 10,000 tokens across treebanks. We exclude L2 learner corpora and code-switched corpora. Korean corpora are excluded due to the lack of verb lemmatization, which is necessary for empirically estimating valency. In total, 79 languages out of the 138 languages remained.

3.2 Quantifying Diversity of Frames and Verbs

Recall that we hypothesize a positive correlation between a verb's frequency and the diversity of frames that it occurs with. In order to test this, we propose two measure of the diversity of frames associated with a verb.

The simplest metric of the diversity of frames associated with a verb is **valency frame count** (VFC): the number of distinct frames that a verb occurs with in the corpus.

By definition, VFC gives equal weight to more or less frequent frames. As discussed in §2, information-theoretic models of language comprehension such as UID predict an effect specifically for the *entropy* of the frames, conditioned on the verb. We formalize this in our second measure. Let R be the set of UD relations under consideration.

Let F be a random variable ranging over subsets of R – that is, over frames. Let be a random variable V ranging over the verb lemmata v in a language. A dependency treebank defines, for each verb v, a probability distribution $P(f|V = v)$ over different frames proportional to their frequencies of occurrence. We formalize the **valency frame entropy** (VFE) as $H[F|V = v]$, the entropy of the frame F conditioned on a verb v:

$$- \sum_{f \subseteq R} P(f|V = v) \log_2 P(f|V = v) \quad (1)$$

Valency frame entropy is equal to $\log(\text{VFC})$ in the special case where all frames that appear with a verb do so at equal frequencies. However, keeping VFC constant, VFE is lowered when the same number of frames appear at more skewed frequencies.

4 Experiment

4.1 Dependency Relations

As we take a broad view on the notion of valency, we correspondingly include a broad range of UD relations (Figure 6). Definitions of valency vary across the literature, but conveniently UD classifies the relevant set of relations. We included the UD v2 relations defined by UD as *core arguments* and *non-core dependents*. Core arguments, including – among others – subjects and objects are most clearly associated with argument structure in typical theories of valency and syntax. Non-core dependents are other possible dependents of a clausal predicate, including – among others – oblique nominals and adverbial modifiers. UD further includes *nominal dependents* – dependents of nouns – which are by definition of no relevance to our study. Finally, UD also has the categories *coordination*, *headless*, *loose*, *special*, and *other*; we do not consider any of these to be relevant: for instance, coordination (*conj* and *cc*) defines a symmetric relationship rather than a hierarchical dependency structure; and *special* lacks cross-lingual applicability. We examine correlations estimated from various subsets in Figure 5.

4.2 Controlling for Confounds via Cross-Entropy and Subsampling

Naively testing our hypothesized relationship faces a **circularity confound**: if there are few observations for a verb in a corpus, the number of distinct frames observed with it will necessarily be small, leading to an underestimation bias for entropy and

frame count. As precisely these verbs are expected to have low frame entropy and low frame count under our hypotheses, it may lead to spurious correlations. Stated differently, low-frequency verbs might have a low measured diversity of frames simply due to lack of observations. We use two strategies to eliminate this confound.

Estimation via Cross-Entropy Our first strategy is to empirically estimate the valency frame entropy using the cross-entropy. That is, for each verb, we randomly split its observations in the corpora into two halves, resulting in two empirical distributions $P(F|V = v)$ and $P(F'|V = v)$. The cross-entropy between them, conditioned on a verb v, is:

$$- \sum_{f \subseteq R} P(xf|V = v) \log_2 P'(f|V = v) \quad (2)$$

We use Laplace smoothing with $\alpha = 1$ over the full set of frames to ensure well-definedness.

How Cross-Entropy Counteracts the Confound Whereas naive estimation would lead to a bias in the direction of our hypothesis (more frequent verbs exhibit higher valency frame entropy), cross-entropy-based estimation reverses the bias, i.e., introduces a *conservative* bias *against* our hypothesis: when a verb has very few observations, the observations in the two halves both are very noisy estimates of the actual distribution, increasing cross-entropy. In contrast, for verbs with many observations, both halves have sufficient data to closely estimate the distribution, eliminating the overestimation bias affecting low-frequency verbs.

In order to more formally understand the impacts of the two methodologies, we consider the simple case of two verbs, each with k frames and identical distributions $P(F|V = v)$, but where v_1 has N observations, and v_2 has $2N$ observations. Importantly, in the limit of an infinite corpus, both verbs would have the same values for both VFC and VFE. However, when N is not much larger than k, there is a substantial amount of probability that not all k frames were observed; that is, the diversity of the frames will be underestimated. Importantly, the probability of such underestimation is larger for the less frequent verb v_1. The entropy of the observed distribution will therefore tend be smaller for v_1 than for v_2, as the observed distribution of valency frames is supported on fewer frames in that case. By underestimating both VFE and VFC more

strongly for low-frequency verbs, this phenomenon introduces a circularity confound in the direction of our proposed relationship.

The bias *reverses* under cross-entropy estimation: cross-entropy (2) reflects the true entropy (1) plus $D_{KL}(P||P')$. By the Law of Large Numbers, this KL Divergence—while always nonnegative—converges to zero as the number of observations increases. The bias introduced under cross-entropy estimation is again stronger for the infrequent verb v_1, but now is an *overestimation* bias, pointing into the direction *opposite* of our proposed relationship.

Cross-entropy estimation controls for the circularity confound in the sense that a systematic pattern in the direction of our hypothesized relationship must reflect a pattern in the underlying true (population-level) VFE values, as the estimation bias points in the opposite direction. However, cross-entropy estimation has two limitations: it replaces one bias by another, and it applies only to VFE, not to VFC. We thus also use a subsampling approach applicable to both measures.

Subsampling   As a second way of mitigating the circularity confound, we performed subsampling such that (i) subsamples of a fixed size (the subsampling threshold) are taken for all verbs with frequency above the threshold, and (ii) verbs with frequency below the subsampling threshold are excluded from the analysis.

Considering again the case of two verbs v_1, v_2 with the same distribution over frames but different counts N, $2N$, the subsampling approach simply caps observations of both verbs at a fixed threshold. Assuming that N exceeds that threshold – so the verbs are included – both verbs are on exactly equal footing in estimation of VFE and VFC – the estimation bias is by construction the same for v_1 and v_2; it depends only on the threshold.

Subsampling is applicable to both VFE and VFC, and eliminates effects of verb frequency on the estimation bias. However, verbs with insufficient observations need to be excluded, reducing the number of verbs for which correlations of these measures with verb frequency can be tested.

We determine the subsampling threshold for each language by applying a ratio of 0.1 to the maximum frequency of any verb in that language, but limit the absolute value to between a ceiling of 25 and a floor of 10 to avoid wide fluctuations between languages, which would have ranged from 2 in Mbya Guarani to 663 in German.

On average for 79 languages, subsampling filters out 91.6% of all verbs (SD = 4.7%). The number of verbs filtered out ranges from Russian (9083 out of 10410 verbs, 87.3%) to Mbya Guarani (108 out of 113 verbs, 95.6%). In cases like the latter, the small number of verbs limits the statistical power of the study. We include only languages where more than 10 verbs remain after subsampling, excluding a further four languages[2].

5   Results

We focus on results with both subsampling and cross-entropy estimation, as these choices lead to the most conservative estimates for testing our hypothesis – indeed, are expected to introduce a conservative bias against the hypothesized relationship. We also assess the effect of subsampling below.

We quantify these relationships using Spearman's rank correlation coefficient (Spearman, 1904). The results show robust correlation between frequency and our valency metrics: when using VFE, we observe at least moderate correlations ($\rho \geq .40$, following Schober et al., 2018) in 66 out of 75 languages and strong to very strong correlations (defined as $\rho \geq .70$) in 18. The remaining 9 languages show only weak correlations[3]. The mean Spearman's ρ is .59, with a standard deviation of .15. Using VFC measure results in similar correlations, marginally stronger than VFE on average across the 75 languages (mean $\rho = .67$, $SD = .14$), of which 71 show at least moderate correlations, and 31 strong to very strong.

Fig. 3a shows how the correlation coefficients vary across the languages and in relation to the sample sizes, using VFE measure. It confirms that larger sample sizes, either measured by the number of verb lexemes observed (shown on the x-axis) or by the overall corpora size in UD (showed color), lead to less cross-lingual variance, as the results for languages with smaller sample size show the greatest variance. A version of the plot using the VFC measure, which shows a similar pattern, is shown in Appendix, Fig. 7a.

At a more granular level, we plot individual verb lexemes by their frequency ranks on the one hand and VFE and VFC on the other in per-language scatter plots. Plots for select languages are shown in

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[2]Manx (10), Mbya Guarani (5), Upper Sorbian (10), Welsh (9)

[3]They are Akkadian (.17), Ancient Hebrew (.27), Bambara (.39), Chinese (.39), Classical Chinese (.39), Japanese (.20), Sanskrit (.38), Turkish (.36), and Urdu (.22).

(a) results using subsampling

(b) results not using subsampling

Figure 3: Scatter plots showing relationship between Spearman's rank correlation between valency frame entropy and frequency rank of verbs and the number of verbs across languages. Results use subsampling in subfigure 3a and full data in subfigure 3b; entropies are estimated as cross-entropies. Languages are represented by their ISO codes. Color shows the total number of tokens for this language in the UD. Note that the y-axes have different ranges in (a) and (b) to maximize visibility of the individual languages.

Figure 4: Scatter plots showing relationship between valency frame entropy or frame count and frequency rank (up to 1000) of verbs. Results use subsampling, the vertical line indicates the subsampling threshold, i.e. only verbs to the left of the line are used in calculating the correlation coefficient in Fig. 3; entropies are estimated as cross-entropies. Subsampling, while controlling for the circularity confound, introduces a flattening of observations at low frequency ranks; see Appendix, Fig. 8 for results for all languages, Fig. 9 for results without subsampling.

Fig. 4; the full set is shown in Appendix B, Fig. 8. Subsampling introduces a flattening effect for verbs with frequency counts above the respective subsampling thresholds, shown as vertical lines in the plots. For some languages, the leveling effect visually obscures the correlation due to the plot's density.

Effects of subsampling Our subsampling procedure as described in §3.2 eliminates the circularity confound by fixing the sample size, albeit at the expense of statistical power due to excluded samples. Results without subsampling validates our cautious approach for VFC: the average Spearman's rho between VFC and frequency rank is a staggering 0.996 (SD=.004), i.e., almost perfect correlations for all languages. Performing subsampling but then including verbs below the cutoff results in similar numbers (mean Spearman's $\rho =$.996, SD = .073 for VFC). This arguably reflects the circularity confound, eliminated using subsampling.

For the VFE measure, cross-entropy estimation alone is enough to remove the circularity confound, even without subsampling (§ 3.2). Indeed, without subsampling, we observe a stronger correlation (mean Spearman's $\rho =$.849, SD = .073), confirming that the results with subsampling are not specific to the higher-frequency verbs remaining under subsampling. Fig 3b shows cross-linguistic variation in the correlation coefficient (VFE) without subsampling. Compared to Fig 3a, we also observe a greater correlation between the sample size (as measured by the number of verbs included) and correlation strength, possibly due to the effect of a large number of infrequent verbs. While subsampling excludes verbs below the cutoff to eliminat the confound, the "long tail" of infrequent verbs is themselves not a problem for our hypothesis. On the contrary, our hypothesis would exactly predict a large number of infrequent verbs, for which little diversity in valency frames is expected.

Impact of UD Relations We next investigate whether the observed relations are driven by specific relations, or reflect an across-the-board statistical pattern. We considered core-only, non-core-only, and randomly sampled subsets (Figure 5). Across random subsets (sizes 5, 10, 15), larger sets lead to stronger correlations (Figure 5). Fig. 5 shows the correlations across randomized subsets. These results indicate that the observed correlation reflects an across-the-board statistical pattern across a larger set of relation types, and not an

Figure 5: Correlations between frequency rank and VFE / VFC, for the randomly sampled subsets of relations, and core-only and non-core-only relations. Results use subsampling.

artifact of any particular relation type.

Next, in order to compare the effects of the specific relations, we fitted a separate linear mixed-effects model for each relation, predicting the regression coefficient from the presence or absence of that relation in a random subset, with random adjustments for each language. We fitted the model on the set of 1000 randomly sampled subsets of size 5. The models are defined formally in Appendix A.

We show the results in Fig. 6, plotting each relation as a linear function with the fixed effect coefficient as the slope and the the fixed effect intercept. The value at UD relation = 1 is therefore the predicted correlation coefficient when the relation is present. It is important to note that a negative slope in the figure only indicates that including the relation *instead of* other relations on average has a negative effect on correlation strength, i.e., it has a *less positive* effect than average on correlation strength. It does not mean that its inclusion has a negative effect on correlation.

Relations do differ in their impact. The presence of nsubj and obl have the strongest positive effect on the correlation strength. The strong effect of the obl relation is as expected, as it encompasses a variety of features often part of key valency frame encoding, including case markings on adjuncts, passive verb use (as it encodes the agent of a passive verb), and dative alternation. The strong positive effect of nsubj is more surprising, but we speculate that it may be due to the variety of non-predicative uses of verbs, which often involve the absence of or different case markings on nsubj. There is little evidence that core and non-core relations may have systematically different effects.

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Figure 6: Effects of the 16 relation labels on VFE correlation strengthon. See Fig. 10 for a version of the plot using VFC. Solid lines indicate core relations; dashed ones indicate non-core relations.

6 Discussion

Results across 79 languages strongly support our hypothesis of a positive correlation between a verb's frequency and the valency frame diversity. It is evidence in support of features of language and lexical structure being shaped by a drive towards efficiency. The quantitative law was supported both by core and non-core dependents, and larger sets of relations strengthened the law. This suggests that, at least for the efficiency considerations studied here, distinctions such as between arguments and adjuncts may not be as clear-cut.

Open Questions and Future Research While the predicted pattern holds across languages, we also observed variation in correlation strength between languages. Understanding whether these differences reflect typologically meaningful variation is an interesting task for future work, beyond our scope here. One possibility is that the UD annotation scheme does not sufficiently account for cross-linguistic differences in argument encoding. One example of this is found in Chinese: verbs that have high frequency but low VFE are often from the class of *coverbs* (e.g., 自 'from', 隨 'follow / with'), which serve semantic functions similar to prepositions while behave syntactically like verbs. They have narrow semantic functions and are associated with a relatively small number of valency frames compared to similarly frequent verbs. This brings into question the strictness of word category boundaries, and fuller picture may need to better situate the verb category within the overall lexicon.

Related Work A key line of computational research on valency has created datasets of the valency frames associated with verbs, such as FrameNet (Baker et al., 1998; Fillmore and Baker, 2015) and VerbNet (Kipper-Schuler, 2005; Kipper et al., 2006, 2008). Mu et al. (2017) automatically classifies verbs in terms of their valency as listed in such resources to recover classifications from the theoretical literature (Levin, 1993). Cross-linguistically, Baker and Lorenzi (2020); Ellsworth et al. (2021) explores the alignment of frames based on FrameNet. In contrast to their approach, our operationalization of valency frame is morphosyntactic in nature, directly in terms of UD relations. As explained in Section 2, this allows us to match valency frames with large scale usage data across many languages and for the full set of verbs appearing in any given treebank – something that is not available when operationalizing frames using curated lexical resources such as FrameNet.

Our work also relates to computational studies of the lexicon's efficiency (e.g. Mahowald et al., 2022; Pimentel et al., 2021). This includes work revisiting Zipf's observation (Piantadosi et al., 2011; Pimentel et al., 2023), morphological regularity (Wu et al., 2019), orthographic forms (Mahowald et al., 2018), and the partitioning of semantic space into word meanings in domains such as color, kinship, and others (e.g. Regier et al., 2015; Kemp et al., 2018; Zaslavsky et al., 2018). In contrast to much of this prior work, our study focuses not on word-internal properties, but on the integration of a word within the sentence.

7 Conclusion

Across 79 languages, we have provided evidence for a quantitative law in the cross-linguistic organization of valency: More frequent verbs are associated with a larger variety of valency frames, operationalized both in terms of count-based or entropy-based metrics. Crucially, we showed that this pattern is not driven by differences in the number of observations between high- and low-frequency verbs, and persists even when equalizing the number of observations available for each verb. Extending a recent line of research studying the lexicon's efficiency for human communication and language use, these results suggest that such considerations apply not just to word-internal structure, but also to the way words are integrated within the sentence.

Limitations

In §2, we derived our prediction from efficiency considerations in both language production and language comprehension. However, our methods did not allow us to determine which of these pressures are ultimately responsible for the observed pattern. Indeed, it is an ongoing debate to what extent apparently efficient properties of language respect optimization for production or comprehension. It is possible that, in the context of valency organization, these perspectives will make distinct predictions. For instance, theories based on comprehension might induce asymmetries between dependents before and after the head. Teasing these apart in order to determine the contributions of production and comprehension is an interesting problem, beyond our scope here.

A second limitation is that, even though we considered data from 79 languages, we are not able to make truly universal claims. Some language families, such as the Indo-European languages, are substantially over-represented in available corpus data, whereas languages from some parts of the world, in particular Africa and indigenous languages of the Americas, are under-represented, both in the number of languages and language families, as well as in the corpora size.

A third limitation is that corpus data is finite and that in particular the circularity confound forced us to introduce biases to the estimation in order to eliminate that confound. While our results unambiguously show that the predicted correlation exists, a more precise estimation will be affected by these factors.

Fourth, we study a very simple operationalization of valency in terms of UD relations, far simpler than the sophisticated representations typically used in formal linguistic theories. This is due to the fact that such sophisticated representations are not currently available at scale and across languages. While we did verify that the pattern held across languages, we opted for cross-linguistic applicability over language-specific analysis.

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A   Linear Mixed-Effect Model

Formally, for the i-th relation across K languages, we fit a linear mixed-effect model with with N different randomly subsampled subsets of relations,

$$\boldsymbol{y} = \alpha^{(i)} + \beta^{(i)} X + Z \boldsymbol{u} + \boldsymbol{\epsilon}$$

where $\boldsymbol{y} \in [-1, 1]^N$ is the outcome variable, i.e. the strength of the rank correlation between verb frequency and valency frame entropy; $X \in \{0, 1\}^N$ indicates, for each of the N subsets, whether the i-th relation is in that subset; $\alpha^{(1)}$ is the fixed effect intercept; $\beta^{(1)}$ is a fixed effect coefficient; Z is a $N \times K$ design matrix of the random effect; u is a $K \times 1$ vector of the random effect of language; and ϵ is a $N \times 1$ column of residuals.

B   Additional Figures

(a) results using subsampling

(b) results not using subsampling

Figure 7: Scatter plots showing relationship between Spearman's rank correlation between valency frame count and frequency rank of verbs and the number of verbs across languages. Results use subsampling in subfigure 7a and full data in subfigure 7b; entropies are estimated as cross-entropies. Color shows the total number of tokens for this language in the UD.

Figure 8: Scatter plots showing relationship between valency frame entropy or frame count and frequency rank of verbs. Results use subsampling, the vertical line showing the subsampling threshold, i.e. only verbs to the left of the line are used in calculating the correlation coefficient in Fig. 1; entropies are estimated as cross-entropies.

Figure 9: Scatter plots showing relationship between valency frame entropy or frame count and frequency rank of verbs. Results do not use subsampling; entropies are estimated as cross-entropies.

Figure 10: Effects of the inclusion of the 16 relation labels on Spearman's rank correlation coefficient between valency frame count and frequency rank, using VFC metric