Forward-Chaining Temporal Point Process

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Abstract

We propose Forward-Chaining Temporal Point Process (FC-TPP), a neuro-symbolic framework that maintains a predicate-level latent state updated via differentiable multi-hop forward chaining. Instead of conditioning directly on raw histories, future events are generated from a logic-completed latent state, enabling constraint-aware and controllable forecasting. Experiments on synthetic and real-world datasets (MIMIC-IV, LogiCity, EPIC-100) show that FC-TPP improves generation and forecasting quality under limited data, achieves stronger rule adherence, and provides interpretable control over predicted trajectories.

1 Introduction

Forecasting future events in complex systems—such as clinical workflows or human activities—requires more than extrapolating statistical patterns. Real-world sequences are often **incomplete**, **sparse**, and **rule-governed**. Most temporal point process (TPP) models learn a hidden state summarizing history, but this state is statistical and does not explicitly track valid system conditions. As a result, forecasts may violate domain rules or fail under limited data.

We argue that forecasting is fundamentally a reasoning problem: predicting future events requires inferring latent conditions and propagating their implications.

To this end, we propose FC-TPP, a framework that integrates:

- a **predicate-level latent state** (interpretable conditions),
- a **multi-hop forward-chaining operator** (reasoning),
- a **TPP decoder** (continuous-time forecasting).

Unlike prior work, FC-TPP follows a **reasoning-before-forecasting paradigm**, ensuring that predictions are always conditioned on a logically consistent latent state.

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2 Background

A temporal point process models events (t_i, m_i) via conditional intensity:

$$\lambda(t, m|\mathcal{H}_t) \tag{1}$$

which defines the distribution of future events. However, standard TPPs rely on latent states that lack explicit semantic meaning. We instead introduce a predicate-based latent state, representing high-level conditions, and update it via forward chaining.

3 Forward-Chaining Latent Temporal Point Processes (FC-TPP)

We consider a marked temporal point process on $\mathbb{R}_+ \times \mathcal{M}$. We denote the history up to time t by $\mathbf{x}_t = \{(t_j, m_j) \mid t_j < t\}$, which serves as the input for updating the latent state. Let $\Delta t_i = t_i - t_{i-1}$ with $t_0 := 0$. FC-TPP generates events sequentially in an autoregressive manner, but each conditional distribution over the next event's time and mark is governed by an explicit *latent symbolic state* that evolves via forward chaining.

3.1 Latent Symbolic State

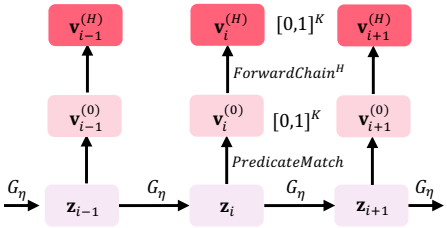

Figure 1. Latent state update in FC-TPP at event index i. Recurrent unit G_η updates the history summary $\mathbf{z}_i$. Given $\mathbf{z}_i$, PredicateMatch produces initial predicate scores $\mathbf{v}_i^{(0)}$, which are refined by H-hop ForwardChain into $\mathbf{v}_i^{(H)}$. Vertical arrows above $\mathbf{z}_{i-1}$ and $\mathbf{z}_{i+1}$ indicate that the same mapping is applied at each step.

FC-TPP maintains two coupled latent representations, as shown in Fig. 1. First, a lower-level recurrent hidden state $\mathbf{z}_t$ summarizes the raw event history up to time t. Second, a higher predicate-level *latent symbolic state*

$$\mathbf{v}_t^{(H)} \in [0, 1]^K$$

captures high-level conditions over a fixed predicate vocabulary $\mathcal{P} = \{P_1, \ldots, P_K\}$, where $[\mathbf{v}_t^{(H)}]_k$ is the current

activation (belief) of predicate P_k. At event times we write $\mathbf{z}_i := \mathbf{z}_{t_i}$ and $\mathbf{v}_i^{(H)} := \mathbf{v}_{t_i}^{(H)}$.

The recurrent state $\mathbf{z}_i$ is updated incrementally as new events arrive. Given the previous state $\mathbf{z}_{i-1}$, the inter-event gap Δt_i, and the new event mark m_i, we apply a GRU-style update:

$$\mathbf{z}_i = G_\eta(\mathbf{z}_{i-1}, \Delta t_i, m_i), \tag{2}$$

where G_η is a lightweight neural recurrent unit parameterized by η.

From $\mathbf{z}_i$ we then construct the predicate-level state in two steps. First, we obtain *initial predicate scores* $\mathbf{v}_i^{(0)} \in [0,1]^K$ by matching $\mathbf{z}_i$ to anchored predicate embeddings and then perform the H-step forward chaining (Sec. 3.2 for the descriptions of the operators):

$$\mathbf{v}_i^{(0)} = \mathrm{PredicateMatch}(\mathbf{z}_i). \tag{3}$$

$$\mathbf{v}_i^{(H)} = \mathrm{ForwardChain}^H\big(\mathbf{v}_i^{(0)}; \Theta_{\mathcal{F}}\big), \tag{4}$$

where $\Theta_{\mathcal{F}}$ is a learnable, embedding-based representation of a set of soft logic rules.

Intuitively, G_η absorbs *local event evidence* into $\mathbf{z}_i$, $\mathrm{PredicateMatch}$ projects that evidence into predicate space to form a higher-level symbolic state $\mathbf{v}_i^{(0)}$, and $\mathrm{ForwardChain}^H$ propagates rule implications to produce a *logically refined*, approximately rule-closed state $\mathbf{v}_i^{(H)}$. Together, η and $\Theta_{\mathcal{F}}$ are the learned parameters governing the latent-state dynamics.

3.2 Neuro-Symbolic Reasoning Layer

The neuro-symbolic reasoning layer consists of two operators: $\mathrm{PredicateMatch}(\cdot)$, and $\mathrm{ForwardChain}^H(\cdot)$. We elaborate on each of them next.

Predicate Matching: We define the operator $\mathbf{v}_i^{(0)} = \mathrm{PredicateMatch}(\mathbf{z}_i)$, which maps the recurrent state $\mathbf{z}_i$ to initial predicate activations $\mathbf{v}_i^{(0)}$. Specifically, for every $P \in \mathcal{P}$ we compute

$$\mathbf{u}_{i,P} = g_P(\mathbf{z}_i) \in \mathbb{R}^d,$$

using small MLPs $\{g_P\}$ or a shared projection with predicate-specific heads. Each predicate P has a fixed *anchored* semantic embedding that forms a set $\boldsymbol{A} = \{\mathbf{a}_P : P \in \mathcal{P}\}$ (see Sec. 4.3 for how to establish the set $\boldsymbol{A}$).

We then measure how well the current event sequence (through $\mathbf{z}_i$) supports predicate P using a cosine-based soft match between $\mathbf{u}_{i,P}$ and its anchor $\mathbf{a}_P$:

$$\mathbf{v}_i^{(0)} = [v_{i,1}^{(0)}, \ldots, v_{i,K}^{(0)}] \in [0,1]^K,\ v_{i,P}^{(0)} = \frac{\cos(\mathbf{u}_{i,P}, \mathbf{a}_P) + 1}{2}. \tag{5}$$

Here the cosine similarity is rescaled into $[0,1]$, so $v_{i,P}^{(0)}$ can be interpreted as a soft confidence score for predicate P at time step i (*before* any rule-based reasoning).

Forward Chaining: We define the operator $\mathbf{v}^{(H)} = \mathrm{ForwardChain}^H\big(\mathbf{v}^{(0)}; \Theta_{\mathcal{F}}\big)$ (to simplify the notaion, we drop time step i here), which recursively updates $\mathbf{v}^{(h)}$ at hop h using the soft rule set $\Theta_{\mathcal{F}}$. One hop corresponds to applying all rules to perform the forward chaining once, propagating evidence from bodies to heads. Repeating this process H times (until reaches the fixed point) yields H-hop reasoning, turning local predicate evidence in $\mathbf{v}_i^{(0)}$ into a multi-step, rule-consistent state $\mathbf{v}_i^{(H)}$.

We use smooth approximations of logical AND and OR. For inputs $u_1, \ldots, u_m$, we define

$$\mathrm{softmin}_\tau(u_1, \ldots, u_m) := -\tau \log \sum_{j=1}^{m} \exp(-u_j/\tau), \tag{6}$$

$$\mathrm{softmax}_\tau(u_1, \ldots, u_m) := \frac{\sum_{j=1}^{m} u_j \exp(u_j/\tau)}{\sum_{j=1}^{m} \exp(u_j/\tau)}, \tag{7}$$

which recover $\min$ and $\max$ as $\tau \downarrow 0$, while remaining smooth for finite τ.

Step 1: Rule Grounding. Each learnable rule parameter $\Theta_f \in \Theta_{\mathcal{F}}$ encodes an abstract symbolic pattern

$$\Theta_f = [\theta_{P_0}, \underbrace{\theta_{P_1}, \ldots, \theta_{P_c}}_{:=\Theta_b}]$$

which is a concatenation of embeddings for its head and body slots. To apply it, we align each slot embedding in the body part of the rule $\theta \in \Theta_b$ with the closest predicate embedding in the anchored predicate set $\boldsymbol{A} = \{\mathbf{a}_P : P \in \mathcal{P}\}$:

$$(\mathbf{a}^*, P^*) = \arg\max_{\mathbf{a}_P \in \boldsymbol{A}} \cos(\mathbf{a}_P, \theta), \quad \theta \in \Theta_b. \tag{8}$$

Here $P^*(\theta)$ is the index of the best-matching anchored predicate, and $\mathbf{a}^*(\theta)$ is its embedding.

Step 2: Body Aggregation (soft AND). Evidence from the body predicates is aggregated into a rule score using $\mathrm{softmin}_\tau$:

$$u_f^{(h)} = \mathrm{softmin}_\tau\Big(\{\cos(\mathbf{a}^*, \theta),\ v_{P^*}^{(h)}\}_{\theta \in \Theta_b}\Big). \tag{9}$$

The rule activation $u_f^{(h)}$ is high only when all body predicates are sufficiently supported: both strong body activations and strong semantic alignment contribute to firing the rule.

Step 3: Head Update (soft OR). If a predicate P is the head of multiple rules, their contributions are aggregated:

$$v_P^{(h+1)} = \mathrm{softmax}_\tau\Big(\{u_f^{(h)} : f \in \mathcal{F},\ \mathrm{head}(f) = P\}\Big). \tag{10}$$

In this update, the head predicate P is strongly activated whenever *any* of its associated rules achieves a high score $u_f^{(h)}$, while $\mathrm{softmax}_\tau$ provides a differentiable approximation to logical OR.

In summary, let $\mathbf{v}^{(h)}$ denote the predicate vector after hop h, the one hop of forward chaining is updating the symbolic latent states using Eqs. (8)-(10). H-hop reasoning is obtained by applying the operation recursively H times.

Choice of H Classical forward chaining iterates until convergence, producing a logically closed set of facts. Our

differentiable operator is a continuous relaxation of this idea: each hop adds one layer of inferred evidence, and H controls the effective reasoning depth. Running to a fixed point would be expensive and can raise the gradient saturation issues for gradient-based training; instead, we fix H to balance expressive multi-hop reasoning with optimization stability (see ablations in the experiments).

The resulting $\mathbf{v}^{(H)}$ serves as a logic-completed latent state that both enforces logical consistency and helps impute missing high-level information.

3.3 Latent Symbolic State-Driven TPP Decoder

The event decoder depends *only* on the latent symbolic state $\mathbf{v}_i^{(H)}$. The next-event distribution factorizes as

$$p_\psi(\Delta t_i, m_i \mid \mathbf{v}_{i-1}^{(H)}) = p_\psi(\Delta t_i \mid \mathbf{v}_{i-1}^{(H)})\, p_\psi(m_i \mid \mathbf{v}_{i-1}^{(H)}). \tag{11}$$

We parameterize $p_\psi(\Delta t_i \mid \mathbf{v}_{i-1}^{(H)})$ using an exponential-family distribution with parameters output by a small network applied to $\mathbf{v}_{i-1}^{(H)}$, and $p_\psi(m_i \mid \mathbf{v}_{i-1}^{(H)})$ as a categorical distribution with softmax logits from $\mathbf{v}_{i-1}^{(H)}$.

4 Learning, Generation, and Prediction

4.1 Learning Objective

Given an observed marked sequence $\mathbf{x} = \{(t_i, m_i)\}_{i=1}^N$, we form inter-event gaps $\Delta t_i = t_i - t_{i-1}$ and run the recurrent and reasoning layers forward to obtain $\mathbf{v}_{i-1}^{(H)}$ for each step i. The parameters $(\eta, \Theta_\mathcal{F}, \psi)$ are learned by maximizing the autoregressive log-likelihood of the next-event factors in Eq. (11):

$$\mathcal{L}(\eta, \Theta_\mathcal{F}, \psi) = \sum_{i=1}^{N} \log p_\psi(\Delta t_i, m_i \mid \mathbf{v}_{i-1}^{(H)}). \tag{12}$$

Since $\mathbf{v}_{i-1}^{(H)}$ is a deterministic function of the history via G_η, PredicateMatch, and ForwardChain^H, gradients back-propagate through the recurrent unit and the neuro-symbolic reasoning layer, encouraging the learned logic rules and predicate activations to be predictive of future times and marks.

4.2 Autoregressive Generation and Prediction.

At test time, FC-TPP defines an autoregressive generator for new TPP samples. Starting from an initial state $(\mathbf{z}_0, \mathbf{v}_0^{(H)})$ (e.g., zeros or derived from context), we iteratively: (i) sample $(\Delta t_i, m_i)$ from $p_\psi(\Delta t_i, m_i \mid \mathbf{v}_{i-1}^{(H)})$, (ii) set $t_i = t_{i-1} + \Delta t_i$ and append the event (t_i, m_i), (iii) update the recurrent state via $\mathbf{z}_i = G_\eta(\mathbf{z}_{i-1}, \Delta t_i, m_i)$, (iv) recompute the symbolic state by $\mathbf{v}_i^{(0)} = \mathrm{PredicateMatch}(\mathbf{z}_i)$ and $\mathbf{v}_i^{(H)} = \mathrm{ForwardChain}^H(\mathbf{v}_i^{(0)}; \Theta_\mathcal{F})$. This procedure is repeated until a time horizon or stopping criterion is reached. Because the decoder always conditions on the rule-completed state $\mathbf{v}_{i-1}^{(H)}$, the same learned logic rules constrain both training and generation, shaping the distribution of emitted event times and types in a structurally consistent way.

4.3 Symbolic Prior Bank (SPB): Anchored Predicate Set & Initial Rules

We initialize the anchored predicate set $\boldsymbol{A} = \{\mathbf{a}_P : P \in \mathcal{P}\}$ by encoding each predicate name (and short description) with a pretrained language model and projecting the resulting embedding into the model's latent dimension via a learned linear map.

Rule embeddings $\Theta_\mathcal{F}$ are initialized from either human-specified templates or LLM-generated candidates, but are always treated as trainable parameters and updated jointly with the recurrent and decoder components during learning.

In addition, we can use prompt-based techniques to iteratively query an LLM for new candidate rules during training, filtering and adding them to $\Theta_\mathcal{F}$ so as to enrich the rule set with higher-quality structures as more data are observed.

5 Experiments

FC-TPP naturally supports both sequence generation and future event prediction, as it models the underlying temporal dynamics through a latent reasoning process. In the following, we empirically validate its effectiveness under both generation and prediction settings.

5.1 Experimental Setup

Datasets We evaluate FC-TPP on six event sequence datasets spanning synthetic, semi-synthetic, and real-world settings: (i) *Synthetic*. **Syn@5** and **Syn@10** sampled from TLPP (Li et al., 2020). (ii) *Semi-Synthetic*. **LogiCity** (Li et al., 2024). (iii) *Real-World*. **MIMIC-IV** (Johnson et al., 2020), **EPIC-Kitchens-100** (Damen et al., 2020), and **IKEA ASM** (Ben-Shabat et al., 2021). All datasets are split into 80%/10%/10% for training, validation, and testing. Additional preprocessing details are provided in Appendix B.1.

Baselines We compare FC-TPP against SOTA baselines from two categories. (i) *Neural TPP models*: **THP** (Zuo et al., 2020), **AVAE** (Mehrasa et al., 2019), **GNTPP** (Lin et al., 2022), and **UFM-TPP** (Shou, 2025). (ii) *LLM-based models*: **SP (Simple-Prompt)** treats the LLM as a sequence sampler and directly prompts it to output complete event sequences in the form "$(t_1, \texttt{event}_1), (t_2, \texttt{event}_2), \ldots$". **QA (Question-Answering)**: the LLM recursively predicts the next event and its timestamp. Finally, **LAMP** (Shi et al., 2023) is also adapted. See implementation details in Appendix B.2.

Evaluation Metrics We evaluate *generation quality* via *in-distribution generation*—For *synthetic* and *semi-synthetic* datasets with oracles rules. We report **KL Divergence**

Table 1. Comparison of different methods about generation performance. Performance metrics are averaged across three different runs, which reported as (Mean ± SD). The best performance is in bold and also colored in purple.

Methods	Synthetic Datasets								Semi-Synthetic Datasets			
	Syn@5				Syn@10				LogiCity			
	KL ↓	QQ-RMSE ↓	MMD ↓	RV ↓	KL ↓	QQ-RMSE ↓	MMD ↓	RV ↓	KL ↓	QQ-RMSE ↓	MMD ↓	RV ↓
THP	$0.84_{\pm 0.06}$	$0.45_{\pm 0.05}$	$0.74_{\pm 0.08}$	$0.48_{\pm 0.03}$	$0.92_{\pm 0.08}$	$0.76_{\pm 0.05}$	$0.80_{\pm 0.05}$	$0.63_{\pm 0.05}$	$1.02_{\pm 0.05}$	$0.87_{\pm 0.04}$	$1.15_{\pm 0.10}$	$0.70_{\pm 0.12}$
AVAE	$0.67_{\pm 0.04}$	$0.38_{\pm 0.03}$	$0.62_{\pm 0.05}$	$0.46_{\pm 0.01}$	$0.79_{\pm 0.05}$	$0.67_{\pm 0.04}$	$0.81_{\pm 0.06}$	$0.58_{\pm 0.05}$	$0.86_{\pm 0.08}$	$0.83_{\pm 0.09}$	$1.05_{\pm 0.05}$	$0.64_{\pm 0.06}$
GNTPP	$0.62_{\pm 0.06}$	$0.37_{\pm 0.03}$	$0.54_{\pm 0.07}$	$0.37_{\pm 0.03}$	$0.71_{\pm 0.07}$	$0.62_{\pm 0.03}$	$0.76_{\pm 0.02}$	$0.45_{\pm 0.04}$	$0.80_{\pm 0.05}$	$0.72_{\pm 0.04}$	$0.94_{\pm 0.04}$	$0.42_{\pm 0.06}$
UFM-TPP	$0.54_{\pm 0.04}$	$0.31_{\pm 0.02}$	$0.48_{\pm 0.03}$	$0.45_{\pm 0.04}$	$0.72_{\pm 0.05}$	$0.64_{\pm 0.06}$	$0.70_{\pm 0.04}$	$0.58_{\pm 0.09}$	$0.75_{\pm 0.07}$	$0.67_{\pm 0.05}$	$0.88_{\pm 0.07}$	$0.83_{\pm 0.04}$
SP	$1.12_{\pm 0.07}$	$0.62_{\pm 0.06}$	$1.24_{\pm 0.11}$	$0.64_{\pm 0.08}$	$1.14_{\pm 0.08}$	$0.78_{\pm 0.07}$	$1.06_{\pm 0.10}$	$0.75_{\pm 0.10}$	$1.05_{\pm 0.09}$	$0.79_{\pm 0.06}$	$1.23_{\pm 0.10}$	$0.78_{\pm 0.03}$
QA	$0.78_{\pm 0.10}$	$0.40_{\pm 0.05}$	$0.71_{\pm 0.12}$	$0.59_{\pm 0.08}$	$0.85_{\pm 0.08}$	$0.72_{\pm 0.06}$	$0.87_{\pm 0.07}$	$0.64_{\pm 0.05}$	$0.96_{\pm 0.04}$	$0.85_{\pm 0.08}$	$1.16_{\pm 0.11}$	$0.72_{\pm 0.07}$
LAMP	$0.51_{\pm 0.03}$	$0.29_{\pm 0.01}$	$0.43_{\pm 0.02}$	$0.41_{\pm 0.03}$	$0.68_{\pm 0.02}$	$0.56_{\pm 0.04}$	$0.64_{\pm 0.05}$	$0.53_{\pm 0.08}$	$0.74_{\pm 0.06}$	$0.70_{\pm 0.04}$	$\mathbf{0.73}_{\pm 0.05}$	$0.47_{\pm 0.00}$
Ours*	$\mathbf{0.42}_{\pm 0.04}$	$\mathbf{0.23}_{\pm 0.04}$	$\mathbf{0.35}_{\pm 0.06}$	$\mathbf{0.28}_{\pm 0.06}$	$\mathbf{0.64}_{\pm 0.06}$	$\mathbf{0.54}_{\pm 0.12}$	$\mathbf{0.58}_{\pm 0.07}$	$\mathbf{0.33}_{\pm 0.00}$	$\mathbf{0.67}_{\pm 0.06}$	$\mathbf{0.65}_{\pm 0.07}$	$0.92_{\pm 0.10}$	$\mathbf{0.31}_{\pm 0.08}$

Methods	Real-World Datasets								
	MIMIC-IV			EPIC-100			IKEA ASM		
	DS ↓	GPTScore ↑	MMD ↓	DS ↓	GPTScore ↑	MMD ↓	DS ↓	GPTScore ↑	MMD ↓
THP	$0.46_{\pm 0.07}$	$0.44_{\pm 0.03}$	$0.78_{\pm 0.06}$	$0.48_{\pm 0.05}$	$0.50_{\pm 0.03}$	$1.46_{\pm 0.08}$	$0.48_{\pm 0.04}$	$0.55_{\pm 0.03}$	$0.82_{\pm 0.07}$
AVAE	$0.46_{\pm 0.02}$	$0.42_{\pm 0.03}$	$0.72_{\pm 0.03}$	$0.45_{\pm 0.06}$	$0.51_{\pm 0.01}$	$1.41_{\pm 0.09}$	$0.48_{\pm 0.03}$	$0.54_{\pm 0.02}$	$0.75_{\pm 0.02}$
GNTPP	$0.38_{\pm 0.03}$	$0.60_{\pm 0.05}$	$0.56_{\pm 0.04}$	$\mathbf{0.37}_{\pm 0.05}$	$\mathbf{0.62}_{\pm 0.02}$	$1.29_{\pm 0.08}$	$0.45_{\pm 0.03}$	$0.58_{\pm 0.02}$	$0.69_{\pm 0.04}$
UFM-TPP	$0.44_{\pm 0.02}$	$0.53_{\pm 0.06}$	$0.65_{\pm 0.03}$	$0.43_{\pm 0.04}$	$0.52_{\pm 0.02}$	$1.16_{\pm 0.08}$	$0.46_{\pm 0.01}$	$0.52_{\pm 0.02}$	$0.71_{\pm 0.03}$
SP	$0.45_{\pm 0.10}$	$0.38_{\pm 0.07}$	$0.94_{\pm 0.07}$	$0.47_{\pm 0.05}$	$0.41_{\pm 0.02}$	$1.63_{\pm 0.10}$	$0.48_{\pm 0.07}$	$0.51_{\pm 0.04}$	$1.38_{\pm 0.13}$
QA	$0.42_{\pm 0.09}$	$0.43_{\pm 0.05}$	$0.71_{\pm 0.06}$	$0.47_{\pm 0.03}$	$0.46_{\pm 0.02}$	$1.47_{\pm 0.14}$	$0.47_{\pm 0.03}$	$0.49_{\pm 0.05}$	$0.79_{\pm 0.07}$
LAMP	$0.41_{\pm 0.02}$	$0.55_{\pm 0.04}$	$0.58_{\pm 0.04}$	$0.45_{\pm 0.02}$	$0.54_{\pm 0.03}$	$1.06_{\pm 0.09}$	$0.39_{\pm 0.02}$	$0.62_{\pm 0.03}$	$0.64_{\pm 0.03}$
Ours*	$\mathbf{0.34}_{\pm 0.03}$	$\mathbf{0.70}_{\pm 0.06}$	$\mathbf{0.48}_{\pm 0.08}$	$0.43_{\pm 0.03}$	$0.58_{\pm 0.02}$	$\mathbf{0.95}_{\pm 0.12}$	$\mathbf{0.34}_{\pm 0.02}$	$\mathbf{0.65}_{\pm 0.06}$	$\mathbf{0.62}_{\pm 0.04}$

Table 2. Comparison of different methods about prediction performance on MIMIC-IV.

Methoods	Metrics	
	MAE	ER%
THP	$3.16_{\pm 0.07}$	$42.15_{\pm 3.25}$
AVAE	$3.41_{\pm 0.13}$	$48.33_{\pm 3.56}$
GNTPP	$3.30_{\pm 0.05}$	$45.72_{\pm 1.04}$
UFM-TPP	$3.25_{\pm 0.17}$	$42.83_{\pm 3.28}$
LAMP	$3.14_{\pm 0.09}$	$38.67_{\pm 1.25}$
SP	$3.72_{\pm 0.18}$	$51.06_{\pm 4.22}$
QA	$3.68_{\pm 0.13}$	$47.28_{\pm 2.46}$
LAMP	$2.96_{\pm 0.12}$	$40.52_{\pm 2.17}$
Ours*	$\mathbf{2.82}_{\pm 0.08}$	$\mathbf{37.50}_{\pm 1.62}$

and **QQ-RMSE** (Xiao et al., 2017) against the true event-time distributions, and **MMD (Maximum Mean Discrepancy)** (Shchur et al., 2020) between generated and real sequences. Moreover, we also consider **RV (Rule Violation Rate)**, directly quantifying logical errors of rule adherence. For *real-world* datasets without ground truth rules, we report proxy realism metrics, including **DS (Discriminator Score)** (Desai et al., 2021)—defined as $(\texttt{accuracy} - 0.5)$ of a two-layer LSTM trained to distinguish real from generated sequences, where values closer to zero indicate higher realism—and **GPTScore** (Fu et al., 2024), which measures the likelihood assigned by a LLM to textualized event sequences. For *prediction tasks*, we consider standard TPP metrics **MAE** and **ER%**. Details are shown in Appendix. B.3.

5.2 Results and Analysis

Analysis 1: Generation Performance and Comparison with Baselines In Tab. 1, on the real-world MIMIC-IV dataset, FC-TPP achieves statistically significant improvements over the strongest neural baseline (GNTPP), with relative gains of 11% in DS, 17% in GPTScore, and 14% in MMD. FC-TPP also attains superior performance on the IKEA ASM dataset. On EPIC-Kitchens-100, FC-TPP performs comparably to GNTPP, with only a small performance gap, suggesting competitive generation quality in more complex and visually grounded action domains.

Analysis 2: Prediction Performance and Comparison with Baselines We evaluate our model on MIMIC-IV using the standard next-event prediction protocol, reporting time prediction error and event type prediction error rate. The results in Tab. 2 show that FC-TPP achieves improved predictive performance over strong neural TPP baselines. Beyond these metrics, FC-TPP additionally provides interpretable predicate states and rules, enabling reasoning-aware predictions, which standard TPP models lack.

6 Conclusion

We introduced FC-TPP, a neuro-symbolic framework for temporal event forecasting that integrates differentiable forward chaining into latent-state dynamics. Unlike conventional approaches that predict directly from history, FC-TPP follows a reasoning-before-prediction paradigm, in which a predicate-level latent state is first updated through multi-hop logical inference and then used to generate future events.

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Appendix

A Prompt Design

Prompt for Querying Initial Rule Set The prompt for querying initial rule set is given below.

```
You are a logic reasoning assistant tasked with extracting high-level predicates and
logic rules based on observed temporal point process events.  Your goal is to identify
high-level predicates based on provided event types and logic rules that can capture
patient physical condition built on extracted predicates to identify the occurrence
patterns in temporal event sequences.  You are given a set of event types and some
of the examples of sequences.  The event type set consists of 38 different clinical
occurred events with clinical meaning.  And event sequences records in the format of
series of events, e.g.  (``event type'' @ ``event time'').

Your Goal:
Extract high-level predicates and generate reasonable and clinical meaningful logic
rules based on given clinical event types.

Instructions:
1.  Think aloud:  Propose reasonable number of high-level predicates.
2.  Think aloud:  Propose 3{5 candidate logic rules.
3.  Explain briefly why each rule might be effective.

Example Input:
Event type set:  [abnormal sysbp, abnormal spo2sao2, abnormal cvp, abnormal svr,
abnormal potassium meql, abnormal sodium, abnormal chloride, abnormal bun, abnormal
creatinine, abnormal crp, abnormal rbc count, abnormal wbc count, abnormal arterial
ph, abnormal arterial be, abnormal arterial lactate, abnormal hco, abnormal svo2 scvo2,
normal sysbp, normal spo2 sao2, normal cvp, normal svr, normal potassium meql, normal
sodium, normal chloride, normal bun, normal creatinine, normal crp, normal rbc count,
normal wbc count, normal arterial ph, normal arterial be, normal arterial lactate,
normal hco3, normal svo2 scvo2, low urine, intravenous fluids, use drugs, survival].


Event Sequences Examples:
Sequence 1:  [abnormal potassium meql @ 0.49, abnormal sodium @ 1.39, normal arterial
ph @ 1.78, ...]
Sequence 2:  [abnormal arterial be @ 1.58, normal arterial ph @ 1.58, abnormal sysbp @
1.62, ...]
Sequence 3:  [survival @ 0.00, abnormal potassium meql @ 0.01, normal arterial lactate
@ 0.01, ...]

Your Tasks:
Extract high-level predicates and generate symbolic rules based on these predicates.
For each rule, include 1{2 lines of explanation.

Output Example:
Extracted predicates:  A, B, C, ...
Generated rules:  IF A AND B AND C AND D, THEN E, ...
Explanation:  Because XXX, XXX...
```

Prompt for Rule Set Refinement The prompt for refine initial/current rule set is given below.

```
You are a logic reasoning assistant tasked with extracting more logic rules except for
the given logic rules.  You are also given some observed temporal point process events
which cannot be explained well by the already provided logic rules.  Your goal is to
identify more different logic rules that can capture patient physical condition built
on previously extracted predicates to identify the occurrence patterns in the provided
temporal event sequences.  You are given some of the sequences that cannot be explained
well by the provided logic rules.  And event sequences records in the format of series
of events, e.g.  (``event time'' @ ``event time'').  Please note that the rules should
build on previous extracted high-level concepts
```

```
Your Goal:
Generate extra reasonable and clinical meaningful logic rules based on previously
extracted high-level predicate and provided sequences which cannot be explained well
by already provided logic rules.

Instructions:
1.  Think aloud:  Propose 1{2 more candidate logic rules.
2.  Explain briefly why each rule might be effective.

Event Sequences Examples that cannot Be Explained Well by the Provided Logic Rules:
Sequence 1:  [abnormal sodium @ 0.01, abnormal chloride @ 0.01, abnormal bun @ 0.01,
...]
Sequence 2:  [abnormal potassium meql @ 0.05, normal arterial ph @ 0.05, abnormal spo2
sao2 @ 0.10, ...]
Sequence 3:  [survival @ 0.00, normal arterial ph @ 1.81, normal arterial be @ 1.81,
...]

Provided Logic Rules:
Rule-1:  IF Cardiovascular Instability AND Electrolyte Imbalance THEN Renal Dysfunction
Rule-2:  IF Respiratory Dysfunction AND Acid{Base Disturbance THEN Cardiovascular
Instability
Rule-3:  IF Electrolyte Imbalance AND Renal Dysfunction THEN Inflammatory / Immune
Response
Rule-4:  IF Cardiovascular Instability AND Treatment Intervention THEN Survival
Rule-5:  IF Respiratory Dysfunction AND Inflammatory / Immune Response THEN Acid{Base
Disturbance

Your Tasks:
Generate symbolic rules that are different with the given logic rules, which can best
explain the given temporal point process sequences.  For each rule, include 1{2 lines
of explanation.

Output Example:
Generated rules:  IF A AND B AND C AND D, THEN E.
Explanation:  Because XXX, XXX...
```

B Dataset, Baselines, and Metrics

B.1 Dataset

We summarize the dataset statistics in Tab. 3 and provide the preprocessing details below.

Synthetic Datasets *(i)* **Syn@5**: We create sequences consisting of 5 event predicates, sampled from a prespecified Temporal Logic Point Process (TLPP) (Li et al., 2020). Specifically, we employ multiple pre-defined logic rules along with their weights to construct the intensity function, and then apply thinning algorithms (Ogata, 1981) to generate new events. To evaluate the scalability of the proposed model, we have created five distinct groups, with the sample size varying from 1000 to 9000 (see Appendix. D.5). In the experiment of main context, we use the dataset with 5000 samples and an average sequence length of 20.56 events. *(ii)* **Syn@10**: We make the synthetic setting more challenging by extending the number of predicates to 10 and further increasing the number of underlying rules, thereby enlarging the complexity of the underlying logical space. This setup enables us to assess the model's capacity to cope with richer and more intricate scenarios, as well as to evaluate its robustness and generalization ability under more demanding conditions. In our main experiments, we employ a dataset comprising 5000 samples with an average sequence length of 38.71 events per sample.

Semi-Synthetic Datasets *(iii)* **LogiCity** (Li et al., 2024): LogiCity is an urban-scale multi-agent simulator grounded in customizable first-order logic (FOL). It models diverse urban entities through semantic and spatial concepts, which are used to formulate FOL rules that govern agent behaviors. In our initial case study, we simulate 8 distinct agents (6 car agents and 2 pedestrian agent), each characterized by predicates such as `IsAtIntersection`, `Stop`, and `CollidingClose`, resulting in a total of 24 predicates. We generate 500 sequences to facilitate evaluation of interpretable rule-based reasoning for generation tasks in dynamic environments. This platform serves as a testbed for neuro-symbolic methods in complex, interactive scenarios.

Real-World Datasets *(iv)* **MIMIC-IV** (Johnson et al., 2020): MIMIC-IV is a publicly available electronic health record dataset comprising patients admitted to the intensive care unit (ICU). We focus on those diagnosed with sepsis—a life-threatening condition and major cause of ICU mortality (Saria, 2018). We extract 2023 sequences with an average event count 33.10. Predicates are primarily categorized into two types: medication-related (e.g., `Use Drugs` or `Intravenous Fluids`) and clinical measurement-related (e.g., `Low Urine`, `Abnormal SpO2SaO2`, or `Abnormal ArterialPH`). These predicates support structured representation and logic-based reasoning for clinical event sequences. *(v)* **EPIC-100** (Damen et al., 2020): This dataset is derived from a large-scale egocentric vision corpus, consisting of unstructured audio-visual recordings captured in natural home environments. It focuses on daily kitchen activities observed over multiple days. From the annotated action sequences, we extract temporal event histories comprising only cooking verbs—intentionally omitting the interacted objects to emphasize verb-centric reasoning. The dataset includes 68 verbs, such as `Put-In`, `Rinse`, `Put-On`, `Pour`, `Stir`, `Peel`, `Chop`, and `Slice`, among others. In total, the dataset contains 8656 sequences, with an average length of 34.84 events per sequence. *(vi)* **IKEA ASM** (Ben-Shabat et al., 2021): This dataset comprises 371 unique assembly configurations of four furniture types—side table, coffee table, TV bench, and drawer—each available in three colors: white, oak, and black. We focus specifically on the TV bench assembly task, which involves 18 action predicates such as `Pick Up Leg`, `Tighten Leg`, and `Attach Shelf to Table`. The subset includes 91 sequences with an average length of 24.46 events per sequence.

Table 3. Dataset statistics. For Epic-Kitchen-100 and IKEA ASM datasets, we scale the time horizon to 0-200.

Category	Dataset	Statistics			
		# Predicates	# Sequences	Events Average Length	Time Horizon
Synthetic	**Syn@5**	5	5000	20.56	20
	Syn@10	10	5000	38.71	20
Semi-Synthetic	**LogiCity**	24	500	56.25	100
Real-World	**MIMIC-IV**	38	2023	33.10	118.42
	Epic-100	68	8656	34.84	1585
	IKEA ASM	18	91	24.46	5955

B.2 Baselines

We choose state-of-the-art baselines considering two different fields:

Neural Temporal Point Process Models *(i)* **THP** (Zuo et al., 2020): THP models event sequences by parameterizing the conditional intensity function with a Transformer encoder that captures long-range temporal dependencies among past events. Event times and types are embedded and processed through self-attention, enabling flexible modeling of history-dependent dynamics without explicit parametric kernels. Event sequences are then generated by sampling from the learned intensity via standard point process simulation. *(ii)* **AVAE** (Mehrasa et al., 2019): The model is a recurrent variational auto-encoder designed for modeling asynchronous action sequences. At each time step, the model utilizes the history of actions and inter-arrival times to generate a distribution over latent variables. A sample from this distribution is then decoded into probability distributions for the inter-arrival time and action label of the next action. To address the limitations of using a fixed prior in the traditional VAE framework, this model incorporates a prior net that enhances the learning process. *(iii)* **GNTPP** (Lin et al., 2022): The model is a comprehensive generative framework for neural temporal point process modeling. It leverages deep generative models as probabilistic decoders, including the temporal conditional diffusion denoising model, temporal conditional VAE, temporal conditional GAN, temporal conditional continuous normalizing flow, and temporal conditional noise score network models. The diverse combinations of encoders and decoders make the GNTPP highly flexible in approximating the target distribution of event occurrence times. For the encoder, the model incorporates both RNN-based approaches and self-attention mechanisms. In our experiments, we select the revised attentive history encoder and the VAE probabilistic decoder. *(iv)* **UFM-TPP** (Shou, 2025): It proposes a unified flow-matching framework for jointly modeling inter-event times and event types (marks), utilizing both continuous and discrete flow matching to achieve integrated generation. This approach enables the model to directly map from simple noise distributions—such as exponential for times and uniform for types—to realistic event sequences, without relying on step-by-step recursion.

LLM-Based Models *(v)* **SP (Simple-Prompt LLM Generation)** (Si et al., 2022): For simple-prompt LLM generation, most existing works on prompt for event generation/extraction aim for teh event extraction (identifying event triggers and

parameters from text); and story generation (generating narrative events (who did what)). In these works, "event" refers to events in the linguistic sense, rather than strictly stochastic modeling of temporal processes. Therefore, they are not directly equivalent to "TPP sequence generation". However, their methodologies can still inspire attempts at TPP generation: Prompt-as-sampler/Template prompting: Like the Simple Event Extraction Framework, this approach dispenses with complex probabilistic modeling and directly uses prompts to ask the LLM to output "$(t_1, \texttt{event type}_1), (t_2, \texttt{event type}_2)$...". *(vi)* **QA (QA-based LLM Generation)** (Castricato et al., 2021): For QA-based Generation, according to the idea of EDGAR (Castricato et al., 2021), the model can recursively generate the next event and its time through question-answering prompts. *(vii)* **LAMP** (Shi et al., 2023): A framework for TPPs that integrates a LLM. In particular, the LLM performs abductive reasoning to support the event sequence model. Guided by a few expert-annotated demonstrations, the LLM learns to suggest plausible causes for each candidate event. A search module then identifies previous events that align with these causes, and a scoring function evaluates whether the retrieved events could indeed lead to the proposed event. To adapt to TPP generation tasks, the original LAMP's proposer—which previously suggested candidate future events—has been replaced with an unconditional or conditional full-sequence candidate generator that produces a complete segment in a single pass. The process retains the LLM's abductive reasoning and re-ranking steps to filter and reorder candidates, ultimately outputting the final sample.

B.3 Computation of Metrics

In-Distribution Generation For fine-tuned generation, we can directly compare the generated sequences with the input sequences, since both are expected to follow similar patterns. Therefore, we can use following metrics to evaluate the quality of the generated sequences:

(i) **KL Divergence**: The synthetic datasets are generated by the temporal logic point process (TLPP). Therefore, we know the ground-truth logic rules and the corresponding rule weights, which allows us to compute the likelihood of each sequence. We first define the input sequence set and the output sequence set as $\boldsymbol{X} = \{\boldsymbol{X_i}\}_{i=1}^n$ and $\boldsymbol{Y} = \{\boldsymbol{Y_j}\}_{j=1}^n$. A sequence $\boldsymbol{S} = \{(t_l, a_l)\}_{l=1}^{N(\boldsymbol{S})}$ on the time horizon $[0, T]$. The event type $a_l \in \{1, ..., K\}$. Then the TLPP intensity for each event type k can be computed as

$$\lambda_k(t|\mathcal{H}_t) = g(\eta_k(t)), \text{ where } \eta_k(t) = b_k + \sum_{f=1}^{F} w_{f,k}\phi_{f,k}(\mathcal{H}_t, t)$$

where $g(\cdot) > 0$, e.g., $g(x) = \exp(x)$. Total intensity can be written as

$$\Lambda(t) = \sum_{k=1}^{K} \lambda_k(t|\mathcal{H}_t)$$

Then compute the log-likelihood for each sequence. For an arbitrary sequence $\boldsymbol{S}$

$$l(\boldsymbol{S}) = \sum_{l=1}^{N(\boldsymbol{S})} \log \lambda_{a_l}(t_l|\mathcal{H}_{t_l}) - \int_0^T \Lambda(t)dt$$

To remove scale effects from sequence length or time window, we compute the per-event normalization and the per-time normalization.

$$l_{\text{per-event}}(\boldsymbol{S}) = \frac{l(\boldsymbol{S})}{N(\boldsymbol{S})} \text{ (if } N(\boldsymbol{S}) > 0), \; l_{\text{per-time}}(\boldsymbol{S}) = \frac{l(\boldsymbol{S})}{T}$$

Take standardization $l_{\text{per-event}}(\boldsymbol{S})/l_{\text{per-time}}(\boldsymbol{S})$ and collect vectors

$$\boldsymbol{l_X} = \{l(\boldsymbol{X}_1), ..., l(\boldsymbol{X}_n)\}, \; \boldsymbol{l_Y} = \{l(\boldsymbol{Y}_1), ..., l(\boldsymbol{Y}_n)\}$$

Estimate the log-likelihood density of two groups using kernel density estimation (KDE)

$$\hat{p}(x) = \frac{1}{nh}\sum_{i=1}^{n} k(\frac{x - l_X^{(i)}}{h}), \; \hat{q}(y) = \frac{1}{nh}\sum_{j=1}^{n} k(\frac{y - l_Y^{(j)}}{h})$$

Then the KL divergence between the two log-likelihood distribution are given by

$$\hat{\text{KL}}(\hat{p}||\hat{q}) = \frac{1}{n}\sum_{i=1}^{n} \log \frac{\hat{p}(l(\boldsymbol{X_i}))}{\hat{p}(l(\boldsymbol{Y_i}))}$$

(ii) **QQ-RMSE** (Xiao et al., 2017): Similarly we can get the KDE or histogram of $\boldsymbol{l_x}$ and $\boldsymbol{l_Y}$ on same axes. Then we compute the empirical quantiles $Q_X(\alpha)$, $Q_y(\alpha)$ for $\alpha \in (0, 1)$. plot pairs $(Q_X(\alpha), Q_Y(\alpha))$. If distributions match, points

lie on $y = x$. Implement discrete sampling by using $\alpha_k = \frac{k}{m+1}, k = 1, ..., m$. Then we can quantify the QQ deviation

$$\text{QQ-RMSE} = \sqrt{\frac{1}{m}\sum_{k=1}^{m}\left(Q_X(\alpha_k) - Q_Y(\alpha_k)\right)^2}$$

(iii) **MMD (Maximum Mean Discrepancy)** (Shchur et al., 2020): We computed MMD (Gretton et al., 2012) between the original sequences and the generated sequences after training. MMD quantifies the dissimilarity between the true data distribution $p^*(\boldsymbol{t})$ and the learned density $p(\boldsymbol{t})$ — lower is better. Now we provide the computation process:

Following above definition, the input sequence set is denoted as

$$\boldsymbol{X} = \{\boldsymbol{X_1}, \boldsymbol{X_2}, ..., \boldsymbol{X_n}\}\,, \text{ each } \boldsymbol{x_i} = [(t_{i1}, a_{i1}), (t_{i2}, a_{i2}), ..., (t_{iL_i}, a_{iL_i})]$$

And the output sequence set is denoted as

$$\boldsymbol{Y} = \{\boldsymbol{Y_1}, \boldsymbol{Y_2}, ..., \boldsymbol{Y_n}\}\,, \text{ each } \boldsymbol{y_j} = \left[(t_{j1}, a_{j1}), (t_{j2}, a_{j2}), ..., (t_{jL_j}, a_{jL_j})\right]$$

For each sequence in input set, we build sequence embedding

$$\boldsymbol{e_{ik}} = [\psi(t_{ik}); \text{emb}(a_{ik})] \in \mathbb{R}^d$$

where $\psi(\cdot)$ is the positional embedding for event time and emb$(\cdot)$ is the one-hot embedding for event type. Take the average of the entire sequence, we obtain

$$\tilde{\boldsymbol{X}}_{\boldsymbol{i}} = \frac{1}{L_i}\sum_{k=1}^{L_i}\boldsymbol{e_{ik}},\ \tilde{\boldsymbol{Y}}_{\boldsymbol{j}} = \frac{1}{L_j}\sum_{k=1}^{L_j}\boldsymbol{e_{jk}}$$

Then, we define the kernel function (RBF kernel, i.e., Gaussian kernel)

$$k(\boldsymbol{A}, \boldsymbol{B}) = \exp\left(-\frac{||\boldsymbol{A} - \boldsymbol{B}||^2}{2\sigma^2}\right)$$

Finally, we can obtain unbiased MMD estimation

$$\text{MMD}^2(\boldsymbol{X}, \boldsymbol{Y}) = \frac{1}{n(n-1)}\sum_{i\neq j}k(\tilde{\boldsymbol{X}}_{\boldsymbol{i}}, \tilde{\boldsymbol{X}}_{\boldsymbol{j}}) + \frac{1}{n(n-1)}\sum_{i\neq j}k(\tilde{\boldsymbol{Y}}_{\boldsymbol{i}}, \tilde{\boldsymbol{Y}}_{\boldsymbol{j}}) - \frac{2}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}k(\tilde{\boldsymbol{X}}_{\boldsymbol{i}}, \tilde{\boldsymbol{Y}}_{\boldsymbol{j}})$$

(iv) **RV (Rule Violation Rate)**: For the given rule set $\mathcal{F}$, the generated event sequences $\boldsymbol{X}$, we define rule trigger count as

$$T = \sum_{x\in\boldsymbol{X}}\sum_{f\in\mathcal{F}}\mathbb{I}\left[\bigwedge_{i=1}^{c}P_i(x) = 1\right]$$

Define rule violation count as

$$V = \sum_{x\in\boldsymbol{X}}\sum_{f\in\mathcal{F}}\mathbb{I}\left[\bigwedge_{i=1}^{c}P_i(x) = 1 \wedge P_0(x) = 0\right]$$

Then, the rule violation rate is defined as

$$RV = \frac{V}{T}$$

Here, we take an example to explain how to define the rule is obeyed. Consider a logic rule as $P_0 \leftarrow P_1 \wedge P_2$, namely P_0 must occur after P_1 and P_2 within Δt. Then, we can define

$$P_0(x) = 1 \Leftrightarrow \exists t_0 \in [\max(t_1, t_2), \max(t_1, t_2) + \Delta_t]$$

(iv) **DS (Discriminator Scores)** (Desai et al., 2021): The generated sequences are used to train the post-hoc sequence classifi-cation models (by optimizing a 2-layer LSTM) to distinguish between sequences from the original and generated sequences. First, each original sequence is labeled "*Real*", and each generated sequence is labeled "*Not Real*". Then, an off-the-shelf classifier is trained to distinguish between the two classes as a standard supervised task. Therefore, we obtain the discrimination scores (`accuracy` - 0.5). A score close to 0 is better, indicating the generated data is hard to distinguish from original data.

(v) **GPTScore** (Fu et al., 2024): A new framework that evaluates event sequence texts with generative pre-training models like GPT-3. It assumes that a generative pre-training model will assign a higher probability of high-quality generated event sequence text following a given instruction and context.

Note that the metrics KL and QQ-RMSE require ground truth from the temporal logic point process model. Consequently, they are only applicable to synthetic datasets.

C Robustness Evaluations in LLM-based Rule Extraction and Evaluation

Compare Different LLMs as Symbolic Prior Bank To compare the quality of rules provided by different LLMs, we consider TinyLlama-1.1B-Chat-v1.0 (Zhang et al., 2024), Gemma-2-2B-IT (Team et al., 2024), Opt family (Zhang et al., 2022) (Opt-125M, Opt-1.5B), and GPT family (OpenAI, 2022): (GPT-3.5, GPT-4o). As shown in Tab. 4, intriguingly, while the capability for logic rule generation generally improves with model scale—with a particularly notable leap from 1B to 10B parameters—we observe that even some smaller LLMs achieve competitive results. For instance, using OPT-1.5B as the rule generator still yields performance on MIMIC-IV that substantially outperforms the baselines in Tab. 1. This demonstrates the compatibility of our approach with a range of pre-trained language models.

Table 4. Compare the effect of rules generated by different LLMs on MIMIC-IV dataset.

Methods	Metrics			
	Loss ↓	DS ↓	GPTScore ↑	MMD ↓
TinyLlama	$162.94_{\pm 21}$	$0.39_{\pm 0.00}$	$0.65_{\pm 0.00}$	$0.62_{\pm 0.02}$
Gemma-2	$162.28_{\pm 27}$	$0.36_{\pm 0.00}$	$0.68_{\pm 0.04}$	$0.54_{\pm 0.02}$
Opt-125M	$163.21_{\pm 25}$	$0.38_{\pm 0.01}$	$0.63_{\pm 0.02}$	$0.54_{\pm 0.02}$
Opt-1.5B	$162.45_{\pm 24}$	$0.34_{\pm 0.01}$	$0.66_{\pm 0.02}$	$0.50_{\pm 0.01}$
GPT-3.5	$162.22_{\pm 16}$	$0.34_{\pm 0.01}$	$0.70_{\pm 0.03}$	$0.50_{\pm 0.03}$
GPT-4o	$\mathbf{161.33}_{\pm \mathbf{0.22}}$	$\mathbf{0.34}_{\pm \mathbf{0.03}}$	$\mathbf{0.70}_{\pm \mathbf{0.06}}$	$\mathbf{0.48}_{\pm \mathbf{0.08}}$

Compare Different LLMs as Judges We employed multiple LLM judges (including GPT-4o and smaller open-source models) to reduce dependency on a single model and reported results with means and variances. Across in-distribution (Tab. 5) evaluations on MIMIC-IV dataset, we observe consistent performance with low variance between different LLMs. This indicates strong agreement in quality assessment across model scales, validating that our results are not reliant on the idiosyncrasies of any single LLM.

Table 5. Robustness evaluation with multiple LLM judges on MIMIC-IV (GPTScore).

Metrics	LLMs used				
	GPT-4o	**GPT-4o + Opt-1.5B**	**GPT-4o + Opt-1.5B + Opt-125M**	**GPT-4o + Opt-1.5B + Opt-125M + Gemma-2**	**GPT-4o + Opt-1.5B + Opt-125M + Gemma-2 + TinyLlama**
GPTScore ↑	$0.71_{\pm 0.00}$	$0.71_{\pm 0.01}$	$0.68_{\pm 0.01}$	$0.68_{\pm 0.02}$	$0.67_{\pm 0.02}$

D Experimental Details

D.1 Visualization of Input and Output Sequence Comparison

In the main text, we report quantitative analyses across multiple evaluation metrics. To complement these results, Fig. 2 presents a representative input–output comparison from the MIMIC-IV dataset after training convergence. The generated sequences basically align with the observed sequences in both event frequency and temporal structure, indicating that our model not only reconstructs the training data effectively but also achieves good-quality sequence generation.

D.2 Representative Queried Rules with Real-World Significance on Real-World Dataset

We present part of representative logic rules queried from LLM for MIMIC-IV, EPIC-Kitchen-100, IKEA ASM real-world datasets in Tab. 6, Tab. 7, and Tab. 8. These logic rules are well-aligned with the domain common sense and realistic constraints of the datasets, and are therefore integrated as domain knowledge into the neuro-symbolic reasoning layer to support structured and interpretable inference.

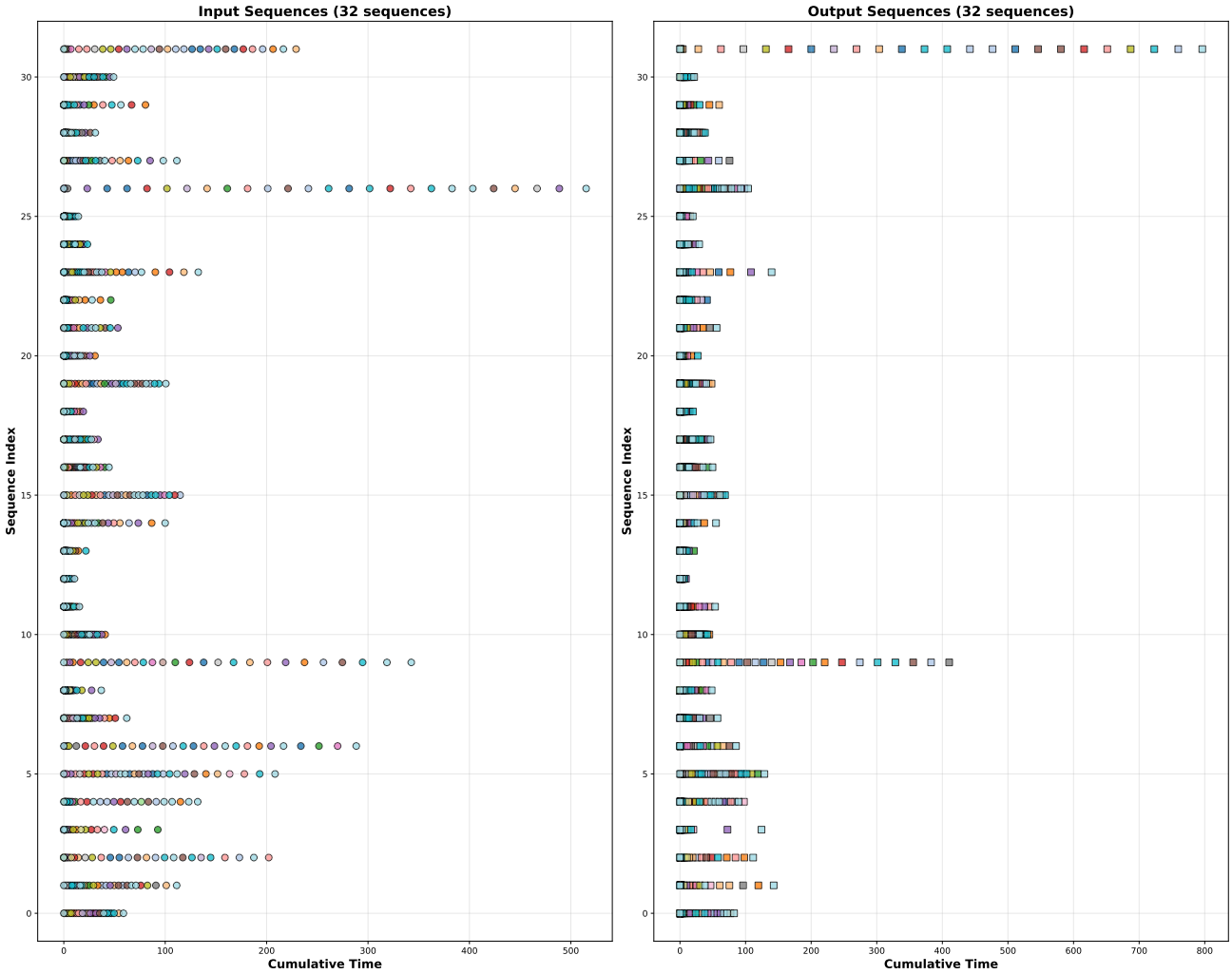

Figure 2. Comparison of model inputs and generated outputs on a representative batch from the MIMIC-IV dataset at convergence.

D.3 Few-Shot Generation for Epic-Kitchen-100 dataset

As presented in Analysis 1 of the main text, our model exhibits slightly lower performance than the GNTPP baseline on the EPIC-Kitchen-100 dataset under full data (100% training) in terms of DS and GPTScore. However, results in Fig. 3 demonstrate stronger data efficiency when less training data is available: with 60% or fewer samples, our model outperforms most baselines on EPIC-Kitchen-100. In the extreme low-data regime using only 20% of the training data, our method substantially surpasses other approaches in sequence generation quality. Specifically, with 20% data, our model achieves DS = 0.45, GPTScore = 0.53, and MMD = 1.25. In comparison, the second-best results are achieved by GNTPP (DS = 0.46, GPTScore = 0.41) and by LAMP (MMD = 1.33). These findings underscore the suitability of our approach for few-shot generation, especially in data-scarce applications such as synthesizing rare disease data.

D.4 Ablation Study

From the results of ablation study for MIMIC-IV dataset in Tab. 9, we found that introducing neural pre-train module enabling finer-grained temporal reasoning and additional improvements, yielding gains of 17% in DS, 7% in GPTScore, and 4% in MMD, along with faster convergence and lower final training loss. The multi-step forward chaining similarly provides consistent gains across all evaluation metrics. The multi-round LLM rule modification and querying module also improves model performance, though to a lesser extent than the first two modules.

D.5 Scalability and Time Efficiency

To evaluate the time efficiency and scalability of our proposed method, we conduct experiments on the synthetic dataset Syn@5 with varying sample sizes: 1000, 3000, 5000, 7000, and 9000. Due to the incorporation of the neuro-symbolic

Table 6. Part of logic rules queried from LLM for MIMIC-IV dataset.

Part of Rules Queried from LLM for MIMIC-IV dataset
Rule-1: `Renal Dysfunction` ← `Cardiovascular Instability` ∧ `Electrolyte Imbalance`
Rule-2: `Cardiovascular Instability` ← `Respiratory Dysfunction` ∧ `Acid{Base Disturbance`
Rule-3: `Inflammatory / Immune Response` ← `Electrolyte Imbalance` ∧ `Renal Dysfunction`
Rule-4: `Survival` ← `Cardiovascular Instability` ∧ `Treatment Intervention`
Rule-5: `Acid{Base Disturbance` ← `Respiratory Dysfunction` ∧ `Inflammatory / Immune Response`
....

Table 7. Part of logic rules queried from LLM for Epic-Kitchen-100 dataset.

Part of Rules Queried from LLM for Epic-Kitchen-100 dataset
Rule-1: `Put-Into (Something)` ← `Open (Something)` ∧ `Take (Something)` ∧ `Cut (Something)`
Rule-2: `Pour-Into (Something)` ← `Turn-On (Something)` ∧ `Put (Something)`
Rule-3: `Turn-Off (Something)` ← `Turn-On (Something)` ∧ `Wash (Something)`
Rule-4: `Chop (Something)` ← `Take (Something)` ∧ `Cut (Something)` ∧ `Put-Down (Something)`
Rule-5: `Grab (Something)` ← `Open (Something)` ∧ `Take (Something)` ∧ `Close (Something)` ∧ `Put-Down (Something)`
....

reasoning layer and multi-round LLM rule refinement, our model exhibits slightly lower time efficiency compared to other deep neural methods. However, this does not constitute a substantial gap—training time per epoch remains comparable to baseline models across all dataset sizes.

Encouragingly, the additional complexity of our reasoning mechanism yields tangible benefits. Consistent with the findings in Analysis 3 in the main context and the results in Appendix. D.3, our model maintains strong performance in few-shot settings, which can also be found in Fig. 4. When the training sample size is reduced to only 1000 samples, our approach still outperforms most baseline models across the generative metrics. Moreover, performance improves steadily as sample size increases, demonstrating favorable scalability.

E Reproducibility Analysis

E.1 Hyper-Parameter Selection

Our model is easy to implement and reproduce the results. We present the selected hyper-parameters on synthetic, semi-syntheti and real-world datasets in Tab. 10. The hyper-parameter selection metric is a trade-off between training converged loss, generation performance, and time efficiency.

E.2 Computing Infrastructure

All synthetic data experiments and real-world data experiments, including the comparison experiments with baselines, are performed on Ubuntu 20.04.3 LTS system with Intel(R) Xeon(R) Gold 6248R CPU @ 3.00GHz, 227 Gigabyte memory.

E.3 Computational and Memory Complexity

Computational Complexity The forward pass complexity is dominated by the encoder and the reasoning layer:

- History encoder: a Transformer over N events: $\mathcal{O}(N^2 \cdot d_e)$.
- Neuro-symbolic reasoning layer: the cost per sequence is $\mathcal{O}(|\mathcal{F}| \cdot H \cdot c)$, where $|\mathcal{F}|$ is the number of rules, H is the number of reasoning hops, and c is the average rule body size. This scales linearly with the key parameters.
- Event decoder: a feed-forward network applied at each of the $\hat{N}$ generated events: $\mathcal{O}(\hat{N})$.

Memory Complexity The memory footprint is primarily determined by storing. The overall memory usage is linear in the sequence length and model dimensions:

- Sequence embeddings: $\mathcal{O}(N \cdot d_e)$ for the encoder outputs.
- Reasoning states: $\mathcal{O}(H \cdot K)$, where K is the number of predicates.

Table 8. Part of logic rules queried from LLM for IKEA ASM dataset.

Part of Rules Queried from LLM for IKEA ASM dataset
Rule-1: `Tighten Leg` ← `Pick Up Leg` ∧ `Align Leg Screw with Table Thread` ∧ `Spin Leg`
Rule-2: `Tighten Leg` ← `Align Leg Screw with Table Thread` ∧ `Spin Leg`
Rule-3: `Rotate Table` ← `Tighten Leg`
Rule-4: `Flip Table` ← `Tighten Leg`
Rule-5: `Attach Shelf to Table` ← `Flip Table` ∧ `Pick Up Shelf`
Rule-6: `Pick Up Leg` ← `Flip Table Top`
....

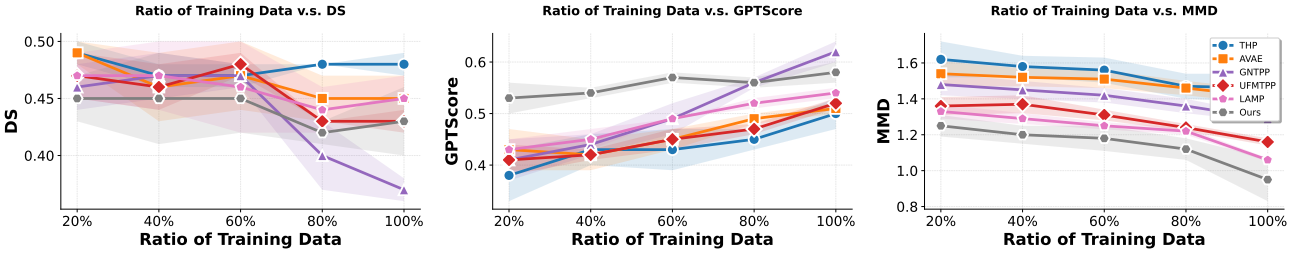

Figure 3. Few-Shot generation comparison for Epic-Kitchen-100 dataset, using MMD as evaluation metric. No comparison for SP and QA because they use pre-trained LLM.

Table 9. Ablation study on MIMIC-IV. We ablate the following modules: *(i)* Neural Pre-Train: if ablated, replace the GRU-style update to a simple MLP update, only take inter-event gap and the event mark as the input (without recurrent state). *(ii)* Forward Reasoning: if ablated, fix the forward reasoning step as 1. *(iii)* Rule Refine: if ablated, remove multi-round LLM rule refinement.

Ablation			**Metrics**			
Neural Pre-Train	**Forward Reasoning**	**Rule Refine**	**Loss ↓**	**DS ↓**	**GPTScore ↑**	**MMD ↓**
✗	✗	✗	$173.08_{\pm 0.16}$	$0.48_{\pm 0.03}$	$0.59_{\pm 0.03}$	$0.57_{\pm 0.04}$
✓	✗	✗	$168.51_{\pm 0.11}$	$0.40_{\pm 0.03}$	$0.63_{\pm 0.03}$	$0.55_{\pm 0.04}$
✓	✓	✗	$162.34_{\pm 0.28}$	$0.37_{\pm 0.02}$	$0.68_{\pm 0.02}$	$0.51_{\pm 0.03}$
✓	✓	✓	$\mathbf{161.33}_{\pm \mathbf{0.22}}$	$\mathbf{0.34}_{\pm \mathbf{0.03}}$	$\mathbf{0.70}_{\pm \mathbf{0.06}}$	$\mathbf{0.48}_{\pm \mathbf{0.08}}$

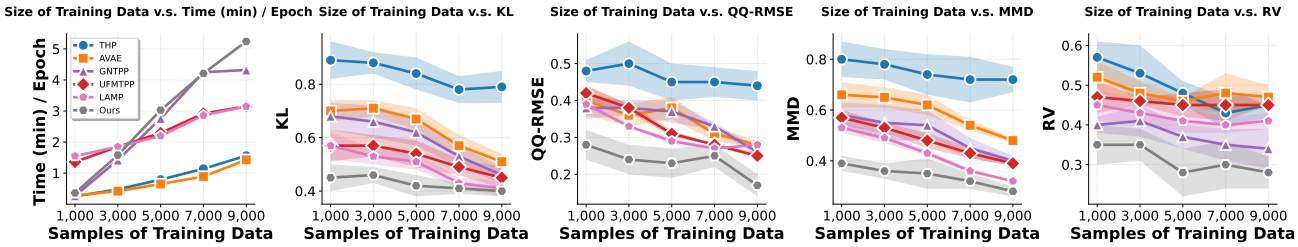

Figure 4. Time efficiency and scalability experiments for Syn@5 dataset. We vary sample size in $\{1000, 3000, 5000, 7000, 9000\}$.

Table 10. Descriptions and values of hyper-parameters used for models trained on the synthetic, semi-synthetic and real-world datasets.

Hyper-Parameters	**Value Used**					
	Syn@5	**Syn@10**	**LogiCity**	**MIMIC-IV**	**EPIC-100**	**IKEA ASM**
Max Epochs	32	32	50	50	64	64
Max Reason Steps (H)	3	3	7	5	5	5
Batch Size	32	32	32	32	64	32
Predicate Embed Size	32	32	64	32	32	32
Hidden Size	32	32	32	32	32	32
# Layers	2	2	2	2	2	2
LLM Rule Extractor	–	–	–	GPT-4o	GPT-4o	GPT-4o
LLM Judge (Evaluation)	–	–	–	Opt-1.5B	Opt-1.5B	Opt-1.5B
Warm-up Learning Rate	False	False	True	True	True	True
Learning Rate	1e-3	1e-3	1e-2	1e-2	1e-2	1e-3
Optimizer	Adam	Adam	Adam	Adam	Adam	Adam