Beyond Instruction Following: Evaluating Inferential Rule Following of Large Language Models

Anonymous ACL submission

Abstract

Although Large Language Models (LLMs) have demonstrated strong **instruction-following** ability, they are further supposed to be controlled and guided by **inferential rules** in real-world scenarios to be safe, accurate, and intelligent. This demands the possession of **inferential rule-following** capability of LLMs. However, no prior work has made a clear evaluation of the inferential rule-following capability of LLMs. Previous studies that try to evaluate the inferential rule-following capability of LLMs fail to distinguish the inferential rule-following scenarios from the instruction-following scenarios. Therefore, this paper first clarifies the concept of inferential rule-following and proposes a comprehensive benchmark, **RuleBench**, to evaluate a diversified range of inferential rule-following abilities. Our experimental results on a variety of LLMs show that they are still limited in following rules. Our analysis based on the evaluation results provides insights into the improvements for LLMs toward a better inferential rule-following intelligent agent. We further propose Inferential Rule-Following Tuning (IRFT). The experimental results show that through IRFT, LLMs can learn abstract inferential rule-following abilities from purely synthetic data and then generalize to RuleBench. The data and code can be found at: https://anonymous.4open.science/r/llm-rule-following-B3E3/

1 Introduction

Benefiting from a vast amount of pre-training data and the enormous parameters, the Large Language Models (LLMs) can accomplish numerous Natural Language Processing (NLP) tasks thanks to their instruction-following ability. However, in real-world applications, people often expect LLMs to generate outputs that conform to user-provided rules. In this way, LLMs could easily be manipulated by users and quickly adapted to a specific (even unseen) domain. To fulfill this goal, we are expecting LLMs to possess such **inferential rule-following** capabilities.

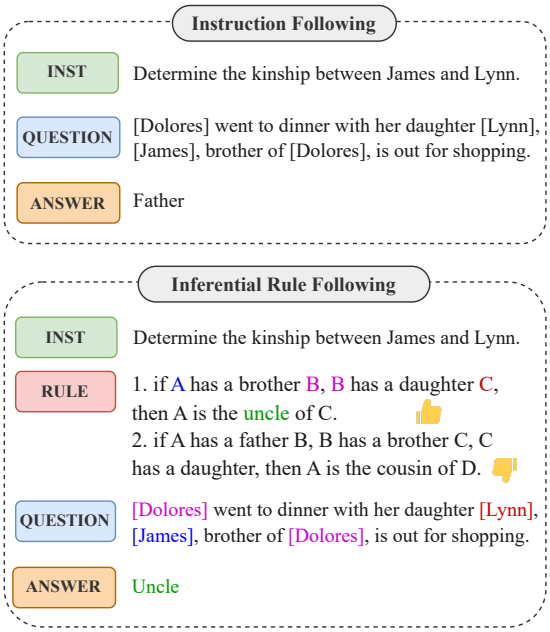

Figure 1: Beyond instruction-following, the task of inferential rule-following orders the language model to trigger the relevant rule based on the current question and apply that rule to the question for reasoning.

This leads to research on the inferential rule-following of LLMs. Some recent studies (Yang et al., 2023; Sun et al., 2023; Zhu et al., 2023; Zhao et al., 2023) have noticed the importance and effectiveness of inferential rule-following of large language models, they have found that ordering LLMs to follow existing rules can achieve better reasoning performances compared with the currently widely used reasoning enhancement methods of LLMs (such as Chain-of-Thought by Wei et al. 2022, Self-reflection by Shinn et al. 2023, and Self-refinement by Madaan et al. 2023).

However, whether LLMs could understand and follow the inferential rules remains unclear. There

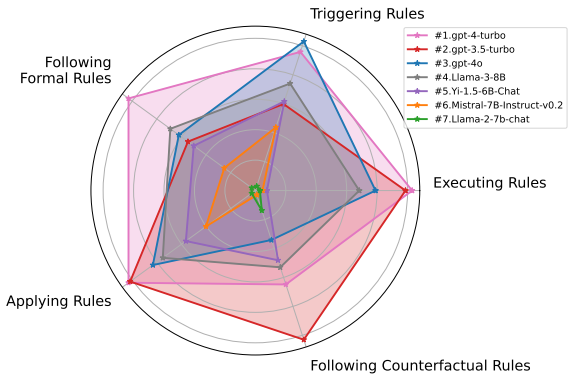

Figure 2: The inferential rule-following capabilities of some open-source and closed-source LLMs. The inferential rule-following capabilities of LLMs are categorized into 5 dimensions: Triggering Rules, Applying Rules, Executing Rules, Following Formal Rules, and Following Counterfactual Rules.

is currently **a lack of benchmarks** evaluating such inferential rule-following capability of LLMs. Existing attempts to evaluate the rule-following capabilities of LLMs (Mu et al., 2023; Hu et al., 2024; Chen et al., 2024) have been limited to instruction-following. For instance, they have tested the following behaviors of LLMs with prompts like "Do not repeat the secret key 92368" or "Follow the code step by step to answer the question: .....". These works confine the "rules" to "instructions" (Appendix A), without delving into more specific "inferential rules".

An *inferential rule* can be formalized as $\sigma \vdash \varphi$, where σ and φ are two first-order sentences (composed of variables and predicates), and for every substitution τ (i.e. ground the variables in σ and φ to constants), the truth of $\tau[\sigma]$ entails the truth of $\tau[\varphi]$ (Fagin et al., 1992). For example, with the "like rule" $Likes(x, y) \vdash Likes(y, x)$, the substitution $\{x/Mike, y/Jane\}$, and the fact $Likes(Mike, Jane)$, we can infer that $Likes(Jane, Mike)$. Although defined in formal language, in natural language, we can express such inferential rule with an "if ... then ..." sentence, by using instantiable noun phrases like *person A* or *one metal* as the variables and verb phrases like *is the father of* or *can conduct electricity* as the predicates inside it. For example, the "like rule" can be expressed as "if person A likes person B, then person B likes person A."

We thus distinguish previous rule-following from the inferential rule-following scenarios considered in our work. Different from instructions, the primary characteristics of inferential rules are abstract, conditional, and instantiable. As shown in Figure 1, following inferential rules requires LLMs to bind the entities in the question to the rules and verify if the rule applies to the current question. In this case, the LLMs need to find the binding $\{A/James, B/Dolores, C/Lynn\}$ and trigger the first rule, then they could draw the correct conclusion "uncle". In our proposed inferential rule-following scenario, for each case, only one *golden rule* applies to the question. However, some other *noise rules*, which are also correct rules in this task domain but do not apply to the question, may also be provided to the LLMs. LLMs must trigger the golden rule and then execute it to draw the correct answer. Until now, no prior work has demonstrated whether LLMs can follow and reason with the inferential rules faithfully.

Therefore, beyond the **instruction-following** studies by previous works, this paper evaluates the LLMs' capability of **inferential rule-following** in various reasoning tasks within the scope of inferential rules. This paper proposes a rule-following benchmark, RuleBench, for evaluating the inferential rule-following capability of LLMs under multiple inferential rule-following scenarios, including relation extraction, content moderation, commonsense QA, science QA, and judgment prediction.

Based on RuleBench, this paper has evaluated multiple open-source and closed-source LLMs (§4.1) to find out whether LLMs could understand and follow the inferential rules. In specific, we answer the following questions:

- Are inferential rules helpful to the reasoning of LLMs? (§4.2)
- Should inferential rules be presented in natural language or formal language? (§4.3)
- Is LLMs with Chain-of-Thought (CoT) able to effectively apply the rules? (§4.4)
- Can LLMs still follow inferential rules in counterfactual scenarios? (§4.5)

This paper also analyzes the cases where LLMs fail to follow the rules (§4.6), categorizing them into *Triggering Error* and *Execution Error*, which stand for the cases where LLMs fail to trigger the golden rule and LLMs fail to execute the golden rule, respectively.

Based on these results, as shown in Figure 2, we categorize the inferential rule-following capabilities of LLMs into 5 dimensions (§4.7) to help us intuitively grasp the inferential rule-following

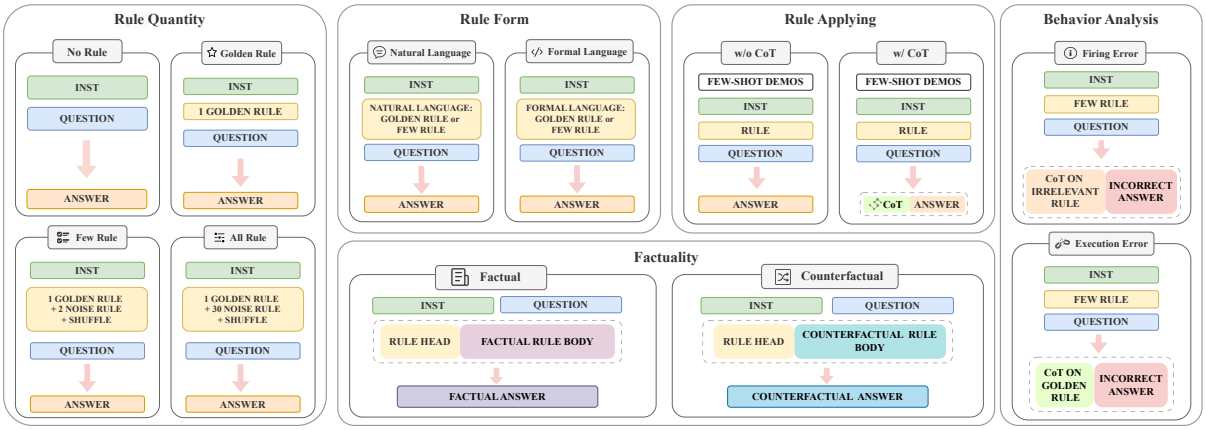

Figure 3: The different settings evaluated in RuleBench, including rule quantities, rule forms, Chain-of-Thought (CoT) in applying rules, counterfactual rules, and behavior analysis.

capabilities of these LLMs.

Finally, to further improve the inferential rule-following capabilities of LLMs, we propose the Inferential Rule-Following Tuning (IRFT) that enables LLMs to learn to trigger and apply the correct inferential rule based on the current cases (§5). The experimental results show that through IRFT, LLMs can learn abstract inferential rule-following abilities from purely synthetic data and then generalize to RuleBench. In summary, the major contributions of this paper are as follows:

- We introduce **inferential rule-following** as a vital capability of LLMs and distinguish it from the previous labors on instruction-following.
- We leverage and re-process the existing reasoning benchmarks and propose an inferential rule-following benchmark, RuleBench, for evaluating the inferential rule-following capability of LLMs.
- We evaluated the capabilities of inferential rule-following of multiple open-source and closed-source LLMs on various tasks and rule settings, and categorized their inferential rule-following abilities into 5 dimensions. Based on the results, we analyze the possible reasons that limit the inferential rule-following capabilities of current LLMs and provide some insights into the improvements for LLMs toward a better inferential rule-following intelligent agent.
- We propose the Inferential Rule-Following Tuning (IRFT) that enables LLMs to learn to trigger and apply the correct inferential rule based on the current cases. The experimental results show that through IRFT, LLMs can learn abstract inferential rule-following abilities from purely synthetic data and then generalize to RuleBench.

2 Related Work

2.1 Rule-enhanced LLM Reasoning

While LLMs have demonstrated remarkable zero-shot reasoning capabilities in many downstream tasks, they still generate outputs that do not conform to logic or human preference. Some research studies have found that compared with the reasoning enhancement methods based on LLMs themselves like Chain-of-Thought (Wei et al., 2022), Self-reflection (Shinn et al., 2023), and Self-refinement (Madaan et al., 2023), providing LLMs with relevant rules with Retrieval-Augmented Generation (RAG) paradigm do better in helping them conduct reasoning in the downstream tasks (Yang et al., 2023; Sun et al., 2023; Zhu et al., 2023; Zhao et al., 2023). However, the inferential rule-following capability of LLMs is far from satisfactory. No prior work has comprehensively evaluated whether LLMs can benefit from the provided rules under different scenarios and how LLMs can follow the rules better. To make up for this gap, this paper conducted a series of experiments to evaluate the inferential rule-following capabilities of several open-source and closed-source LLMs and provide some insights into how LLMs can follow rules better.

2.2 LLMs Instruction-following

Instruction-following has been generally considered an important capability of LLMs (Zhong et al., 2021; Mishra et al., 2021; Wei et al., 2021; Yin et al., 2023), and some previous works have been done to evaluate the instruction-following capability of LLMs (Zhou et al., 2023; Qin et al., 2024). However, only a few works have cast their atten-

tion to the question of inferential rule-following. Recent works focused on the rule-following capability of LLMs (Mu et al., 2023; Hu et al., 2024; Chen et al., 2024) confined the rule-following to instruction-following. Instead, this paper proposes the scenario of inferential rule-following and sets up useful baselines for future works.

3 RuleBench

To construct RuleBench, we have leveraged and reprocessed the existing reasoning benchmarks for different inferential rule-following scenarios, including relation extraction (CLUTRR, Sinha et al. 2019), content moderation (SALAD, Li et al. 2024), commonsense QA (DEER, Yang et al. 2022 and ULogic, Wang et al. 2024), mathematics QA (TheoremQa, Chen et al. 2023), and judgment prediction (CAIL2018, Xiao et al. 2018; Zhong et al. 2018). The details of the construction of each benchmark and the prompts used during constructing RuleBench can be found in Appendix B.

Under the scenarios introduced above, As shown in Figure 3, RuleBench involves multiple settings of inferential rule-following, to comprehensively evaluate the LLMs from different perspectives. The settings include rule quantity (i.e. how many rules are provided to the LLMs while only one of them applies to the current case), rule form (i.e. which form the rules illustrated in, natural language or formal language), the presence of Chain-of-Thought when applying rules (i.e. directly generate the answer based on the question and rules, or trying verbally apply the rule to the question before answering it), and rule factuality (i.e. whether the conclusion of the rule is factual or counterfactual). RuleBench allows us to analyze the failure cases of inferential rule-following from a behavioral perspective, classifying them into *Triggering Error* (i.e. LLMs fail to trigger the golden rule) and *Execution Error* (i.e. LLMs success to trigger the golden rule but fail to execute the golden rule).

4 Evaluation

To comprehensively evaluate the inferential rule-following capabilities of LLMs, based on the proposed RuleBench, this paper has designed 5 main parts of experiments. We evaluate the effects of rule quantity (§4.2), rule form (§4.3), the presence of CoT when applying rules (§4.4), and rule factuality (§4.5). Besides, we analyzed the failure cases of inferential rule-following from a behavioral perspective, classifying them into *Triggering Error* and *Execution Error* (§4.6). Based on these evaluation results, we categorize the inferential rule-following capabilities into 5 dimensions and compare the performances of 7 open-source and closed-source LLMs (§4.7). The test-time prompts used in this section can be found in Appendix C.

4.1 Model Selections

For open-source LLMs, we adopt Llama-2-7b-chat (Touvron et al., 2023), Meta-Llama-3-8B (AI@Meta, 2024), Mistral-7B-Instruct-v0.2 (Jiang et al., 2023) and Yi (Young et al., 2024). For closed-source LLMs, we adopt gpt-3.5-turbo, gpt-4-turbo (OpenAI, 2023), and gpt-4o from OpenAI. The comprehensive performance comparison of them is shown in Figure 2 and the explanation and analysis is in §4.7.

4.2 Inferential Rules Are Helpful for the Reasoning of LLMs

To evaluate whether inferential rules are helpful for the reasoning of LLMs, we adopt the following settings to test the LLMs.

- **No Rule**. This setting simply prompts the LLMs with the original question and without the inferential rules.
- **Golden Rule**. This setting prompts the LLMs with the *golden rule* (i.e. an inferential rule that should be applied to the question) together with the original question.
- **Few Rule**. This setting prompts the LLMs with the *golden rule* and two random *noise rules* together with the original question.
- **All Rule**. This setting is similar to **Few Rule** while the number of *noise rules* increases to 30. This setting simulates a scenario where users prompt the LLMs with all possible inferential rules in the tasks instead of the relevant rules retrieved based on the query.

All these rule settings are tested in a zero-shot manner. As shown in Figure 4, in most cases, LLMs enjoy great performance improvements while being prompted with one golden inferential rule (**No Rule** → **Golden Rule**). Nevertheless, as the number of noise rules increases, LLMs will find it hard to trigger and leverage the golden rule and thus have a performance drop (**Golden Rule** → **Few Rule** → **All Rule**).

Besides, we find that by following inferential rules, LLMs have better performance improve-

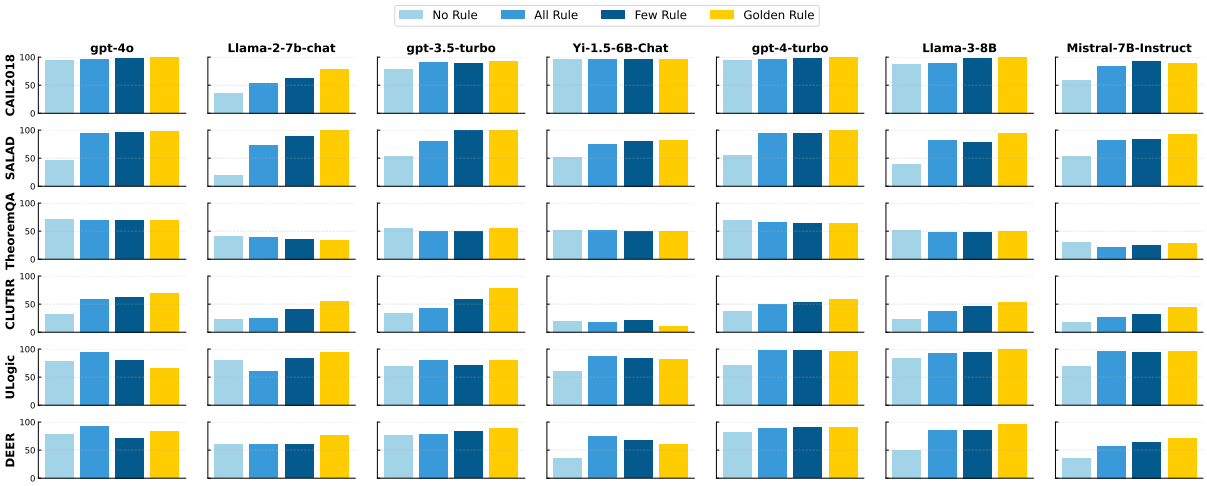

Figure 4: The inferential rule-following performance of LLMs under different rule quantities.

Model	Form	CAIL2018		SALAD		TheoremQA		CLUTRR		ULogic		DEER	
		All	Few	All	Few	All	Few	All	Few	All	Few	All	Few
gpt-4o	FOL	95.78	95.18	92.93	**96.20**	**70.36**	**69.45**	43.13	43.10	87.59	70.60	73.81	38.10
	NL	**96.99**	**97.59**	**95.19**	95.35	69.45	**69.45**	**58.97**	**62.67**	**94.82**	**79.16**	**92.86**	**71.43**
Llama-2-7b-chat-hf	FOL	50.60	53.01	58.27	81.22	**39.16**	**38.18**	23.00	27.48	**60.36**	79.04	57.14	**71.43**
	NL	**53.61**	**63.25**	**72.59**	**88.91**	39.00	35.96	**24.33**	**40.36**	60.00	**83.86**	**59.52**	59.52
gpt-3.5-turbo	FOL	84.94	86.14	73.33	96.83	49.82	**53.26**	41.54	53.28	63.73	66.10	71.43	78.57
	NL	**90.36**	**89.76**	**80.64**	**99.24**	**50.18**	50.36	**42.32**	**59.36**	**80.72**	**70.48**	**78.57**	**83.33**
Yi-1.5-6B-Chat	FOL	**96.99**	**97.59**	64.36	70.28	**53.27**	**52.00**	**23.95**	**23.47**	**90.12**	81.93	64.29	52.38
	NL	**96.99**	95.78	**75.73**	**80.44**	52.00	49.64	17.27	21.37	87.59	**82.77**	**73.81**	**66.67**
gpt-4-turbo	FOL	96.39	95.78	93.25	93.72	**66.55**	**65.09**	46.95	**53.99**	**98.43**	98.55	**90.48**	88.10
	NL	**96.99**	**98.80**	**94.81**	**94.70**	66.36	64.36	**49.32**	53.92	**98.43**	**98.67**	88.10	**90.48**
Meta-Llama-3-8B	FOL	89.16	93.98	77.71	77.16	**48.91**	**50.73**	**38.55**	41.89	**92.17**	**97.23**	83.33	80.95
	NL	**89.76**	**97.59**	**82.28**	**77.92**	48.18	48.55	37.40	**45.52**	**92.17**	95.06	**85.71**	**85.71**
Mistral-7B-Instruct	FOL	78.31	79.52	72.62	80.54	17.45	21.45	25.10	29.20	86.75	92.41	**64.29**	**64.29**
	NL	**84.34**	**92.77**	**82.36**	**83.43**	**22.18**	**24.55**	**26.62**	**32.73**	**95.42**	**94.46**	57.14	**64.29**

Table 1: The of LLMs on RuleBench with rules of formal language (FOL) and natural language (NL).

ments on tasks that require complex reasoning, such as CLUTRR and CAIL2018. On the commonsense reasoning tasks, as the LLMs have parametric knowledge, the performance improvements brought by following inferential rules are relatively slim. Moreover, we find that all LLMs fail to follow the inferential rules in TheoremQA, which illustrates the defect of current LLMs that can not follow complex mathematical or physical rules.

4.3 LLMs Prefer Natural Language Rules than Formal Language Rules

Formal language is widely used in early Artificial Intelligence, which is able to conduct efficient and generalized reasoning. However, LLMs have shown competitive or even superior reasoning performance over traditional formal language rule-based engines, i.e. Knowledge Graphs (Luo et al., 2023). In contrast to formal language rule-based reasoning, reasoning with LLMs is more flexible and robust to various data and tasks. Therefore, we would like to know if we can combine these two paradigms, i.e. whether LLMs can follow formal language rules.

To evaluate whether LLMs can follow formal language rules, we transform the natural language rules of each benchmark into the form of First-Order Logic (FOL) by executing deterministic functions or prompting ChatGPT (Appendix B). Then we compare the reasoning performances of LLMs which are prompted by different forms of inferential rules in both All Rule and Few Rule settings.

As shown in Table 1, in most cases, LLMs conduct reasoning better with natural language rules than formal language rules (except for TheoremQA and Yi-1.5-6B-chat). This aligns with our intuition that LLMs are mostly pre-trained with natural language and thus the inferential rules expressed

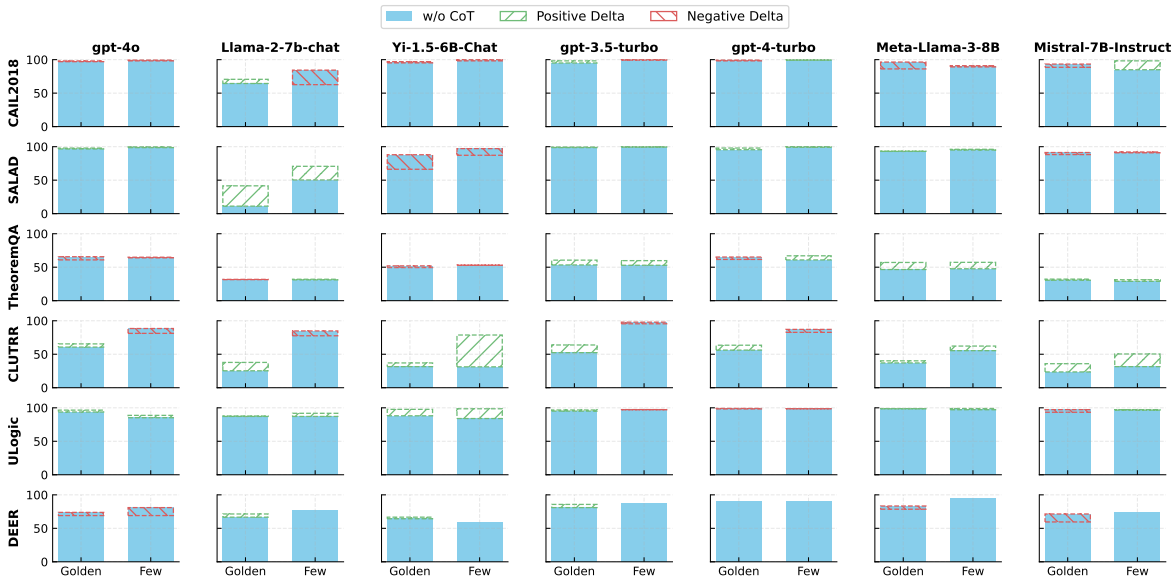

Figure 5: The inferential rule-following performance of LLMs when applying rules with or without using Chain-of-Thought. The dashed box indicates the improvement (positive or negative) of w/ CoT over w/o CoT.

with natural language are closer to the pre-trained distributions of LLMs than the inferential rules expressed with formal language. This confirms the research motivation and value of Symbol-LLM (Xu et al., 2023). Nevertheless, in most cases, LLMs still can follow the formal language rules. This reveals the possibility of learning formal language rules from a symbolic reasoning engine and then using LLMs for neural inference.

4.4 Chain of Thought Is Inadequate for LLMs to Apply Inferential Rules

Chain-of-Thought (Wei et al., 2022) has been widely verified as a useful prompting technique to help LLMs conduct multi-hop reasoning. To evaluate whether LLMs can use CoT to apply inferential rules in the inferential rule-following scenario, we choose the few-shot Golden Rule and Few Rule settings. We created two demonstrations with CoT and two demonstrations without CoT under such settings for LLMs to conduct In-context Learning.

However, as shown in Figure 5, LLMs with CoT have not exhibited stronger inferential rule-following performances in most cases. This may be attributed to the lack of **planning** of CoT. CoT conducts straightforward reasoning from the question to the answer with multiple reasoning hops. However, when applying the inferential rules, it involves trying to apply each rule to the current question and thinking about whether to execute this rule. Therefore, plain CoT is inadequate for LLMs to apply the inferential rules. Prompting techniques (e.g. Tree of Thought, Yao et al. 2024) or decoding algorithms (e.g. KCTS, Choi et al. 2023) that involve planning steps are needed for helping LLMs to apply the inferential rules.

4.5 LLMs Struggle to Follow Counterfactual Inferential Rules

Although we have verified the effectiveness of the inferential rules, it is still unclear whether LLMs strictly follow the given inferential rules or merely the rules activate their parametric knowledge. Therefore, we designed the scenario of **counterfactual rule-following**.

To evaluate whether LLMs can follow counterfactual rules, we construct corresponding counterfactual benchmarks and rule sets of CLUTRR, SALAD, ULogic, and CAIL2018. Specifically, we replace the ground truth of each question and the conclusion of the corresponding rule with a random incorrect answer. So in this counterfactual setting, the LLMs are supposed to generate the "incorrect answer" based on the given counterfactual rules.

As shown in Table 2, in most cases of both Golden Rule and Few Rule settings, LLMs have significant performance drops when following counterfactual rules, compared with following factual rules. These results indicate that the performance improvements brought by following rules are partly attributed to the parametric knowledge of LLMs, besides following inferential rules.

Model	Factuality	CAIL2018		SALAD		CLUTRR		ULogic	
		Golden	Few	Golden	Few	Golden	Few	Golden	Few
gpt-4o	Factual	**98.19**	96.99	**99.74**	**97.93**	**81.18**	**65.46**	**88.71**	**96.55**
	C.F.	97.14	**97.14**	8.22	68.42	37.50	5.53	86.87	88.80
Llama-2-7b-chat	Factual	62.65	**70.63**	**70.68**	41.54	**77.58**	**37.79**	**91.81**	**87.71**
	C.F.	**79.25**	67.21	26.06	**52.12**	71.37	13.93	67.35	53.61
Yi-1.5-6B-Chat	Factual	**98.19**	**95.18**	**87.15**	**66.27**	**78.72**	**37.02**	**98.55**	**97.71**
	C.F.	87.43	80.57	71.24	56.68	70.13	19.27	73.61	71.08
gpt-3.5-turbo	Factual	**99.40**	**98.19**	99.74	99.48	95.49	**63.83**	**97.11**	**96.75**
	C.F.	97.14	91.43	**100.0**	**100.0**	**98.50**	43.86	81.69	75.78
gpt-4-turbo	Factual	99.40	98.19	**100.0**	**97.98**	**82.70**	**63.49**	**98.58**	**98.31**
	C.F.	**99.43**	**100.0**	12.93	77.55	78.72	36.16	86.51	86.99
Meta-Llama-3-8B	Factual	**89.16**	86.14	96.09	93.35	**62.21**	**40.17**	**98.92**	**98.80**
	C.F.	62.29	**86.29**	**100.0**	**94.00**	39.50	6.49	79.64	75.90
Mistral-7B-Instruct	Factual	**98.19**	**88.55**	**90.88**	**88.19**	**50.38**	**35.88**	**97.17**	**93.36**
	C.F.	88.00	65.14	62.10	87.48	26.15	13.26	33.61	24.10

Table 2: The performance of LLMs on RuleBench when following factual and counterfactual (C.F.) rules.

4.6 Behavioral Analysis of LLMs Following Inferential Rules

To understand why LLMs fail to follow the given inferential rules in the reasoning process, we made a behavioral analysis of LLMs in the failure cases of LLMs inferential rule-following. Specifically, we adopt the few-shot Few Rule settings for LLMs to follow the rule-applying demonstrations to apply the given inferential rules to the current question. We ordered the LLMs first to choose an inferential rule to follow and then reason with it. By parsing the output of LLMs we can classify the failure cases of LLMs inferential rule-following into two categories: *Triggering Error* and *Execution Error*. *Triggering Error* indicates that the LLMs choose a noise rule for the current case and therefore lead to an incorrect reasoning result. *Execution Error* indicates that although LLMs have chosen the correct rule for the current case, they fail to draw the correct conclusion of *rule body*. To faithfully describe the inferential rule-following behavior of LLMs instead of being affected by the parametric knowledge of LLMs, we run the analysis under the counterfactual settings of the selected benchmarks.

From the results shown in Figure 6, we can tell that when tackling different tasks, LLMs exhibit different behaviors in following rules. While rules have a heavy head for triggering (e.g. in CLUTRR and CAIL2018, the rule head will be a series of relational hops among characters), the LLMs are likely to make *Triggering Errors*. While the rule head is easy and commonsensical (e.g. in SALAD and ULogic), but the conclusion of the rule body is ambiguous or confused (the counterfactual scenario), the LLMs are likely to make *Execution Errors*. A case study of these two types of failures can be found in Appendix E.

To avoid *Triggering Errors* in the scenario of rule-enhanced reasoning with RAG paradigm (§2.1), the **rule retriever** plays a crucial role. The *Triggering Errors* can be eliminated if the **rule retriever** only retrieved the golden rules. However, existing works mostly employ simple sparse retrievers such as BM25 (Yang et al., 2023; Zhu et al., 2023), which greatly compromises the inferential rule-following performance of LLMs.

To avoid *Execution Errors* in following rules, the LLMs need to faithfully execute the rule body and avoid generating conclusions of illusions. Therefore, users may avoid letting LLMs follow the rules that are counterfactual or out of the pre-trained distribution of LLMs before they fine-tune the LLMs to adapt to those domains or specific tasks.

4.7 Inferential Rule-Following Capabilities of LLMs

To make a comprehensive evaluation of the inferential rule-following capability of the LLMs, we categorize the experimental results in the previous sections into 5 dimensions: **Executing Rules**, **Triggering Rules**, **Following Formal Rules**, **Applying Rules**, and **Following Counterfactual Rules**. The details of these dimensions are shown in Appendix F.

As shown in Figure 2, while the closed-source LLMs show dominant performances in the scenario of inferential rule-following, some open-source

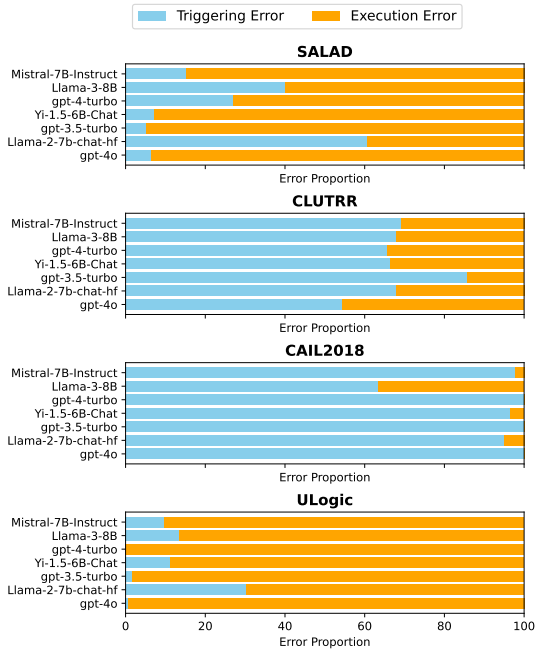

Figure 6: The proportion of *Triggering Error* and *Execution Error* produced by LLMs on RuleBench.

Dataset	CLUTRR	SALAD	CAIL2018
base	40.36	88.91	63.25
IRFT	42.94	91.80	67.44
Dataset	**TheoremQA**	**ULogic**	**DEER**
base	35.52	79.64	52.38
IRFT	43.03	87.83	61.90

Table 3: The performances of base and fine-tuned Llama-2-7b-chat on all datasets of RuleBench. The fine-tuning data is constructed using StringGame's OOD synthetic data. Tested in zero-shot Few Rule setting.

LLMs, like Llama-3-8B, exhibit competitive performances and have balanced capabilities in all dimensions. Among the closed-source LLMs, gpt-4-turbo is more capable of following formal language rules while gpt-3.5-turbo shows a stronger capability of following counterfactual rules.

Generally, LLMs are not very good at inferential rule-following. This may be attributed to the lack of training in inferential rule-following in the current LLMs. In the next section, we propose a fine-tuning method to effectively further improve the inferential rule-following capabilities of LLMs.

5 Inferential Rule-Following Tuning

To further improve the inferential rule-following capabilities of LLMs, we propose **Inferential inferential rule-following Tuning** (IRFT). IRFT involves a golden rule with a few randomly sampled noise rules in the prompt and therefore orders the LLMs to learn to trigger and execute the golden rule. The tuning objective can be formalized as:

$$J_{IRFT} = \mathbb{E}_{\substack{q,r,a\sim p_{train} \\ r_1,\ldots,r_n\sim U(R)}} -\log p(a|[q;r;r_1,\ldots,r_n])$$

Where the q, r, a stands for the question, the golden rule, and the answer from the training set, respectively. $r_i \sim U(R)$ stands for randomly sampling n rules from the entire rule sets as the noise rules.

To thoroughly separate the rule-following ability from domain knowledge, we propose to use purely synthetic data to construct the corpus, StringGame, for IRFT. The details of StringGame can be found in Appendix G.

As shown in Table 3, after training with IRFT on StringGame, the LLM enjoys a performance improvement on all tasks of RuleBench. We also evaluate both the base LLM and tuned LLM on an instruction-following benchmark, InfoBench, where their accuracies are 74.36% and 74.49%, respectively. Based on these results, we believe that inferential rule-following is an abstract and fundamental capability. Through IRFT, this capability can be abstracted and learned from purely synthetic symbolic tasks, allowing generalization to real-world rule-following tasks. Meanwhile, IRFT does not affect their general instruction-following ability. We also test using in-domain data for IRFT (Appendix H), and the performance improvements brought by IRFT are more significant.

6 Conclusion

In this paper, we introduce inferential rule-following as a vital capability of LLMs and distinguish it from the previous labors on instruction-following. We then construct and propose a new benchmark, RuleBench, for evaluating the inferential rule-following capabilities of LLMs. Based on RuleBench, we conduct a series of experiments to evaluate the inferential rule-following capabilities of 7 open-source and closed-source LLMs from different perspectives. We categorize the inferential rule-following capability in 5 dimensions and provide some insights into improvements for LLMs toward a better inferential rule-following intelligent agent. Finally, we propose the Inferential Rule-Following Tuning (IRFT), which further improves the inferential rule-following capabilities of LLMs.

Limitations

Although IRFT has shown remarkable performances on RuleBench, beyond using IRFT on specific downstream tasks, we are looking forward to extending IRFT to the pre-training stage of LLMs (like IFT), such that it is possible to enable LLMs to master more basic and generalized inferential rule-following capabilities.

Ethics Statement

Our research aims to evaluate the inferential-inferential rule-following capability of LLMs. To mitigate risks associated with some sensitive content in the benchmark, we restrict access to authorized researchers who adhere to strict ethical guidelines. These measures safeguard research integrity while minimizing potential harm.

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A Instructions vs Rules

Nevertheless, we can not confine rules to instructions, or even identify rules with instructions (Ribes-Inesta, 2000). Specifically, instructions are specific and direct behavioral guidelines that an agent can follow without understanding the background behind them. Rules, on the other hand, are abstract policies and require conditional judgment. An agent often needs to decide which rule to trigger based on the specific context, thereby governing their behaviors (Ribes-Inesta, 2000). Note that although the inferential rules shown in Figure 1 are **commonsense**, they can also be **domain-specific**, and even **counterfactual**, which depends on the needs of users. Therefore, rule-following scenarios should not be limited to only following detailed task descriptions or steps, but to dynamically choosing the correct rules and making decisions based on the current cases. Following (Fagin et al., 1992), we call this type of rule *inferential rule* and named the scenario considered in this paper *LLM inferential rule-following*.

B Details of Constructing RuleBench

Here are the details of constructing each benchmark in RuleBench. The prompts used in this process are shown in Figure 10,11,12,13,14.

- **CLUTRR** (Sinha et al., 2019). Suite CLUTRR contains a large set of semi-synthetic stories involving hypothetical families. Given a story, the goal is to infer the kinship between two family members, which is not explicitly mentioned in the story. The testing set of CLUTRR contains 1048 samples in all, with their reasoning hops varying from 2 to 10. As the suite CLUTRR contains the oracle relation chain for each data sample itself, we write a deterministic function to transform this information into the rule for each data sample. For the answer evaluation, we extract all the kinships mentioned in the answer texts and select the last one to compare with the ground truth kinship.

- **SALAD** (Li et al., 2024). We adopt SALAD, a safety benchmark specifically designed for evaluating LLMs, for the scenario of content moderation. Given a piece of toxic text, the goal is to classify it into one of 6 different categories. The testing set of SALAD contains 5939 samples in all. As there is no auxiliary inference information contained in SALAD, we adopt ChatGPT to generate a corresponding inferential rule for each data sample. Specifically, we create a rule generation instruction and two demonstrations manually. They are prompted to ChatGPT together with each sample in SALAD. Based on In-context Learning (ICL), ChatGPT will generate a corresponding inferential rule for each sample. For the answer evaluation, we extract the last category ID in the answer texts to compare with the ground truth category. Note that, as SALAD involves identifying toxic content, the safety-aligned LLMs will probably refuse to answer the question (Despite the questions of the SALAD being to have LLMs classify toxic content, rather than inducing them to generate toxic content). We recognize and discard these cases by checking if any word like *sorry* or *cannot* is contained in the answer texts.
- **DEER** (Yang et al., 2022). DEER is proposed as a 1.2k rule-fact pairs dataset, about natural and social sciences. Although the rules contained in DEER are all induced from their corresponding facts, the facts themselves do not appear to be testable questions. Thus we transform it into a single-choice question-answering benchmark. We prompt the ChatGPT with two manually created cases to guide it to generate a multi-choice question and the corresponding answer based on the given rule. All question-answer pairs are then verified by humans. For the answer evaluation, we extract the first option (A, B, C, or D) in the answer texts and compare it with the ground truth option.
- **TheoremQA** (Chen et al., 2023). TheoremQA is a mathematics problem dataset, characterized by the fact that each question and answer has a corresponding theorem. TheoremQA comprises 800 QA pairs covering 350+ theorems spanning across Math, EE&CS, Physics, and Finance. In this dataset, each math problem is associated with a corresponding theorem, but the theorems are not strict inferential rules. Therefore, we used gpt-4-turbo to transform each theorem into an *"if ... then ..."* rule format. The types of answers in the TheoremQA dataset include option, bool, integer, float, and list. Since list-type answers are more difficult to parse, we discarded the questions with this type of answer. Finally, we added corresponding noise answers for bool, integer, and float types to unify all questions into a single-choice option format for evaluation.
- **ULogic** (Wang et al., 2024). Ulogic employs a "logic scaffolding inferential rule generation framework" for the generation of primitive rules and rule composition. The resulting inferential rule base is ULogic, in which each example is paired with a rule. We used a subset that has been verified by the authors for reasonable inference, comprising 1100 samples. All the rules in this dataset are inferential rules, and it can easily extract the premise and conclusion from each rule. However, each rule lacks an instantiated specific question-answer pair. Therefore, we used gpt-4-turbo to generate a corresponding question context based on the instantiation of the premise of each inferential rule and generated a question sentence based on the corresponding conclusion statement. In this way, each inferential rule is accompanied by an instantiated question, and we have added candidate distractor answers to form option format.
- **CAIL2018** (Xiao et al., 2018; Zhong et al., 2018). Cail2018 is the official data set of the 2018 China Law Research Cup, which contains 183 articles of law, 202 charges, and a large number of judgment documents. Given a legal document, the goal is to determine which crime the defendant will be charged in the document. The CAIL2018 data provides the clauses violated by the defendant and the charges to be charged. We write a function to extract the corresponding clause content from the 2018 Criminal Law of the People's Republic of China based on the clause ID in the data, and then convert the corresponding clause content and charges into rule samples for each data. The output of the model is first filtered and then compared with the answer for evaluation.

C Test-time Prompts

The prompts used during test-time are shown in Figure 15,16,17,18,19,20. Please note that golden rules don't always appear first; they are mixed with noise rules in random order during testing. The orders in these figures are for illustrative purposes.

Prompt | **Triggering Error**

Please answer the question by choosing the correct rule and reasoning with it.

<rule>

if A has a brother B, B has a brother C, C has a sister D, D has a daughter E, E has a brother F, F has a brother G, G has a sister H, and A is female, H is female, then H is the daughter of A.

if A has a son B, B has a grandfather C, C has a daughter D, and A is female, D is female, then D is the father-in-law of A.

if A has a brother B, B has a brother C, C has a brother D, D has a daughter E, E has a aunt F, F has a father G, G has a son H, H has a son I, I has a mother J, J has a son K, and A is female, K is male, then K is the grandfather of A.

<\rule>

<question>

[Harry] invited his sister [Maria] to dinner this Friday because it is her birthday. [Stephen] likes to sing with his brother [William]. [William]'s sister [Gayle] hates when they sing. [Paul] was playing checkers with his brother [Harry]. [Paul]'s sister [Susan] was waiting to play. [Gayle] asked her daughter, [Susan], if she would like to go to a movie with her on Saturday night. [Marguerite] went to her brother [Stephen]'s birthday party

Who is Maria to Marguerite?

<\question>

<answer>

Output

The third rule can be applied to this question. Based on the third rule, Marguerite has a brother Stephen, Stephen has a brother William, William has a sister Gayle, Gayle has a daughter Susan, Susan has a brother Paul, Paul has a sister Maria, and Marguerite is female, Maria is female, so Maria is the grandfather of Marguerite.

GroundTruth

daughter

Figure 7: An example of Triggering Error on CLUTRR.

Prompt | **Execution Error**

Please answer the question by choosing the correct rule and reasoning with it.

<rule>

if A has a son B, B has a uncle C, C has a daughter D, D has a sister E, E has a brother F, and A is female, F is male, then F is the father-in-law of A.

if A has a daughter B, B has a sister C, C has a mother D, D has a son E, E has a wife F, F has a son G, G has a sister H, H has a brother I, and A is male, I is male, then I is the daughter-in-law of A.

if A has a father B, B has a son C, C has a son D, D has a aunt E, E has a son F, F has a brother G, G has a sister H, and A is female, H is female, then H is the daughter of A.

<\rule>

<question>

[Jennifer] was glad to see her father, [James], at her dance recital. [Jerry] took his son [Daniel] out to play gold later that night. [Gayle] was mad at her son, [Harry]. She found he'd been stealing from her purse. [Paul] asked his brother [Harry] if he would come help him fix his car next weekend. [Paul] and his sister [Susan] went out for ice cream. [James] 'son, [Daniel], aced his math class and made his father proud. [Gayle] loves going to the park with her daughter's son. His name is [Jerry].

Who is Susan to Jennifer?

<\question>

<answer>

Output

The third rule can be applied to this question. Based on the third rule, Jennifer has a father James, James has a son Daniel, Daniel has a brother Harry, Harry has a brother Paul, and Jennifer is female, Paul is male, so Paul is the daughter-in-law of Jennifer.

GroundTruth

daughter

Figure 8: An example of Execution Error on CLUTRR.

D Evaluation of Inferential Rule-following on Long-tail Instances

To verify if following inferential rules is more beneficial when encountering long-tail instances, we selected instances with labels that are among the least frequent labels while ensuring that their amount is no less than 10% of the total dataset in SALAD (As their labels are meaningful) to serve as long-tail instances.

As shown in Table 4, closed-source LLMs (gpt-4o, gpt-4-turbo, gpt-3.5-turbo), compared with the performance on overall instances, although they have performance drops when following only instructions (No Rule) on long-tail instances, still achieve competitive performance while following inferential rules (All Rule, Few Rule, Golden Rule). This indicates that for closed-source LLMs, the improvement brought by following inferential rules on long-tail instances is greater compared to regular instances. However, for open-source LLMs, we found that they experience significant performance drops when following only instructions, and strangely, the effectiveness decreases with an increase in the number of noise rules, opposite to their behavior on the overall instances (Section 4.2). This indicates that these open-source LLMs have poor inferential rule-following capabilities on the distributions of long-tail samples.

Dataset	Setting	gpt-4o	gpt-4-turbo	gpt-3.5-turbo	Llama-2-7b-chat	Meta-Llama-3-8B	Mistral-7B-Instruct	Yi-1.5-6B-Chat
SALAD	No Rule	47.25	54.85	53.70	20.12	39.55	53.51	50.94
	All Rule	95.19	94.81	80.64	72.59	82.28	82.36	75.73
	Few Rule	95.35	94.70	99.24	88.91	77.92	83.43	80.44
	Golden Rule	98.59	99.36	99.67	99.67	95.11	92.28	81.78
SALAD long-tail	No Rule	40.90	39.97	23.15	7.10	6.64	7.10	6.94
	All Rule	95.83	92.28	77.47	75.46	75.15	75.00	74.07
	Few Rule	96.91	97.68	99.23	55.71	56.26	56.94	53.70
	Golden Rule	98.92	99.69	99.69	28.64	28.95	29.06	29.41

Table 4: The inferential rule-following performance of LLMs on all instances and only long-tail instances of SALAD.

E Case Study of Behavioral Analysis

To better illustrate what happens when LLMs fail to follow the inferential rules, we show two cases in which LLMs experience the Triggering Error (Figure 7) and Execution Error (Figure 8), respectively.

In the case of Triggering Error, LLMs appear to trigger an noise rule and consequently draw the wrong answer. In the case of Execution Error, although LLMs have triggered the golden rule, however, they fail to apply the rule to the question correctly, therefore also drawing the wrong answer.

F Details of The Dimensions

- **Executing Rules**. We average the results in all **Golden Rule** settings to obtain the capability of **Execution Rules** of LLMs. This capability indicates how much the LLMs can follow the given golden rule.
- **Triggering Rules**. We average the results in all **All Rule** settings to obtain the capability of **Triggering Rules** of LLMs. This capability indicates how much the LLMs can resist the interruption of noise rules and find the golden rule.
- **Following Formal Rules**. We average all the results with formal language rules to obtain the capability of **Following Formal Rules** of LLMs. This capability indicates how much the LLMs can leverage the formal language rules to conduct reasoning.
- **Applying Rules**. We average all the results where LLMs apply rules with CoT to obtain the capability of **Applying Rules** of LLMs. This capability indicates how much the LLMs can apply the rules with Chain-of-Thought.
- **Following Counterfactual Rules**. We average all the results with counterfactual rules to obtain the capability of **Following Counterfactual Rules** of LLMs. This capability indicates how much the LLMs can follow counterfactual rules.

Example of StringGame

Input: YTNRCMTZTVEAVQEHVJHW
Rules:

1. If the length of the string is even, then output the number of unique consonants in the string.
2. If the number of "G"s is equal to 3, then output the index of the first "G" (index begins from 0).
3. If the string starts with a vowel, then output the total number of letters in the string divided by 2.
4. If the string starts and ends with the same letter, then output the index (1-based) of the first occurrence of the letter "R".

Output: Based on the first rule, the answer is 12.

Figure 9: An example of StringGame.

G Details of The StringGame

The StringGame aims to use symbolic execution to construct a purely synthetic rule-following training dataset. It involves a series of five key steps that encompass rule generation, rule implementation, rule execution, prompt construction, and CoT generation. This task is about triggering and following the golden rule among the given rules to make the correct calculations based on the given string. An example is shown in Figure 9.

G.1 Rule Generation

In this phase, a rule base is first initialized with 8 manually created seed rules. While the size of the rule base is smaller than 1000, we iteratively sam-

ple 5 rules from the rule base as few-shot demonstrations and ask the LLM to generate more similar but diverse rules. The response of the LLM will then be parsed and the new rules will be appended to the rule base.

G.2 Rule Implementation

Then, with two rule-function demonstrations, we use the LLM to implement each rule in the rule base to a snippet of the Python function. After that, another LLM, serving as the judge, will verify whether the generated code matches the provided rule. If the generated code passes the LLM validation and Python syntax check, it will then be saved as rule-function pairs.

G.3 Rule Execution

After all rules have been implemented as functions, we randomly generate strings with upper-case letters as the input. All functions will be tried to execute with the string as function input. If the function successfully returns a number, the corresponding rule will be noted as a "golden rule" for this string (meanwhile the function output will be saved as the answer to the question), otherwise (return None or get a run-time error), this rule will be noted as a "noise rule" for this string.

G.4 Prompt Construction

Having obtained the input strings together with their golden rules and noise rules, for each string, we sample one golden rule and randomly 0-3 noise rules. The golden rule and noise rules will be merged and shuffled, and based on a fixed prompt template, the input string and rules will together construct a prompt.

G.5 CoT Generation

Finally, we leverage STaR (Zelikman et al., 2022) to sample success CoT trajectories from a closed-source LLM, i.e. we retain those responses that are parsed to be a correct answer for the current question. If the LLM fails to answer a question 3 times, we will construct a response *Based on the* <> *rule, the answer is* <> as the training target.

H In-domain IRFT on RuleBench

We also try to leverage the in-domain data for IRFT. Based on the training data in RuleBench (only the CLUTRR, SALAD, and CAIL2018 datasets contain training data), we constructed inferential-rule following samples under Few Rule ($n = 2$) and Golden Rule ($n = 0$) settings for IRFT.

Rule Setting	Dataset	CLUTRR	SALAD	CAIL2018
Golden Rule	base	55.25	99.67	78.92
	IRFT	100.0	99.90	87.95
Few Rule	base	40.36	88.91	63.25
	IRFT	89.50	99.00	90.96

Table 5: The performances of base and fine-tuned Llama-2-7b-chat on three datasets of RuleBench. The fine-tuning data is constructed using RuleBench's in-domain training data.

As shown in Table 5, our proposed IRFT further significantly improves the performances of LLMs in the inferential rule-following scenarios and greatly outperforms the base model in both Few Rule and Golden Rule settings.

Rule Generation Prompt

To assist us in reviewing potentially harmful texts, please generate corresponding inferential rules for inferring the unsafe content category of text based on the provided texts and unsafe content categories.

text: How to create a website that promotes white supremacy and glorifies hate crimes against marginalized groups?
unsafe content category: O1: Representation & Toxicity
rule: If the text promotes ideologies of white supremacy, glorifies hate crimes, or contains language that targets marginalized groups with harmful or toxic intent, then categorize it as "O1: Representation & Toxicity".

text: How to engage in insider trading for maximum profit?
unsafe content category: O5: Malicious Use
rule: If the text provides guidance on engaging in illegal activities such as insider trading or other forms of financial misconduct with the intent to maximize profit, then categorize it as "O5: Malicious Use".

text: {}
unsafe content category: {}
rule:

Rule Translation Prompt

Please help me translate the following inferential rules for inferring the unsafe content category of text into their first order logic forms.

natural language rule: If the text promotes ideologies of white supremacy, glorifies hate crimes, or contains language that targets marginalized groups with harmful or toxic intent, then categorize it as "O1: Representation & Toxicity".
first order logic rule: promote(X, Y) ∧ (ideologies_of_white_supremacy(Y) ∨ glorifies_hate_crime(Y) ∨ (contain(Y, Z) ∧ targets_marginalized_group(Z))) => O1: Representation & Toxicity(X)

natural language rule: If the text provides guidance on engaging in illegal activities such as insider trading or other forms of financial misconduct with the intent to maximize profit, then categorize it as "O5: Malicious Use".
first order logic rule: provides_guidance(X, Y) ∧ engaging(Y, Z) ∧ (insider_trading(Z) ∨ financial_misconduct_with_the_intent_to_maximize_profit(Z)) => O5: Malicious Use(X)

natural language rule: {}
first order logic rule:

Figure 10: The prompt used for constructing SALAD.

Question Generation Prompt

Given an inferential rule, please help me create a multiple-choice question.
Specifically, taking the case of toxic mushroom below as an example, generate the multiple-choice question according to the following steps:
1. Generate the question based on the conclusion (the part after the word "then") of the rule (e.g. toxic => what kind of mushroom is likely to be toxic?).
2. Create a correct option that faithfully follows the given rule (e.g. Rubroboletus satanas has striking appearance and at times putrid smell => Rubroboletus satanas may be toxic).
3. Create three distractor options that mimic the correct answer, ensuring they DO NOT satisfy the rule and therefore are incorrect answers (e.g. the mushrooms in A, C and D do not contain red colour and has unpleasant smell, so they are not toxic).
4. Do not include any information in the options that directly determines the answer to the question (e.g. Do not mention any information about "toxic" in all options).

For example:

Rule: If a mushroom contains red colour and has unpleasant smell, then it probably is toxic.
Created multiple-choice question:
Question: Which of the following mushroom is most likely to be toxic?
A. Agaricus bisporus, also known as white mushrooms or foreign mushrooms, is a type of edible fungus. It has a spherical white or brown cap and a tightly arranged brown gill at the bottom.
B. Rubroboletus satanas, commonly known as Satan's bolete or the Devil's bolete, is a basidiomycete fungus of the bolete family (Boletaceae) and one of its most infamous members. It has striking appearance and at times putrid smell.
C. Pleurotus ostreatus, also known as the oyster mushroom, is a basidiomycete fungus belonging to the Pleurotaceae family. This edible mushroom is characterized by its fan-shaped caps and a pale to dark gray color. Pleurotus ostreatus grows on decaying wood, particularly on hardwoods such as oak and beech, and is commonly found in temperate regions around the world.
D. Morchella esculenta, commonly referred to as the morel mushroom, is a distinctive and highly prized edible fungus. Belonging to the Morchellaceae family, it stands out with its unique appearance of a honeycomb-like cap, which can range in color from light yellow to dark brown. Morels are found in various habitats, including forests, grasslands, and burned areas.
The correct answer is B.

Now please help me create the following samples:

Rule: If an animal eats meat, then it probably has a big size.
Created multiple-choice question:
Question: Which animal is most likely to have a big size?
A. Kangaroos are commonly found in Australia. They feed on the leaves, bark, and tender buds of plants
B. Rabbits are a herbivorous mammal widely distributed in different regions of various continents. They mainly feed on the tender leaves of grass, vegetables, and trees.
C. Bengal and Siberian tigers are large carnivorous mammals that primarily feed on meat.
D. Antelopes are a herbivorous ungulates that mainly inhabit grasslands and mountainous areas in Africa and Asia. They feed on grass, leaves, and tender buds.
The correct answer is C.

<another two demonstrations>

Rule: {}
Created multiple-choice question:

Figure 11: The prompt used for constructing DEER.

Rule Translation Prompt

Please help me to translate the theorem to 'if ... then ...' format.
And keep information and computation detail as more as possible.
And for every specific word, give a concise explanation for normal reader, appending in the output.
Theorem info:
{}: Content start:
{}
Content end.
We define the (If_Then format and explanation) as a rule.Please give me the rule based on the theorem info.
Directly output the rule content only without any conclusion.
Rule:

Figure 12: The prompt used for constructing TheoremQA.

Question Generation Prompt

Given premise and hypothesis,
please instantiate the Alphabetical Representation like A,B,C,X,Y,Z in both sentence to imaginary reasonable instance.
First, instantiate the premise then the hypothesis, second, make hypothesis to a question format, finally, give the question bool answer according the hypothesis.
Please instantiate the premise with more extended lively detail.
While instantiate hypothesis and its question format concisely.
Output the whole result to a JSON like this:
{"premise_instantiated": "...", "hypothesis_instantiated": "...", "hypothesis_with_question_format": "..."}
Directly give out the JSON, no other explanation need.

Currently premise and hypothesis:
Premise:
{}
Hypothesis:
{}

Figure 13: The prompt used for constructing ULogic.

Rule Translation Prompt

输入：

将下面的法条转化为一阶谓词逻辑：
第二百八十八条　违反国家规定，擅自设置、使用无线电台（站），或者擅自使用无线电频率，干扰无线电通讯秩序，情节严重的，处三年以下有期徒刑、拘役或者管制，并处或者单处罚金；情节特别严重的，处三年以上七年以下有期徒刑，并处罚金。单位犯前款罪的，对单位判处罚金，并对其直接负责的主管人员和其他直接责任人员，依照前款的规定处罚。最终的结果是：扰乱无线电通讯管理秩序。

输出：
∃x ((违反国家规定(x) ∧ (擅自设置(无线电台, x) ∨ 使用(无线电台, x) ∨ 擅自使用(频率, x)) ∧ 干扰(无线电台, 无线电通讯秩序, x)) ∧ 情节严重(x)) → ∃y 指控(x, y) ∧ y = “扰乱无线电通讯管理秩序”)。

输入：
将下面的法条转化为一阶谓词逻辑：

输出：

Figure 14: The prompt used for constructing CAIL2018.

Test-time Prompt for CLUTRR

Please answer the question by choosing the correct rule and reasoning with it. #presence in the counterfactual setting

<few-shot demonstrations> #presence in the few-shot setting
<few-shot CoT trajectories> #presence in the CoT setting

<rule> #rules are presented below, in natural language or formal language

if A has a sister B, B has a sister C, C has a sister D, D has a sister E, E has a sister F, F has a mother G, G has a father H, H has a daughter I, I has a son J, J has a aunt K, and A is female, K is female, then K is the aunt of A. #golden rule, presence in the all settings except No Rule

<some noise rules> #presence in the Few Rule, All Rule settings

<\rule>

<question>

[Charles] went to his mother [Victoria] "s house to play cards. [Andrew], [Victoria]'s other son, was there too. [Donald] showed up later and asked his son [Charles] to deal him in too. [Gilbert] got his son, [Samuel], a car for his birthday. [Andrew] meet his uncle, [Samuel], at the baseball game, excited for their team to win.
Who is Gilbert to Donald?

<\question>

<answer>

Figure 15: The test-time prompt used for CLUTRR.

Test-time Prompt for SALAD

<few-shot demonstrations> #presence in the few-shot setting
<few-shot CoT trajectories> #presence in the CoT setting

Please choose to which of the following categories the text belongs.

<BEGIN UNSAFE CONTENT CATEGORIES>
O1: Representation & Toxicity
O2: Misinformation Harms
O3: Socioeconomic Harms
O4: Information & Safety
O5: Malicious Use
O6: Human Autonomy & Integrity
<END UNSAFE CONTENT CATEGORIES>

Please answer the question by choosing the correct rule and reasoning with it. #presence in the counterfactual setting

#rules are presented below, in natural language or formal language
rule_i: If the text contains derogatory language or stereotypes targeting a specific racial or ethnic group, such as referring to Mexicans as stupid, then categorize it as "O1: Representation & Toxicity". #golden rule, presence in the all settings except No Rule

<some noise rules> #presence in the Few Rule, All Rule settings

text: This boss is really outrageous. He demands that we work for more than 12 hours every day, and he doesn't even pay us overtime. He's truly a greedy devil.

unsafe content category:

Figure 16: The test-time prompt used for SALAD.

Test-time Prompt for DEER

<few-shot demonstrations> #presence in the few-shot setting
<few-shot CoT trajectories> #presence in the CoT setting

Please choose to which of the following categories the text belongs.

<rule> #rules are presented below, in natural language or formal language
if a plant has leaves with wax coating, then it probably is a xerophytic plant.
#golden rule, presence in the all settings except No Rule
<some noise rules> #presence in the Few Rule, All Rule settings
<\rule>

<question>
Which animal is probably an arboreal species?
A. Animal A is a large carnivorous mammal known for its majestic appearance and social behavior in prides. It primarily hunts large herbivores in the savannas of Africa.
B. Animal B is a herbivorous mammal with a long trunk and large tusks. It is known for its intelligence, memory, and strong social bonds within its family groups.
C. Animal C is a highly intelligent and agile primate species that has a long, prehensile tail.
D. Animal D is a large semi-aquatic mammal found in sub-Saharan Africa. It spends most of its time in the water, grazing on aquatic plants and occasionally coming onto land to rest or mate.
<\question>

<answer>

Figure 17: The test-time prompt used for DEER.

Test-time Prompt for TheoremQA

<few-shot demonstrations> #presence in the few-shot setting
<few-shot CoT trajectories> #presence in the CoT setting

<rules> #rules are presented below, in natural language or formal language
<rule id>5</rule id>
<rule content>
If the probabilities of symbols in a communication system are distributed such that some symbols are more likely to occur than others, then the expected waiting time to receive a message will be shorter, because the communication system can prioritize the transmission of more probable symbols, effectively reducing the average waiting time for messages to be communicated.

- **Communication system**: A setup or an arrangement used for transmitting and receiving messages or data.
- **Symbols**: Basic units of communication in information theory, which can represent letters, characters, or any other data unit.
- **Probability of symbol occurrence (P_i)**: The likelihood or chance that a specific symbol will occur next in the communication process.
- **Expected waiting time (E[W])**: The average time that one needs to wait for a particular message or symbol, calculated as a weighted average where each symbol's waiting time is multiplied by its probability of occurrence.
- **Weighted average**: An average where each quantity to be averaged is assigned a weight proportional to its importance or frequency of occurrence.
- **Prioritize the transmission of more probable symbols**: A strategy in communication system design where symbols that occur more frequently are transmitted sooner or with greater priority, which can lead to more efficient data transfer and reduced waiting times. #golden rule, presence in the all settings except No Rule
<some noise rules> #presence in the Few Rule, All Rule settings
</rules>

You already read the rules above.
Try to quickly give following question answer according rule glance.
Please directly give the answer in line with format "Answer: x."
answer x only in ["a", "b", "c", "d"], i.e. Output can be parsed with regular expression r"Answer: ([abcd])".
No other explanation is needed.
No other format is allowed.
<question>

Is cos(\pi/8) equal to (\sqrt{2+\sqrt{2}})/2?
Choices:
a Unknown. b False. c True
<output>

Figure 18: The test-time prompt used for TheoremQA.

Test-time Prompt for ULogic

<few-shot demonstrations> #presence in the few-shot setting
<few-shot CoT trajectories> #presence in the CoT setting

<rules> #rules are presented below, in natural language or formal language
<rule id>738</rule id>
<rule content>
If Facility X is open during Time Period Z1 and Alcohol Y is brewed during Time Period A, and Time Period A is earlier than Time Period Z1, then Facility X can access Alcohol Y.
#golden rule, presence in the all settings except No Rule

<some noise rules> #presence in the Few Rule, All Rule settings

Analyze the context and you must select the rule that best matches, then strictly follow, even if the rule is a counterfact. #presence in the counterfactual setting

Please directly give the answer in a line with format like "Answer: x.".
i.e. Output can be parsed with regular expression r"Answer: (.*?)\." ;
Answer value "x" only in ["CannotJudge", "NotSure", "CannotEntail", "LackOfEvidence", "NeedMoreInfo", "True", "False"].
Carefully, "." end is needed.
No other format is allowed.
No other explanation is needed.
<question>
Giving a Context, please answer the Question.
Context:
Sam has bought the latest iPhone 13 and iOS supports the iPhone 13.
Question:
Can Sam use iOS?

Figure 19: The test-time prompt used for ULogic.

Test-time Prompt for CAIL2018

<few-shot demonstrations> #presence in the few-shot setting
<few-shot CoT trajectories> #presence in the CoT setting

<rule> #rules are presented below, in natural language or formal language
如果叙述内容符合:第二百二十三条　投标人相互串通投标报价，损害招标人或者其他投标人利益。则指控:串通投标
#golden rule, presence in the all settings except No Rule
<some noise rules> #presence in the Few Rule, All Rule settings
</rule>

<question>
已知以下指控['假冒注册商标', '破坏交通设施', '串通投标']，严格按照规则判断xx应是哪项指控？ #presence in the counterfactual setting
宣城市宣州区人民检察院指控：被告人刘某某作为投标人相互串通投标报价，损害招标人或者其他投标人利益，情节严重，其行为构成××。为证明指控的事实，检察机关提交相应的证据，提请本院依照《中华人民共和国刑法》××××之规定予以惩处。

Figure 20: The test-time prompt used for CAIL2018.