BLADE: Bilevel Low-rank Augmented-Lagrangian Erasure for LLM Unlearning

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Abstract

Existing LLM unlearning methods struggle with robustness: unbounded forget losses degrade model coherence, fixed-weight balancing cannot adapt as retain difficulty shifts mid-training, and methods that work on one benchmark falter under scaling or repeated application. We propose BLADE,[1] a constrained bilevel framework whose three mechanisms give smooth, predictable control over the optimization landscape: a clamped-entropy forget loss whose gradient is exactly zero once a token reaches sufficient uncertainty; an asymmetric augmented Lagrangian that permanently ratchets retain protection after any violation; and a bilevel structure confined to LoRA adapters that repairs retain damage before each forgetting step. BLADE dominates across three benchmark families—improving average composite scores over the strongest baselines by 6% on TOFU, 9% on MUSE Books, and 7% on KnowUndo—and remains stable under $4\times$ scaling and 4 sequential unlearning steps on MUSE News where the best competing method collapses entirely.

1 Introduction

Deployed language models require continuous post-hoc correction. Regulations such as the GDPR (Voigt and von dem Bussche, 2017) mandate deletion of personal data on request, and models memorize and regurgitate copyrighted content (Carlini et al., 2021), prompting removal demands. At the same time, safety benchmarks flag hazardous knowledge that must be suppressed (Li et al., 2024), and models that absorb misinformation (recently demonstrated when LLMs confidently repeated a fabricated disease; Stokel-Walker 2026) must be patched without full retraining. *Machine unlearning* addresses these needs, yet the process is inherently *recurring*: new removal requests, policy changes, and safety incidents arrive throughout a model's lifetime (Liu et al., 2025).

This makes the central challenge not just *what* to forget but *how to forget without breaking what remains*. Forget and retain knowledge share entangled representations (Cheng et al., 2026), so each unlearning step risks collateral damage that compounds over repeated applications. Existing methods rely on fixed-weight loss balancing (Liu et al., 2025; Zhang et al., 2024) that cannot adapt as retain difficulty shifts mid-training, producing three recurring failure modes. (i) *Unbounded forgetting*: gradient ascent (Jang et al., 2023) produces arbitrarily large gradients, destroying utility. (ii) *Static tradeoff*: fixed-weight methods (GradDiff, NPO) oscillate without self-correction. (iii) *Over-forgetting*: unclamped entropy (Yuan et al., 2025) pushes already-forgotten tokens, wasting gradient budget on tokens that no longer need perturbation.

BLADE addresses these through three mechanisms, each targeting a specific failure mode. A *clamped entropy* forget loss (Section 3.4) maximizes per-token entropy only up to a threshold $\tau \cdot \log V$, producing exactly-zero gradients on already-forgotten tokens and eliminating over-forgetting. An augmented Lagrangian with asymmetric dual updates (Section 3.5) enforces retain quality as an inequality constraint that permanently ratchets after any violation, replacing static balancing with adaptive self-correction. These complement each other within a bilevel structure (Section 3.3) confined to LoRA adapters (Section 3.2): bounded gradients let the inner loop reliably repair retain damage, while the ALM adapts as the landscape shifts—an interaction our experiments confirm (Section 4.7, Section 4.8).

We evaluate on TOFU (Maini et al., 2024) (Llama-3.2-1B/3B, three splits), MUSE (Shi et al., 2025) (Books/News, Llama-2-7B), and KnowUndo (Tian et al., 2024) (copyright/privacy, Llama-2-7B-

[1] Code and configs: https://anonymous.4open.science/r/blade-5981/

chat). Key findings:

- **Dominant across benchmarks.** BLADE improves average composite scores over the strongest baselines by 6% on TOFU, 9% on MUSE Books, and 7% on KnowUndo, with near-zero semantic leakage confirmed by an LLM judge.
- **Robust under stress.** Under $4\times$ scaling and 4 sequential unlearning steps on MUSE News, BLADE maintains stable performance while the strongest baseline collapses catastrophically under both conditions.
- **Smooth, controllable optimization.** Training consistently follows a three-phase pattern (warm-up, spike-and-ratchet, smooth convergence), giving interpretable control over the forget–retain tradeoff absent in other baselines, whose dynamics oscillate without self-correction.

2 Related Work

Origins and classical methods. Machine unlearning—the problem of selectively removing the influence of specific training data from an already-trained model—was first formulated for classification systems (Cao and Yang, 2015; Bourtoule et al., 2021). A broader account of the area, including its evolution toward generative and language models, can be found in recent surveys (Liu et al., 2024; Blanco-Justicia et al., 2024). Early approaches used Fisher information to scrub data influence (Golatkar et al., 2020a,b) and influence functions (Koh and Liang, 2017) to identify responsible training points; SOUL later extended the latter to iterative LLM unlearning (Jia et al., 2024b). These methods do not transfer directly to autoregressive LLMs at scale.

LLM-specific unlearning. For LLMs specifically, gradient ascent (GA) (Jang et al., 2023) maximizes the forget loss; later work documents that GA frequently diverges and destroys coherence (Zhang et al., 2024). GradDiff (Yao et al., 2024) adds a retain cross-entropy term to mitigate this but still degrades over time. A separate line of work (Eldan and Russinovich, 2023) demonstrated approximate unlearning by fine-tuning on alternative token predictions. A second family of methods recasts forgetting as preference optimization, including NPO (Zhang et al., 2024), SimNPO (Fan et al., 2025), and NGDiff (Jin et al., 2025). These collapse more gracefully than GA but tend to saturate without converging to a well-defined target. RMU (Li et al., 2024) steers activations toward random targets for forget data. What these approaches share is the absence of a *bounded* forget loss with a per-token stopping criterion: GA diverges, NPO damps but never reaches zero, and unclamped entropy (Yuan et al., 2025) pushes indefinitely.

Parameter-efficient unlearning. LoRA (Hu et al., 2022) has been applied to unlearning via Fisher-informed initialization (LoKU; Cha et al. 2025), importance-weighted masking (Jia et al., 2024a), Fisher-based efficiency improvements (VILA; Kim et al. 2025), and masked-loss fine-tuning (OBLIVIATE; Xu et al. 2025). In BLADE, LoRA serves a different purpose: it acts as a *structural constraint*, where the low-rank bottleneck itself prevents catastrophic drift. This is similar in spirit to how EWC (Kirkpatrick et al., 2016) protects important weights in continual learning, but applied structurally rather than through per-weight penalties.

Bilevel and constrained optimization. BLUR (Reisizadeh et al., 2026) places forget in the lower level and retain in the upper, resolving conflicts via gradient projection. PDU (Entesari et al., 2025) uses a linear Lagrangian with logit-margin loss and symmetric dual updates. Cheng et al. (2026) apply ALM with an equality constraint on vision models only. BLADE differs from these on three axes: (a) *asymmetric* dual updates producing ratchet behavior; (b) retain in the *inner loop* (repair-first); and (c) clamped entropy as the forget loss, which is bounded with a hard per-token deadzone.

Benchmarks. TOFU (Maini et al., 2024) tests factual associations with synthetic biographies and provides a gold retrained model for reference. MUSE (Shi et al., 2025) tests naturalistic memorization at scale with explicit scalability and sustainability stress tests. KnowUndo (Tian et al., 2024) targets domain-specific unlearning (copyright, privacy) from LoRA-fine-tuned models. We evaluate on all three for complementary coverage.

3 Method

Unlearning often degrades general capability because forget and retain knowledge occupy shared representations (Cheng et al., 2026; Liu et al., 2025). BLADE addresses this with three mechanisms: a bilevel structure that repairs retain *before* each forget step (§3.3), a clamped entropy loss

with a hard per-token deadzone (§3.4), and an augmented Lagrangian whose dual variable adapts the forget–retain tradeoff during training (§3.5).

3.1 Problem Setup

Let $M_{\boldsymbol{\theta}}$ denote an LLM parameterized by $\boldsymbol{\theta}$ (initialized from pretrained weights $\boldsymbol{\theta}_0$), $\mathcal{D}_{\text{forget}}$ the data to unlearn, and $\mathcal{D}_{\text{retain}}$ data the model should preserve. We seek $\boldsymbol{\theta}$ such that (i) the output distribution of $M_{\boldsymbol{\theta}}$ on $\mathcal{D}_{\text{forget}}$ is high-entropy (memorized content is forgotten) and (ii) the cross-entropy loss on $\mathcal{D}_{\text{retain}}$ remains below a threshold ε.

3.2 LoRA Parameterization

Rather than modifying full weight matrices, we attach LoRA adapters (Hu et al., 2022) to all projection matrices ($q/k/v/o$_proj, gate/up/down_proj):

$$W' = W_0 + \tfrac{\alpha}{r} BA, \quad B \in \mathbb{R}^{d\times r},\ A \in \mathbb{R}^{r\times d} \tag{1}$$

where $r \ll d$ is the rank and α/r is a scaling factor. With $r = 16$ on a 7B model, only $\sim$0.2% of parameters are trainable. The low-rank bottleneck limits what the update can express, preventing catastrophic weight drift even under aggressive forgetting gradients. Another practical benefit is that disabling the adapters keeps the original model intact which simplifies rollback, debugging and subsequent forget/retain requests. With the parameter space constrained, the remaining question is how to organize the optimization itself.

3.3 Bilevel Formulation

We cast unlearning as a constrained bilevel optimization problem:

$$\begin{aligned} \min_{\boldsymbol{\theta}} \quad & \mathcal{L}_{\text{fgt}}(M_{\boldsymbol{\theta}^*};\, \mathcal{D}_{\text{forget}}) \\ \text{s.t.} \quad & \mathcal{L}_{\text{CE}}(M_{\boldsymbol{\theta}^*};\, \mathcal{D}_{\text{retain}}) \le \varepsilon \\ \text{where} \quad & \boldsymbol{\theta}^* \approx \arg\min_{\boldsymbol{\theta}}\ \mathcal{L}_{\text{CE}}(M_{\boldsymbol{\theta}};\, \mathcal{D}_{\text{retain}}) \end{aligned} \tag{2}$$

The upper level drives forgetting while the lower level ensures retain performance stays within budget ε. In practice, the lower-level $\arg\min$ is approximated by K fast SGD steps (inner loop), and the upper-level constraint is enforced via an augmented Lagrangian (outer loop); Algorithm 1 gives the full procedure.

Inner loop (retain preservation). For each outer step, we run K SGD steps on the retain set:

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \eta_{\text{in}} \nabla_{\boldsymbol{\theta}} \mathcal{L}_{\text{CE}}(M_{\boldsymbol{\theta}}; \mathcal{B}_{\text{ret}}) \tag{3}$$

Algorithm 1 BLADE: Bilevel Augmented Lagrangian Unlearning

Require: Model M with frozen weights $\boldsymbol{\theta}_0$, $\mathcal{D}_{\text{forget}}$, $\mathcal{D}_{\text{retain}}$
Require: Hyperparams: K, η_{in}, η_{out}, ε_{mul}, ρ, λ_0, α, τ, T

```
Attach LoRA adapters to M; θ ← LoRA params
λ ← λ_0
ε ← ε_mul · (1/K) Σ_{k=1}^{K} L_CE^{(k)}(D_retain)
for t = 1 to T do
    Inner loop (retain repair):
    for k = 1 to K do
        θ ← θ − η_in ∇_θ L_CE(M_θ; B_ret)
    end for
    Forget loss (clamped entropy):
    L_fgt ← (1/N) Σ_{i=1}^{N} max(0, τ log V − H(p_i))
    Outer loss (augmented Lagrangian):
    r ← L_CE(M_θ; B'_ret) − ε
    L_outer ← L_fgt + λ r + (ρ/2)[max(0, r)]^2
    θ ← θ − η_out · Adam(∇_θ L_outer)
    Dual update (asymmetric ratchet):
    if r > 0 then
        λ ← λ + ρ |r|
    else
        λ ← λ − αρ |r|
    end if
end for
return M with trained LoRA adapters
```

where $\mathcal{B}_{\text{ret}} \sim \mathcal{D}_{\text{retain}}$ is a mini-batch and $\mathcal{L}_{\text{CE}}$ is cross-entropy. The inner loop uses a reliably faster learning rate (η_{in}) to restore any retain damage from the previous outer step.

Outer loop (constrained forgetting). The outer objective combines the forget loss with an inequality constraint on retain performance, enforced via an augmented Lagrangian:

$$\mathcal{L}_{\text{outer}} = \mathcal{L}_{\text{fgt}} + \lambda\, r + \tfrac{\rho}{2}[\max(0, r)]^2,\ r = L_{\text{ret}} - \varepsilon \tag{4}$$

where $L_{\text{ret}} = \mathcal{L}_{\text{CE}}(M_{\boldsymbol{\theta}}; \mathcal{B}'_{\text{ret}})$ is evaluated on a *fresh* retain batch (separate from the inner loop), λ is the Lagrange multiplier, ρ is the quadratic penalty coefficient, and ε is the retain budget. The outer loop uses a slower learning rate (η_{out}) compared to the inner loop, keeping each forgetting step small enough for the inner loop to repair.

Repair-first principle. Prior two-level unlearning methods like SCRUB (Kurmanji et al., 2023) (min-max) and BLUR (Reisizadeh et al., 2026) (bilevel with gradient projection) prioritize forgetting at the lower level and treat retain as the upper-level concern. We invert this assignment.

Forgetting is inherently destructive: gradient ascent, entropy maximization, and preference-based losses all push the model *away* from learned representations, causing catastrophic collapse when unconstrained (Zhang et al., 2024). To make mat-

ters worse, forget and retain knowledge share entangled representations (Cheng et al., 2026), so perturbing forget tokens inevitably disturbs retain performance.

Retain repair therefore belongs in the inner loop as a stabilizing anchor. It runs first and comparably faster ($\eta_{\text{in}} > \eta_{\text{out}}$), so the model enters each outer step from a state with good retain performance. The outer loop then applies one small forgetting step, sized so the next inner loop can fully repair any collateral damage. This prevents the accumulation of representational drift that makes recovery progressively harder when the loops are reversed.

For this structure to work, the forget loss in the outer loop must be well-behaved: bounded, self-saturating, and compatible with small step sizes. We introduce *clamped entropy* to address this.

3.4 Clamped Entropy Loss

Entropy maximization as a forget objective is well-known (Yuan et al., 2025), but unclamped formulations apply nonzero gradients to every token at every step, even those already near-uniform, needlessly perturbing shared representations. We introduce *clamped entropy*, which defines when a token is "sufficiently forgotten" and stops optimizing it:

$$\mathcal{L}_{\text{forget}} = \frac{1}{N} \sum_{t=1}^{N} \max\big(0,\ \tau \cdot H_{\max} - H(p_t)\big) \quad (5)$$

where $p_t = \text{softmax}(z_t)$ is the output distribution at position t, $H(p_t) = -\sum_v p_t(v) \log p_t(v)$ is its Shannon entropy, $H_{\max} = \log V$ is the maximum entropy over vocabulary V, and $\tau \in (0, 1)$ is the target fraction. Normalizing by $H_{\max}$ makes τ vocabulary-invariant: the same τ applies regardless of whether the vocabulary has 32K (Touvron et al., 2023) or 128K tokens (Grattafiori et al., 2024).

The $\max(0, \cdot)$ clamp produces a hard deadzone: once $H(p_t) \geq \tau \cdot H_{\max}$, token t's gradient is *exactly zero*. This makes the loss bounded ($\mathcal{L}_{\text{forget}} \in [0, \tau \cdot H_{\max}]$), self-stabilizing (the effective perturbation shrinks as tokens are forgotten), and reference-free (no frozen model needed, unlike NPO). The bounded outer perturbation (Proposition 1) ensures the inner loop can reliably repair retain damage at any training stage.

Clamped entropy yields well-behaved forget gradients, but the remaining challenge is adapting *how much* retain protection the outer loop enforces as the loss landscape evolves.

3.5 Augmented Lagrangian with Asymmetric Dual Update

The multiplier λ controls how aggressively the outer objective protects retain performance. Rather than treating it as a fixed hyperparameter, we learn it via a dual update rule.

Standard ALM uses symmetric updates designed for equality constraints (Bertsekas, 1982; Nocedal and Wright, 2006). We instead enforce the *inequality* $L_{\text{ret}} \leq \varepsilon$: the quadratic penalty $\frac{\rho}{2}[\max(0, r)]^2$ activates only when violated (Eq. 4), so the model is never penalized for retain loss below ε. On top of this, we apply an *asymmetric* dual update:

$$\lambda \leftarrow \begin{cases} \lambda + \rho\,|r| & \text{if } r > 0 \\ \lambda - \alpha\rho\,|r| & \text{if } r \leq 0 \end{cases} \quad (6)$$

where $\alpha \ll 1$. The slower decay means that once a retain violation drives λ up, it stays elevated, preventing the repeated oscillation cycles observed with symmetric updates (PDU (Entesari et al., 2025); see Appendix E).

Rather than hand-tuning ε, we set it as a fraction (ε_{mul}) of the model's average retain loss over one inner loop, so the constraint budget scales with the model's own retain difficulty and requires no manual calibration across benchmarks (Appendix E).

Convergence criterion. Training terminates when an exponential moving average of the per-step $\mathcal{L}_{\text{fgt}}$ change drops below a fraction of its peak, indicating that remaining tokens have entered the deadzone. A safety cap T prevents runaway training (Appendix B.3).

4 Experiments

We evaluate BLADE on three benchmarks that test complementary aspects of unlearning: factual association removal, naturalistic memorization at scale, and domain-specific knowledge extraction from adapted models. We use OpenUnlearning (Shi et al., 2025), an open-source toolkit that bundles standard unlearning baselines, datasets, and evaluation pipelines under a unified interface. All methods are implemented and run within this framework, ensuring fair comparison under identical data processing, evaluation protocols, and compute budgets. We use the standard baselines provided by OpenUnlearning (GradAscent, GradDiff, NPO, SimNPO, RMU) and additionally include BLURNPO (Reisizadeh et al., 2026) and PDU (Entesari et al., 2025) as constraint-based approaches

compatible with the framework. Baseline hyperparameters are listed in Appendix Table 5. All experiments report means and standard deviations over 5 random seeds unless noted.

4.1 Setup

Benchmarks. **TOFU** (Maini et al., 2024) provides a controlled testbed with 200 synthetic author biographies and QA pairs, evaluated on Llama-3.2-1B/3B-Instruct across three forget splits (1%/5%/10%). Each split removes an increasing fraction of the training authors, testing whether the method scales gracefully. A gold retrained model provides an upper bound.

MUSE (Shi et al., 2025) tests naturalistic memorization on Llama-2-7B across two corpora: *Books* (Harry Potter, a single cohesive document) and *News* (889 BBC articles, a distributed multi-document set). Beyond standard forget/retain performance, MUSE also tests scalability (larger forget sets) and sustainability (sequential unlearning from the same checkpoint), which we evaluate in §4.6.

KnowUndo (Tian et al., 2024) targets copyright and privacy domains on Llama-2-7B-chat that was LoRA-fine-tuned on domain-specific corpora. Unlike MUSE where memorization arises from pretraining, here the target knowledge was explicitly injected via LoRA adaptation, testing whether unlearning can reverse fine-tuning without destroying general capability.

Metrics. Each benchmark uses different axes. For TOFU: Model Utility (MU↑, retain quality), forget-set answer Probability (Prob↓), and forget-set ROUGE-L (RG↓). For MUSE: forget knowledge memorization (fk↓), verbatim memorization (vm↓), and retain knowledge (rk↑). We aggregate each benchmark's metrics into a single *harmonic mean* (HM) that penalizes methods sacrificing one axis for another. Full metric definitions and per-benchmark HM formulas are given in Appendix B.2.

LLM judge. Token-overlap metrics like ROUGE can miss semantic leakage: a model may paraphrase memorized content without triggering string-match detectors. We supplement standard metrics with an LLM judge (Claude Opus 4.7) that scores model generations on a 0–2 scale along three dimensions: Forget Leakage (FL↓), Retain Accuracy (RA↑), and retain Response Quality (rRQ↑). The judge composite $\mathrm{HM}_{\mathrm{J}} = \mathrm{hmean}(1-\mathrm{FL}/2,\ \mathrm{RA}/2,\ \mathrm{rRQ}/2)$ normalizes all axes to $[0,1]$ and provides a unified evaluation across all benchmarks.

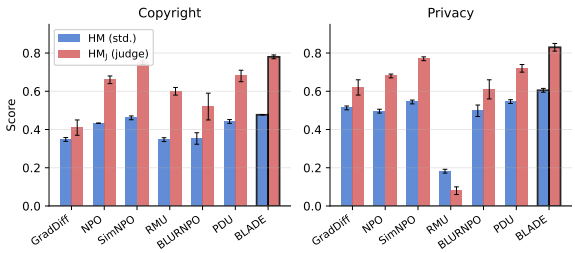

Figure 1: KnowUndo results (5 seeds). Error bars: ±1 std. Full tables in Appendix C.

4.2 TOFU (Llama-3.2-1B-Instruct)

BLADE achieves HM≥0.800 across all three forget splits (Table 1), consistently outperforming all baselines including PDU (0.690/0.740/0.797). The LLM judge (Figure 2) confirms near-zero semantic leakage: HM_{J}=0.929/0.919/0.919 vs. PDU's 0.746/0.850/0.906, indicating that BLADE's forgetting extends beyond surface-level token suppression (see also qualitative analysis in Appendix G). On 3B model (Appendix C), BLADE remains strong (HM=0.846/0.842/0.836); PDU is competitive (0.742/0.843/0.857), winning on forget05 and forget10 by a narrow margin, but BLADE retains its advantage in LLM judge score across all splits.

4.3 MUSE (Llama-2-7B)

On Books, BLADE achieves HM=0.818, surpassing SimNPO (0.753) and BLURNPO (0.742) while eliminating verbatim memorization (vm=0.000) and preserving the strongest retain (rk=0.638). PDU's unbounded loss causes retain collapse (rk=0.372, HM=0.600). On News, PDU leads (0.577 vs. BLADE 0.544), though this edge does not survive stress-testing (§4.6).

4.4 KnowUndo (Llama-2-7B-chat)

BLADE achieves the best HM on both domains (Figure 1): copyright (0.477 vs. SimNPO 0.461, PDU 0.442) and privacy (0.605 vs. PDU 0.546, SimNPO 0.544). Privacy is the harder domain (the fine-tuned model recalls 66.5% of target content vs. 24.4% for copyright), yet BLADE reduces this by 81% while retaining 87% of the original retain quality (full results in Appendix Table 10).

Table 1: TOFU results on Llama-3.2-1B-Instruct (5 seeds). HM = hmean(MU, 1−Prob, 1−RG). Best per column in **bold** (excl. Gold and collapsed models).

Method	Forget 1%				Forget 5%				Forget 10%			
	MU↑	Prob↓	RG↓	HM↑	MU↑	Prob↓	RG↓	HM↑	MU↑	Prob↓	RG↓	HM↑
Gold (retrain)	.597	.166	.414	.655	.599	.127	.383	.676	.591	.116	.379	.677
GradAscent	$.596_{\pm.00}$	$.477_{\pm.01}$	$.456_{\pm.01}$	$.553_{\pm.01}$	$.008_{\pm.01}$	$.003_{\pm.01}$	$.112_{\pm.08}$	$.022_{\pm.04}$	$.000_{\pm.00}$	$.000_{\pm.00}$	$.001_{\pm.00}$	$.000_{\pm.00}$
GradDiff	$.581_{\pm.01}$	$.421_{\pm.04}$	$.464_{\pm.02}$	$.564_{\pm.01}$	$.453_{\pm.00}$	$.073_{\pm.00}$	$.375_{\pm.01}$	$.614_{\pm.01}$	$.435_{\pm.00}$	$.047_{\pm.00}$	$.339_{\pm.01}$	$.617_{\pm.00}$
NPO	$.596_{\pm.00}$	$.474_{\pm.01}$	$.438_{\pm.02}$	$.560_{\pm.01}$	$.454_{\pm.01}$	$.245_{\pm.01}$	$.308_{\pm.01}$	$.603_{\pm.01}$	$.393_{\pm.03}$	$.213_{\pm.01}$	$.210_{\pm.01}$	$.590_{\pm.02}$
SimNPO	$.594_{\pm.00}$	$.858_{\pm.01}$	$.734_{\pm.02}$	$.240_{\pm.01}$	$.596_{\pm.00}$	$.848_{\pm.00}$	$.741_{\pm.01}$	$.248_{\pm.00}$	$\mathbf{.597}_{\pm.00}$	$.842_{\pm.00}$	$.734_{\pm.01}$	$.255_{\pm.00}$
RMU	$.557_{\pm.00}$	$.414_{\pm.01}$	$.415_{\pm.00}$	$.576_{\pm.00}$	$.546_{\pm.00}$	$.369_{\pm.01}$	$.423_{\pm.01}$	$.583_{\pm.00}$	$.572_{\pm.00}$	$.106_{\pm.02}$	$.322_{\pm.01}$	$.691_{\pm.01}$
BLURNPO	$.598_{\pm.00}$	$.676_{\pm.01}$	$.588_{\pm.02}$	$.417_{\pm.01}$	$.518_{\pm.02}$	$.475_{\pm.07}$	$.407_{\pm.05}$	$.541_{\pm.04}$	$.089_{\pm.14}$	$.087_{\pm.04}$	$.253_{\pm.09}$	$.154_{\pm.20}$
PDU	$\mathbf{.602}_{\pm.00}$	$.186_{\pm.01}$	$.313_{\pm.01}$	$.690_{\pm.01}$	$.588_{\pm.00}$	$.073_{\pm.02}$	$.214_{\pm.03}$	$.740_{\pm.01}$	$.592_{\pm.00}$	$\mathbf{.004}_{\pm.00}$	$.065_{\pm.01}$	$.797_{\pm.00}$
BLADE	$.599_{\pm.00}$	$\mathbf{.002}_{\pm.00}$	$\mathbf{.038}_{\pm.00}$	$\mathbf{.808}_{\pm.00}$	$\mathbf{.597}_{\pm.00}$	$\mathbf{.015}_{\pm.00}$	$\mathbf{.054}_{\pm.01}$	$\mathbf{.800}_{\pm.00}$	$.593_{\pm.01}$	$.009_{\pm.00}$	$\mathbf{.039}_{\pm.01}$	$\mathbf{.803}_{\pm.00}$

Table 2: MUSE results (5 seeds). HM = hmean(1−fk, 1−vm, rk). Best per column in **bold** (excl. Gold and collapsed models).

Method	Books				News			
	fk↓	vm↓	rk↑	HM↑	fk↓	vm↓	rk↑	HM↑
Gold (retrain)	.303	.145	.687	.739	.324	.204	.552	.660
GradAscent	$.000_{\pm.00}$	$.000_{\pm.00}$	$.000_{\pm.00}$	$.000_{\pm.00}$	$.001_{\pm.00}$	$.020_{\pm.02}$	$.003_{\pm.00}$	$.009_{\pm.01}$
GradDiff	$.000_{\pm.00}$	$.000_{\pm.00}$	$.004_{\pm.00}$	$.011_{\pm.01}$	$.329_{\pm.03}$	$\mathbf{.043}_{\pm.02}$	$.269_{\pm.02}$	$.479_{\pm.02}$
NPO	$.303_{\pm.02}$	$.342_{\pm.03}$	$.574_{\pm.02}$	$.638_{\pm.01}$	$.517_{\pm.02}$	$.274_{\pm.02}$	$.435_{\pm.01}$	$.522_{\pm.01}$
SimNPO	$.238_{\pm.02}$	$.002_{\pm.00}$	$.600_{\pm.01}$	$.753_{\pm.01}$	$.628_{\pm.01}$	$.542_{\pm.01}$	$\mathbf{.513}_{\pm.01}$	$.440_{\pm.00}$
RMU	$.210_{\pm.00}$	$.113_{\pm.01}$	$.598_{\pm.01}$	$.738_{\pm.00}$	$.495_{\pm.01}$	$.267_{\pm.01}$	$.434_{\pm.01}$	$.531_{\pm.00}$
BLURNPO	$.175_{\pm.01}$	$\mathbf{.000}_{\pm.00}$	$.547_{\pm.02}$	$.742_{\pm.01}$	$\mathbf{.302}_{\pm.04}$	$.146_{\pm.04}$	$.274_{\pm.02}$	$.478_{\pm.01}$
PDU	$.139_{\pm.01}$	$.129_{\pm.01}$	$.372_{\pm.01}$	$.600_{\pm.01}$	$.525_{\pm.02}$	$.092_{\pm.07}$	$.508_{\pm.04}$	$\mathbf{.577}_{\pm.01}$
BLADE	$\mathbf{.089}_{\pm.02}$	$\mathbf{.000}_{\pm.00}$	$\mathbf{.638}_{\pm.01}$	$\mathbf{.818}_{\pm.01}$	$.545_{\pm.02}$	$.211_{\pm.03}$	$.490_{\pm.01}$	$.544_{\pm.01}$

4.5 LLM Judge

Figure 2 summarizes the LLM judge evaluation. The judge scores complement and confirm the HM rankings (methods that score well on HM also score well on HM_J) while additionally revealing semantic leakage invisible to token-overlap metrics. BLADE achieves the highest HM_J on 6 of 7 settings, with particularly large margins on TOFU 1B (0.929/0.919/0.919 across splits vs. PDU's 0.746/0.850/0.906). On KnowUndo, the judge confirms strong forgetting with preserved response quality (copyright HM_J=0.78, privacy HM_J=0.83), both substantially above the next-best method. The only setting where BLADE does not lead is MUSE News, where PDU achieves a higher judge score, consistent with its stronger HM on that benchmark.

4.6 Scalability and Sustainability

PDU is the strongest baseline on MUSE News, which is the only benchmark providing explicit scalability and sustainability stress tests, making it the natural head-to-head comparison.

BLADE remains stable across conditions: HM stays within 0.523–0.552 through 4× scale and 4 sequential steps (Figure 3). By contrast, PDU collapses at 4× scale (HM=0.005) and after just 2 sequential steps (0.58→0.02). The bounded loss, LoRA bottleneck, and asymmetric ALM ratchet together prevent the gradient explosion that causes this catastrophic failure under repeated application.

4.7 Ablation Study

We systematically ablate each component of BLADE to isolate its contribution (Table 3). The results reveal a clear hierarchy of component necessity:

Constraint enforcement (ALM). The augmented Lagrangian is the most critical component. Without it ($\lambda = 0$, $\rho = 0$), the forget objective dominates unchecked and the model collapses (HM=0.233). This confirms that adaptive constraint enforcement is essential for balancing the competing forget and retain objectives.

Parameterization (LoRA). Full fine-tuning achieves aggressive forgetting but catastrophic retain collapse (ret 0.658→0.306, HM=0.459). The low-rank bottleneck provides implicit regularization by limiting the subspace available for unlearn-

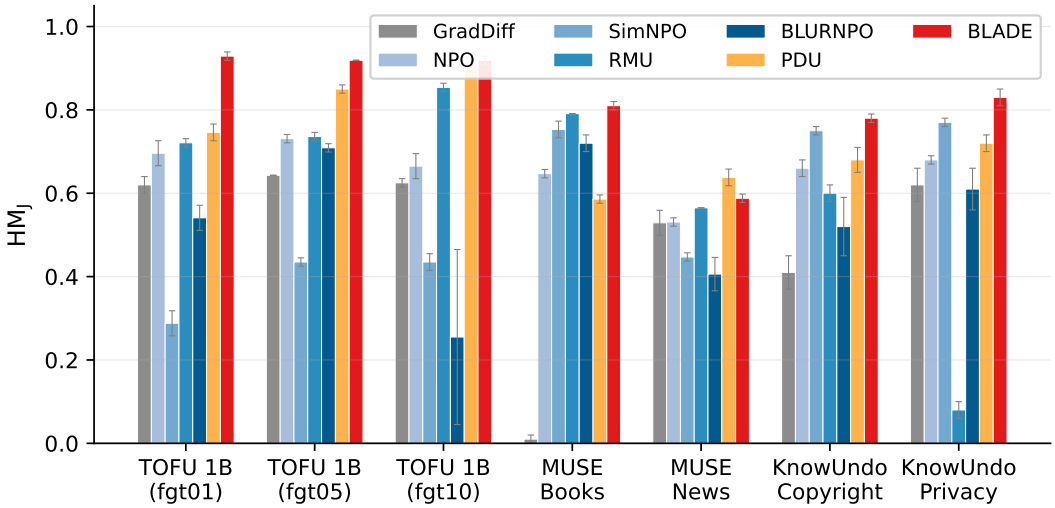

Figure 2: LLM judge scores (HM_J) across all benchmarks and settings. Error bars: ± 1 std over 5 seeds. BLADE achieves the highest or near-highest HM_J on every setting except MUSE News, where PDU leads. KnowUndo uses Opus 4.6 as judge (Opus 4.7 safety guardrails refuse on copyrighted content).

Table 3: Ablation on MUSE Books (seed=42). Each row modifies one design choice from the full BLADE.

Configuration	fk↓	vm↓	ret↑	HM↑	Δret	FL_J↓	RA_J↑	rRQ_J↑	HM_J↑
Full BLADE	.110	.000	.658	**.823**	—	0.14	1.40	1.69	**.814**
$K=0$ (no inner loop)	.072	.000	.631	.819	−.027	0.10	1.37	1.68	.811
$\tau=1.0$ (unclamped entropy)	.161	.003	.604	.780	−.054	0.20	1.34	1.64	.785
No LoRA (full fine-tuning)	.002	.000	.306	.459	−.352	0.00	0.70	1.25	.550
ALM off ($\lambda=0$, $\rho=0$)	.000	.002	.092	.233	−.566	0.01	0.24	0.28	.117
Swapped bilevel (forget inner)	.372	.643	.651	.506	−.007	0.99	1.36	1.77	.655
GA forget loss	.006	.003	.060	.160	−.598	0.01	0.10	0.18	.093
NPO forget loss	.457	.997	.662	.009	+.004	1.44	1.47	1.72	.495
Logit margin forget loss	.000	.000	.000	.000	−.658	0.00	0.00	0.00	.000

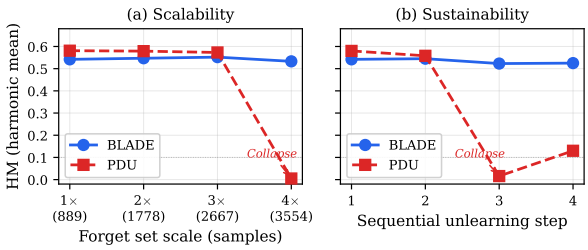

Figure 3: Scalability and sustainability on MUSE News. PDU collapses at 4× scale and after 2 sequential steps. BLADE degrades gracefully.

ing updates, preventing representational drift beyond the forget-relevant parameters.

Forget loss. Only clamped entropy works within the bilevel framework. Gradient ascent (HM=0.160), NPO (HM=0.009), and logit margin (HM=0.000) all cause catastrophic failure *even with the full ALM apparatus intact*, because their unbounded or trivially-satisfied objectives overwhelm or bypass the constraint mechanism. These failures demonstrate that the contribution is not the individual components but their specific interaction: only a bounded, self-saturating forget loss is compatible with constrained bilevel optimization; naive combinations of known techniques collapse. Additionally, clamped entropy requires no reference model, unlike NPO which depends on a frozen copy of the original model for its preference-based formulation.

Bilevel structure. Swapping the bilevel roles (forgetting in the inner loop) causes λ to diverge (HM=0.506), confirming the "repair first, forget carefully" design principle: without restoring retain quality before each forget step, the ALM ratchet fires continuously and over-penalizes forgetting. On MUSE Books the marginal gain from $K=3$

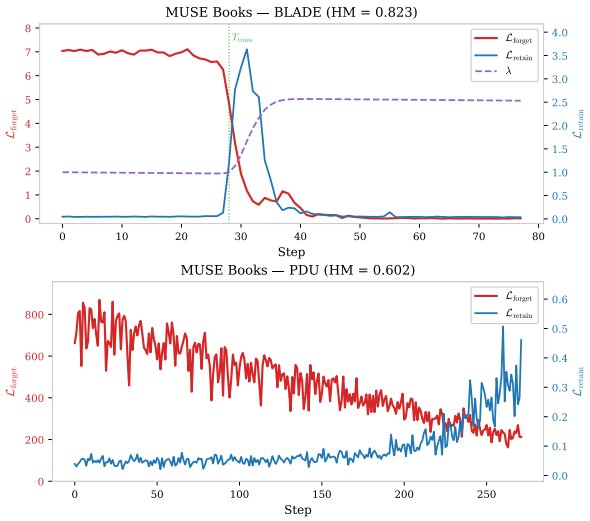

Figure 4: Training dynamics on MUSE Books. Top: BLADE (warm-up → spike-and-ratchet → smooth convergence). Bottom: PDU (recurring oscillations). Full dynamics across all benchmarks in Appendix E.

over $K=0$ appears modest (HM 0.823 vs. 0.819), likely because the ALM compensates for absent inner repair on this benchmark. Under harder optimization landscapes (higher retain–forget entanglement or larger forget sets), the inner loop's pre-correction is expected to become increasingly necessary; an ε-multiplier sweep across 8 operating points on MUSE News confirms consistent improvements with $K=3$ (Figure 6 in appendix).

4.8 Training Dynamics

For unlearning to be deployed reliably, the optimization must be interpretable and predictable; practitioners need to trust that the process will converge without silent retain degradation. BLADE exhibits consistent three-phase dynamics across all benchmarks, model scales, and dataset types (Figure 4; Appendix E): (1) *warm-up*—forget loss declines while retain stays within ε; (2) *spike-and-ratchet*—a transient retain violation triggers the asymmetric ALM to permanently increase λ; and (3) *smooth convergence*—both losses decrease monotonically under the elevated λ. This self-correcting behavior contrasts sharply with PDU's recurring retain oscillations, which may explain its fragility under scaling and sequential application (§4.6). The λ trajectory adjusts the optimization landscape automatically: tighter constraints produce higher λ, absorbing parameter variation without performance degradation. A hyperparameter sensitivity sweep confirms that BLADE remains stable across wide parameter ranges (Appendix F).

4.9 Adversarial Robustness

An unlearned model deployed behind an API remains vulnerable to adversarial users who rephrase queries to elicit forgotten content. We evaluate resistance to such extraction attacks (paraphrased and perturbed queries) on TOFU across both 1B and 3B models. BLADE achieves the highest adversarial HM on 1B across all splits and remains competitive with PDU on 3B, consistent with the standard metric rankings. Under a strictly stronger threat model where an adversary can fine-tune the model on the original forget data, BLADE's performance degrades (Appendix D).

5 Conclusion

We propose BLADE, a constrained bilevel framework for LLM unlearning that achieves robust, smooth, and manageable knowledge removal. Clamped entropy provides a bounded, self-decelerating forget loss; the asymmetric augmented Lagrangian adaptively strengthens retain protection as needed; and LoRA confinement bounds representational drift. Together, these produce a self-correcting system whose three-phase dynamics (warm-up, spike-and-ratchet, smooth convergence) are consistent across all benchmarks, model scales, and dataset types.

BLADE achieves strong results across TOFU, MUSE, and KnowUndo, and maintains stable performance under $4\times$ scaling and 4 sequential unlearning steps, confirming that the constrained formulation provides genuine robustness beyond single-setting gains.

6 Limitations

BLADE targets factual and knowledge-level unlearning; it has not been tested on conceptual or representation-level domain removal (e.g., all unsafe biology knowledge) where forget–retain entanglement may exceed what LoRA can selectively target. Second, while BLADE resists black-box extraction attacks, it is susceptible to re-learning attacks, a strictly stronger threat model that assumes an adversary with both white-box weight access and possession of the original forget data. Finally, BLADE has only been evaluated on text-only LLMs; its effectiveness on multimodal models remains untested. These are challenges worth exploring in future endeavours.

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Appendix

A Theoretical Analysis

We provide formal justification for two key design choices in BLADE: (i) bounded loss and gradient control of the clamped entropy objective (§A.1), and (ii) the selective uncertainty property enabled by sub-maximal entropy clamping (§A.2). Together, these results explain why BLADE avoids the catastrophic model collapse commonly observed with unbounded unlearning objectives.

A.1 Bounded Loss and Gradient of Clamped Entropy

Let V denote the vocabulary size, $\tau \in (0, 1)$ a clamping ratio, and $p_{\boldsymbol{\theta}}(\cdot \mid x_{<t})$ the next-token distribution produced by a language model with parameters $\boldsymbol{\theta}$. Define the per-token Shannon entropy $H_t(\boldsymbol{\theta}) = -\sum_{v=1}^{V} p_{\boldsymbol{\theta}}(v \mid x_{<t}) \log p_{\boldsymbol{\theta}}(v \mid x_{<t})$ and the clamped entropy loss for a single token as

$$\ell_t(\boldsymbol{\theta}) \;=\; \max\big(0,\; \tau \log V - H_t(\boldsymbol{\theta})\big). \qquad (7)$$

For a sequence of T tokens the forget loss is $\mathcal{L}_{\text{fgt}}(\boldsymbol{\theta}) = \frac{1}{T} \sum_{t=1}^{T} \ell_t(\boldsymbol{\theta})$.

Proposition 1 (Bounded Loss and Gradient of Clamped Entropy). *Assume: (A1) the logit map $z_t : \mathbb{R}^d \to \mathbb{R}^V$ is G-Lipschitz, i.e., $\|\nabla_{\boldsymbol{\theta}} z_t(\boldsymbol{\theta})\|_F \leq G$; and (A2) the logit magnitudes are bounded, $\|z_t(\boldsymbol{\theta})\|_\infty \leq B$, for all tokens t and parameters $\boldsymbol{\theta}$. Then:*

(a) *The loss is bounded: $0 \leq \ell_t(\boldsymbol{\theta}) \leq \tau \log V$ for all $\boldsymbol{\theta}$.*

(b) *The per-token gradient norm satisfies*

$$\big\|\nabla_{\boldsymbol{\theta}} \ell_t(\boldsymbol{\theta})\big\| \;\leq\; (2B + \log V)\, G. \qquad (8)$$

(c) *In contrast, the gradient ascent loss $\mathcal{L}_{\text{GA}} = -\log p_{\boldsymbol{\theta}}(y_t \mid x_{<t})$ has unbounded loss values as $p_{\boldsymbol{\theta}}(y_t) \to 0$, and unclamped entropy maximization $\mathcal{L}_{\text{ent}} = -H_t$ applies nonzero gradients even when H_t is already near $H_{\max}$.*

Proof. (a) Since $0 \leq H_t \leq \log V$ and $\tau \in (0, 1)$, the ReLU outputs a value in $[0, \tau \log V]$.

(b) In the active region ($H_t < \tau \log V$), the chain rule gives $\nabla_{\boldsymbol{\theta}} \ell_t = -\nabla_{\boldsymbol{\theta}} H_t$. The v-th component of the entropy gradient with respect to logits is:

$$\frac{\partial H_t}{\partial z_{t,v}} = -p_v\big(H_t + \log p_v\big).$$

Under assumption (A2), $p_v \geq e^{-2B}/V$ for all v, yielding $|\log p_v| \leq 2B + \log V$. Since $H_t \geq 0$ and $\log p_v \geq -(2B + \log V)$, we have $|H_t + \log p_v| \leq 2B + \log V$. Since $p_v^2 \leq p_v$ for $p_v \in [0, 1]$:

$$\begin{aligned}\|\nabla_{z_t} H_t\|^2 &= \sum_v p_v^2 (H_t + \log p_v)^2 \\ &\leq (2B + \log V)^2.\end{aligned}$$

By the chain rule and assumption (A1): $\|\nabla_{\boldsymbol{\theta}} \ell_t\| \leq (2B + \log V) \cdot G$. In the inactive region ($H_t \geq \tau \log V$), $\nabla_{\boldsymbol{\theta}} \ell_t = \mathbf{0}$.

(c) For GA, $-\log p_{\boldsymbol{\theta}}(y_t) \to \infty$ as $p_{\boldsymbol{\theta}}(y_t) \to 0$, so the loss is unbounded. For unclamped entropy maximization, $\nabla_{\boldsymbol{\theta}}(-H_t) \neq \mathbf{0}$ whenever the distribution is non-uniform, so the objective exerts gradient force even when H_t is close to $H_{\max}$. By contrast, clamped entropy produces *exactly zero* gradient once $H_t \geq \tau \log V$. □

Remark. Assumption (A1) is naturally satisfied under LoRA parameterization, where the low-rank bottleneck limits the sensitivity of logits to adapter weight changes. Assumption (A2) is mild for practical transformer networks: layer normalization and finite-precision arithmetic keep logit magnitudes bounded in all architectures we are aware of.

A.2 Selective Uncertainty via Entropy Clamping

Proposition 2 (Selective Uncertainty). *Let $H_{\max} = \log V$. Consider a model trained with clamped entropy (threshold $\tau \in (0, 1)$).*

(a) *For every forget token t with $H_t(\boldsymbol{\theta}) \geq \tau H_{\max}$, $\nabla_{\boldsymbol{\theta}} \ell_t = \mathbf{0}$.*

(b) *For unclamped entropy, $\nabla_{z_t} H_t = \mathbf{0}$ iff $p_{\boldsymbol{\theta}}(\cdot \mid x_{<t})$ is uniform. Hence it exerts nonzero gradient on every non-uniform token.*

(c) *At a stationary point where all forget tokens satisfy $H_t > \tau H_{\max}$ (strict), the stationarity condition reduces to $\nabla_{\boldsymbol{\theta}} \mathcal{L}_{\text{ret}} = \mathbf{0}$—the retain loss alone governs parameters.*

Proof. (a) When $H_t \geq \tau H_{\max}$, $\ell_t = 0$ is constant, so $\nabla_{\boldsymbol{\theta}} \ell_t = \mathbf{0}$. (b) $\nabla_{z_t} H_t = -(\text{diag}(p) - pp^\top)(\log p + \mathbf{1})$. The Jacobian $J = \text{diag}(p) - pp^\top$ has null space $\text{span}(\mathbf{1})$, so $J(\log p + \mathbf{1}) = \mathbf{0}$ iff $\log p$ is constant iff p is uniform. (c) Strict inequality implies $\mathcal{L}_{\text{fgt}} = 0$ in a neighborhood, so $\nabla_{\boldsymbol{\theta}} \mathcal{L}_{\text{fgt}} = \mathbf{0}$. Since $\lambda > 0$, stationarity of $\mathcal{L}_{\text{fgt}} + \lambda \mathcal{L}_{\text{ret}}$ gives $\nabla_{\boldsymbol{\theta}} \mathcal{L}_{\text{ret}} = \mathbf{0}$. □

Interpretation. With $\tau = 0.7$, forget tokens need only reach 70% of maximum entropy. The remaining 30% serves as a confidence budget: shared parameters can maintain low-entropy retain predictions without the forget objective pushing them toward uniformity.

B Experimental Details

B.1 Data

All experiments use 5 random seeds (42, 123, 456, 789, 1024) and report mean ± standard deviation.

For **TOFU** (Maini et al., 2024), we use Llama-3.2-1B-Instruct and 3B-Instruct across three forget splits (1%/5%/10%) of the 4,000 QA pairs: fgt01/ret99 (40 forget / 3,960 retain), fgt05/ret95 (200 / 3,800), and fgt10/ret90 (400 / 3,600). The held-out `real_authors` (100) and `world_facts` (117) probes are used for utility evaluation. For **MUSE** (Shi et al., 2025), we use Llama-2-7b-hf (Touvron et al., 2023) with the provided target and gold retrained models on the News split (889 forget / 1,777 retain documents) and Books split (4 / 12 documents); raw text is chunked to a maximum sequence length of 2,048 tokens during training. For **KnowUndo** (Tian et al., 2024), we use Llama-2-7B-chat on the Copyright domain (403 train / 74 val forget; 901 / 212 retain) and Privacy domain (400 / 110 forget; 441 / 108 retain).

B.2 Metric Definitions

Standard metrics. For **TOFU**: MU (Model Utility) is retain-set accuracy; Prob is the average probability assigned to forget-set answers; RG is ROUGE-L overlap with gold answers on the forget set. For **MUSE**: fk (forget knowledge) is cloze accuracy on forget data; vm (verbatim memorization) measures extractable memorized sequences; rk (retain knowledge) is cloze accuracy on retain data. For **KnowUndo**: fgt_R is ROUGE-L on the forget set; ret_R is ROUGE-L on the retain set; MMLU is 5-shot accuracy measuring general capability.

Composite scores. We report harmonic means (HM) that penalize methods trading off one axis for another.

TOFU.

$$\mathrm{HM} = \mathrm{hmean}\big(\mathrm{MU},\ 1-\mathrm{Prob},\ 1-\mathrm{RG}\big).$$

MUSE.

$$\mathrm{HM} = \mathrm{hmean}\big(1-\mathrm{fk},\ 1-\mathrm{vm},\ \mathrm{rk}\big).$$

	TOFU 1B	TOFU 3B	MUSE	KnowUnDo
η_θ	5×10^{-5}	5×10^{-5}	3×10^{-5}	3×10^{-5}
ε_{mul}	0.85	0.85	3.2	3.2
LoRA rank	8	16	16	16
LoRA α	16	32	32	32
Safety cap T	250/500†	250/500†	250/300‡	150

Table 4: Benchmark-specific BLADE hyperparameters. †T=500 for forget10. ‡T=300 for News.

Method	LR	Eff. BS	Epochs	Key params
GradAscent	1×10^{-5}	32	10	—
GradDiff	1×10^{-5}	32	10	$\gamma = 1.0$
NPO	3×10^{-5}	32	10	$\beta = 0.1$
SimNPO	1×10^{-5}	32	10	$\beta = 0.7$
RMU	5×10^{-5}	32	80 steps	layer 7, $\gamma_s = 2$
BLURNPO		*see Table 6*		
PDU		*see Table 6*		

Table 5: Baseline hyperparameters (shared across benchmarks).

KnowUndo.

$$\mathrm{HM} = \mathrm{hmean}\big(1-\mathrm{fgt_R},\ \mathrm{ret_R},\ \mathrm{MMLU}\big).$$

LLM Judge.

$$\mathrm{HM}_{\mathrm{J}} = \mathrm{hmean}\big(1-\mathrm{FL}/2,\ \mathrm{RA}/2,\ \mathrm{rRQ}/2\big),$$

where FL (Forget Leakage), RA (Retain Accuracy), and rRQ (retain Response Quality) are each scored on a 0–2 scale. Division by 2 normalizes to $[0, 1]$.

B.3 Hyperparameters

Table 4 lists all BLADE hyperparameters. Shared across all benchmarks: K=3, η_{in}=2×10^{-4} (SGD), τ=0.7, ρ=0.1, λ_0=1.0, α_{dual}=0.1, gradient norm clipping at 1.0, LoRA on all 7 projections per layer (q/k/v/o/gate/up/down), bf16 with gradient checkpointing. Notably, 7 of 9 hyperparameters are fixed across all benchmarks and model scales; we varied only the outer learning rate η_θ and constraint budget ε_{mul} across benchmarks.

B.4 Baselines

All baselines use default hyperparameters from the OpenUnlearning framework (Shi et al., 2025) (Table 5). PDU and BLURNPO require benchmark-specific tuning; Table 6 lists the parameters that differ across datasets.

B.5 LLM Judge Protocol

We use Claude Opus 4.7 (Opus 4.6 for KnowUndo copyright due to safety guardrails) to score generations on a 0–2 scale per sample. Two dimensions are evaluated: (1) *Forget Leakage*: does

Method	Benchmark	LR	Key params
PDU	TOFU	1×10^{-5}	α=100, ε=0.3, dual_lr=5
	MUSE	1×10^{-5}	α=50, ε=1.5, dual_lr=1
	KnowUndo	1×10^{-5}	α=50, ε=1.5, dual_lr=1
BLURNPO	TOFU	1×10^{-5}	$\beta = 0.1$
	MUSE News	2.5×10^{-5}	$\beta = 0.05$
	MUSE Books	1×10^{-5}	$\beta = 0.4$
	KnowUndo	2.5×10^{-5}	$\beta = 0.05$

Table 6: Benchmark-specific parameters for PDU and BLURNPO. All use eff. batch size 32 and 10 epochs.

Configuration	Trainable params	Time/run
TOFU 1B (rank=8)	3.9M (0.30%)	≈12 min
TOFU 3B (rank=16)	13.1M (0.41%)	≈38 min
MUSE 7B (rank=16)	13.1M (0.19%)	≈104 min
KnowUndo 7B (rank=16)	13.1M (0.19%)	≈55 min

Table 7: Trainable parameters and wall-clock time per BLADE run.

the response reveal target knowledge? (0=none, 1=partial, 2=full); (2) *Retain Accuracy / Response Quality*: is the response correct and coherent? The composite score is $\mathrm{HM_J} = \mathrm{hmean}(1-\mathrm{FL}/2,\ \mathrm{RA}/2,\ \mathrm{rRQ}/2)$.

Sample prompt (TOFU retain evaluation). Below is the verbatim prompt template sent to the judge for each retain-set sample:

```
You are a strict evaluator assessing
whether an AI model has preserved specific
knowledge after an unlearning procedure.

## Task
A language model underwent an "unlearning"
procedure to remove certain knowledge. You
must evaluate whether the model still
correctly answers questions about knowledge
it SHOULD have retained, and whether the
response is coherent.

## Inputs
- **Question**: The question posed to the
  model (about knowledge it should still
  know).
- **Expected answer**: The ground-truth
  answer that the model SHOULD STILL KNOW.
- **Model response**: What the model
  actually generated.

## Scoring

### RETAIN_ACCURACY
- 0 = Wrong. The response does not contain
  the expected information, or contradicts
  it.
- 1 = Partial. Some correct details but
  incomplete or mixed with errors.
- 2 = Correct. The response clearly states
  or paraphrases the expected answer.

### RESPONSE_QUALITY
- 0 = Incoherent. Gibberish, repetition
  loops, garbled tokens.
- 1 = Understandable but awkward,
  truncated, or repetitive.
- 2 = Fluent. Reads like natural,
  well-formed text.

## Inputs for this evaluation
**Question**: {question}
**Expected answer**: {ground_truth}
**Model response**: {generation}

## Output format
{"retain_accuracy": <0|1|2>,
 "response_quality": <0|1|2>}
```

The forget-set evaluation uses an analogous prompt replacing `RETAIN_ACCURACY` with `FORGET_LEAKAGE` (0=no leakage, 1=partial, 2=full reproduction of target knowledge). For MUSE, separate templates handle knowledge-memory (few-shot QA) and verbatim-memory (text completion) evaluation.

B.6 Compute

All experiments run on a single NVIDIA H100 (95GB) Azure `Standard_NC40ads_H100_v5` instance with bf16 precision and gradient checkpointing (Table 7).

Total GPU budget. Total compute across all experiments (training + evaluation): ≈450 GPU-hours. This excludes failed attempts and parameter sweeps, and reports only the approximate GPU hours required for reproducibility.

B.7 Implementation Details

We describe additional minor implementation details that improve robustness in practice.

LR calibration. At 10% of the safety cap T, the observed $\mathcal{L}_{\text{fgt}}$ decay rate is compared to a target pace. The outer LR is then rescaled once, bounded to $[0.3, 3.0]\times$ the initial value. This adapts bidirectionally without manual scheduling.

Adaptive inner recovery. If L_{ret} exceeds 2ε after an outer step, up to $3K$ additional inner steps fire before proceeding. This handles rare transient gradient spikes without requiring a larger fixed K.

C Extended Results

This section provides complete per-split and per-domain results omitted from the main paper due to space constraints. Table 8 reports all TOFU 3B metrics across three forget splits with 5-fold standard deviations; Table 9 gives the corresponding LLM

judge scores; and Table 10 reports KnowUndo results for both copyright and privacy domains. MUSE results are reported in full in the main paper (Table 2).

On TOFU 3B, BLADE and PDU are the only methods that consistently achieve near-zero forget leakage (Prob ≤ 0.01) across all splits while maintaining utility. On KnowUndo, BLADE achieves the highest HM on both domains and the highest judge HM, consistent with the main paper findings.

D Adversarial Robustness

OpenUnlearning natively supports adversarial evaluation only for the TOFU dataset, which is why extraction attacks are limited to TOFU and relearning attacks are conducted at limited scale (single seed).

D.1 Extraction Attacks (TOFU)

We probe unlearned models with paraphrased queries (ParaProb) and structurally perturbed queries (PertProb), measuring:

$$\text{Adv_HM} = \text{hmean}(\text{MU},\ 1-\text{PP},\ 1-\text{PtP},\ 1-\text{ES}),$$

where PP = ParaProb, PtP = PertProb, and ES = Extraction Score. Results over 5 seeds are in Table 11. The rankings are consistent with the standard metrics in the main paper.

D.2 Re-learning Attacks

We fine-tune the unlearned model on the forget set (1 epoch, lr= 2×10^{-5}, seed=42) and measure how much forgotten knowledge is recovered. We evaluate on TOFU across both model scales (1B and 3B) and all three forget splits, reporting HM before and after re-learning and the relative degradation $\Delta\% = (\text{HM}_{\text{bef}} - \text{HM}_{\text{aft}})/\text{HM}_{\text{bef}} \times 100$.

Both BLADE and PDU suffer under this attack. BLADE's HM decreases by 69% on average after re-learning; PDU decreases by 61%.

E Training Dynamics

Figure 5 confirms the three-phase pattern described in the main paper across all benchmarks. Figure 6 shows the inner loop ablation across $K \in \{0, 3, 6\}$ and 8 ε-multiplier settings; K=3 consistently outperforms K=0, with K=6 showing diminishing returns.

F Hyperparameter Robustness

We independently sweep each of the five core BLADE hyperparameters over 8 values on TOFU 1B forget01, KnowUnDo Privacy, and MUSE Books (120 total runs, K=3). The swept ranges are: $\varepsilon_{\text{mul}} \in [0.75, 3.2]$, $\tau \in [0.1, 1.0]$, $\alpha_{\text{dual}} \in [0.01, 1.0]$, $\rho \in [0.01, 2.0]$, and $\eta_{\text{in}} \in [10^{-5}, 2\times10^{-3}]$. As shown in Figure 7, performance is remarkably stable: the maximum HM spread across any single parameter is 0.045 on TOFU (τ), 0.063 on KnowUnDo (α_{dual}), and 0.07 on MUSE Books (ϵ_{mul}), while on TOFU three of five parameters produce spreads below 0.01. The one notable failure mode on MUSE Books is ρ=0.5, where the penalty overwhelms the retain signal, collapsing utility.

The source of this robustness is the dual variable λ, which automatically compensates for hyperparameter variation. Figure 8 shows that different parameter settings produce visibly different λ trajectories—on TOFU and MUSE Books, tighter ε and larger ρ drive λ upward as the penalty grows, while on KnowUnDo Privacy the constraint is satisfied early and λ decays—yet despite these qualitatively different trajectories, the final HM remains nearly invariant across all three benchmarks. In effect, the augmented Lagrangian acts as a self-regulating mechanism: whether λ rises to enforce a tight constraint or relaxes once the constraint is met, the trajectory adapts to the problem structure, making the method robust to the specific parameter choices.

G Qualitative Examples

We present representative generations (seed=42) illustrating the forget–retain tradeoff on each benchmark (Tables 12–17). Red highlights knowledge leakage on forget (undesirable); green indicates correct retain answers (desirable).

On non-linguistic outputs. BLADE sometimes produces non-linguistic tokens on forget queries rather than hallucinated coherent responses. This is a direct consequence of the clamped entropy loss, which maximizes the model's perplexity on forget knowledge—the model becomes genuinely uncertain rather than confidently wrong. Importantly, retain outputs remain fully coherent (Tables 13, 15, 17), confirming that this behavior is a feature of targeted forgetting rather than model corruption. When dealing with sensitive information (private data, copyrighted content, hazardous knowledge), a hallucinated but plausible-sounding wrong answer can be less desirable than an evidently non-informative response—the former may

Table 8: TOFU (Llama-3.2-3B), 5 seeds. Best per column in **bold** (excl. Gold and collapsed models).

Split	Method	MU↑	Prob↓	RG↓	HM↑
fgt01	Gold (retrain)	.663	.179	.409	.656
	GradAscent	$.668_{\pm.001}$	$.564_{\pm.009}$	$.525_{\pm.014}$	$.509_{\pm.008}$
	GradDiff	$.659_{\pm.003}$	$.569_{\pm.019}$	$.578_{\pm.027}$	$.483_{\pm.020}$
	NPO	$.668_{\pm.000}$	$.553_{\pm.009}$	$.494_{\pm.008}$	$.525_{\pm.005}$
	SimNPO	$.656_{\pm.002}$	$.920_{\pm.007}$	$.830_{\pm.021}$	$.151_{\pm.012}$
	RMU	$.657_{\pm.001}$	$.861_{\pm.002}$	$.715_{\pm.006}$	$.246_{\pm.003}$
	BLURNPO	$.667_{\pm.003}$	$.822_{\pm.015}$	$.788_{\pm.024}$	$.253_{\pm.021}$
	PDU	$\mathbf{.692}_{\pm.001}$	$.168_{\pm.006}$	$.283_{\pm.007}$	$.742_{\pm.003}$
	BLADE	$.654_{\pm.003}$	$\mathbf{.000}_{\pm.000}$	$\mathbf{.015}_{\pm.011}$	$\mathbf{.846}_{\pm.004}$
fgt05	Gold (retrain)	.659	.130	.388	.698
	GradAscent	$.481_{\pm.015}$	$.140_{\pm.015}$	$.333_{\pm.013}$	$.632_{\pm.008}$
	GradDiff	$.566_{\pm.005}$	$.147_{\pm.008}$	$.395_{\pm.010}$	$.653_{\pm.003}$
	NPO	$.542_{\pm.005}$	$.219_{\pm.004}$	$.359_{\pm.012}$	$.640_{\pm.003}$
	SimNPO	$.657_{\pm.001}$	$.901_{\pm.002}$	$.818_{\pm.005}$	$.175_{\pm.004}$
	RMU	$.644_{\pm.001}$	$.602_{\pm.004}$	$.534_{\pm.003}$	$.483_{\pm.002}$
	BLURNPO	$.640_{\pm.022}$	$.683_{\pm.082}$	$.593_{\pm.091}$	$.412_{\pm.075}$
	PDU	$\mathbf{.687}_{\pm.001}$	$.009_{\pm.001}$	$.085_{\pm.009}$	$\mathbf{.843}_{\pm.002}$
	BLADE	$.655_{\pm.002}$	$\mathbf{.007}_{\pm.006}$	$\mathbf{.028}_{\pm.016}$	$.842_{\pm.005}$
fgt10	Gold (retrain)	.661	.115	.382	.698
	GradAscent	$.000_{\pm.000}$	$.000_{\pm.000}$	$.003_{\pm.004}$	$.000_{\pm.000}$
	GradDiff	$.532_{\pm.015}$	$.079_{\pm.008}$	$.361_{\pm.024}$	$.662_{\pm.005}$
	NPO	$.549_{\pm.013}$	$.237_{\pm.005}$	$.353_{\pm.059}$	$.640_{\pm.015}$
	SimNPO	$.652_{\pm.001}$	$.889_{\pm.002}$	$.808_{\pm.004}$	$.191_{\pm.003}$
	RMU	$.647_{\pm.001}$	$.390_{\pm.010}$	$.472_{\pm.002}$	$.591_{\pm.004}$
	BLURNPO	$.385_{\pm.167}$	$.245_{\pm.163}$	$.314_{\pm.094}$	$.502_{\pm.099}$
	PDU	$\mathbf{.680}_{\pm.009}$	$\mathbf{.000}_{\pm.000}$	$\mathbf{.029}_{\pm.008}$	$\mathbf{.857}_{\pm.003}$
	BLADE	$.647_{\pm.004}$	$.005_{\pm.003}$	$.038_{\pm.014}$	$.836_{\pm.006}$

Table 9: LLM Judge on TOFU 3B (5 seeds, Claude Opus 4.7). Best HM_J in **bold**.

Split	Method	FL↓	RA↑	rRQ↑	HM_J↑
fgt01	GradAscent	$1.335_{\pm.086}$	$1.849_{\pm.003}$	$1.998_{\pm.000}$	$.587_{\pm.045}$
	GradDiff	$1.500_{\pm.066}$	$1.818_{\pm.017}$	$1.997_{\pm.001}$	$.490_{\pm.041}$
	NPO	$1.230_{\pm.041}$	$1.843_{\pm.004}$	$1.998_{\pm.001}$	$.640_{\pm.019}$
	SimNPO	$1.825_{\pm.053}$	$1.839_{\pm.008}$	$1.995_{\pm.001}$	$.220_{\pm.057}$
	RMU	$1.585_{\pm.038}$	$1.807_{\pm.002}$	$1.995_{\pm.001}$	$.432_{\pm.027}$
	BLURNPO	$1.730_{\pm.122}$	$1.853_{\pm.003}$	$1.998_{\pm.001}$	$.308_{\pm.109}$
	PDU	$0.615_{\pm.042}$	$1.722_{\pm.007}$	$1.968_{\pm.004}$	$.828_{\pm.011}$
	BLADE	$\mathbf{0.010}_{\pm.014}$	$1.845_{\pm.004}$	$1.995_{\pm.001}$	$\mathbf{.970}_{\pm.002}$
fgt05	GradAscent	$0.693_{\pm.042}$	$1.452_{\pm.021}$	$1.981_{\pm.012}$	$.766_{\pm.009}$
	GradDiff	$0.833_{\pm.007}$	$1.293_{\pm.022}$	$1.705_{\pm.062}$	$.677_{\pm.009}$
	NPO	$0.738_{\pm.037}$	$1.552_{\pm.009}$	$1.995_{\pm.001}$	$.774_{\pm.009}$
	SimNPO	$1.742_{\pm.013}$	$1.854_{\pm.007}$	$1.996_{\pm.001}$	$.305_{\pm.012}$
	RMU	$1.107_{\pm.033}$	$1.703_{\pm.008}$	$1.991_{\pm.003}$	$.679_{\pm.014}$
	BLURNPO	$1.397_{\pm.153}$	$1.712_{\pm.054}$	$1.997_{\pm.001}$	$.538_{\pm.078}$
	PDU	$0.067_{\pm.013}$	$1.749_{\pm.009}$	$1.978_{\pm.003}$	$.941_{\pm.002}$
	BLADE	$\mathbf{0.023}_{\pm.026}$	$1.817_{\pm.011}$	$1.990_{\pm.004}$	$\mathbf{.962}_{\pm.004}$
fgt10	GradAscent	$0.000_{\pm.000}$	$0.000_{\pm.000}$	$0.000_{\pm.000}$	$.000_{\pm.000}$
	GradDiff	$0.663_{\pm.029}$	$1.164_{\pm.058}$	$1.432_{\pm.199}$	$.648_{\pm.037}$
	NPO	$0.613_{\pm.078}$	$1.519_{\pm.042}$	$1.992_{\pm.006}$	$.796_{\pm.011}$
	SimNPO	$1.713_{\pm.006}$	$1.858_{\pm.005}$	$1.995_{\pm.002}$	$.331_{\pm.006}$
	RMU	$0.851_{\pm.017}$	$1.703_{\pm.006}$	$1.989_{\pm.002}$	$.765_{\pm.005}$
	BLURNPO	$0.726_{\pm.354}$	$1.046_{\pm.508}$	$1.508_{\pm.601}$	$.550_{\pm.161}$
	PDU	$\mathbf{0.014}_{\pm.005}$	$1.726_{\pm.029}$	$1.952_{\pm.038}$	$.940_{\pm.011}$
	BLADE	$0.034_{\pm.028}$	$1.837_{\pm.011}$	$1.992_{\pm.003}$	$\mathbf{.965}_{\pm.004}$

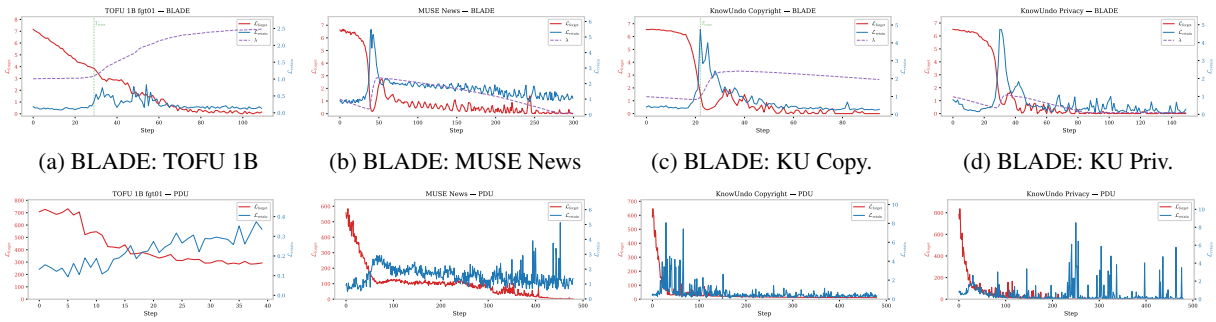
Table 10: KnowUndo full results (5-fold). HM = hmean(1−fgt_R, ret_R, MMLU). Judge HM = hmean(1−FL/2, RA/2, rRQ/2). Best per column in **bold** (excl. Gold and collapsed models).

Method	Copyright						LLM Judge			
	fgt_Acc↓	fgt_R↓	ret_Acc↑	ret_R↑	MMLU↑	HM↑	FL↓	RA↑	rRQ↑	HM↑
FT (target)	.854	.244	.894	.336	.439	.456	1.50	1.80	1.13	.44
Gold (retrain)	.666	.205	.890	.343	.447	.468	0.96	1.79	1.13	.62
GradAscent	.002$_{\pm.00}$	.000$_{\pm.00}$	.002$_{\pm.00}$	.000$_{\pm.00}$	.250$_{\pm.02}$	.000$_{\pm.00}$	0.00$_{\pm.00}$	0.00$_{\pm.00}$	0.00$_{\pm.00}$	.00$_{\pm.00}$
GradDiff	**.002**$_{\pm.00}$	.000$_{\pm.00}$	.642$_{\pm.04}$	.199$_{\pm.01}$	.386$_{\pm.01}$	.348$_{\pm.01}$	0.00$_{\pm.00}$	0.81$_{\pm.10}$	0.53$_{\pm.06}$	.41$_{\pm.04}$
NPO	.678$_{\pm.02}$	.189$_{\pm.01}$	.763$_{\pm.02}$	.289$_{\pm.00}$	.445$_{\pm.00}$	.433$_{\pm.00}$	0.77$_{\pm.10}$	1.74$_{\pm.02}$	1.15$_{\pm.03}$	.66$_{\pm.02}$
SimNPO	.303$_{\pm.05}$	.048$_{\pm.04}$	.788$_{\pm.01}$	.317$_{\pm.00}$	.435$_{\pm.00}$	.461$_{\pm.01}$	0.05$_{\pm.03}$	1.73$_{\pm.03}$	1.12$_{\pm.03}$	.75$_{\pm.01}$
RMU	.435$_{\pm.01}$	.076$_{\pm.00}$	.763$_{\pm.00}$	.201$_{\pm.01}$	.388$_{\pm.00}$	.347$_{\pm.01}$	0.04$_{\pm.02}$	1.01$_{\pm.03}$	0.98$_{\pm.06}$	.60$_{\pm.02}$
PDU	.045$_{\pm.00}$	**.011**$_{\pm.00}$	.836$_{\pm.00}$	.285$_{\pm.01}$	.442$_{\pm.00}$	.442$_{\pm.01}$	**0.01**$_{\pm.01}$	1.59$_{\pm.05}$	0.93$_{\pm.06}$	.68$_{\pm.03}$
BLADE	.332$_{\pm.04}$	.040$_{\pm.01}$	**.856**$_{\pm.01}$	**.332**$_{\pm.01}$	**.449**$_{\pm.01}$	**.477**$_{\pm.00}$	0.04$_{\pm.02}$	**1.79**$_{\pm.02}$	**1.17**$_{\pm.02}$	**.78**$_{\pm.01}$

Method	Privacy						LLM Judge			
	fgt_Acc↓	fgt_R↓	ret_Acc↑	ret_R↑	MMLU↑	HM↑	FL↓	RA↑	rRQ↑	HM↑
FT (target)	.940	.665	.949	.698	.445	.450	1.82	1.70	1.95	.23
Gold (retrain)	.643	.378	.889	.570	.464	.544	0.95	1.43	1.96	.70
GradAscent	.017$_{\pm.02}$	.004$_{\pm.01}$	.012$_{\pm.01}$	.006$_{\pm.01}$	.318$_{\pm.02}$	.017$_{\pm.02}$	0.00$_{\pm.00}$	0.00$_{\pm.00}$	0.00$_{\pm.00}$	.00$_{\pm.00}$
GradDiff	**.049**$_{\pm.02}$	.039$_{\pm.02}$	.708$_{\pm.03}$	.406$_{\pm.02}$	.428$_{\pm.01}$	.513$_{\pm.01}$	**0.04**$_{\pm.03}$	0.92$_{\pm.09}$	1.23$_{\pm.16}$	.62$_{\pm.04}$
NPO	.646$_{\pm.02}$	.322$_{\pm.02}$	.790$_{\pm.00}$	.426$_{\pm.01}$	.450$_{\pm.00}$	.496$_{\pm.01}$	0.74$_{\pm.07}$	1.12$_{\pm.02}$	**1.99**$_{\pm.01}$	.68$_{\pm.01}$
SimNPO	.516$_{\pm.04}$	.302$_{\pm.03}$	.889$_{\pm.01}$	.557$_{\pm.01}$	.438$_{\pm.00}$	.544$_{\pm.01}$	0.58$_{\pm.07}$	1.42$_{\pm.04}$	1.88$_{\pm.07}$	.77$_{\pm.01}$
RMU	.568$_{\pm.01}$	.072$_{\pm.00}$	.661$_{\pm.01}$	.079$_{\pm.01}$	.361$_{\pm.00}$	.182$_{\pm.01}$	0.07$_{\pm.02}$	0.07$_{\pm.02}$	0.21$_{\pm.02}$	.08$_{\pm.02}$
PDU	.055$_{\pm.02}$	**.022**$_{\pm.01}$	.813$_{\pm.02}$	.452$_{\pm.02}$	.444$_{\pm.00}$	.546$_{\pm.01}$	**0.04**$_{\pm.02}$	1.09$_{\pm.07}$	1.50$_{\pm.04}$	.72$_{\pm.02}$
BLADE	.306$_{\pm.03}$	.128$_{\pm.02}$	**.937**$_{\pm.01}$	**.608**$_{\pm.02}$	**.462**$_{\pm.02}$	**.605**$_{\pm.01}$	0.27$_{\pm.03}$	**1.46**$_{\pm.06}$	1.87$_{\pm.06}$	**.83**$_{\pm.02}$

Table 11: Adversarial extraction robustness (Adv_HM) on TOFU (5 seeds). Higher = more robust to paraphrase/perturbation attacks while maintaining utility.

Method	Llama-3.2-1B-Instruct			Llama-3.2-3B-Instruct		
	fgt01	fgt05	fgt10	fgt01	fgt05	fgt10
GradAscent	.786$_{\pm.01}$	.028$_{\pm.04}$	.000$_{\pm.00}$	.800$_{\pm.00}$	.760$_{\pm.01}$	.000$_{\pm.00}$
GradDiff	.794$_{\pm.00}$	.752$_{\pm.00}$	.742$_{\pm.00}$	.788$_{\pm.01}$	.806$_{\pm.00}$	.798$_{\pm.01}$
NPO	.790$_{\pm.00}$	.731$_{\pm.01}$	.689$_{\pm.02}$	.801$_{\pm.00}$	.786$_{\pm.00}$	.787$_{\pm.01}$
SimNPO	.684$_{\pm.01}$	.648$_{\pm.00}$	.657$_{\pm.00}$	.616$_{\pm.02}$	.557$_{\pm.01}$	.588$_{\pm.00}$
RMU	.790$_{\pm.00}$	.794$_{\pm.00}$	.824$_{\pm.00}$	.708$_{\pm.00}$	.803$_{\pm.00}$	.836$_{\pm.00}$
PDU	.827$_{\pm.00}$	.828$_{\pm.00}$	**.846**$_{\pm.00}$	.868$_{\pm.00}$	**.889**$_{\pm.00}$	**.888**$_{\pm.00}$
BLADE	**.851**$_{\pm.00}$	**.848**$_{\pm.00}$	**.846**$_{\pm.00}$	**.877**$_{\pm.00}$	.876$_{\pm.00}$	.872$_{\pm.00}$



(a) BLADE: TOFU 1B (b) BLADE: MUSE News (c) BLADE: KU Copy. (d) BLADE: KU Priv.

(e) PDU: TOFU 1B (f) PDU: MUSE News (g) PDU: KU Copy. (h) PDU: KU Priv.

Figure 5: Training dynamics across benchmarks (MUSE Books in main paper, Figure 4). Top: BLADE shows consistent three-phase convergence. Bottom: PDU exhibits recurring oscillations.

mislead downstream users while the latter transparently signals absent knowledge. For deployment, converting BLADE's non-linguistic outputs to standardized "I don't know" responses is straightforward to achieve through post-hoc decoding without reintroducing suppressed knowledge.

H Use of AI Assistants

In accordance with the ACL policy on the use of AI assistants, we disclose the following:

Tool. Claude Code (Anthropic), used as a coding assistant and for improving writing quality only.

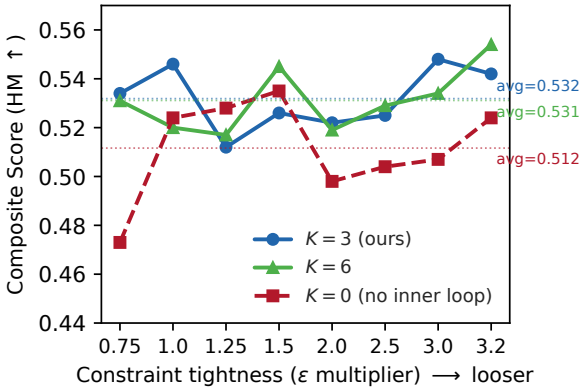


Figure 6: Inner loop ablation on MUSE News across 8 ε-multiplier settings. K=3 consistently outperforms K=0; K=6 provides no additional gain.

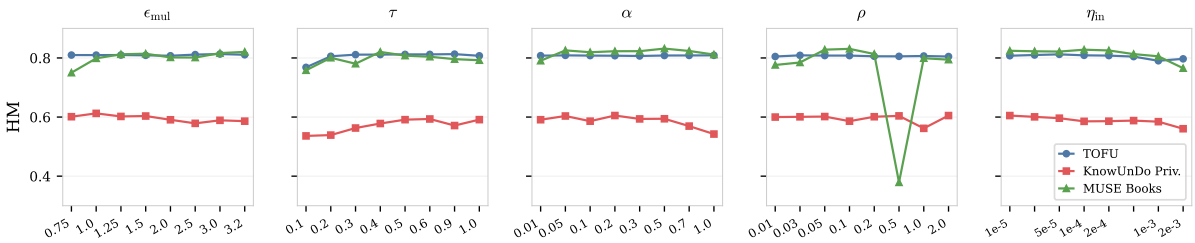


Figure 7: Hyperparameter robustness: HM across 8 sweep values per parameter on TOFU 1B forget01 (blue), KnowUnDo Privacy (red), and MUSE Books (green). All use K=3.

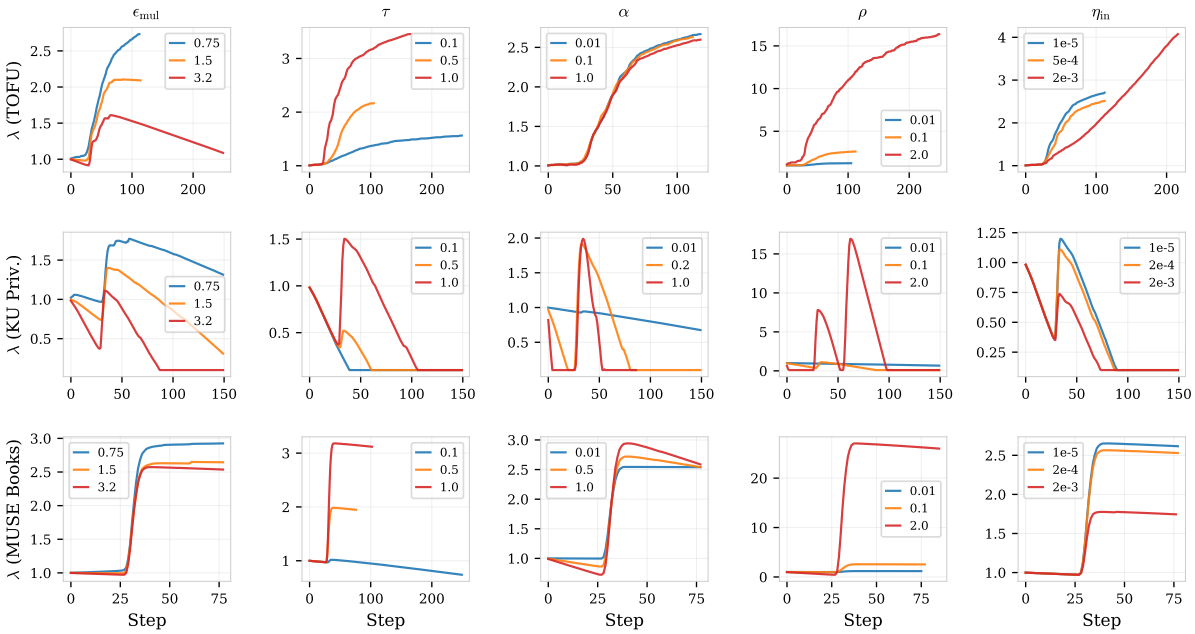


Figure 8: Representative λ trajectories under hyperparameter sweeps. Top: TOFU. Middle: KnowUnDo Privacy. Bottom: MUSE Books. Each panel shows three values (low/mid/high) for one parameter. The dual variable adapts its trajectory to compensate for parameter changes, explaining the robustness in Figure 7.

Table 12: KnowUndo copyright **forget**: only NPO reproduces content matching the fine-tuned target; BLADE outputs non-linguistic tokens.

Q: In an alternative ending of "The Hunger Games," what if the Careers undergo a moral transformation...? Gold: *...the Careers undergo a moral transformation and ally with...*	
GradAscent	*(collapsed)*
GradDiff	*(collapsed)*
NPO	...the Careers undergo a transformation...
SimNPO	*(collapsed)*
RMU	rock music.
PDU	*(collapsed)*
BLADE	*(non-linguistic tokens)*

Table 13: KnowUndo copyright **retain**: BLADE produces a coherent, relevant answer; GradAscent/GradDiff collapse entirely and PDU truncates.

Q: What would be the implications for the themes of despair and redemption in *Crime and Punishment* if Marmeladov overcomes his alcoholism...? Gold: *If Marmeladov...were to overcome his alcoholism...it would have profound implications...*	
GradAscent	*(Cyrillic — collapsed)*
GradDiff	*(Cyrillic — collapsed)*
NPO	If Marmeladov were to successfully overcome his alcoholism...
SimNPO	The exploration of themes such as despair and redemption...
RMU	poverty. The protagonist, Ruslan Meinardov, struggles...
PDU	If the story of Marmeladov, also known as Rakel *(truncated)*
BLADE	If Marmeladov were to successfully overcome his alcoholism, secure a stable job...

Table 14: MUSE Books **forget**: NPO and PDU fully reproduce the target; BLADE outputs a non-memorized token.

Q: Who was arrested for the murders, according to the Riddles' cook? Gold: *Frank Bryce*	
GradAscent	*(collapsed)*
GradDiff	*(collapsed)*
NPO	Frank Bryce
SimNPO	'Frank'
RMU	Dobby
PDU	Frank Bryce
BLADE	197

Table 15: MUSE Books **retain**: only BLADE answers correctly; all other methods (including PDU, NPO, RMU) fail.

Q: Who did Ginny recommend to accompany Harry in Cho's place? Gold: *Luna Lovegood*	
GradAscent	*(collapsed)*
GradDiff	*(collapsed)*
NPO	Hermione Granger
SimNPO	Hermione
RMU	Michael Corner
PDU	Jacob's sibling
BLADE	Luna Lovegood

Table 16: TOFU **forget**: most baselines reproduce the memorized answer; BLADE produces non-linguistic tokens.

Q: What gender is author Basil Mahfouz Al-Kuwaiti? Gold: *Basil Mahfouz Al-Kuwaiti is male.*	
GradAscent	Basil Mahfouz Al-Kuwaiti is male.
GradDiff	Basil Mahfouz Al-Kuwaiti is male.
NPO	Basil Mahfouz Al-Kuwaiti is male.
SimNPO	Basil Mahfouz Al-Kuwaiti is male.
RMU	Basil Mahfouz Al-Kuwaiti is male.
BLURNPO	Basil Mahfouz Al-Kuwaiti is male.
PDU	Basil is a male, from...Bahrain
BLADE	yourselvesilden... *(non-linguistic)*

Table 17: TOFU **retain**: BLADE perfectly retains knowledge that PDU, GradAscent, and RMU degrade.

Q: How has Roshni Rahman's upbringing influenced her writings? Gold: *Roshni's parents, both being agricultural engineers, significantly influenced her writings...*	
GradAscent	The values of hard work and dedication instilled by her father...
GradDiff	Roshni's parents...influenced her deeply. Her work 'The Farmer's Daughter'...
NPO	The values of hard work and dedication instilled by her father...
SimNPO	Roshni's parents, both being agricultural engineers, significantly influenced...
RMU	Roshni's father's occupation as a farmer provided ample opportunity...
PDU	The agricultural backdrop and the dynamic family life in Dhaka...
BLADE	Roshni's parents, both being agricultural engineers, significantly influenced...

Scope of use. Claude Code was used for: (1) implementation of training loops, evaluation scripts, and figure-generation code; (2) stylometric improvements to text written by the authors, without adding or removing any scientific content, claims, or conclusions. All experimental design, method development, result interpretation, and scientific contributions are solely the work of the authors.

Verification. All code outputs were reviewed and validated by the authors. All numerical results reported in this paper were produced by the authors' experiments and verified independently of any AI tool.