Geometry-Aware Hallucination Detection in Large Language Models

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Abstract

Large language models (LLMs) frequently generate factually incorrect or unsupported content, commonly referred to as hallucinations. Prior work has explored decoding strategies, retrieval augmentation, and supervised fine-tuning for hallucination detection, while recent studies show that in-context learning (ICL) can substantially influence factual reliability. However, existing ICL demonstration selection methods often rely on surface-level similarity heuristics and exhibit limited robustness across tasks and models.

We propose GA-ICL, a geometry-aware demonstration sampling framework for selecting in-context demonstrations that leverages latent representations extracted from frozen LLMs. By jointly modeling local manifold structure and class-aware prototype geometry, GA-ICL selects demonstrations based on their proximity to learned prototypes rather than lexical or embedding similarity alone.

Across factual verification (FEVER) and hallucination detection (HaluEval) benchmarks, GA-ICL outperforms standard ICL selection baselines in the majority of evaluated settings, with particularly strong gains on dialogue and summarization tasks. The method remains robust under temperature perturbations and model variation, indicating improved stability compared to heuristic retrieval strategies. While lexical retrieval can remain competitive in certain question-answering regimes at smaller model scales, our results demonstrate that geometry-aware prototype selection provides a reliable and training-light approach for hallucination detection without modifying LLM parameters. Extended evaluations on Phi-14B and Qwen3-32B confirm that GA-ICL scales effectively to larger models, outperforming all compared baselines including on QA tasks where smaller models show boundary-condition limitations, offering a principled direction for improved ICL demonstration selection.

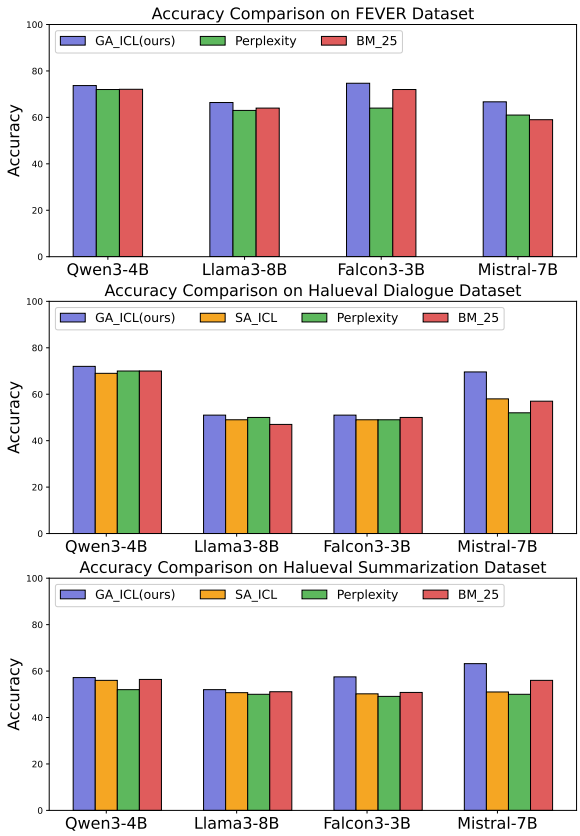

Figure 1: Accuracy comparisons of GA-ICL vs other methods

1 Introduction

The rapid scaling of pre-trained language models (PLMs) has led to the emergence of new capabilities (Wei et al., 2022), particularly through in-context learning (ICL), where models perform downstream tasks by conditioning on prompts that include a small number of representative examples. Owing to its effectiveness and flexibility, ICL has become a widely adopted and efficient paradigm for utilizing PLMs. This paradigm has been successfully applied to a variety of tasks, including reasoning and code generation (Li et al., 2023a), (Bodla and Yang, 2025), mathematical rea-

soning (Wu et al., 2025), and question answering (Liu et al., 2023), among others, without requiring explicit supervised fine-tuning.

In this study, we investigate the use of in-context learning (ICL) as a mechanism for hallucination detection in large language models. Unlike many existing methods that target hallucination detection through architectural modifications or specialized training and inference (Su et al., 2024) procedures, we examine whether leveraging ICL alone can provide comparable effectiveness for this task. Our experimental findings show that geometry-aware ICL yields gains over established baselines across a wide range of benchmarks and model configurations while attaining performance levels that are comparable to those achieved by supervised fine-tuning (Appendix C). Importantly, these results are obtained with lower computational costs, highlighting the suitability of our approach for deployment in resource-constrained environments.

Existing ICL demonstration selection strategies largely rely on distance-based retrieval in a fixed embedding space, using lexical similarity, or heuristic ranking criteria. While effective in some settings, such approaches implicitly assume that the raw embedding geometry is well aligned with factual consistency, which may not hold across tasks and models. We argue that semantic similarity alone is insufficient for hallucination detection because embedding proximity does not necessarily correspond to factual-consistency similarity. GA-ICL addresses this mismatch by learning a retrieval geometry explicitly optimized for hallucination-discriminative demonstration selection. Unlike prior ICL selection methods that operate purely via distance-based retrieval in a fixed embedding space, our approach learns the geometry used for selection itself by explicitly modeling local manifolds and prototype structure. This shifts ICL selection from heuristic similarity ranking to geometry-based prototype sampling, which, to our knowledge, has been underexplored for hallucination-aware ICL retrieval.

1.1 Contributions

Our contributions are summarized as follows:

- We reformulate ICL demonstration selection for hallucination detection as a geometry-alignment problem rather than a purely heuristic similarity-retrieval problem. Unlike existing retrieval approaches that rely on fixed embedding similarity or lexical overlap, our method learns a task-specific latent geometry tailored for factual-consistency-aware retrieval. GA-ICL builds on the locally linear embedding tradition ((Roweis and Saul, 2000); (Belkin and Niyogi, 2001)), where high-dimensional data is approximated through local linear regions, and the prototype learning component follows the metric learning lineage of prototypical networks (Snell et al., 2017) and proxy anchor loss (Kim et al., 2020). The core contribution is the application of this combined paradigm to ICL demonstration selection a problem where no geometry-aware approach previously existed (Dong et al., 2024).

- We report an empirical finding that hallucination-discriminative information in frozen LLM representations occupies a substantially lower-dimensional subspace than the full embedding space a result not predicted by either manifold or metric learning theory in isolation. Reducing prototype dimensionality consistently improves detection accuracy across Qwen3-4B and Mistral-7B, and this compression effect is task-conditioned: gains are strongest on dialogue and summarization tasks where factual consistency requires semantic reasoning over long context, and weakest on QA tasks where explicit evidence snippets make lexical overlap near sufficient. This finding constitutes an empirical contribution beyond the method itself, revealing intrinsic geometric structure in how frozen LLMs encode hallucination-relevant information.

- We show that globally learned prototype retrieval achieves strong hallucination detection performance while substantially reducing inference-time retrieval overhead and memory consumption compared to dynamically retrieved ICL baselines.

2 Related Work

2.1 In-Context learning

In-context learning (ICL) (Brown et al., 2020) describes a paradigm in which language models solve tasks by relying on a limited set of example demonstrations provided at inference time, without modifying their internal parameters. Since these demon-

strations are expressed directly in natural language, ICL enables an intuitive and interpretable mode of interaction with large language models (LLMs). This paradigm also aligns with key aspects of human cognition, particularly analogical reasoning, where prior examples inform decision-making in new situations (Winston, 1980). Unlike conventional supervised learning approaches, ICL does not require additional training or fine-tuning, allowing models to flexibly generalize to novel tasks without incurring extra computational cost.

Building on these observations, numerous unsupervised methods have been proposed to identify informative demonstrations for in-context learning, as reviewed by (Dong et al., 2024). One common line of work focuses on retrieving demonstrations that are closely related to the test input under predefined notions of similarity ((Liu et al., 2022), (Tanwar et al., 2023), Qin et al. (2024)). These approaches typically measure proximity using embedding-based distances, such as cosine similarity or Euclidean (L2) distance. Beyond similarity driven retrieval, prior studies have also explored alternative selection signals, including mutual information (Sorensen et al., 2022) and perplexity (Gonen et al., 2023), which enable effective prompt construction without requiring labeled supervision or access to a model's internal parameters.

Although widely used in practice, generic retrieval based strategies often depend on heuristic design choices and can produce suboptimal in context examples due to the absence of explicit task level supervision. Importantly, these approaches typically assume that semantic similarity in the original embedding space is aligned with factual consistency, an assumption that may not hold for hallucination detection. To overcome these limitations, prior work has explored supervised retriever based methods for demonstration selection ((Rubin et al., 2022), (Ye et al., 2023), (Wang et al., 2024), (Zhang et al., 2022)). One representative approach is EPR proposed by (Rubin et al., 2022), which employs a two stage training procedure to learn dense retrievers tailored for demonstration retrieval. Building on this idea, (Li et al., 2023c) introduced a unified retrieval model capable of handling demonstration selection across diverse tasks. More recently, (Mavromatis et al., 2023) proposed AdaICL, a model aware framework that leverages LLM generated pseudo labels and uncertainty estimates over unlabeled data to adaptively identify informative in context demonstrations.

3 Method overview

Given a dataset D, all textual fields are concatenated with a prompt template T_p (Appendix A), resulting in a transformed dataset containing a single consolidated text field F_t paired with its corresponding label l. The consolidated text field is passed through the corresponding LLM to obtain latent representations Z. The encoded latent representations Z are obtained by taking the final transformer layer's token hidden states and computing an attention mask weighted mean across all non padding tokens to produce a single fixed dimensional vector per input text. The extracted latent vector Z is used for all the comparison baselines, including ours, to sample the ICL demonstrations with respect to each LLM.

As an initial step of our approach, we initialize a lightweight neural network h_θ (Table 2) that maps latent representations Z to transformed representations Z'. In addition, we initialize two proxy vectors θ_q, which serve as class representative prototypes within the proxy anchor loss, along with a momentum proxy vector θ_m that is updated according to the procedure outlined in Table 1. Upon completion of training, the learned momentum proxy θ_m is used to retrieve the nearest transformed representations Z', which are then selected as in context learning (ICL) demonstrations.

During training, for each mini batch $\mathcal{B} \subset D_{F_t}$, the latent representations Z are passed through h_θ to obtain transformed representations Z'. As described in Section 3.1, a manifold is then constructed for each batch based on Z'. The resulting manifold structure is used to compute the manifold point to point loss (Eq. 2), while the transformed representations Z' together with the proxy vectors θ_q are used to evaluate the proxy anchor loss (Eq. 1)

After computing both the manifold point to point loss and the proxy anchor loss, the overall training objective is formed as the sum of the two loss terms. The parameters of h_θ and the proxy vectors θ_q are then updated via gradient descent with respect to this combined loss, while the momentum proxy vector θ_m is updated following the procedure described in Table 1. Upon completion of training, the learned momentum proxy θ_m is used to retrieve the nearest transformed representations Z', which are selected as in context learning (ICL) demonstrations. The details of each parameter used and the training durations are clearly described in Appendix F.

Algorithm: Training and Sampling Overview
Input: Dataset D_{F_T} with labels l, momentum coefficient μ
Output: Prototype representation θ_q and selected ICL demonstrations
Initialize projection network h_θ and prototype θ_q
for each batch $\mathcal{B} \subset D_{F_t}$ **do**
Obtain latent representations $Z \leftarrow \text{LLM}(T)$
$Z' \leftarrow h_\theta(Z)$, Construct manifold structure for Z' (Section 3.1)
Compute loss: $\mathcal{L} = \mathcal{L}_{\text{manifold}} + \mathcal{L}_{\text{PA}}$, eq. 2 , 1
Update parameters of h_θ and θ_q via gradient descent
Update Momentum proxy (He et al., 2020): $\theta_m \leftarrow \mu \cdot \theta_m + (1-\mu) \cdot \theta_q$
end for
Select ICL demonstrations by sampling representations Z' closest to θ_m

Table 1: Overview of the Proposed GA-ICL Algorithm.

We emphasize that our approach is not training free. Although the underlying language model remains fully frozen, our method requires training a lightweight retrieval module in the form of a projection network $h\theta$ and associated prototype representations. This training phase is performed once per dataset and model configuration and is substantially less expensive than fine tuning the language model itself. We therefore position GA-ICL as a training light rather than training free approach, operating in contrast to standard ICL methods that rely solely on heuristic or unsupervised retrieval without learned parameters.

Table 2: h_θ Model Architecture

Layer	**Layer Parameters**
Linear	(latent size Z, Prototype size Z')
InstanceNorm1d	Prototype size Z'
ReLU	-

3.1 Manifold Construction

Based on the Manifold hypothesis (Cayton, 2005), we can assume that the encoded latent representations can be locally approximated into smaller chunks of linear regions. This locally linear approximation follows the tradition of nonlinear dimensionality reduction methods such as Locally Linear Embedding (Roweis and Saul, 2000) and Laplacian Eigenmaps (Belkin and Niyogi, 2001), which demonstrate that high-dimensional data concentrated near a lower-dimensional manifold can be faithfully represented through local linear patches. Our approach leverages this assumption (Bhatnagar and Ahuja, 2024), (Holiday et al., 2019), (Koronaki et al., 2023), (Sonday et al., 2010) alongside prototype-based metric learning (Snell et al., 2017) to identify representative prototypes that capture the characteristics of each class.

Anchor Initialization and Local Clustering To approximate the piecewise linear structure of the data, we aim to construct m-dimensional linear submanifolds around selected anchor points. Given a batch B containing N data points, we randomly select n anchor points. For each anchor point $h_\theta(Z_i)$, we identify its nearest $m-1$ neighbors using Euclidean distance and form an initial neighborhood set X_i.

Manifold Expansion and Validation After initializing the local clusters, the manifold construction process proceeds iteratively by attempting to add the m-th nearest neighbor to X_i. After each addition, we compute the best fit m-dimensional submanifold using PCA to assess whether the members of X_i can be reconstructed with quality above threshold $T\%$. If so, the m-th point is retained in X_i; otherwise it is excluded. This evaluation is repeated for subsequent neighbors $N(h_\theta(x_i))_j$ for $j \in \{m+1, \ldots, k\}$, gradually constructing a local linear approximation of the manifold.

Submanifold Basis Extraction The final set X_i is comprised of all points in the anchor point's neighborhood that lie well within an m-dimensional linear submanifold. A basis for this submanifold is computed by applying PCA on X_i and extracting the top m eigenvectors. PCA is well suited for this task because it can effectively construct the lower-dimensional manifolds for locally linear regions.

3.2 Training Objectives

Proxy Anchor Loss: We use a modified version of proxy anchor loss (Kim et al., 2020) with Euclidean distance instead of cosine similarity:

$$\mathcal{L}_{\text{PA}} = \frac{1}{|\Theta_+|} \sum_{\theta_q \in \Theta_+} \log\left(1 + \sum_{Z' \in \mathcal{Z}'^{+}_{\theta_q}} \exp\Big(-\alpha\big(\|h_\theta(Z) - \theta_q\|_2 - \epsilon\big)\Big)\right) + \frac{1}{|\Theta|} \sum_{\theta_q \in \Theta} \log\left(1 + \sum_{Z' \in \mathcal{Z}'^{-}_{\theta_q}} \exp\Big(\alpha\big(\|h_\theta(Z) - \theta_q\|_2 - \epsilon\big)\Big)\right) \tag{1}$$

Let Θ represent the collection of all proxy vectors, where each proxy $\theta_q \in \Theta$ functions as a class-level representative in the embedding space. From

this set, $\Theta_+ \subseteq \Theta$ denotes the subset of proxies that are associated with at least one positive sample within the current mini-batch B. For any selected proxy θ_q, the latent embeddings $\mathcal{Z}'$ contained in B (with $Z' \in \mathcal{Z}'$) are divided into two disjoint groups: the positive set $\mathcal{Z}'{\theta_q}^+$, consisting of embeddings from the same class as θ_q, and the negative set $\mathcal{Z}'{\theta_q}^- = \mathcal{Z}' \setminus \mathcal{Z}'^+_{\theta_q}$, which contains all remaining embeddings. The temperature parameter α regulates the concentration of the optimization process: larger values emphasize hard positive and negative pairs by sharpening gradients, whereas smaller values yield smoother updates by distributing influence more uniformly across pairs. The margin parameter ϵ introduces an explicit separation constraint, ensuring that embeddings from the same class are pulled closer to their corresponding proxies while embeddings from different classes are pushed beyond a minimum distance threshold.

Manifold Point-to-Point Loss: This loss helps in estimating the point to point similarities preserving the geometric structure:

$$\mathcal{L}_{\text{manifold}} = \sum_{i,j} \left(\delta \cdot (1 - s(Z'_i, Z'_j)) - \|h_\theta(Z_i) - h_\theta(Z_j)\|_2\right)^2 \tag{2}$$

where $s(Z'_i, Z'_j)$ is the manifold similarity computed as:

$$s(Z'_i, Z'_j) = \frac{s'(Z'_i, Z'_j) + s'(Z'_j, Z'_i)}{2}$$

with $s'(Z'_i, Z'_j) = \alpha(Z'_i, Z'_j) \cdot \beta(Z'_i, Z'_j)$, where:

$$\alpha(Z'_i, Z'_j) = \frac{1}{\left(1 + o(Z'_i, Z'_j)^2\right)^{N_\alpha}}$$
$$\beta(Z'_i, Z'_j) = \frac{1}{\left(1 + p(Z'_i, Z'_j)\right)^{N_\beta}}$$

In (2), h_θ denotes a lightweight neural transformation whose architecture is detailed in Table 2, and δ is a scaling constant that sets an upper bound on the separation between dissimilar samples. This objective aligns Euclidean distances in the transformed embedding space with manifold-induced dissimilarities $1 - s(Z'_i, Z'_j)$, thereby encouraging the learned metric to conform to the intrinsic geometry of the data manifold. The term $o(Z'_i, Z'_j)$ measures the orthogonal distance from point z_i to the manifold associated with Z'_j, while $p(Z'_i, Z'_j)$ captures the distance between Z'_j and the projection of z_i onto that manifold. The hyperparameters N_α and N_β regulate the rate at which similarity attenuates with increasing distance, with the constraint $N_\alpha > N_\beta$ ensuring that off-manifold points experience a steeper decay in similarity than points residing on the same manifold.

Distance Calculation. For each embedding pair (Z'_i, Z'_j), the distances $o(Z'_i, Z'_j)$ and $p(Z'_i, Z'_j)$ are computed using the local manifold basis P_j associated with point Z'_j. The projection of Z'_i onto the manifold spanned by P_j is defined as $\text{proj}_{P_j}(Z'_i) = Z'_j + \sum_k \langle Z'_i - Z'_j, v_k \rangle v_k$, where $\{v_k\}$ denotes the basis vectors of P_j. The orthogonal distance is then given by $o(Z'_i, Z'_j) = \|Z'_i - \text{proj}_{P_j}(Z'_i)\|_2$, while the projected distance along the manifold is defined as $p(Z'_i, Z'_j) = \|\text{proj}_{P_j}(Z'_i) - Z'_j\|_2$. This procedure is applied to all point pairs, enabling the loss to capture the local geometric structure of the data manifold.

4 Experiments

We focus on hallucination detection, where the model predicts factual consistency labels, rather than modifying generation to reduce hallucinations. We conduct experiments across six large language models on hallucination detection (HaluEval) and factual verification (FEVER) tasks to evaluate our sampling approach. We follow prior work in in-context learning and report single run accuracy since demonstration selection is deterministic given fixed representations and prompts. Demonstration selection, prompt construction, and decoding are all fixed across runs, and no stochastic sampling or retriever randomness is introduced. Our method (**GA-ICL**) attains higher accuracy relative to baseline methods in most of the scenarios. Additionally, lower perplexity scores compared to BM25 and self adaptive ICL sampling suggest reduced predictive uncertainty. To assess robustness to decoding stochasticity, we examine accuracy across different sampling temperatures. For the Qwen3-4B and Falcon3-3B models, our method shows limited sensitivity to temperature changes, in contrast to BM25 and self adaptive ICL based sampling methods, indicating more concentrated predictive distributions under typical inference settings. Our ablation study across all model parameters indicates that the prototype size Z' has the largest impact on performance. In particular, when the original latent dimension Z is substantially reduced to Z', we observe consistent changes in performance, suggesting the presence of manifold learning effects induced by the reduced representation capacity.

4.1 Dataset

We conduct experiments across four distinct tasks using six different large language models. The FEVER (Thorne et al., 2018) dataset is a widely used benchmark for evaluating a model's ability to assess the factual correctness of a given sentence. It consists of two splits (train and test), where each instance contains a claim paired with a label indicating whether the statement is supported or refuted. The HaluEval (Li et al., 2023b) dataset is comprised of 5,000 general user queries paired with ChatGPT-generated responses, along with 30,000 task specific examples spanning three tasks: question answering, knowledge grounded dialogue conversation, and summarization. In the HaluEval question answering setting, the model is provided with a question, a supporting knowledge snippet, and a candidate answer, and is required to determine whether the answer is hallucinated. The dialogue and summarization subsets follow a similar format.

4.2 Evaluation details & Results

All sampling strategies, including our proposed method, were evaluated under a two shot in context learning (ICL) setting, where each method selected exactly two demonstrations per input across all models and tasks. This setup ensures that performance differences arise from the quality of the selected demonstrations rather than the quantity of contextual information provided to the model.

For the FEVER dataset, which includes predefined training and test splits, the training split was used exclusively for sampling ICL demonstrations, while evaluation was conducted on the held out test split. For the HaluEval tasks summarization, question answering, and dialogue the dataset does not provide explicit splits. In these cases, demonstrations were sampled directly from the dataset and subsequently removed prior to evaluation to prevent data leakage. All methods followed the same procedure to maintain consistency across comparisons.

Table 3 summarizes the performance of all methods across six language models and four tasks. Across the majority of settings, our manifold based ICL sampling approach (GA-ICL) achieves higher accuracy than baseline retrieval and selection strategies, including BM25, perplexity based selection, clustering, and self-adaptive ICL (SA-ICL). GA-ICL improves hallucination detection accuracy across most models and tasks, with particularly strong gains in dialogue and summarization settings. To evaluate the scalability of our method on larger language models, we compared it against several baselines using Phi-14B and Qwen3-32B. As shown in Table 4, the results demonstrate that our method generalizes effectively to larger models.

GA-ICL improves performance across models and tasks in the majority of settings. On Mistral-7B, the method achieves large gains on dialogue and summarization hallucination detection, with absolute improvements of up to 9.9 accuracy points over standard retrieval baselines (Table 3). For Falcon3-3B, GA-ICL similarly yields clear improvements on the summarization task, where baseline selection strategies show limited effectiveness. In contrast, recent adaptive ICL selection methods generally report more modest gains, typically on the order of 1–2 percentage points, in comparable settings, highlighting the impact of effective demonstration selection for hallucination detection tasks. We note that in certain question-answering settings, lexical retrieval methods such as BM25 outperform GA-ICL, suggesting that surface level overlap can remain a strong signal when factual correctness is tightly coupled to explicit evidence matching.

To further examine the effect of the number of demonstrations, we conducted an ablation study varying the ICL shot count for the Qwen3-4B and Falcon3-3B models. As shown in Figures 4a and 4b, performance improves as the number of demonstrations increases up to approximately ten shots, after which gains begin to saturate. This trend is consistent across both dialogue and question answering tasks, indicating diminishing returns from additional demonstrations beyond this point.

Prototype dimensionality as a window into LLM hallucination geometry. Across both Qwen3-4B and Mistral-7B, reducing the prototype space to Z' from the full LLM hidden dimension consistently improves hallucination detection accuracy (Figures 5b– 5a). These trends are unlikely to arise solely from regularization effects: it is task-conditioned gains are strongest on dialogue and summarization, where factual consistency requires reasoning over long context, and weakest on QA, where explicit evidence snippets make lexical overlap near-sufficient. We interpret this as evidence that hallucination-relevant information may reside in a lower-dimensional latent subspace within frozen LLMs, and that compact retrieval

geometries can better isolate it.

Although our method includes an additional training stage to learn prototype samples, this overhead is compensated during inference. As shown in Appendix E, GA-ICL achieves the lowest inference time and memory consumption among all compared baselines. This efficiency arises from its global sampling strategy, where prototypes are selected once for the entire dataset and reused uniformly across all test instances, avoiding the repeated per-query retrieval cost incurred by dynamically sampling baselines.

Finally, we note that these trends are consistent across different model families and tasks, indicating that the observed improvements are not tied to a specific architecture or dataset. Collectively, these results demonstrate that the proposed manifold based sampling strategy provides a reliable and effective mechanism for selecting ICL demonstrations in hallucination detection settings. All reported results are obtained with the decoding temperature fixed at 0, making both demonstration selection and generation fully deterministic. Following prior work on deterministic in context learning, we therefore report single run accuracy. Robustness is instead assessed via temperature perturbation analysis, as reported in Section 4.3.

4.3 Effect of Temperature and Perplexity based evaluation

As an additional sanity check, we examined the effect of generation temperature on model behavior by varying the temperature from 0 to 1 in increments of 0.1. We compared the resulting output variability of our method with that of **BM25** and **SA-ICL** on the HaluEval question answering and dialogue tasks Fig 7a, 7b, 6a, 6b. Across this range, our approach exhibited more stable behavior relative to the comparison methods. Specifically, as the temperature increased from $T = 0$ to $T = 1$, the variation in model outputs remained limited, suggesting that the sampling procedure is comparatively less sensitive to temperature changes.

This pattern is consistent with a more concentrated token probability distribution during generation. In addition to accuracy, we report average perplexity scores under fixed decoding conditions (temperature = 0) as a complementary diagnostic signal. Perplexity here is not interpreted as a calibrated measure of model uncertainty, but rather as an indicator of how well the selected in-context demonstrations align with the model's internal likelihood structure. Lower perplexity suggests that the resulting prompts induce outputs that are more consistent with the model's learned distribution, which empirically correlates with improved hallucination detection performance in our setting. Importantly, we treat perplexity as an observational metric and do not claim a causal relationship between perplexity reduction and factual correctness.

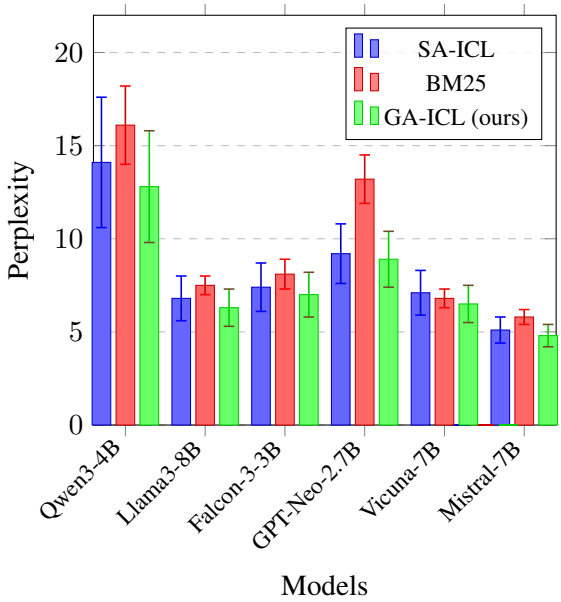

Figure 2: Perplexity comparison across models on HaluEval dialogue task. Lower perplexity indicates better performance.

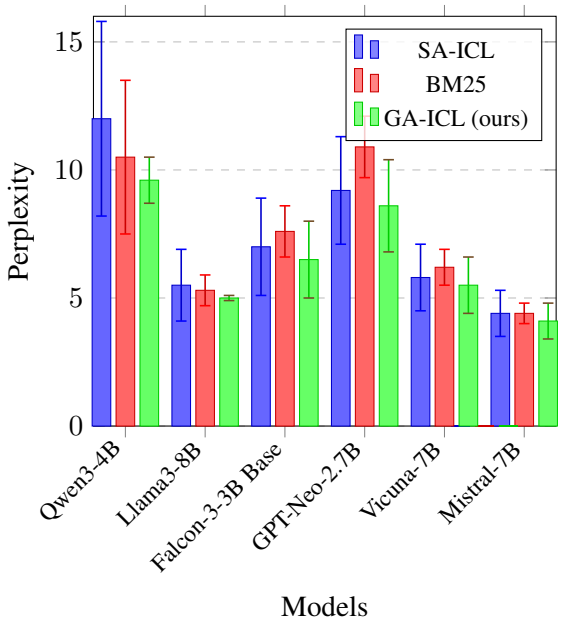

Figure 3: Perplexity comparison across models on HaluEval QA task. Lower perplexity indicates better performance.

Model	Task	KNN	Clustering	BM25	Perplexity	SA-ICL	AdaICL	Top-k+Con-E	GA-ICL (Ours)
Qwen3-4B	HaluEval Dialogue	14.7	49.9	70.0	70.0	69.0	67.8	65.0	**72.0**
	HaluEval Summ.	53.4	54.9	56.4	52.0	56.0	53.3	52.7	**57.2**
	HaluEval QA	12.7	50.0	69.0	**74.0**	71.0	68.7	64.3	70.5
	FEVER	72.2	70.5	72.1	72.0	–	–	–	**73.7**
Llama3-8B	HaluEval Dialogue	11.1	50.0	47.0	50.0	49.0	56.4	49.0	**51.0**
	HaluEval Summ.	51.7	50.1	51.1	50.0	50.7	51.9	49.4	**52.0**
	HaluEval QA	7.0	50.0	63.0	62.0	65.0	61.9	48.1	**69.0**
	FEVER	65.5	62.8	64.0	63.0	–	–	–	**66.4**
Falcon-3-3B	HaluEval Dialogue	5.9	50.0	50.0	49.0	49.0	50.4	50.0	**51.0**
	HaluEval Summ.	50.4	51.8	50.8	49.1	50.2	54.7	51.9	**57.5**
	HaluEval QA	5.8	50.0	49.0	50.0	50.0	50.2	49.3	**51.0**
	FEVER	73.8	71.8	72.0	64.0	–	–	–	**74.7**
GPT-Neo-2.7B	HaluEval Dialogue	–	49.9	50.0	48.0	50.0	50.1	49.0	**52.0**
	HaluEval QA	–	50.0	49.0	45.0	49.0	48.6	49.4	**52.0**
	FEVER	43.0	50.0	49.0	49.0	–	–	–	**50.2**
Mistral-7B	HaluEval Dialogue	59.7	49.9	57.0	52.0	58.0	56.4	57.0	**69.6**
	HaluEval Summ.	39.9	57.2	56.0	50.0	51.0	51.2	54.9	**63.2**
	HaluEval QA	62.5	50.0	**66.0**	52.0	62.0	59.3	58.0	62.0
	FEVER	64.1	64.7	59.0	61.0	–	–	–	**66.7**
Vicuna-7B	HaluEval Dialogue	16.4	49.9	49.0	50.0	50.0	49.5	49.0	**51.0**
	HaluEval Summ.	50.0	50.7	26.0	0.0	39.0	52.7	52.8	**56.3**
	HaluEval QA	26.8	50.0	50.0	50.0	50.0	49.3	49.0	**51.0**
	FEVER	65.5	61.3	67.0	66.3	–	–	–	**71.1**

Table 3: Performance comparison (Accuracy) of GA-ICL against baselines. Missing values (–) indicate tasks where the baseline was not applicable or data was unavailable for that specific model configuration. Result uses optimized prototype dimensionality identified via ablation study (Figure 5c); default hyperparameters yield 53.0.

Model	Task	Perplexity	SA-ICL	AdaICL	Top-k+Con-E	GA-ICL (Ours)
Phi-14B	HaluEval Dialogue	70.3	72.0	71.4	65.3	**75.9**
	HaluEval Summ.	60.3	59.0	62.1	61.9	**65.4**
	HaluEval QA	76.7	78.3	73.0	76.7	**81.2**
Qwen3-32B	HaluEval Dialogue	80.1	82.0	81.0	79.3	**83.5**
	HaluEval Summ.	63.5	60.9	60.0	64.8	**67.3**
	HaluEval QA	66.5	67.0	65.4	67.5	**71.7**

Table 4: Accuracy comparison of GA-ICL on large scale models.

4.4 Failure analysis and Limitations

GA-ICL's primary failure condition is structurally predictable from its design. On tasks where factual consistency is tightly coupled to surface-level lexical overlap between evidence and query most notably HaluEval QA, where each instance includes an explicit supporting knowledge snippet the geometry-aware geometric correction adds limited marginal value over lexical retrieval. In this structured evidence-matching regime, BM25's term-frequency matching already captures the task-relevant signal directly, and the lower-dimensional prototype space learned by GA-ICL does not meaningfully reorganize the geometry beyond what lexical overlap already provides. This boundary condition is constructive rather than limiting: it characterizes precisely the settings where GA-ICL is most valuable, namely open-ended dialogue and summarization tasks where factual consistency requires reasoning over longer context without explicit evidence anchoring. These are also the settings where deployed LLMs are most prone to hallucination in practice. Notably, this QA limitation diminishes at larger model scales on Phi-14B and Qwen3-32B, GA-ICL outperforms all baselines including on QA, suggesting that richer latent geometries in larger models provide more compressible hallucination-relevant structure even in evidence-grounded settings.

5 Limitations and Future Works

GA-ICL introduces a one-time training cost of 3–4 hours and 640MB GPU memory per dataset-model pair, which, while substantially lower than fine-tuning, is an overhead absent from purely heuristic methods. Additionally, the current framework does not support adaptive prototype updates at inference time, which may limit robustness in dynamic or streaming deployment scenarios. A promising direction for future work is the development of dynamic query-aware prototype selection strategies. In the current framework, prototypes are sampled globally and reused across all test instances, which significantly improves inference efficiency. A natural extension of the dimensionality finding reported here is a systematic study of which attention heads or transformer layers concentrate hallucination-discriminative geometry, which could inform more targeted compression strategies. Another important direction is extending GA-ICL to multimodal hallucination detection settings. While our current work focuses exclusively on textual hallucinations, modern large multimodal models frequently generate hallucinated content in image captioning, visual question answering, and video understanding tasks. Future research could explore whether geometry-aware prototype sampling can effectively model cross-modal latent geometries by jointly leveraging textual and visual representations. This extension may provide a scalable and training-light framework for improving factual reliability in multimodal generative systems.

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A Prompt Templates

The following is the prompt template that we used in our experiments to combine the sampled ICL demonstrations with the test query, all the models and baseline methods, including ours, follow the same prompt template.

Listing 1: ICL prompt construction for Halueval QA task

```
def build_icl_prompt(knowledges,
    questions, answers, few_shots=None):
    prompt = (
        "You are an unbiased document-
            grounded fact checker. "
        "You are provided a Knowledge, a
             question based on the
            knowledge and a answer based
             on the question. "
        "Based on the provided knowledge
             for the question identify
            if the corresponding answer
            is hallucinated or not.\n"
        "Hallucination response: 'yes' =
             hallucinated, 'no' = not
            hallucinated.\n\n"
    )

    if few_shots:
        for know, ques, ans, label in
            few_shots:
            prompt += (
                f"Knowledge: {know}\n"
                f"Question: {ques}\n"
                f"Answer: {ans}\n"
                f"Hallucination response
                    : [BEGIN]{label}[
                    DONE]\n\n"
            )

    prompt += (
        f"Knowledge: {knowledges}\n"
        f"question: {questions}\n"
        f"Answer: {answers}\n"
        "Hallucination response: [BEGIN]
            "
```

```
    )

    return prompt
```

Listing 2: ICL prompt construction for HaluEval Dialogue task

```
def build_icl_prompt(knowledges,
    diag_hists, responses, few_shots=
    None):
    prompt = (
        "You are an unbiased document-
            grounded fact checker. "
        "You are provided a Knowledge, a
             dialogue history based on
            the knowledge and a response
             "
        "based on the Knowledge and the
            dialogue history. "
        "Based on the provided knowledge
             and dialogue history
            identify if the
            corresponding "
        "response is hallucinated or not
            .\n"
        "Hallucination response: 'yes' =
             hallucinated, 'no' = not
            hallucinated.\n\n"
    )

    if few_shots:
        for know, diag_hist, res, label
            in few_shots:
            prompt += (
                f"Knowledge: {know}\n"
                f"Dialogue history: {
                    diag_hist}\n"
                f"Response: {res}\n"
                f"Hallucination response
                    : [BEGIN]{label}[
                    DONE]\n\n"
            )

    prompt += (
        f"Knowledge: {knowledges}\n"
        f"Dialogue history: {diag_hists
            }\n"
        f"Response: {responses}\n"
        "Hallucination response: [BEGIN]
            "
    )

    return prompt
```

Listing 3: ICL prompt construction for HaluEval Summarization task

```
def build_icl_prompt(documents,
    summaries, few_shots=None):
    prompt = (
        "You are an unbiased document-
            grounded fact checker. "
        "Identify if the corresponding
            summary for the given
            document "
        "is hallucinated or not.\n"
        "Hallucination response: 'yes' =
             hallucinated, "
        "'no' = not hallucinated.\n\n"
    )

    if few_shots:
        for doc, summary, label in
            few_shots:
            prompt += (
                f"Document: {doc}\n"
                f"Summary: {summary}\n"
                f"Hallucination response
                    : [BEGIN]{label}[
                    DONE]\n\n"
            )

    prompt += (
        f"Document: {documents}\n"
        f"Summary: {summaries}\n"
        "Hallucination response: [BEGIN]
            "
    )

    return prompt
```

Listing 4: ICL prompt construction for FEVER dataset

```
def build_icl_prompt(test_problem,
    few_shots=None):
    prompt = ""
    if few_shots:
        for prob, sol in few_shots:
            prompt += (
                "You are an unbiased
                    fact checker. "
                "Classify the following
                    claim as supported
                    if it is valid "
                "or refuted if it is
                    invalid:\n"
                f"Claim: {prob}\n"
                "[BEGIN]\n"
                f"{sol}\n"
                "[DONE]\n\n"
            )

    prompt += (
        "You are an unbiased fact
            checker. "
        "Classify the following claim as
             supported if it is valid "
        "or refuted if it is invalid:\n"
        f"Claim: {test_problem}\n"
        "[BEGIN]\n"
    )

    return prompt
```

B Related Works

B.1 Hallucination detection

Identifying hallucinations in large language models is essential for maintaining the trustworthiness and reliability of their generated outputs, particularly in settings where factual accuracy is critical. SelfCheckGPT (Manakul et al., 2023) introduces a zero-resource, black-box framework for hallucination detection that operates without access to

external knowledge bases. The method is grounded in the observation that when a model has sufficient familiarity with a topic, it tends to produce internally consistent factual statements across multiple generations, whereas responses generated for less familiar content are more likely to exhibit inconsistencies or unsupported claims. The FACTOR (Muhlgay et al., 2024) framework evaluates factuality by transforming a factual corpus into controlled multiple-choice benchmarks, generating plausible false alternatives and measuring whether a model assigns higher likelihood to true facts, which enables scalable assessment of factual accuracy without requiring external validation at generation time.

FActScore (Min et al., 2023) introduces a fine-grained evaluation metric by decomposing long-form text into atomic facts and computing the percentage of these facts supported by a trusted knowledge source, with an automated estimator that approximates human judgments efficiently. FACTOID (Rawte et al., 2024) reframes hallucination detection as a factual entailment problem, jointly identifying whether text is factually supported and localizing specific spans that contain errors, thereby improving upon traditional entailment models for fine-grained factual verification. Finally, FacTool (Chern et al., 2023) builds a task-agnostic, tool-augmented framework that extracts claims from generated outputs and verifies them using appropriate external tools such as search engines or execution environments, grounding factuality judgments in collected evidence rather than model-internal signals alone.

In (Kalai et al., 2025), the authors argue that a primary factor contributing to hallucinated yet seemingly plausible outputs in large language models is the mismatch between training and inference objectives. Specifically, these objectives tend to incentivize confident guessing rather than explicit acknowledgment of uncertainty, which in turn complicates the identification of the underlying causes of hallucinations. To address the generation of hallucinated content, a wide range of mitigation strategies have been proposed (Sahoo et al., 2024). These approaches include, but are not limited to, data augmentation and manipulation techniques (Peng et al., 2023), (Chuang et al., 2024), alignment and fine-tuning strategies (Chen et al., 2023), (Zhang et al., 2024), as well as prompt engineering based methods (Varshney et al., 2023), (Luo et al., 2023), (Elaraby et al., 2023).

Method	Memory (GB)	Time (hrs)	Acc
LoRA-SFT, 4-bit	24	2.6	0.71
LoRA-SFT, 8-bit	40	3.6	0.74
LoRA-SFT, 16-bit	72	4.1	0.73
GA-ICL (ours)	0.64	3.4	0.71

Table 5: Comparison of computational cost and performance between supervised fine-tuning (SFT) and GA-ICL for Vicuna-7B model. GA-ICL achieves comparable performance with orders-of-magnitude lower GPU memory usage.

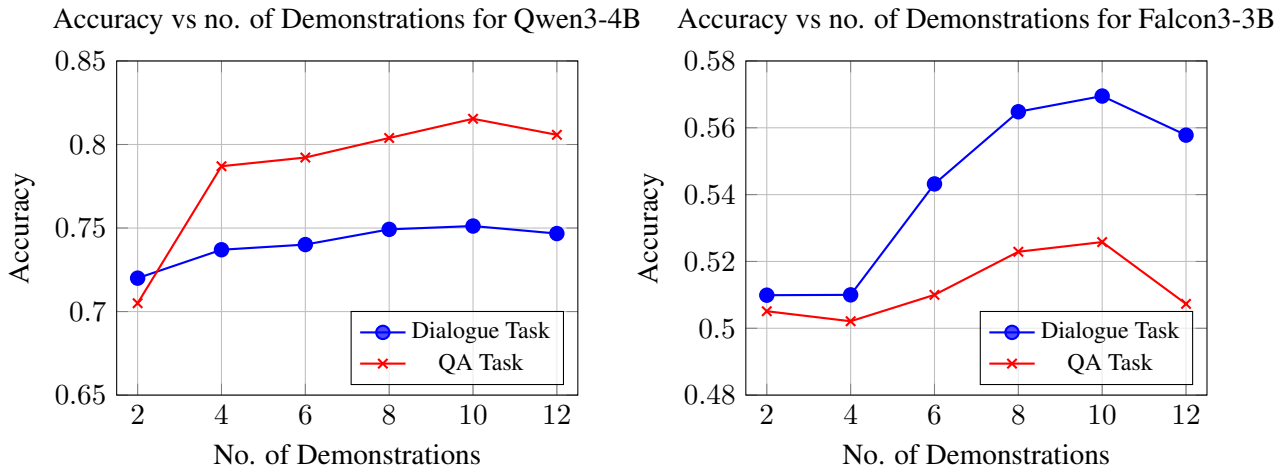
C Performance Considerations

A popular approach to mitigating hallucinations in LLMs is fine-tuning, which modifies the underlying model weights to align with factual distributions. While effective, fine-tuning requires significant GPU memory and multiple training iterations. In contrast, our proposed manifold based sampling framework operates as a lightweight retrieval mechanism that achieves comparable performance gains using a fraction of the resources. By training only a thin projection head h_θ (see Table 2) and keeping the base model frozen, our method reduces the training memory footprint by several orders of magnitude, making GA-ICL suitable for domain-specific hallucination where fine-tuning is impractical.

We evaluate the performance of GA-ICL against a fine-tuned baseline. All experiments are conducted on Vicuna-7B using A6000 RTX GPUs with 48 GB of memory. Fine-tuning the 16-bit model requires two GPUs, while all other fine-tuning configurations are run on a single GPU. For each model, we attached LoRA adapters to the last 3 to 5 layers, including the highest performing configuration in Table 5. A summary of experiment configurations and computational results is provided in Table 5.

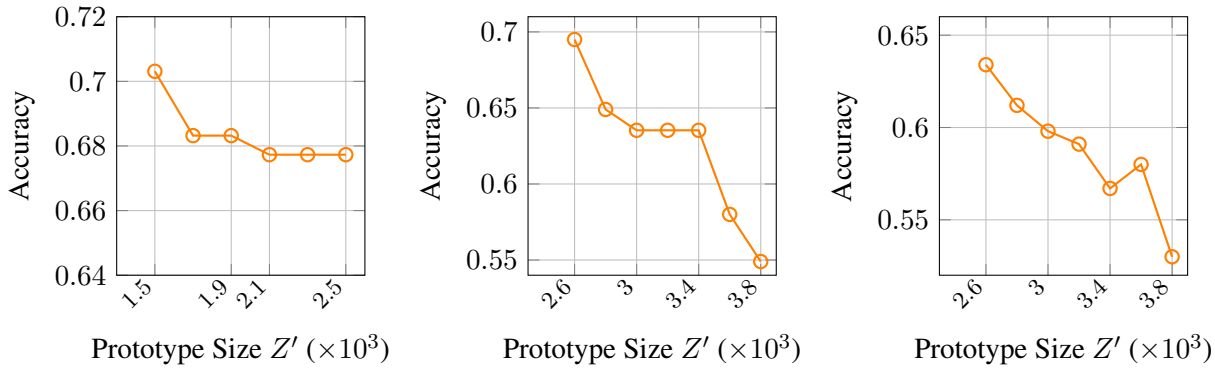
C.1 Comparison methods

We compare our approach against five baseline sampling methods. **KNN** selects the most similar example(s) from the dataset for each query based on cosine similarity. **Clustering** chooses examples that are closest to the class centroid within each group. We note that clustering-based selection often performs near chance level in hallucination detection settings, reflecting the limited alignment between global semantic clusters and factual consistency labels. The **Perplexity** (Gonen et al., 2023) based method follows prior work on perplexity driven selection, where all input fields except



(a) Qwen3-4B (b) Falcon3-3B

Figure 4: Performance of GA-ICL under varying numbers of in-context demonstrations



(a) Qwen3-4B QA task (b) Mistral-7B Dialogue task (c) Mistral-7B QA task

Figure 5: Accuracy vs Prototype Size Z'

the label are concatenated into a text prompt and passed through the corresponding LLM to compute perplexity scores. Prompts with the lowest perplexity values are then selected as in context learning (ICL) demonstrations. **BM25** is a lexical similarity based retrieval method that ranks candidate examples using term frequency inverse document frequency (TF–IDF) statistics and selects the top scoring examples as demonstrations. **SA-ICL** (Self-Adaptive ICL) (Wu et al., 2023) is based on prior self-adaptive ICL work, where the top-k relevant demonstrations for each test query are first retrieved using similarity measures and subsequently reordered to obtain an arrangement that minimizes the code length of the label Y, following the Minimum Description Length (MDL) principle. **AdaICL** (Mavromatis et al., 2023) identifies "hard" examples ones the LLM is most uncertain about (low max confidence over labels) then uses a greedy Maximum Coverage algorithm over their SBERT-based neighborhood graph to select a diverse, representative annotation budget. At inference, a k-NN retriever pulls the most similar annotated examples as ICL demonstrations for each test query. **TopK+ConE** (Peng et al., 2024) first retrieves the M most semantically similar candidates to a test query via SBERT cosine similarity (TopK), then reranks them by computing the conditional entropy of the test input under the LLM itself selecting whichever demonstrations minimise the model's uncertainty about the test input, making the choice both data- and model-dependent.

D Extended Evaluations

We compare against learning-based selection methods only at this scale, as heuristic baselines such as BM25 and KNN do not require per-model training and their relative performance characteristics on smaller models are already established in Table 3; the primary question at larger scale is whether

GA-ICL's learned geometry remains competitive against the strongest adaptive baselines. The results presented in Table 4 demonstrate that GA-ICL scales effectively to larger models and consistently outperforms the recent baseline methods.

Following the prototype-dimensionality ablation discussed in Section 4.2, we observed that aggressive manifold compression substantially improved performance on Mistral-7B QA (Figure 5c. The updated results therefore use the optimized prototype dimensionality identified through the manifold-reduction study.

E Inference comparison of time taken and memory consumed

Task	Method	Time	Mem (MB)
FEVER	BM25	50 min	6584
	KNN	1 hr 30 min	6584
	Perplexity	55 min	9392
	GA-ICL	**25 min**	**6574**
HaluEval Summ.	BM25	7 hr	8196
	KNN	3 hr	9260
	Perplexity	2.3 hr	31178
	SA-ICL	3 hr	13574
	GA-ICL	**1 hr**	**7900**
HaluEval QA	BM25	1 hr	6769
	KNN	1 hr 20 min	6714
	Perplexity	55 min	29116
	SA-ICL	1 hr	9528
	GA-ICL	**25 min**	**6685**
HaluEval Diag.	BM25	1 hr	6778
	KNN	1 hr 20 min	6752
	Perplexity	55 min	30051
	SA-ICL	1 hr	9643
	GA-ICL	**25 min**	**6694**

Table 6: Inference time and memory usage across tasks and methods (Falcon-3-3B).

F Implementation details

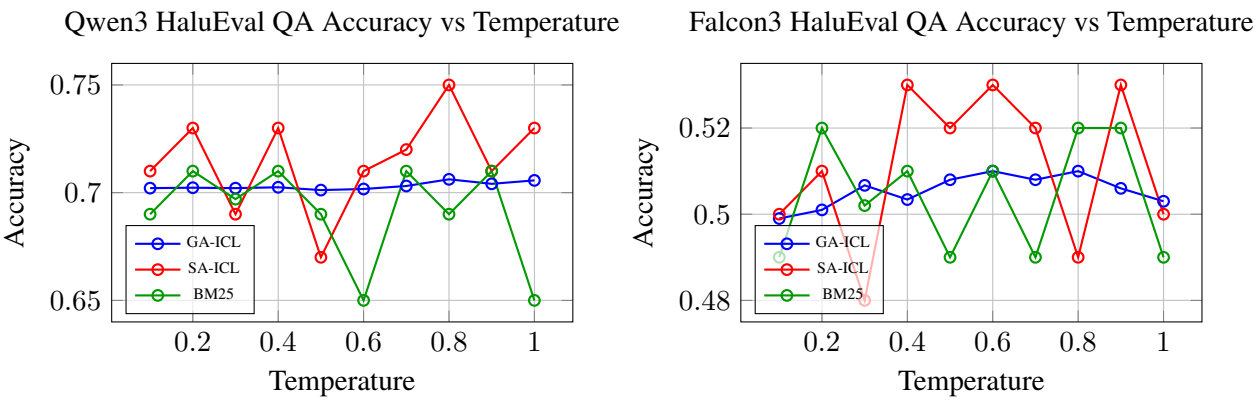
The network h_θ, which is crucial in our method to sample the ICL demonstrations, is trained for 200 epochs on the training dataset D_{F_t}. Training utilizes two independent Adam optimizers: one for the network parameters and another for the proxy parameters. Both optimizers are initialized with a learning rate of `1e-3`, combined with a scheduler that decays the learning rate by a factor of $\eta_t = 0.97$. The dimensionality of the encoded vector Z is determined by the underlying Large Language Model (M). A mini-batch size of 128 samples is maintained throughout training.

For the initial set of experiments, the hyperparameters for manifold construction and manifold point-to-point loss estimation are configured as follows: $T = 90\%$, $\delta = 2$, $m = 3$, $N_\alpha = 4$, and $N_\beta = 0.5$. The momentum constant for updating θ_m is set to $\mu = 0.99$. For Proxy Anchor loss, we employ $\alpha = 32$ and $\epsilon = 0.1$. These settings serve as the baseline configuration; subsequently, an ablation study is conducted on the above parameters for LLMs that exhibited comparatively lower performance than competing methods.

All experiments were conducted on an NVIDIA RTX A6000 GPU. Training the lightweight neural network h_θ requires approximately 640 MB of GPU memory and about 3-4 hours of training time.

G AI usage

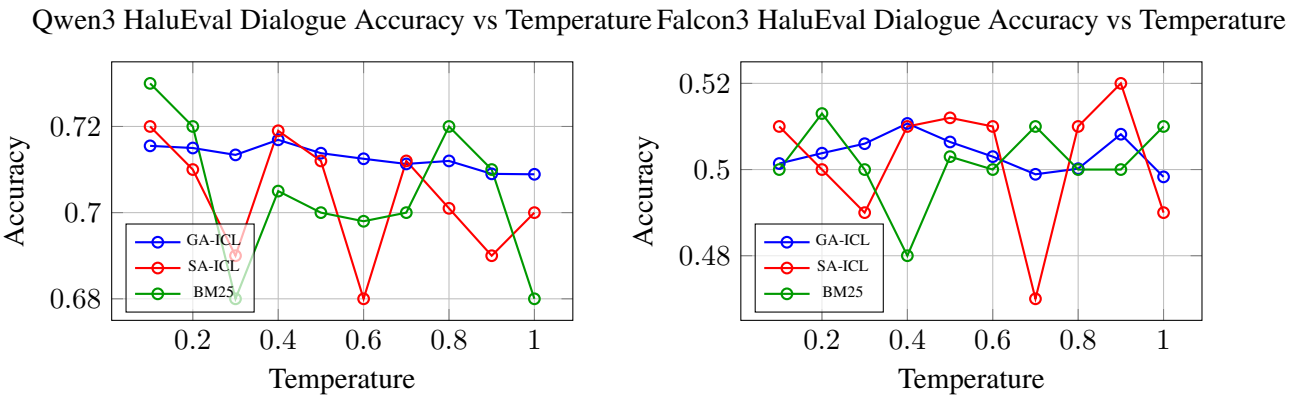
We have used AI tools(Chatgpt) for writing assistance. We did not use it during any of the ideation or experimentation phase.



(a) Qwen3 HaluEval QA

(b) Falcon3 HaluEval QA

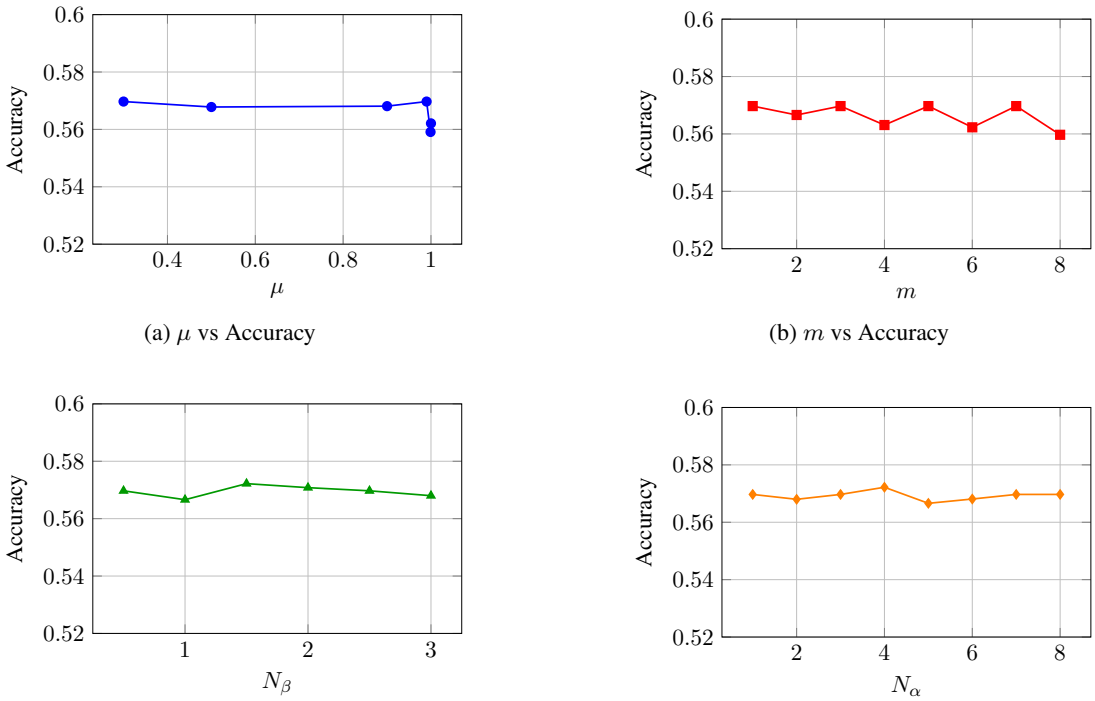
Figure 6: Accuracy vs Temperature comparison for Qwen3 and Falcon3 models on HaluEval QA dataset



(a) Qwen3 HaluEval Dialogue task

(b) Falcon3 HaluEval Dialogue task

Figure 7: Accuracy vs Temperature comparisons for Qwen3 and Falcon3 models on HaluEval Dialogue dataset



(a) μ vs Accuracy

(b) m vs Accuracy

(c) N_β vs Accuracy

(d) N_α vs Accuracy

Figure 8: Hyperparameter tuning for Qwen3-4B on Summarization task