FlipTS: Failure-to-Success Mathematical Self-Refinement via Trajectory-Aware Sampling in End-to-End Online RL

Anonymous EMNLP submission

Abstract

Self-Refinement has emerged as a promising paradigm for improving Large Language Model (LLM) performance by iteratively refining its response at inference time. Existing training methods typically treat refinement as a decoupled task and rely on pre-generated responses, which detaches training from the model's own reasoning trajectories and limits its performance. In this paper, we propose **FlipTS** (**F**ailure-to-Success **L**earning for self-ref**I**nement with **P**otential-guided **T**rajectory-aware **S**ampling), an end-to-end reinforcement learning (RL) framework that optimizes the entire self-refinement loop online. We identify a key bottleneck in this setting: the sparsity of informative *Failure-to-Success* (*F2S*) refinement behaviors. **FlipTS** addresses this by utilizing a non-stationary Bayesian model to estimate data potential and applying trajectory-aware sampling to enrich training with valuable refinement signals. Experiments on Qwen3-4B-Instruct-2507, Qwen3-1.7B and LLaMA-3.1-8B-Instruct across challenging mathematical benchmarks demonstrate that **FlipTS** consistently outperforms offline and online RL baselines. Notably, it exhibits robust generalization to factuality, scientific reasoning and legal reasoning tasks without domain-specific tuning. Our code will be open-sourced.

1 Introduction

LLMs can improve their outputs at inference time through *Self-Refinement* (see Figure 1). *Self-Refinement* allows the model to iteratively improve its responses and correct errors, effectively exploiting additional computation at inference time (Madaan et al., 2023; Shinn et al., 2023; Khatri et al., 2025). However, its efficacy is constrained by the model's limited verification and reasoning capabilities (Zhang et al., 2025). This motivates explicit training to strengthen self-correction.

Recent advancements highlight the potential of outcome-based end-to-end RL (Guo et al., 2025; Wang et al., 2025; Xue et al.). Despite these successes, this paradigm remains underexplored for *Self-Refinement*, where the multi-turn process can be optimized as a unified trajectory. Existing approaches (Zeng et al., 2025; Huang et al., 2024; Hu et al., 2025) typically formulate refinement as an isolated, single-step task, relying predominantly on synthetic trajectories or offline generated responses for training. While straightforward, such methods suffer from a distribution mismatch between training data and actual inference trajectories. As a result, the learned policies may overfit to fixed refinement patterns and generalize poorly to spontaneous self-correction.

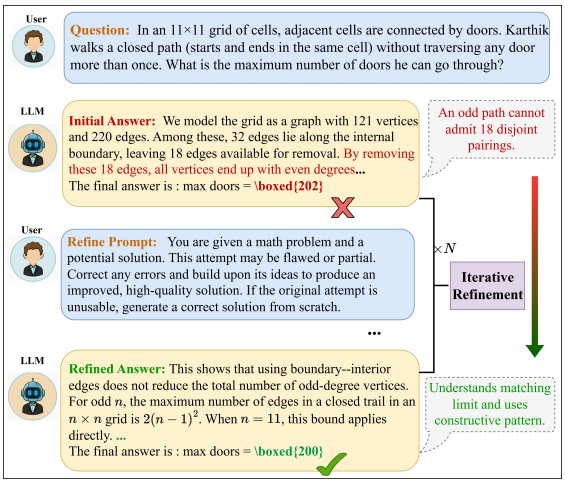

Figure 1: An example of *Self-Refinement*. The model diagnoses flaws in its initial answer and iteratively corrects them to achieve ground truth.

Applying end-to-end RL to *Self-Refinement* poses non-trivial challenges, primarily stemming from the sparsity of high-value behaviors. Specifically, *Failure-to-Success* (*F2S*) trajectories, where the model corrects an initially wrong response, are exceptionally rare under naive sampling. Instead, positive rollouts are predominantly composed of *Success-to-Success* (*S2S*) instances, where the model generates a correct initial response and

merely maintains it. While *S2S* trajectories provide positive reward signals, they lack explicit error-correction actions and are therefore less effective for learning refinement.

To address these challenges, we propose **FlipTS** (**F**ailure-to-Success **L**earning for self-ref**I**nement with **P**otential-guided **T**rajectory-aware **S**ampling), a novel framework that applies end-to-end RL to *Self-Refinement*. Our approach generates and optimizes complete multi-turn self-refinement trajectories online, enabling on-policy optimization and reducing the distribution shift.

Furthermore, we tackle the sparsity issue with a potential-guided trajectory-aware sampling strategy. We model the occurrence of successful correction for each query as a non-stationary Bernoulli process. Leveraging the Beta-Bernoulli conjugate prior for Bayesian estimation, we employ Thompson Sampling to actively prioritize queries with high refinement potential. **FlipTS** achieves superior performance while adhering to simple outcome-based rewards, effectively circumventing the need for manual reward engineering and mitigating the risk of reward hacking. Our method and subsequent analysis underscore the pivotal role of rollout distribution in RL training: sufficient exposure to high-value trajectories serves as both a prerequisite and a powerful catalyst for the emergence of advanced behaviors.

To evaluate the efficacy of **FlipTS**, we conduct extensive training and evaluate models across challenging benchmarks: AIME25, HMMT (Feb 2025), BRUMO25, and CMIMC25 (Balunović et al., 2025). Experimental results demonstrate that **FlipTS** consistently outperforms competitive baselines, including step-level RL trained on offline data and trajectory-level RL with dynamic filtering. Our work offers three key contributions:

1. We present an end-to-end RL framework for multi-turn *Self-Refinement*, enabling models to evolve intrinsic correction capabilities with complete on-policy trajectories rather than offline curated data.

2. We identify the efficiency bottleneck arising from the sparsity of high-value behaviors and propose a potential-guided Thompson Sampling strategy to address it.

3. We demonstrate that our method enhances performance on challenging mathematical benchmarks, while significantly improving out-of-distribution generalization on factuality, scientific reasoning and legal reasoning evaluations.

2 Related Work

Self-Correction and Self-Refinement in LLMs. LLMs can boost their reasoning skills simply by critiquing and refining their own outputs, no extra training required. Early approaches primarily focused on inference-time prompting strategies. (Madaan et al., 2023) demonstrated that LLMs can improve their answers through iterative feedback loops without updating model parameters. Moreover, (Wang et al., 2024; Shridhar et al., 2024) further revealed that *Self-Refinement* can serve as an intrinsic mechanism for continuous reasoning enhancement and model self-evolution. However, studies like (Huang et al., 2023; Liu et al., 2025) suggest that LLMs often struggle to self-correct intrinsic reasoning errors without specific training, particularly in mathematical reasoning.

Online vs. Offline RL Learning for Reasoning. Existing training-based *Self-Refinement* methods are predominantly *offline*: they rely on static datasets synthesized before training and cannot adapt to the model's evolving error patterns. One line of work uses SFT on teacher-generated refinement trajectories, e.g., (Hu et al., 2025; Zhang et al., 2024) trains on math problems with refined answers produced by a stronger model. Another employs offline RL, optimizing aggregation or selection policies over pre-collected multi-solution traces, as in (Lee et al., 2024; Zeng et al., 2025). Consequently, these methods suffer from distribution shift and tend to imitate fixed, off-policy behaviors rather than correcting their own on-policy errors. In contrast, our fully online RL framework updates exclusively from on-policy trajectories, closing the *Self-Refinement* learning loop and enabling the model to initiate corrections autonomously.

Data Efficiency and Trajectory Selection. A key challenge in online RL for reasoning is the scarcity of informative feedback signals. Existing methods struggle to adapt to the evolving policy: Kumar et al. (2024) attempts to mitigate sparsity via static, offline pre-screening, which inevitably leads to a growing data-policy mismatch due to distributional shifts as training progresses. Conversely, while methods like DAPO (Yu et al., 2025) perform online trajectory filtering, they require prohibitively

expensive large-scale rollouts. To efficiently navigate this trade-off between stability and cost, we introduce an online Bayesian mechanism, inspired by Qu et al. (2025), that dynamically estimates the informativeness of trajectories. By applying Thompson Sampling, our approach actively prioritizes *Failure-to-Success* trajectories, as these are most likely to yield a high learning signal. This maximizes information gain and sample efficiency without incurring significant inference overhead.

3 Methodology

3.1 End-to-End RL for *Self-Refinement*

We treat the iterative *Self-Refinement* process as a multi-turn generation task. For a given question q, the process initiates by generating an initial draft solution. In subsequent turns, the model conditions on the original question and the previous solution to produce a refined response. This cycle repeats for a fixed number of turns T, yielding a trajectory of reasoning steps where each iteration aims to improve the correctness of the preceding output.

MDP Formulation. We formalize this process as a Markov Decision Process (MDP). A complete episode is represented as a trajectory τ containing a sequence of states, actions, and immediate rewards:

$$\tau = \big((s_1, a_1, r_1), (s_2, a_2, r_2), \ldots, (s_T, a_T, r_T)\big).$$

Here, the state and action spaces are defined dynamically based on the turn t. At the initial turn ($t = 1$), the state is the question itself, $s_1 = q$. For refinement turns ($t > 1$), the state encapsulates the context of the previous attempt, defined as $s_t = (q, a_{t-1})$. The action a_t corresponds to the generated solution at step t. Crucially, r_t denotes the immediate reward received after action a_t, which evaluates the quality of the current step.

RLVR with Trajectory-level GRPO. Following the spirit of Reinforcement Learning with Verifiable Rewards (RLVR), we define the immediate reward r_t based on the objective correctness of the solution. Formally,

$$r_t = \begin{cases} 1, & \text{if } a_t \text{ is correct,} \\ -1, & \text{otherwise.} \end{cases}$$

Specifically, we extract the answer within `\boxed` from a_t and evaluate it against the ground truth using the `math_verify`[1] toolkit of version 0.7.0.

[1]https://github.com/huggingface/Math-Verify

The trajectory reward $R(\tau)$ is defined as the reward of the final output, i.e., $R(\tau) = r_T$.

We employ Group Relative Policy Optimization (GRPO) (Shao et al., 2024) for RL training. For each question q, a group of G trajectories $\tau_i = \{(s_t^{(i)}, a_t^{(i)}, r_t^{(i)})\}_{t=1}^{T}$ is sampled from an old policy π_{old}. The advantage for the i-th trajectory is computed by normalizing the rewards within the group:

$$A_i = \frac{R(\tau_i) - \text{mean}(\{R(\tau_j)\}_{j=1}^{G})}{\text{std}(\{R(\tau_j)\}_{j=1}^{G})}.$$

Then the trajectory-level GRPO objective is formulated as follows, omitting the KL divergence constraint:

$$\mathcal{J}_{GRPO}(\theta) = \mathbb{E}_{q\sim\mathcal{D},\{\tau_i\}\sim\pi_{\text{old}}(\cdot|q)}\Big[\frac{1}{GT}\sum_{i=1}^{G}\sum_{t=1}^{T} \frac{1}{|a_t^{(i)}|}\sum_{l=1}^{|a_t^{(i)}|}\min\{\rho_{t,l}^{(i)}A_i, \text{clip}(\rho_{t,l}^{(i)}, 1\pm\epsilon)A_i\}\Big], \quad (1)$$

where ϵ is the clip ratio and $\rho_{t,l}^{(i)}$ is the importance sampling ratio for l-th token in solution $a_t^{(i)}$,

$$\rho_{t,l}^{(i)} = \frac{\pi_\theta\big(a_{t,l}^{(i)} \mid s_t^{(i)}, a_{t,<l}^{(i)}\big)}{\pi_{\text{old}}\big(a_{t,l}^{(i)} \mid s_t^{(i)}, a_{t,<l}^{(i)}\big)}.$$

3.2 Sparsity of High-value Behaviors

Applying end-to-end RL to complex processes is fundamentally limited by the intrinsic sparsity of high-value behaviors. Trajectories that exhibit desirable capabilities, such as error recovery, are rare compared to trivial solutions. Without sufficient exposure to these informative examples, the optimization algorithm lacks the gradient signals needed to reinforce targeted skills.

This bottleneck is particularly acute in the iterative *Self-Refinement* process. In this context, *Failure-to-Success* (*F2S*) instances are extremely rare under natural sampling, leaving the model with few positive examples from which to learn effective error-correction strategies. Two factors compound this sparsity. First, LLMs exhibit a strong *self-consistency bias* (Xu et al., 2024), favoring initial responses over critical revision. Second, *misaligned problem difficulty* further reduces useful signals: tasks are often too easy (solved on the first attempt) or too hard (unsolvable even with refinement). As shown in Figure 3, with a horizon

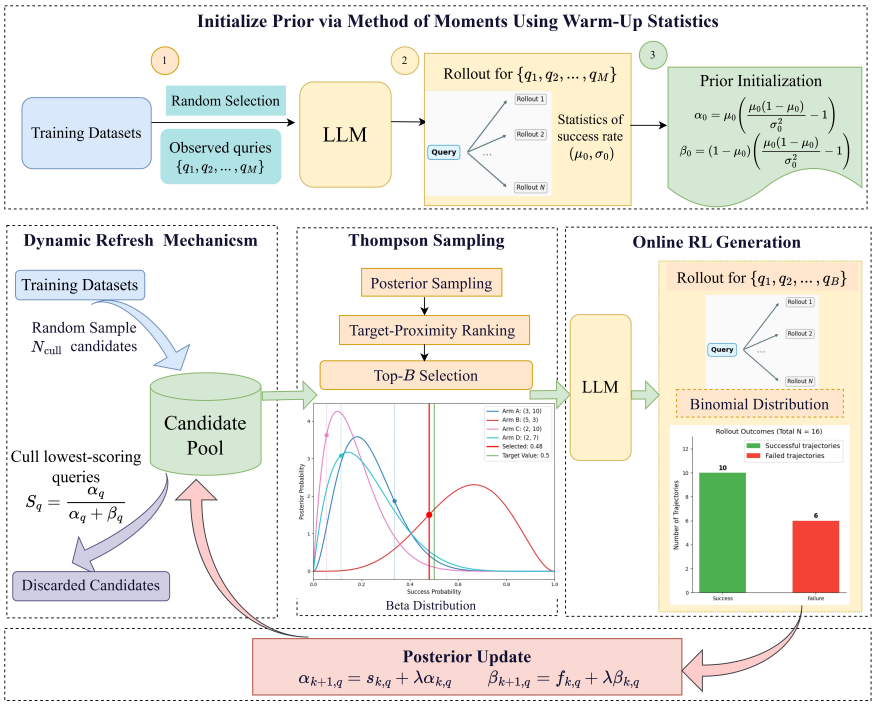

Figure 2: Framework of the Proposed Method.

of $T = 2$, valid corrections appear in only about 5% trajectories. This extreme sparsity dilutes the training signal and hampers policy optimization.

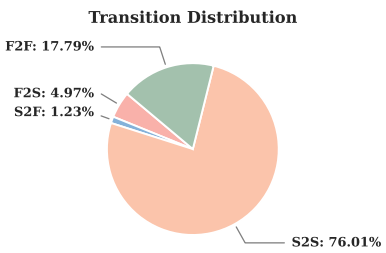

Figure 3: Proportion of four transition types during first 50 training steps of Trajectory GRPO.

3.3 Active Data Selection via Thompson Sampling

To address the sparsity of informative refinement signals, we propose an active data selection strategy that prioritizes *Failure-to-Success* (*F2S*) trajectories, where an initially incorrect solution is corrected during refinement. For a refinement horizon of $T = 2$, these trajectories correspond to the reward pattern $r_1 = -1$ followed by $r_2 = 1$.

We formulate trajectory selection as a non-stationary multi-armed bandit (NS-MAB) problem, where each question q is treated as an arm and the reward reflects its likelihood of producing *F2S* behavior. Assuming *F2S* transitions follow a Bernoulli process with unknown probability $\xi_{k,q}$ at training step k, we model the uncertainty using a Beta-Bernoulli framework:

$$\xi_{k,q} \sim \text{Beta}(\alpha_{k,q}, \beta_{k,q}),$$

where $\alpha_{k,q}$ and $\beta_{k,q}$ denote the effective pseudo-counts of observed successes and failures.

Prior Initialization and Posterior Update. To mitigate cold-start issues, we estimate an informative global prior from a warm-up phase of the first W training steps using uniformly sampled questions. For each question q_i, we compute its empirical *F2S* probability $\hat{\xi}_i$ from collected rollouts. We then fit a Beta prior via the Method of Moments:

$$\alpha_0 = \mu_0\kappa, \quad \beta_0 = (1-\mu_0)\kappa, \tag{2}$$

where μ_0 and σ_0^2 are the sample mean and variance of $\{\hat{\xi}_i\}$, and $\kappa = \mu_0(1-\mu_0)/\sigma_0^2 - 1$. All questions are initialized with $\alpha_{0,q} = \alpha_0$ and $\beta_{0,q} = \beta_0$.

At training step k, let $s_{k,q}$ and $f_{k,q}$ denote the counts of trajectories with and without *F2S* transitions for question q. The posterior is updated as:

$$\begin{aligned} \alpha_{k+1,q} &= s_{k,q} + \lambda\alpha_{k,q}, \\ \beta_{k+1,q} &= f_{k,q} + \lambda\beta_{k,q}, \end{aligned} \tag{3}$$

where $\lambda \in (0, 1]$ is an exponential decay factor for handling non-stationarity. At training step k, we

construct the training batch $\mathcal{B}_k$ by leveraging the posterior to identify questions whose estimated *F2S* probabilities are closest to a target ξ_{target}. Using Thompson Sampling, we draw

$$\tilde{\xi}_{k,q} \sim \text{Beta}(\alpha_{k,q}, \beta_{k,q})$$

for each question q, and select the top-B questions minimizing the absolute deviation:

$$\mathcal{B}_k = \text{Top-B}_{q \in \mathcal{D}} \left(-\left| \tilde{\xi}_{k,q} - \xi_{\text{target}} \right| \right). \quad (4)$$

3.4 Scalable Exploration via a Dynamic Candidate Pool

Applying Thompson Sampling directly to a massive dataset $\mathcal{D}$ with $|\mathcal{D}| \gg B$ is inefficient, as large candidate spaces amplify statistical outliers and lead to stale posterior estimates due to sparse observations. To address this, we restrict Thompson Sampling to a dynamically maintained candidate pool $\mathcal{C} \subset \mathcal{D}$ with $B < M \ll |\mathcal{D}|$. After warm-up, the pool is initialized by uniformly sampling M questions from $\mathcal{D}$. During training, once the fraction of observed questions exceeds a threshold η, we refresh the pool by removing a fraction γ of questions whose posterior means deviate most from the target ξ_{target}, and replace them with newly sampled questions. This strategy improves exploration efficiency while mitigating posterior staleness and pool saturation. The complete workflow is provided in Appendix I.

4 Experiments

4.1 Experimental Settings

We implement **FlipTS** and all baselines using the `verl-agent` framework (Feng et al., 2025). Experiments are conducted on **Qwen3-4B-Instruct-2507**, **Qwen3-1.7B** (Team, 2025), and **Llama3.1-8B-Instruct** (Dubey et al., 2024). All models are trained on a mixture of **DeepMath** (He et al., 2025) and the level-1/2 subset of **AM** (Ji et al., 2025).

For challenging mathematical reasoning, we evaluate Qwen models on **AIME25**, **HMMT (Feb 2025)**, **BRUMO25**, and **CMIMC25** (Balunović et al., 2025), reporting mean accuracy over 128 independently sampled trajectories, denoted as $\text{Acc}_{\text{avg}}@128$. For Llama models, we evaluate on **MATH500** (Lightman et al., 2024) with $\text{Acc}_{\text{avg}}@8$. To assess out-of-distribution generalization, we further evaluate on **TruthfulQA** (Lin et al., 2022), **SuperGPQA** (Du et al., 2025), and the **Gaokao-Physics** and **Law-Analytics** subsets from **AGIEval** (Zhong et al., 2023). Unless otherwise specified, OOD results are reported using $\text{Acc}_{\text{avg}}@128$. Detailed benchmark statistics are provided in Appendix C.

Training follows a strict on-policy setting without KL regularization. We use learning rate $1\text{e}{-6}$, batch size 16, group size 16, and trajectory length $T = 2$. For **FlipTS**, we set $\xi_{\text{target}} = 0.5$, pool size $M = 2000$, refresh threshold $\eta = 0.3$, and culling ratio $\gamma = 0.25$. These hyperparameters are chosen without exhaustive tuning; sensitivity analyses are provided in Appendix G. Additional implementation details, including model-specific decoding budgets and temperatures, are summarized in Appendix D.

4.2 Main Results and Comparative Analysis

We compare against three competitive baselines: (1) **Step-level GRPO**, which applies single-turn GRPO to refinement tasks with offline-generated initial solutions; (2) **Trajectory GRPO**, the end-to-end RL algorithm in Section 3.1 without active data selection or the dynamic candidate pool; and (3) **Dynamic-filter GRPO**, a Trajectory GRPO variant that dynamically filters trajectories with zero advantage. Unlike Dynamic-filter GRPO, which requires extra online rollouts to maintain the same effective batch size after filtering, FlipTS updates the candidate pool without additional rollout cost and incurs negligible memory overhead. More baseline details are given in Appendix E.

Table 1 summarizes the performance of different training strategies applied to Qwen3-4B-Instruct-2507 on four challenging mathematical reasoning benchmarks using 10-turn self-refinement.

Step-level GRPO provides only marginal gains over the base model, indicating limitations of offline-synthesized solutions. Trajectory GRPO improves accuracy through multi-step reasoning but is restricted by uniform sampling. Dynamic-filter GRPO further boosts performance via online filtering, but requires $4\times$ generated trajectories per valid update. In contrast, FlipTS achieves the highest accuracy with the same rollout budget as Trajectory GRPO ($1\times$). By selectively reinforcing high-value trajectories using trajectory-value guided sampling and Bayesian estimation, FlipTS balances exploration and exploitation efficiently, achieving superior performance without extra computational overhead.

Figure 4 shows the evaluation performance of **FlipTS** under iterative self-refinement. Although

Table 1: $\mathbf{Acc}_{\text{avg}}$@128 (%) of Qwen3-4B-Instruct under 10-turn *Self-Refinement* across four benchmarks. 'Rollouts' indicates the relative number of generated trajectories required to obtain one update during training.

Method	AIME25	HMMT (Feb 2025)	BRUMO25	CMIMC25	Average ↑	Rollouts ↓
w/o training	52.50	36.88	63.59	36.29	47.31	N/A
Step-level GRPO	55.45	37.86	63.10	37.75	48.29	1×
Trajectory GRPO	58.41	40.13	65.42	39.34	50.82	1×
Dynamic-filter GRPO	59.74	42.71	66.54	40.04	52.36	4×
FlipTS	**62.76**	**42.73**	**67.11**	**40.08**	**53.17**	1×

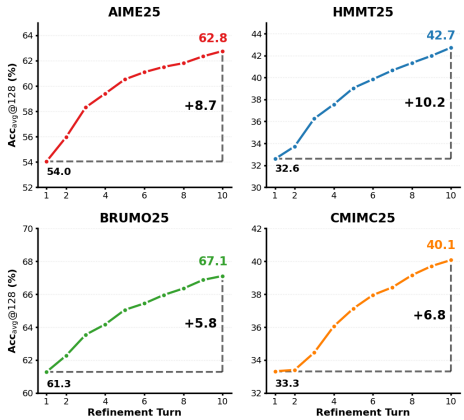

Figure 4: Acc-Avg@128 of FlipTS on AIME25, HMMT25, BRUMO25, and CMIMC25 across refinement turns.

trained with a short horizon ($T = 2$), the model consistently improves over up to 10 refinement turns, suggesting that FlipTS effectively learns a transferable self-correction capability.

Compared with both offline baselines (e.g., Trajectory GRPO) and online filtering methods (e.g., Dynamic-filter GRPO), FlipTS achieves stronger refinement performance without additional rollout cost. These results demonstrate that targeted trajectory refinement is more effective and efficient than uniform sampling or difficulty-based filtering.

4.3 Performance of the Thinking Model

Table 2 reports the refinement performance of Qwen3-1.7B (thinking mode) before and after iterative refinement. Since Thinking models typically generate substantially longer reasoning trajectories, we focus on the refinement gain brought by a single refinement stage and report the results after one refinement.

The untrained model exhibits a moderate intrinsic self-revision ability, improving by 1.87 points on average, from 30.12% to 31.99%. Trajectory GRPO increases the average gain to 3.01 points, corresponding to a 61% larger refinement gain than the untrained baseline. This confirms that trajectory-level RL can strengthen refinement behavior. However, FlipTS further enlarges the gain to 4.75 points, which is 2.55 times the untrained gain and 1.58 times the Trajectory GRPO gain. By emphasizing failure-to-success trajectories, FlipTS provides more informative learning signals for correcting initially imperfect reasoning paths, resulting in stronger self-refinement gains.

4.4 Can FlipTS Rescue Models without Intrinsic Self-Refinement Ability?

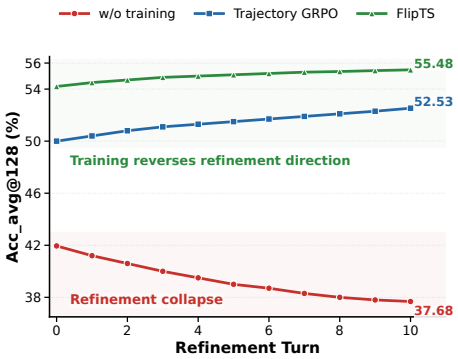

Figure 5: Iterative self-refinement performance of LLaMA3.1-8B-Instruct across refinement turns. The vanilla model exhibits refinement collapse, while Trajectory GRPO reverses the degradation trend through trajectory-level reinforcement learning. FlipTS further improves both initialization and long-horizon refinement stability.

We further examine whether FlipTS can improve models with weak intrinsic self-refinement capability. As shown in Figure 5, vanilla LLaMA3.1-8B-Instruct exhibits refinement collapse: accuracy drops from 41.95% at initialization to 37.68% at Turn 10. This suggests that self-refinement may accumulate reasoning errors when the model lacks an explicit self-correction mechanism.

Trajectory GRPO reverses this trend, improv-

Table 2: $\mathbf{Acc}_{\text{avg}}$@128 (%) of Qwen3-1.7B (thinking mode) before and after *Self-Refinement* across four benchmarks. "Init" denotes the initial response, while "Refine" denotes the response after 2-turn refinement. Δ_{avg} denotes the average refinement gain across four benchmarks.

Method	AIME25		HMMT (Feb 2025)		BRUMO25		CMIMC25		Refinement
	Init	Refine	Init	Refine	Init	Refine	Init	Refine	Gain Δ_{avg}
w/o training	35.34	36.09	20.73	21.74	46.09	49.64	18.32	20.47	+1.87
Trajectory GRPO	35.96	37.68	23.36	25.18	45.21	50.99	20.25	22.97	+3.01
FlipTS	34.90	**38.33**	23.88	**27.19**	44.97	**53.28**	19.12	**23.07**	**+4.75**

ing Turn-10 accuracy to 52.53%, which indicates that trajectory-level reinforcement learning can induce basic self-correction behavior. FlipTS further strengthens this process through trajectory-aware sampling, reaching the best refinement performance of 55.48% and improving over vanilla refinement by 17.80 percentage points. These results show that prioritizing failure-to-success trajectories can transform unreliable refinement into stable improvement.

4.5 Mechanism Analysis of FlipTS

To better understand why FlipTS works, we analyze both its architectural components and its sampling objective. We first ablate the dynamic candidate pool and Thompson Sampling, and then compare the proposed *Failure-to-Success* (*F2S*) target with a *Final Success* (*Final-S*) target. The results are summarized in Table 3.

4.5.1 Ablation of Architectural Components

We consider three variants: *w/o Dynamic Pool*, which applies Thompson Sampling over the full training set; *w/o Thompson Sampling*, which keeps the dynamic pool but replaces *F2S*-targeted Thompson Sampling with random sampling; and *w/o Both*, which reduces the method to standard Trajectory GRPO.

The results show that both components are crucial to FlipTS. Removing either the dynamic candidate pool or Thompson Sampling substantially reduces the average score from 53.17 to 50.33 and 50.68, respectively, making both variants comparable to Trajectory GRPO. This suggests that neither component alone provides a consistent advantage over standard trajectory-level RL; the gain of FlipTS comes from their joint effect.

Without the dynamic candidate pool, Thompson Sampling is applied over the full training set, where sparse observations, stale posterior estimates, and noisy high-variance samples can make selection ineffective, as discussed in Section 3.4 and Appendix B. Conversely, without Thompson Sampling, the dynamic pool merely restricts training to a smaller candidate space but does not actively identify questions with suitable *F2S* likelihoods. Thus, Thompson Sampling discovers informative self-correction examples, while the dynamic pool makes this discovery reliable and scalable under large-scale non-stationary RL training.

4.5.2 Which Trajectories Drive Effective *Self-Refinement*?

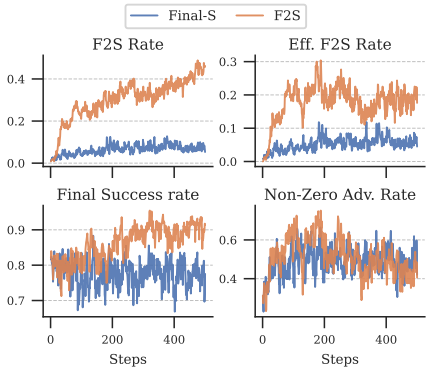

Figure 6: Training dynamics of *F2S* and *Final-S* sampling. (a) F2S Rate: proportion of *F2S* trajectories per batch. (b) Effective F2S Rate: proportion of *F2S* trajectories with non-zero advantage. (c) Final Success Rate: proportion of trajectories achieving the correct final answer. (d) Non-zero Advantage Rate: proportion of trajectories with non-zero advantage.

Prior methods such as DAPO (Yu et al., 2025) and MoPPS (Qu et al., 2025) typically select questions based on the *Final Success* rate, aiming to filter out trivial samples with zero advantage while retaining questions of moderate difficulty. To examine whether final correctness or explicit correction behavior is more useful for refinement learning, we compare FlipTS with a *Final-S*-targeted variant. As shown in Table 3, replacing the *F2S* target with *Final-S* reduces the average accuracy from 53.17 to 51.54, indicating that successful self-correction provides a more informative signal than final cor-

Table 3: Mechanism analysis of FlipTS. We study both component ablations and the effect of sampling targets.

Variant	AIME25	HMMT (Feb 2025)	BRUMO25	CMIMC25	Average ↑
Component ablation					
FlipTS (F2S Target)	**62.76**	**42.73**	**67.11**	40.08	**53.17**
w/o Dynamic Pool	57.68	38.52	65.78	39.34	50.33
w/o Thompson Sampling	59.95	39.11	64.22	39.45	50.68
w/o Both (Trajectory GRPO)	58.41	40.13	65.42	39.34	50.82
Sampling target ablation					
Final-S Target	58.88	40.91	66.30	**40.49**	51.54

Table 4: $\mathbf{Acc}_{\text{avg}}$@128 (%) of Qwen3-4B-Instruct under 10-turn *Self-Refinement* on three challenging OOD benchmarks.

Method	TruthfulQA		Gaokao-physics		SuperGPQA		Law-Analytics	
	Init	Refine	Init	Refine	Init	Refine	Init	Refine
w/o training	81.73	84.33	90.04	91.88	36.13	45.30	81.55	86.59
Trajectory GRPO	80.80	84.99	93.04	93.73	44.07	47.00	84.09	88.52
FlipTS	**84.05**	**86.08**	**93.84**	**94.65**	**44.19**	**47.46**	**85.47**	**89.56**

rectness alone.

Figure 6 further explains this gap. The *F2S*-guided strategy maintains a substantially higher proportion of *F2S* trajectories, both in raw counts and among non-zero-advantage samples, increasing the density of explicit correction signals with negligible additional overhead. Although the *Final-S* strategy tends to select harder prompts with lower final accuracy, both strategies yield comparable non-zero-advantage rates. This suggests that simply increasing task difficulty or filtering zero-advantage samples is insufficient; effective self-refinement training benefits more from enriching the training distribution with successful correction behaviors.

4.6 Out-of-Distribution Performance

We further evaluate whether the refinement behavior learned from mathematical reasoning transfers to out-of-domain tasks. As shown in Table 4, FlipTS consistently improves both initialization and long-horizon refinement performance across factual verification, STEM-style reasoning, and legal analysis benchmarks.

FlipTS achieves clear gains on TruthfulQA (Lin et al., 2022), suggesting improved suppression of hallucinated or self-inconsistent reasoning trajectories. More pronounced improvements are observed on reasoning-intensive STEM benchmarks, including SuperGPQA (Du et al., 2025) and Gaokao-Physics (Zhong et al., 2023), where solving requires multi-step inference and verification of intermediate results. Moreover, Law-Analytics (Zhong et al., 2023) evaluates legal reasoning, and FlipTS still brings consistent improvements. These results suggest that the learned refinement behavior is not limited to mathematical reasoning, but transfers to diverse OOD settings where reliable correction of intermediate reasoning steps are required.

5 Conclusion

This work introduces **FlipTS**, an end-to-end online RL framework that enhances iterative *Self-Refinement* in LLMs and addresses key limitations of offline methods. FlipTS combines Bayesian trajectory-value estimation with targeted Thompson Sampling to prioritize promising trajectories, effectively mitigating the sparsity of informative refinement behaviors. FlipTS reverses **refinement collapse**, where untrained models degrade over multiple self-refinement steps due to the lack of an internal verification mechanism. Through principled training and targeted trajectory sampling, FlipTS stabilizes iterative refinement, enabling the model to reliably correct its own errors and progressively improve reasoning. Results demonstrate that FlipTS achieves strong performance on challenging mathematical reasoning benchmarks and generalizes effectively to OOD tasks, highlighting its robust and transferable reasoning capabilities.

6 Limitations

Our analysis highlights the importance of rollout distribution in RL training, particularly in terms of the model's exposure to high-value trajectories. In the present work, training is restricted to a short horizon of $T = 2$, consisting of an initial response followed by a single refinement step, primarily due to limitations in available computational resources. This setting allows us to observe basic self-refinement behavior. However, how extending RL training to longer, multi-turn rollouts would influence trajectory coverage or reasoning dynamics remains a worthwhile direction for investigation. Systematic investigation of such settings may help clarify under what conditions rollout length affects the emergence and utilization of high-value trajectories.

Our current work focuses on enhancing the efficiency of sequential, iterative *Self-Refinement*. Moving forward, exploring parallel self-evolving strategies is crucial for scalability. Future research will investigate integrating our trajectory-aware sampling principles with parallel refinement techniques. In particular, we aim to develop policies that can concurrently generate and evaluate multiple candidate solutions or reasoning chains, while supporting efficient online RL training.

Furthermore, we address the bottleneck of sparse high-value trajectories through active online sample selection. While this strategy effectively prioritizes informative data, the optimization process still relies on relatively coarse trajectory-level rewards. In future work, we will integrate our sampling framework with finer-grained step-level reward shaping and dynamic advantage optimization to jointly improve online sample selection and credit assignment, thereby alleviating reward sparsity and enabling more robust self-correction in complex reasoning scenarios.

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A Notations and Symbols

Table 5 summarizes the primary symbols, variables, and parameters used in the **FlipTS** framework, categorized by trajectory dynamics, RL optimization, and adaptive sampling mechanism.

Notation	Description
Problem & Trajectory Formulation	
q, y	Input query and its corresponding ground-truth answer.
τ	Multi-turn *Self-Refinement* trajectory $\{(s_t, a_t, r_t)\}_{t=1}^{T}$.
T	Maximum number of turns per episode.
s_t, a_t, r_t	State, action (solution), and verifiable reward at turn t.
RL Optimization & Dynamics	
$\pi_\theta, \mathcal{D}$	Policy network and the global training dataset.
$R(\tau), A_i$	Trajectory-level return and group-relative advantage.
G	Group size (number of sampled trajectories per query).
$\mathcal{J}_{\text{GRPO}}$	Objective function for trajectory-level GRPO.
Active Selection via Thompson Sampling	
$\xi_{k,q}$	*F2S* transition probability for question q at step k.
α, β	Shape parameters (pseudo-counts) of the Beta distribution.
λ	Decay factor addressing non-stationarity of the policy.
ξ_{target}	Target *F2S* probability for prioritizing trajectories.
$\mathcal{C}, M$	Candidate pool and its capacity ($M = \lvert\mathcal{C}\rvert$).
$\mathcal{O}$	Set of observed questions within the candidate pool.
η	Saturation threshold for triggering a pool refresh.
γ	Fraction of candidates to be culled during replenishment.
S_q	Posterior mean used for candidate filtering.

Table 5: Summary of notations and definitions.

B Theoretical Analysis of Thompson Sampling Collapse Due to Large-Scale Datasets

In this section, we provide a formal justification for the adoption of Dynamic Candidate Pool. We analyze the asymptotic behavior of Thompson Sampling when the dataset size N is much larger than the sampling batch B, and show that, despite the presence of informative samples, the selection mechanism effectively degenerates into uniform random sampling: a phenomenon we term *Thompson Sampling Collapse*.

Theorem 1 (Asymptotic Collapse of Thompson Sampling Precision). Let $\mathcal{D}$ be a dataset of cardinality N, partitioned into two disjoint sets:

- $\mathcal{O}$: The set of **Observed** samples with cardinality $|\mathcal{O}| = K \ll N$.
- $\mathcal{U}_N$: The set of **Unobserved** samples with cardinality $|\mathcal{U}_N| = N - K \approx N$.

Statement: As $N \to \infty$ with fixed batch size B, the expected number of Observed samples in the batch approaches zero, causing the batch precision to converge to the prior sparsity ρ.

Proof. **1. The Budget Constraint**
The selection of the top-B samples defines a selection window

$$\mathcal{W}_N = [x^* - \delta_N, x^* + \delta_N].$$

The total count B comprises selected observed samples (M_{obs}) and unobserved samples (M_{un}). Using the Law of Large Numbers to approximate discrete counts with probability integrals:

$$\begin{aligned} B &= \underbrace{K \int_{\mathcal{W}_N} f_{post}(x)\, dx}_{\mathbb{E}[M_{obs}]} \\ &+ \underbrace{(N-K) \int_{\mathcal{W}_N} f_{prior}(x)\, dx}_{\mathbb{E}[M_{un}]} \end{aligned} \tag{5}$$

2. Asymptotic Shrinkage of δ_N
As $N \to \infty$, the unobserved component $(N - K)$ dominates the budget equation. To keep the total sum finite (B), the integration interval δ_N must shrink to zero, i.e., $\delta_N \to 0$. By the Mean Value Theorem for small intervals, we can approximate:

$$B \approx 2\,\delta_N \Big[K \cdot f_{\text{post}}(x^*) + (N-K) \cdot f_{\text{prior}}(x^*)\Big].$$

Since $N \gg K$, the contribution from the unobserved portion dominates:

$$B \approx 2\,\delta_N (N-K)\, f_{\text{prior}}(x^*),$$

Thus, the batch density is governed almost entirely by the prior of the unobserved population.

3. Asymptotic Dilution of Informative Samples
We now determine the expected number of high-value observed samples that successfully enter the batch by substituting δ_N back into the expectation term:

$$\begin{aligned} \mathbb{E}[M_{obs}] &\approx K \cdot 2\delta_N \cdot f_{post}(x^*) \\ &\approx \tfrac{K \cdot B}{N} \cdot \underbrace{\frac{f_{post}(x^*)}{f_{prior}(x^*)}}_{\text{Density Ratio}}. \end{aligned} \tag{6}$$

The observed samples typically exhibit **Posterior Concentration** (low variance) due to evidence updates, whereas unobserved samples follow a diffuse

prior. This implies that the local density at the target is topically higher for observed samples:

$$f_{post}(x^*) \geq f_{prior}(x^*) > 0.$$

The density ratio remains a finite constant. As the dataset size scales ($N \to \infty$), the linear growth of N in the denominator overwhelms the density benefit:

$$\lim_{N\to\infty} \mathbb{E}[M_{obs}] = 0. \tag{7}$$

Therefore, the batch becomes statistically saturated with unobserved samples ($\mathcal{B} \approx \mathcal{B}_{un} \subset \mathcal{U}_N$). Since the unobserved set is a random subset inheriting the global signal sparsity, the final batch precision reverts to ρ. □

Remark 1. This theoretical insight directly motivates our sampling design. In practice, datasets are large ($N \sim 10^5$–10^7), while batch sizes are small ($B \sim 2^4$–2^7). Under this regime ($N \gg B$), Theorem 1 shows that standard Thompson Sampling suffers from **asymptotic dilution of informative samples**, effectively degenerating into uniform random sampling and failing to leverage newly discovered high-value trajectories. To mitigate this, we introduce a **Candidate Pool** of manageable size (e.g., $M = 2000$), ensuring the ratio B/M remains sufficiently large. This preserves the concentration of the posterior density $f_{\text{post}}(x^*)$ around informative samples, thereby maintaining the efficacy of Thompson Sampling for active exploitation.

C Dataset Details and Statistics

In this section, we provide detailed documentation of the datasets used for training, validation, and benchmarking. The primary domains cover competition-level mathematics, scientific reasoning, and AI safety/trustworthiness.

C.1 Training Dataset

Our training corpus is constructed by uniformly mixing the following mathematical reasoning datasets: (1) **DeepMath** (He et al., 2025): Contains 103k samples; (2) **AM-Math-Difficulty-RL** (Ji et al., 2025): We utilize the subset of difficulty level-1 (98k) and level-2 (101k). The total size of the mixed training dataset is approximately 302k samples.

C.2 Evaluation Benchmarks

We employ a diverse set of benchmarks to evaluate mathematical capability and out-of-distribution (OOD) generalization. Table 6 summarizes the statistics, domains, and evaluation settings for each dataset. Note that for TruthfulQA, we adhere to the new, improved multiple-choice version[2] recommended by the authors. For SuperGPQA, we use a fixed random subset of 100 questions for evaluation.

D Implementation Details

For training, we set the maximum output length to 8192 tokens for Instruct models and 32768 tokens for Thinking models. During evaluation, the maximum output length is set to 16384 tokens for Instruct models and 38912 tokens for Thinking models. We use temperature 1.0 for Instruct models and temperature 0.6 for Thinking models in both training-time sampling and evaluation.

E Baseline Details

We detail the baselines and summarize their key differences and characteristics in Table 7.

Step-level GRPO. This baseline treats solution generation and refinement as decoupled single-turn tasks. We randomly sample 20,000 questions and use the original model to generate one initial solution for each. The training is conducted via single-turn GRPO on a dataset comprising two tasks: (1) generating a correct solution directly from the question, and (2) generating a refined solution given the pre-generated initial response as context. This baseline serves to isolate the benefits of end-to-end online optimization.

Trajectory GRPO. This baseline implements the end-to-end RL framework detailed in Section 3.1, optimizing self-refinement with complete multi-turn trajectories. Unlike our proposed method, it omits the Thompson Sampling mechanism and instead adheres to the original dataset distribution for rollout generation.

Dynamic-filter GRPO. This baseline integrates the dynamic filtering spirit of DAPO into the Trajectory GRPO framework. Instead of following the original DAPO formulation strictly, it utilizes online rejection sampling to filter out trajectories with non-positive advantages. This ensures a fair comparison by applying DAPO's adaptive sampling logic to the same trajectory-level search space as

[2] https://www.alignmentforum.org/posts/Bunfwz6JsNd44kgLT/new-improved-multiple-choice-truthfulqa

Table 6: Statistics of evaluation datasets.

Category	Dataset	Size (Questions)	Evaluation Setting
Validation	AIME24[3], AIME25, HMMT (Feb 2025) (Balunović et al., 2025)	30 each	Acc_{avg}@8
Math Benchmark	AIME25, HMMT (Feb 2025), BRUMO25 (Balunović et al., 2025)	30 each	Acc_{avg}@128
	CMIMC25 (Balunović et al., 2025)	40	Acc_{avg}@128
	MATH500 (Lightman et al., 2024)	500	Acc_{avg}@8
Science	Gaokao-physics (Rein et al., 2024)	174	Acc_{avg}@128
	SuperGPQA (Subset) (Du et al., 2025)	100	Acc_{avg}@128
Law	Law-Analytics (Zhong et al., 2023)	240	Acc_{avg}@128
Factuality	TruthfulQA (Lin et al., 2022)	817	Acc_{avg}@128

[3] https://huggingface.co/datasets/AI-MO/aimo-validation-aime

Table 7: Comparison of baseline methodologies across core algorithmic dimensions. E2E RL: End-to-End RL optimization of self-refinement; Sample Pruning: Online filtering of zero-advantage trajectories.

Method	E2E RL Paradigm	Thompson Sampling	Sample Pruning	Compute Efficiency	Gradient Fidelity
Step-level GRPO	×	×	×	High	Moderate
Trajectory GRPO	✓	×	×	High	Moderate
Dynamic-filter GRPO	✓	×	✓	Low[†]	High
FlipTS (Ours)	✓	✓	×	High	High

[†] Dynamic-filter GRPO increases the rollout-to-update ratio through extra online sampling and rejection.

our method. However, it incurs significant compute overhead from oversampling and rejection.

F Computation Budget

Our experimental framework and algorithms are implemented as modifications to the open-source `verl-agent` library, utilizing RL training for model optimization. All experiments were conducted on a dedicated compute node equipped with 8× NVIDIA GPUs (80GB VRAM), operating with CUDA 12.2. The computational cost for training to approximately 800 update steps is as follows:

- **Qwen3-4B-Instruct:** Completing 800 update steps requires approximately 3.6 days of continuous wall-clock time, largely due to the iterative generation and self-refinement cycles.
- **Llama-3.1-8B-Instruct:** Under identical configurations, the training process for 800 steps requires approximately 2.5 days.

G Sensitivity Analysis of Hyperparameters

In this section, we conduct several experiments to investigate the impact of key hyperparameters and design choices within our dynamic candidate pool framework. To provide a comprehensive view of the training dynamics, we visualize the validation scores throughout the optimization process. The validation set comprises three challenging benchmarks: AIME24, AIME25, and HMMT (Feb 2025), evaluated under a self-refinement process with a horizon of $T = 2$. We report performance using the Average Accuracy over 8 sampled trajectories (denoted as Acc_{avg}@8).

G.1 Target *Failure-to-Success* Probability

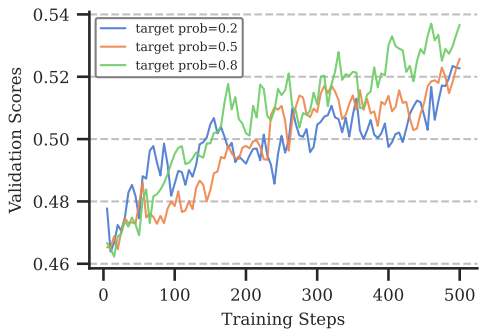

Figure 7: Validation scores for target Failure-to-Success probability $\xi_{\text{target}} \in \{0.2, 0.5, 0.8\}$ over training steps.

We investigate the impact of the target *Failure-to-Success* probability ξ_{target}. We evaluate the model's performance with $\xi_{\text{target}} \in \{0.2, 0.5, 0.8\}$, and the corresponding validation scores are presented in

Figure 7. The setting of $\xi_{\text{target}} = 0.8$ consistently yields superior performance compared to lower values of 0.5 and 0.2. This empirical evidence validates the hypothesis that exposing the model to a higher density of Failure-to-Success transitions is essential for mastering *Self-Refinement*. Thus, prioritizing these trajectories through the sampling mechanism serves as a crucial driver for performance improvements.

G.2 Size of Candidate Pool M

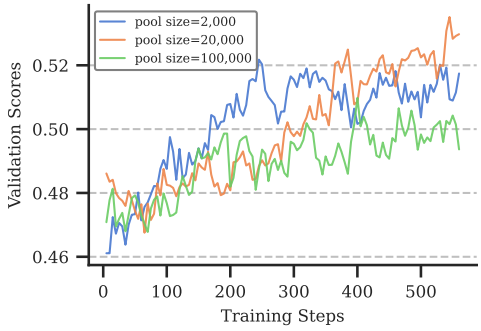

Figure 8: Validation scores for pool sizes $M = 2,000$ and $M = 20,000$ over training steps.

To analyze the sensitivity of our method to the pool size M, we compare the validation scores of $M \in \{2000, 20000, 100000\}$ in Figure 8. The results indicate a trade-off between convergence speed and final performance. A smaller pool ($M = 2,000$) accelerates early-stage training by encouraging rapid exploitation, whereas a moderately larger pool ($M = 20,000$) exhibits a slower initial learning curve but fosters more thorough exploration, ultimately yielding superior asymptotic performance. This suggests that M can be adjusted to balance computational efficiency against peak accuracy depending on the available budget. Notably, an excessively large pool ($M = 100,000$) leads to significant performance degradation. This observation empirically validates the necessity of the candidate pool mechanism itself, aligning with our discussion in Section 3.4 and corroborate the theoretical proofs provided in Appendix B.

H Artifact Licenses and Usage Compliance

This work strictly complies with the licenses and terms of service for all utilized artifacts. Our usage is confined to academic, non-commercial research.

- Permissive Licenses (MIT, Apache-2.0, CC-BY-4.0, ODC-By): We adhere to the attribution and open-source terms for DeepMath, TruthfulQA, SuperGPQA, AIME24, and AGIEval, from which Gaokao-Physics and Law-Analytics are derived. For AGIEval, we follow its MIT license for the repository and comply with the licenses and terms of the original datasets as required by AGIEval. Our codebase utilizes verl-agent and `math_verify` under the Apache-2.0 license.
- Non-Commercial Licenses (CC-BY-NC-4.0, CC-BY-NC-SA-4.0): We utilize AM-Math-Difficulty-RL, AIME25, HMMT (Feb 2025), BRUMO25, and CMIMC25 strictly for non-commercial research purposes, ensuring no violation of the "NC" (Non-Commercial) or "SA" (Share-Alike) clauses.

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Furthermore, we confirm that, to the best of our knowledge, none of the utilized datasets contain personally identifiable information (PII). All datasets are used solely for academic, non-commercial research and evaluation purposes, in accordance with their corresponding licenses and terms of use.

I Algorithm

Algorithm 1 summarizes our proposed method, **FlipTS**, which integrates dynamic Bayesian posterior estimation with targeted Thompson sampling to guide end-to-end RL for *Self-Refinement*. It maintains a candidate pool of questions and continuously updates belief parameters based on observed trajectory outcomes. By sampling trajectories whose estimated refinement potential is closest to a target probability ξ_{target}, **FlipTS** focuses training on the model's zone of proximal development. A refresh-and-cull mechanism ensures pool diversity and mitigates posterior staleness under non-stationary learning dynamics.

J Self-Refinement Prompt Template

Our self-refinement process is structured as a multi-turn dialogue. The specific prompt configurations for each stage of the process are detailed below.

System Message (All Turns) For every turn in the conversation (both initial generation and subsequent refinements), the system message remains

constant to enforce step-by-step reasoning and correct output formatting:

System:
```
Please reason step by step, and
put your final answer within
\boxed{}.
```

Turn 1: Initial Generation In the first turn, the user message consists solely of the math problem. The model generates the initial candidate solution based on this input.

User:
```
{question}
```

Subsequent Turns: Refinement For all subsequent turns, the user message follows a structured refinement template. This template feeds the problem and the previous candidate solution (which may be flawed) back into the model, instructing it to critique and improve the answer.

User:
```
You are given a math problem and
a potential solution.
This attempt may be flawed or
partial.
Correct any errors and build upon
its ideas to produce an improved,
high-quality solution.
If the original attempt is
unusable, generate a correct
solution from scratch.
End with the final result in
\boxed{}.
```

```
Problem:
{question}
```

```
Candidate solution (may contain
mistakes):
-- Candidate --
{candidate_answer}
```

```
Now refine the candidate into an
improved solution.
Provide clear reasoning and
end with the final answer in
\boxed{}.
```

Here, `{question}` is the original problem statement, and `{candidate_answer}` is the model's output from the immediate previous turn.

Algorithm 1: FlipTS

Input: Dataset $\mathcal{D}$; Batch size B; Decay factor λ; Target probability ξ_{target}; Warm-up steps W; Pool size M; Refresh threshold η; Culling fraction γ; Initial Policy π_{θ_0}.

Output: Optimized Policy π_{θ_N}

for $k = 1, 2, \ldots, N$ **do**
  `// Thompson Sampling for Questions`
  if $k \leq W$ **then**
    Sample batch $\mathcal{B}_k$ uniformly from $\mathcal{D}$;
  end
  else
    Sample a proxy probability $\tilde{\xi}_{k,q} \sim \text{Beta}(\alpha_{k,q}, \beta_{k,q})$ for each $q \in \mathcal{C}$;
    Select batch $\mathcal{B}_k$ as the top-B questions in $\mathcal{C}$ minimizing $|\tilde{\xi}_{k,q} - \xi_{target}|$ (Eq. 4);
  end
  `// RL Training (Trajectory-level GRPO)`
  Generate trajectories $\{\tau_i\}_{i=1}^{B \times G}$ using π_{θ_k} for questions in $\mathcal{B}_k$;
  Update θ_k to θ_{k+1} by maximizing $\mathcal{J}_{GRPO}(\theta)$ (Eq. 1);
  `// Bayesian Estimation`
  Count the number of trajectories with and without *F2S* transition s_q, f_q; compute $\hat{\xi}_q = s_q/G$;
  if $k = W$ **then**
    Compute prior parameters α_0, β_0 using the empirical probabilities $\{\hat{\xi}_q\}$ from the first W steps (Eq. 2); initialize $\alpha_{k,q} \leftarrow \alpha_0, \beta_{k,q} \leftarrow \beta_0, \forall q \in \mathcal{D}$;
  end
  if $k > W$ **then**
    Update $\alpha_{k+1,q} \leftarrow s_q + \lambda\alpha_{k,q}$, $\beta_{k+1,q} \leftarrow f_q + \lambda\beta_{k,q}$ for each $q \in \mathcal{B}_k$ (Eq. 3);
  end
  `// Dynamic Pool Mechanism`
  if $k = W$ **then**
    Initialize candidate pool $\mathcal{C}$ by uniformly sampling M questions from $\mathcal{D}$;
    Initialize observed questions in pool $\mathcal{O} = \emptyset$;
  end
  if $k > W$ **then**
    Update observed questions: $\mathcal{O} \leftarrow \mathcal{O} \cup \mathcal{B}_k$;
    if $|\mathcal{O}|/M \geq \eta$ **then**
      Calculate scores $S_q = \alpha_{k,q}/(\alpha_{k,q} + \beta_{k,q})$ for $q \in \mathcal{O}$;
      Identify cull set $\mathcal{Q}_{rem} \subset \mathcal{O}$ with $\lfloor \gamma M \rfloor$ questions with largest $|S_q - \xi_{\text{target}}|$;
      Uniformly sample a new set $\mathcal{Q}_{new}$ from $\mathcal{D} \setminus \mathcal{C}$ with $|\mathcal{Q}_{rem}| = |\mathcal{Q}_{new}|$;
      Update pool $\mathcal{C} \leftarrow (\mathcal{C} \setminus \mathcal{Q}_{rem}) \cup \mathcal{Q}_{new}$; $\mathcal{O} \leftarrow \mathcal{O} \setminus \mathcal{Q}_{rem}$
    end
  end
end
return π_{θ_N}