CDQuant: Greedy Coordinate Descent for Accurate LLM Quantization

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Abstract

Quantization has emerged as a key technique for compressing large models with minimal impact on performance. The recent GPTQ algorithm, a post-training quantization (PTQ) method, has proven highly effective for compressing LLMs, sparking a wave of research that leverages GPTQ as a core component. Recognizing the pivotal role of GPTQ in the PTQ landscape, we introduce CDQuant, a simple and scalable alternative to GPTQ with improved performance. CDQuant uses greedy coordinate descent to minimize the layer-wise reconstruction loss to achieve high-quality quantized weights. Our algorithm is easy to implement and scales efficiently to models with hundreds of billions of parameters. We perform extensive evaluation on Gemma, and PaLM2 model families, and demonstrate that CDQuant consistently outperforms GPTQ in 2-4 bit weight quantization. Moreover, CDQuant improves the performance of state-of-the-art PTQ techniques such as QuIP and FrameQuant when used as a replacement for their GPTQ component, resulting in further gains in quality.

1 Introduction

Large language models (LLMs) have shown remarkable ability to handle various language tasks [Touvron et al., 2023, OpenAI, 2023, Google, 2023], but their widespread adoption is hampered by their substantial computational and memory demands. To tackle this challenge, researchers have explored techniques like quantization, pruning, and distillation, with quantization being a particularly promising avenue for reducing model size and inference time without significantly sacrificing performance [Miao et al., 2023]. Quantization techniques broadly fall into two categories: post-training quantization (PTQ) and quantization-aware training (QAT). QAT, while potentially yielding better results, poses a significant challenge for LLMs due to the immense resources required to train these large models. As a result, a growing body of research has focused on PTQ, which is generally less computationally intensive and can be applied to large, pre-trained models.

A seminal work in PTQ for LLMs is GPTQ [Frantar et al., 2022], which introduced a one-shot weight quantization approach that minimizes the following layer-wise output reconstruction loss: $\|X(W - \hat{W})\|_F^2$, where $X = [\mathbf{x}_1, \mathbf{x}_2 \dots \mathbf{x}_n]$ represents the layer inputs and $W, \widehat{W} \in \mathbb{R}^{d_{\text{in}} \times d_{\text{out}}}$ are the original and quantized weight matrices, respectively. This method has sparked a wave of research in PTQ. Subsequent works have built upon GPTQ, addressing its limitations and improving it's performance. For instance, SpQR [Dettmers et al., 2023] and OWQ [Lee et al., 2023] improve GPTQ by explicitly addressing outlier weights, maintaining them in full precision, while quantizing the remaining weights using GPTQ. This hybrid approach improves quantization accuracy, particularly at lower bit-widths. Another line of research focuses on transforming the weight space before applying GPTQ. For instance, QuIP [Chee et al., 2024], FrameQuant [Adepu et al., 2024] transform the weights into a more quantization-friendly space and apply GPTQ in the transformed space. AWQ [Lin et al., 2023], SmoothQuant [Xiao et al., 2023] reduce the effect of activation outliers by performing feature scaling before quantizing the weights using standard techniques.

Given the central role GPTQ plays in the landscape of PTQ methods for LLMs, improving its quality directly translates to better quantization across numerous techniques that build upon it. Consequently, in this work, we revisit the core optimization problem addressed by GPTQ and introduce a novel coordinate descent based algorithm, CDQuant, to minimize the objective. CDQuant is an iterative optimization technique that greedily selects coordinates to descend along in each iteration. This approach contrasts with GPTQ, which cycles through the coordinates only *once*, in a predetermined order, often leading to suboptimal solutions for the layer-wise objective (see Section 2 for more details). CDQuant is simple to implement and scales effectively to models with billions of parameters. We further extend CDQuant to group/sub-channel quantization, a technique gaining popularity of late [Dettmers et al., 2021, Dettmers and Zettlemoyer, 2023].

Extensive experiments on Gemma [Mesnard et al., 2024], and PaLM2 [Anil et al., 2023] model families demonstrate that CDQuant consistently outperforms GPTQ across various model sizes and quantization precision levels (2-4 bits). Notably, for INT2 quantization of PaLM2-Otter, CDQuant achieves a 10% perplexity reduction compared to GPTQ. Furthermore, integrating CDQuant with QuIP [Chee et al., 2024] and FrameQuant [Adepu et al., 2024] improves their performance by $\sim 5\%$ for INT2 quantization of the Gemma-2 27B model, showcasing its potential as a drop-in replacement for GPTQ in existing PTQ techniques.

2 Related Work

The literature on quantization is huge. In this section, we only review the works that are related to quantization of large pre-trained models, and those that are related to our work.

GPTQ. Inspired by the Optimal Brain Surgeon (OBS) framework for pruning [LeCun et al., 1989], Optimal Brain Quantization (OBQ) [Frantar and Alistarh, 2022] emerged as a promising post-training quantization (PTQ) technique. However, OBQ remains computationally expensive and struggles to scale effectively to large language models (LLMs). This is because of a complicated step involved in OBQ which requires updating all the unquantized coordinates in each step (see Equation 2 in Frantar et al. [2022]). Implementing this step in GPUs is extremely slow because of the sheer number of gathers/scatters that need to be performed at each step. Frantar et al. [2022] subsequently introduced GPTQ, which employs heuristics to accelerate the OBQ algorithm. Specifically, GPTQ updates coordinates once in a cyclic manner rather than the greedy approach used in OBQ. This modification significantly speeds up the algorithm but comes at the cost of reduced quality. In our work, we address this performance drop by proposing greedy coordinate descent algorithms that are both straightforward and easy to implement.

Other Weight-only Quantization Techniques. Recent works on post-training quantization have sought to improve upon GPTQ. One popular strategy here is to identify and isolate problematic weights before applying GPTQ. For example, SpQR [Dettmers et al., 2023] isolates outlier weights and keeps them at full precision, quantizing only the remaining values using GPTQ. Similarly, OWQ [Lee et al., 2023] identifies weights that are highly sensitive to small perturbations and excludes them from the GPTQ quantization process. Another promising direction involves transforming the weights into a different basis before quantization. Both QuIP [Chee et al., 2024], and FrameQuant [Adepu et al., 2024] take this route and perform quantization in the transformed space using GPTQ. In all these approaches, we could potentially substitute GPTQ with CDQuant and expect to see further improvements in performance (see Section 4 for empirical evidence). Other promising methods such as AWQ [Lin et al., 2023], AffineQuant [Ma et al., 2024], suppress the effect of outliers in input activations by adjusting their scale and transferring it to weights. They then perform the standard MinMax quantization on the rescaled weights. One could replace MinMax with GPTQ, CDQuant to improve the performance of these techniques (see Table 2 for a comparison of CDQuant with AWQ). For further discussion on related works, please refer to Appendix C.

3 CDQuant

Notation. Throughout the paper, we denote vectors by bold faced letters ($\mathbf{a}$), and matrices by capital letters (A). $\|\mathbf{a}\|_2 = \sqrt{\sum_i \mathbf{a}_i^2}$ is the Euclidean norm and $\|A\|_F = \sqrt{\sum_{i,j} A_{ij}^2}$ is the Frobenius norm of a matrix. $\text{diag}(\mathbf{a})$ represents a diagonal matrix with $\mathbf{a}$ as its diagonal entries. $d_{\text{in}}, d_{\text{out}}$ denote the input, output dimensions of a layer. $W \in \mathbb{R}^{d_{\text{in}} \times d_{\text{out}}}$ is the weight matrix of the layer, and

$X = [\mathbf{x}_1, \mathbf{x}_2 \dots \mathbf{x}_n] \in \mathbb{R}^{n \times d_{\text{in}}}$ is the matrix containing n datapoints that are sent as input to the layer. $H = X^T X$ is the Hessian matrix for our objective in Equation (1). c denotes the number of bits of precision used in quantization. In quantization, we aim to represent W as $Q \times \text{diag}(\mathbf{a}) + \mathbf{1}\mathbf{b}^T$, where $\mathbf{a}, \mathbf{b} \in \mathbb{R}^{d_{\text{out}}}$ represent the scale and bias parameters and $Q \in \{0, 1, \dots 2^c - 1\}^{d_{\text{in}} \times d_{\text{out}}}$ is the quantized matrix.

Many existing PTQ techniques, including GPTQ, aim to solve the following layer-wise optimization objective: $\min_{\mathbf{a},\mathbf{b},Q} \|X(W - Q \times \text{diag}(\mathbf{a}) - \mathbf{1}\mathbf{b}^T)\|_F^2$. Observe that this problem breaks down into d_{out} independent problems across the output dimension. So, in the sequel, we focus on the following problem of quantizing a d_{in}-dimensional vector

$$\min_{a,b,\mathbf{q}} \|X(\mathbf{w} - a\mathbf{q} - b)\|_2^2. \tag{1}$$

For a fixed (a, b), this problem is called *Integer Linear Regression* problem. It turns out, finding optimal solutions to this problem is NP-hard [Chrétien and Corset, 2009, Park and Boyd, 2018]. So, several works have designed heuristics to solve this problem [Nagel et al., 2020, Li et al., 2021, Frantar and Alistarh, 2022, Frantar et al., 2022, Hubara et al., 2021]. Within the context of LLMs, GPTQ is perhaps the most popular among these techniques, as it scales efficiently to models with billions of parameters [Frantar et al., 2022]. In this work, we aim to improve upon GPTQ by designing better heuristics, while maintaining its scalability aspect. To this end, we rely on performing coordinate descent on objective (1), which we describe below.

3.1 Greedy Coordinate Descent

In this section, we assume we have suitable values for scale (a) and bias (b) parameters already at hand, and focus on optimizing $\mathbf{q}$. For a more in-depth discussion of how we determine these values, please refer to the final part of the section. As the name suggests, in greedy coordinate descent, at each round, we find the coordinate that leads to the biggest reduction in the objective and descend along that coordinate. Letting $\mathcal{L}(\mathbf{q}) := \|X(\mathbf{w} - a\mathbf{q} - b)\|_2^2$ be the objective in Equation (1), we try to find a coordinate i and value r, such that updating the ith coordinate to r gives the biggest reduction in loss

$$\min_{i,r} \mathcal{L}(\mathbf{q} + (r - q_i)\mathbf{e}_i) - \mathcal{L}(\mathbf{q}),$$

where q_i is the ith element of $\mathbf{q}$, and $\mathbf{e}_i$ is the standard basis vector with 1 in i position and 0 everywhere else. Luckily for us, this can be implemented extremely efficiently as we have analytical expressions for the objective. In particular, one can easily show that

$$\mathcal{L}(\mathbf{q} + (r - q_i)\mathbf{e}_i) - \mathcal{L}(\mathbf{q}) = (r - q_i)^2 H_{i,i} + (r - q_i)\mathbf{g}_i,$$

where $H, \mathbf{g}$ are the Hessian and gradient of $\mathcal{L}$ evaluated at $\mathbf{q}$; this follows from the fact that $\mathcal{L}$ is a quadratic function in $\mathbf{q}$. Algorithm 1 describes this procedure. Observe that *line 5* is the only computationally non-trivial step of the algorithm. It requires finding the minimum value among $d_{\text{in}} \times 2^c$ possibilities, a task that is well-suited for parallelization on GPUs.

Extension to Block Coordinate Descent. A natural extension to Algorithm 1 is block coordinate descent (BCD), where multiple coordinates are updated simultaneously in each iteration. While a greedy approach to BCD could, in principle, optimize the objective much better, the computational cost becomes prohibitive. Specifically, updating k coordinates at a time necessitates evaluating $(d_{\text{in}} \times 2^c)^k$ possible combinations of coordinates and their corresponding values. To address this, we propose a randomized BCD strategy (see Algorithm 2 in Appendix), which partitions the coordinates into random d_{in}/k blocks and searches only over these blocks. This significantly reduces the search space to a more manageable $d_{\text{in}}/k \times 2^{kc}$ possibilities, making the algorithm practical for larger models. In our experiments, we primarily use $k = 2$ and $c \in \{2, 3, 4\}$.

Initializing $a, b, \mathbf{q}$. To initialize $a, b, \mathbf{q}$ in Algorithms 1, 2, we introduce a technique called Optimal Weight Clipping (OWC), which draws inspiration from the Learnable Weight Clipping (LWC) mechanism used in OmniQuant [Shao et al., 2023]. In OWC, we quantize weight $\mathbf{w}$ as follows

$$\mathbf{q} = \text{clamp}\left(\left\lfloor \frac{\mathbf{w} - b}{a} \right\rceil, 0, 2^c - 1\right), \quad a = \frac{\gamma(\max(\mathbf{w}) - \min(\mathbf{w}))}{2^c - 1}, \quad b = \min(\mathbf{w}). \tag{2}$$

Algorithm 1 Greedy Coordinate Descent (CD)

1: **Input:** T - coordinate descent steps, X - input data matrix, $\mathbf{w}$ - vector to be quantized, a - scale, b - bias, $\mathbf{q}_0$ - initial estimate
2: Compute Hessian H as: $H \leftarrow X^T X$
3: Compute gradient $\mathbf{g}$ as: $\mathbf{g} \leftarrow 2H(\mathbf{q}_0 - a^{-1}(\mathbf{w} - b))$
4: **for** $t \in [1 : T]$ **do**
5: Find the coordinate that leads to the largest reduction in loss

$$i^*, r^* = \underset{i \in \{0,1,\ldots d_{\text{in}}-1\}, r \in \{0,1,\ldots 2^c-1\}}{\arg\min} (r - q_{t-1,i})^2 H_{i,i} + (r - q_{t-1,i})\mathbf{g}_i$$

6: Update gradient $\mathbf{g}$ as

$$\mathbf{g} \leftarrow \mathbf{g} + 2(r^* - q_{t-1,i^*})H_{i^*,\cdot},$$

where $H_{i^*,\cdot}$ is the i^* column of H
7: Update $\mathbf{q}_{t-1}$ as

$$\mathbf{q}_t \leftarrow \mathbf{q}_{t-1} + (r^* - q_{t-1,i^*})\mathbf{e}_{i^*},$$

where $\mathbf{e}_{i^*}$ is the standard basis vector with 1 in i^* position and 0 everywhere else
8: **end for**

Here, $\gamma \in [0, 1]$ represents the clipping strength. We determine the optimal γ by minimizing the following layer-wise loss:

$$\min_{\gamma \in [0,1]} \|X(\mathbf{w} - a\mathbf{q} - b)\|_2^2.$$

Note that while not explicitly stated, both $\mathbf{q}, a$ in the above objective are implicitly dependent on γ. This optimization can be efficiently solved using a simple grid search. In contrast, LWC [Shao et al., 2023] optimizes a different objective function, focusing on end-to-end quantization of entire transformer block using gradient-based techniques. It is worth noting that setting $\gamma = 1$ in OWC recovers the widely used MinMax quantization scheme, which is used in many existing quantization methods, including GPTQ [Frantar et al., 2022], SmoothQuant[Xiao et al., 2023]. However, MinMax quantization is susceptible to outlier weights, and a smaller γ often yields superior results. In our experiments, we observed that OWC provides a much better initialization compared to MinMax quantization, for both GPTQ and CDQuant, suggesting its broader applicability.

3.2 Extension to Sub-channel Quantization

In this section, we consider sub-channel (or group) quantization, which is a more fine-grained quantization that divides the weight vector $\mathbf{w}$ into multiple groups and assigns a quantization scale to each group [Dettmers et al., 2021, Dettmers and Zettlemoyer, 2023]. Letting g be the group size, we divide weight $\mathbf{w}$ into d_{in}/g groups $\{\mathbf{w}^{(0)}, \mathbf{w}^{(1)}, \ldots \mathbf{w}^{(d_{\text{in}}/g-1)}\}$ each of size g, and quantize $\mathbf{w}^{(i)}$ as $a^{(i)}\mathbf{q}^{(i)} + b^{(i)}$. To learn the optimal parameters for this sub-channel quantization, we solve the following optimization problem:

$$\min_{\{a^{(i)}, b^{(i)}, \mathbf{q}^{(i)}\}_{i=0}^{d_{\text{in}}/g-1}} \left|\left| \sum_{i=0} X^{(i)}(\mathbf{w}^{(i)} - a^{(i)}\mathbf{q}^{(i)} - b^{(i)}) \right|\right|_2^2. \tag{3}$$

Here, $X^{(i)}$ represents the columns of X corresponding to the indices within group i. To solve this optimization problem, we employ a coordinate descent approach, similar to Algorithms 1 and 2 described earlier. That is, given initial values for the scaling and bias parameters, we iteratively optimize the quantized representation $\mathbf{q}$ using coordinate descent. Due to space constraints, we present the resulting algorithms in Appendix F (see Algorithms 3, 4).

Initialization. Next, we tackle the initialization of parameters $\{a^{(i)}, b^{(i)}, \mathbf{q}^{(i)}\}_{i=0}^{d_{\text{in}}/g-1}$ for our coordinate descent procedure. Our approach draws inspiration from the OWC algorithm described above, adapting its core idea to this problem. In essence, we reframe the initialization problem as one of selecting optimal clipping strengths $\{\gamma^{(i)}\}_{i=0}^{d_{\text{in}}/g-1}$ for groups $\{0, \ldots d_{\text{in}}/g - 1\}$. This leads us to the following problem

$$\min_{\gamma^{(0)}, \ldots \gamma^{(d_{\text{in}}/g-1)}} \left|\left| \sum_{i=0} X^{(i)}(\mathbf{w}^{(i)} - a^{(i)}\mathbf{q}^{(i)} - b^{(i)}) \right|\right|_2^2, \tag{4}$$

where

$$\mathbf{q}^{(i)} = \text{clamp}\left(\left\lfloor \frac{\mathbf{w}^{(i)} - b^{(i)}}{a^{(i)}} \right\rceil, 0, 2^c - 1\right),\ a^{(i)} = \frac{\gamma^{(i)}(\max(\mathbf{w}^{(i)}) - \min(\mathbf{w}^{(i)}))}{2^c - 1},\ b^{(i)} = \min(\mathbf{w}^{(i)}).$$

While not explicitly stated, both $\mathbf{q}^{(i)}, a^{(i)}$ implicitly depend on the clipping strength $\gamma^{(i)}$. We use greedy coordinate descent to optimize Equation (4). In each iteration, we update the $\gamma^{(i)}$ that leads to biggest drop in loss (lines 10-14 of Algorithm 5). This procedure, which we call OWC-CD, is described in Algorithm 5 in Appendix.

4 Experiments

In this section, we first establish that CDQuant achieves superior quantization quality compared to GPTQ. Subsequently, we show its versatility by successfully integrating it as a drop-in replacement for GPTQ within existing PTQ methods such as AWQ [Lin et al., 2023], QuIP [Chee et al., 2024], FrameQuant [Adepu et al., 2024].

Setup. We first present the experimental setup and then move on to our results. In all the experiments in this section, the attention layers are quantized to INT8 using MinMax quantization, and low-bit (2-4 bits) quantization schemes are only applied to the feed-forward layers (FFN). See Appendix H.3 for experiments with quantized attention and FFN layers. Our experiments leverage two families of language models: the open-source Gemma models [Mesnard et al., 2024] and the proprietary PaLM 2 models [Anil et al., 2023].

Baselines. Since our primary goal is to demonstrate that CDQuant gets improved performance over GPTQ, we have chosen it as the primary baseline in most of our experiments. To further demonstrate the our technique can be used as a plug-and-play replacement for GPTQ, we also include comparisons with AWQ, QuIP, FrameQuant. For all our experiments, we initialize GPTQ with OWC, and run GPTQ for $T = d_{\text{in}}$ steps. To ensure stability and generalization, GPTQ regularizes its Hessian matrix by adding a scaled identity matrix (λI). Tuning this λ for every (model, layer) pair is infeasible. So, we determine a single optimal value using the PaLM2-Gecko model, and apply it universally.

CDQuant. We evaluate both the coordinate descent variants described in Section 3.1: CD (Algorithm 1), BCD (Algorithm 2). For per-channel quantization, we initialize CD with OWC, and for sub-channel quantization, we additionally include initialization with OWC-CD. BCD is always initialized with CD in our experiments. Unless otherwise stated, both CD and BCD are run for $T = d_{\text{in}}$ iterations, OWC-CD is run for d_{in}/g iterations, where g is the group size. Similar to GPTQ, we regularize the Hessian matrix used in CD, BCD by adding λI. We determine a reasonable value for λ using the PaLM2-Gecko model, and use it in all our experiments.

Evaluation. Following recent works [Frantar et al., 2022, Ma et al., 2024], we evaluate all algorithms using two key metrics: perplexity and downstream task performance. For Gemma models, following Frantar et al. [2022], we calculate perplexity on C4's [Raffel et al., 2019] validation set. For PaLM2 models, we calculate perplexity using a 100 million token subset derived from the PaLM2 training mixture. Please refer to Appendix H.2 for further details on the downstream evaluations.

Training. All techniques are calibrated using 1280 data points, where each data point has 2048 tokens. For OWC, we use a grid size of 50 to find the most optimal γ. We used 8 Nvidia H100 GPUs for quantizing the models.

4.1 Results

Comparison with GPTQ. Table 1 presents the INT2 perplexity numbers for different quantization techniques applied to FFN layers. It can be seen that both our CD and BCD methods have a clear advantage over GPTQ, leading to lower perplexity scores for all models and quantization levels. For example, on PaLM-2 Otter, we see almost 10% improvement in perplexity over GPTQ. For results on 3,4 bit quantization and downstream evaluations, please refer to Appendix H.1, H.2, H.3. We also observe that coordinate descent techniques are better at optimizing the layer-wise objective in Equation (1), than GPTQ. For instance, the average objective value (relative to all 0's solution) for the GPTQ solution for the 1st feed-forward layer is 0.164, whereas for CD it is 0.158, and for BCD(k=2) it is 0.157.

Table 1: Table presents the perplexity evaluations for GPTQ, CD, BCD for INT2 quantization of FFN weights. wx, ay, gz in the config column corresponds to x-bit weights, y-bit activations and group/sub-channel size of z.

Method	Epochs	Gemma-1 7B	Gemma-2 9B	Gemma-2 27B	PaLM2-Otter	PaLM2-Bison
w16a16		10.384	10.683	8.682	5.984	5.298
w2a16g128						
GPTQ	-	375.153	13.785	12.181	10.816	7.230
CD	1	75.55	13.709	11.966	9.917	7.123
BCD(k=2)	1	**68.732**	**13.662**	**11.873**	**9.822**	**7.094**

Table 2: Table presents the perplexity evaluations for GPTQ, CD, BCD for INT3 quantization of FFN layers with AWQ.

Method	Gemma-1 7B		Gemma-2 9B		PaLM2-Otter	
	w3a16	w3a16g128	w3a16	w3a16g128	w3a16	w3a16g128
GPTQ + AWQ	25.695	14.539	11.331	11.158	7.381	6.519
CD + AWQ	19.703	13.254	**11.255**	11.123	7.244	6.475
BCD(k=2) + AWQ	**18.137**	**13.205**	11.265	**11.119**	**7.211**	**6.474**

Versatility of CDQuant. A major strength of our algorithm is its versatility. It seamlessly replaces GPTQ in any quantization technique that relies on it. To illustrate this, we focus on the AWQ, QuIP and FrameQuant (the latter two are state-of-the-art PTQ techniques). We demonstrate that our algorithm, when layered on top of the aforementioned algorithms, surpasses the performance of GPTQ layered on top of them. Tables 2, 3 present the result from this experiment. It can be seen that both CD and BCD provide 5% boost in performance for QuIP and FrameQuant.

Runtime. Table 4 shows the quantization runtime of GPTQ, CD and BCD on Gemma-2 27B model using 8 H100s. For attention weight quantization, CD is comparable to GPTQ in speed, while BCD is $2\times$ slower. For FFN1 (FFN2) quantization, CD is $5\times$ ($2\times$) slower than GPTQ, whereas BCD is an order of magnitude slower than GPTQ. For quantization of the entire model, CD is $2\times$ slower than GPTQ (because quantizing FFN2 takes up most of the time). In Appendix H.4, we provide simple strategies to speed up both CD and BCD.

Table 4: Runtime comparison of CDQuant, GPTQ for Gemma-2 27B. Note that FFN1 and FFN1-Gate have the exact same runtimes for all compared methods, hence we report only FFN1's runtime.

Config	Method	8 H100s		
		FFN1 (mins.)	FFN2 (mins.)	Attn. (mins.)
w3a16	GPTQ	0.3	7.37	1.18
	CD	1.23	14.1	0.51
	BCD(k=2)	6.03	54.94	2.11
w4a16	GPTQ	0.3	7.35	1.18
	CD	1.65	17.87	0.64
	BCD(k=2)	14.54	133.34	4.64

5 Conclusion and Future Work

In this work, we developed a coordinate descent framework (CDQuant) for quantization of LLMs. CDQuant is a simple and effective alternative to GPTQ, that consistently outperformed it on PaLM2 models. The simplicity of our algorithm makes it a seamless substitute for GPTQ in various algorithmic contexts where GPTQ currently functions as a sub-routine. Our future work aims to further improve the performance of CDQuant. In particular, we aim to speed up our BCD algorithm, and make it as fast as CD. Furthermore, we will focus on developing layer-wise loss functions that are more closely aligned with end-to-end loss, thereby reducing the performance gap between full-precision and quantized models.

Table 3: Table presents the perplexity evaluations for INT2 per-channel quantization of FFN weights with QuIP and FrameQuant where GPTQ, CD and BCD and used as subroutines.

	Method	Gemma-1 7B	Gemma-2 9B	Gemma-2 27B
	w16a16	10.384	10.683	8.682
QuIP	GPTQ	25.941	13.695	11.909
	CD	24.025	13.172	**11.479**
	BCD(k=2)	**19.167**	**13.144**	11.547
FrameQuant	GPTQ	26.262	12.941	10.958
	CD	19.726	12.748	10.785
	BCD(k=2)	**18.242**	**12.674**	**10.608**

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A Broader Impact

We introduce a Coordinate Descent based approach, CDQuant, for compressing Large Language Models. Our method uses only a small amount of data for calibration. We do not foresee any ethical implications arising from the technical aspects of our approach. However, compressing LLMs may give rise to bias effects, a study of which seems essential given the extensive use of LLMs. Our work may be of assistance to such studies. Also, since quantization allows for easier deployment of LLMs, it could have potential societal implications which seem difficult to predict.

B Limitations

- While both CD and BCD outperformed GPTQ in our experiments, BCD achieves slightly better performance than CD. However, BCD is not as fast as CD and can be expensive for large models. In future, we aim to develop techniques to speed up BCD.
- Our algorithms still don't bridge the gap between QAT and PTQ, especially on smaller models. To bridge this gap, we believe one should move away from the ℓ_2^2 surrogate loss that is being considered by most of the existing work. Instead, we should design surrogate losses that are more closely aligned with end-to-end loss.

C Related Work

Detailed comparison with OBQ. At a high level, both OBQ, and our technique use greedy strategies to quantize the weights. However, OBQ is extremely slow and doesn't even scale to models with a few million parameters. The primary reason for this unreasonable slowness is because of a more complicated step involved in OBQ which requires updating all the unquantized coordinates in each step (please refer to equation 2 in Frantar and Alistarh [2022]). Implementing this step in GPUs is extremely slow because of the sheer number of gathers/scatters that need to be performed at each step. In contrast, CDQuant doesn't have this step. At each step of coordinate descent, we only update the single coordinate that leads to the biggest drop in performance. As a result, CDQuant is orders of magnitude faster than OBQ. To quantify this last statement, note that GPTQ paper only ran OBQ for models with a few hundred millions of parameters and showed that OBQ is almost 120x slower than GPTQ (note that this difference will only grow with larger models because OBQ will require even more gathers/scatters). In fact, in the OBQ paper [Frantar and Alistarh, 2022], in Table 6, OBQ takes 65 minutes to quantize a ResNet50 model with 25M parameters. That being said, scaling it to models with billions of parameters seems extremely difficult. In contrast, CD is only 5x slower than GPTQ, and gives improved performance (see our results for INT2 quantization on PaLM2 and Gemma). While BCD is 28x slower than GPTQ, its slowness mainly comes from FFN2. To overcome this, one could use CD for FFN2 and use BCD for the rest of the layers.

Detailed comparison with GPTQ. GPTQ, as discussed in Section 2, is a heuristic designed to accelerate OBQ by replacing its greedy strategy with a single, predetermined cycle through all coordinates. While this approach improves speed, it often sacrifices accuracy. Although multiple cycles could potentially mitigate this performance drop, our experiments revealed no significant improvement. In contrast, we propose a simple yet effective greedy coordinate descent strategy that achieves superior accuracy while maintaining the computational efficiency of GPTQ.

QuantEase: While working on our paper, we came across QuantEase [Behdin et al., 2023], a parallel research effort sharing similar goals as ours for improving GPTQ. While both methods leverage the concept of coordinate descent, QuantEase adopts a cyclic approach for weight updates, whereas our work employs a greedy strategy. Although our experiments (Appendix D) indicate comparable performance between the two methods, our work extends beyond QuantEase by introducing specialized algorithms for group/sub-channel quantization, not just full-channel quantization. Additionally, we develop novel block coordinate descent algorithms that further improve the performance, and integrate CDQuantinto SOTA methods like AWQ, QuIP and FrameQuant. That being said, both these works (QuantEase and ours) collectively highlight the potential of coordinate descent algorithms in outperforming GPTQ.

Weight+Activation Quantization. SmoothQuant [Xiao et al., 2023], OS+[Wei et al., 2023, 2022] have a similar flavour as AWQ, but quantize both weights and activations after scaling them ap-

propriately. OmniQuant [Shao et al., 2023] performs quantization of the entire transformer block in a single shot. This encompasses both activation and weight quantization. Furthermore, it subsumes SmoothQuant, OS+ by using both feature scaling and outlier suppression. QLLM [Liu et al., 2023] tackles the issue of outliers in the activations by splitting the outlier features into multiple sub-channels and then recombining them, effectively reducing their influence. QLLM also incorporates a fine-tuning step at the end, introducing low-rank weights into each layer of the LLM. LLM.int8() [Dettmers et al., 2022] quantizes both acativations and weights to 8-bits and also identifies outliers and stores them in full precision.

D Comparison to QuantEase

In this section, we compare our per-channel quantization results with those of QuantEase, a parallel study to ours. It's important to note that QuantEase does not have a publicly available implementation. So, we implemented the cyclic coordinate descent strategy used by QuantEase, and used the same initialization and regularization strength as our algorithms (although the QuantEase paper doesn't provide these details). We then ran QuantEase for the recommended number of iterations specified in the paper (20 epochs or $T = 20d_{\text{in}}$ iterations). The findings are presented in Table 5. On PaLM2-Gecko, PaLM2-Otter, and PaLM2-Bison, both these approaches have similar performance. These results collectively highlight the potential of coordinate descent algorithms in outperforming GPTQ, especially for low bit quantization.

Table 5: Comparison of QuantEase with CD and BCD for FFN quantization.

Config	Method	PaLM2-Gecko	PaLM2-Otter	PaLM2-Bison
w3a16	GPTQ	11.347	7.176	5.774
	QuantEase (epochs=20)	10.731	6.996	5.741
	CD	10.920	7.002	5.739
	BCD(k=2)	10.898	6.979	5.733
w4a16	GPTQ	8.764	6.249	5.417
	QuantEase (epochs=20)	8.670	6.197	5.408
	CD	8.694	6.195	5.407
	BCD(k=2)	8.691	6.192	5.405

E CDQuant for per-channel quantization

In this section, we present the block coordinate descent algorithm for per-channel quantization.

Algorithm 2 Block Coordinate Descent with Random Blocks (BCD)

1: **Input:** T - coordinate descent steps, k - block size, X - input data matrix, $\mathbf{w}$ - vector to be quantized, a - scale, b - bias, $\mathbf{q}_0$ - initial estimate
2: Compute Hessian H as: $H \leftarrow X^T X$
3: Compute gradient $\mathbf{g}$ as: $\mathbf{g} \leftarrow 2H(\mathbf{q}_0 - a^{-1}(\mathbf{w} - b))$
4: **for** $t \in [1 : T]$ **do**
5: Randomly partition the set $\{0, 1, \ldots d_{\text{in}} - 1\}$ into d_{in}/k blocks, each of size k
6: Find the block that leads to the largest reduction in loss

$$i^*, r^* = \underset{\substack{i \in \{0,1,\ldots d_{\text{in}}/k-1\}, \\ r \in \{0,1,\ldots 2^c-1\}^k}}{\arg\min} (r - \mathbf{q}_{t-1,i})^T H_{i,i} (r - \mathbf{q}_{t-1,i}) + (r - \mathbf{q}_{t-1,i})^T \mathbf{g}_i,$$

where $\mathbf{q}_{t-1,i}, H_{i,i}$ are the sub-vector, sub-matrix of $\mathbf{q}_{t-1}, H$ corresponding to block i.
7: Update gradient $\mathbf{g}$ as

$$\mathbf{g} \leftarrow \mathbf{g} + 2H_{i^*,\cdot}(r^* - \mathbf{q}_{t-1,i^*}),$$

8: Update $\mathbf{q}_{t-1}$ as

$$\mathbf{q}_{t-1,i^*} \leftarrow r^*, \quad \mathbf{q}_t \leftarrow \mathbf{q}_{t-1},$$

9: **end for**

F CDQuant for sub-channel quantization

In this section, we extend CDQuant to sub-channel quantization. To simplify the explanation in this section, we introduce a slightly modified notation. Given a weight vector $\mathbf{w}$, we represent it's sub-channel quantization as $\mathbf{a} \odot \mathbf{q} + \mathbf{b}$, where $\odot$ is the elementwise multiplication, and $\mathbf{a}, \mathbf{b} \in \mathbb{R}^{d_{\text{in}}}$ are the scale, and bias parameters that satisfy the following constraints: $\mathbf{a}_k = \mathbf{a}_l, \mathbf{b}_k = \mathbf{b}_l$, for any two indices k, l that fall in the same group. With this notation, for any given $\mathbf{a}, \mathbf{b}$, the optimization problem in Equation (3) can be rewritten as

$$\min_{\mathbf{q}} \|X(\mathbf{w} - \mathbf{a} \odot \mathbf{q} - \mathbf{b})\|_2^2. \tag{5}$$

Letting $D_{\mathbf{a}} = \text{diag}(\mathbf{a}), \tilde{X} = XD_{\mathbf{a}}, \tilde{\mathbf{w}} = D_{\mathbf{a}}^{-1}\mathbf{w}, \tilde{\mathbf{b}} = D_{\mathbf{a}}^{-1}\mathbf{b}$, the above problem can be further rewritten as

$$\min_{\mathbf{q}} \|\tilde{X}(\tilde{\mathbf{w}} - \mathbf{q} - \tilde{\mathbf{b}})\|_2^2. \tag{6}$$

Observe that this problem is the same as the per-channel quantization problem described in Equation (1), but with modified parameters $\tilde{X}, \tilde{\mathbf{w}}, \tilde{\mathbf{b}}$. So extending CDQuant to sub-channel quantization simply involves running Algorithms 1, 2 with these modified parameters. Algorithms 3, 4 present the resulting algorithms.

Algorithm 3 Greedy Coordinate Descent (CD)

Input: T - coordinate descent steps, X - input data matrix, $\mathbf{w}$ - vector to be quantized, a - scale, b - bias, $\mathbf{q}_0$ - initial estimate
$\tilde{X} \leftarrow X\text{diag}(\mathbf{a}), \tilde{\mathbf{w}} = \text{diag}(\mathbf{a})^{-1}\mathbf{w}, \tilde{\mathbf{b}} = \text{diag}(\mathbf{a})\mathbf{b}$
Compute Hessian H as: $H \leftarrow \tilde{X}^T\tilde{X}$
Compute gradient $\mathbf{g}$ as: $\mathbf{g} \leftarrow 2H(\mathbf{q}_0 - (\tilde{\mathbf{w}} - \tilde{\mathbf{b}}))$
for $t \in [1 : T]$ **do**
    Find the coordinate that leads to the largest reduction in loss

$$i^*, r^* = \underset{i \in \{0,1,\ldots d_{\text{in}}-1\}, r \in \{0,1,\ldots 2^c-1\}}{\arg\min} (r - \mathbf{q}_{t-1,i})^2 H_{i,i} + (r - \mathbf{q}_{t-1,i})\mathbf{g}_i$$

    Update gradient $\mathbf{g}$ as

$$\mathbf{g} \leftarrow \mathbf{g} + 2(r^* - \mathbf{q}_{t-1,i^*})H_{i^*,\cdot},$$

    where $H_{i^*,\cdot}$ is the i^* column of H
    Update $\mathbf{q}_{t-1}$ as

$$\mathbf{q}_t \leftarrow \mathbf{q}_{t-1} + (r^* - \mathbf{q}_{t-1,i^*})\mathbf{e}_{i^*},$$

    where $\mathbf{e}_{i^*}$ is the standard basis vector with 1 in i^* position and 0 everywhere else
end for

G Coordinate Descent for Optimal Weight Clipping

In this section, we present the OWC based initialization for initializing CD, BCD for sub-channel quantization.

Algorithm 4 Block Coordinate Descent with Random Blocks (BCD)

Input: T - coordinate descent steps, k - block size, X - input data matrix, $\mathbf{w}$ - vector to be quantized, a - scale, b - bias, $\mathbf{q}_0$ - initial estimate

$\tilde{X} \leftarrow X\text{diag}(\mathbf{a}), \tilde{\mathbf{w}} = \text{diag}(\mathbf{a})^{-1}\mathbf{w}, \tilde{\mathbf{b}} = \text{diag}(\mathbf{a})\mathbf{b}$

Compute Hessian H as: $H \leftarrow \tilde{X}^T\tilde{X}$

Compute gradient $\mathbf{g}$ as: $\mathbf{g} \leftarrow 2H(\mathbf{q}_0 - (\tilde{\mathbf{w}} - \tilde{\mathbf{b}}))$

for $t \in [1 : T]$ **do**

  Randomly partition the set $\{0, 1, \ldots d_{\text{in}} - 1\}$ into d_{in}/k blocks, each of size k

  Find the block that leads to the largest reduction in loss

$$i^*, r^* = \underset{\substack{i\in\{0,1,\ldots d_{\text{in}}/k-1\},\\ r\in\{0,1,\ldots 2^c-1\}^k}}{\arg\min} (r - \mathbf{q}_{t-1,i})^T H_{i,i}(r - \mathbf{q}_{t-1,i}) + (r - \mathbf{q}_{t-1,i})^T \mathbf{g}_i,$$

  where $\mathbf{q}_{t-1,i}, H_{i,i}$ are the sub-vector, sub-matrix of $\mathbf{q}_{t-1}, H$ corresponding to block i.

  Update gradient $\mathbf{g}$ as

$$\mathbf{g} \leftarrow \mathbf{g} + 2H_{i^*,\cdot}(r^* - \mathbf{q}_{t-1,i^*}),$$

  Update $\mathbf{q}_{t-1}$ as

$$\mathbf{q}_{t-1,i^*} \leftarrow r^*, \quad \mathbf{q}_t \leftarrow \mathbf{q}_{t-1},$$

end for

Algorithm 5 Coordinate Descent for Optimal Weight Clipping (OWC-CD)

Input: T - coordinate descent steps, g - group size, X - input data matrix, $\mathbf{w}$ - weight vector, Γ- grid of possible values for clipping strength

for $\beta \in \Gamma$ **do**

  for $i \in [0 : d_{\text{in}}/g - 1]$ **do**

    Compute quantization residual $\Delta(i, \beta)$ for group i with clipping strength β as:

$$\Delta(i, \beta) \leftarrow \mathbf{w}^{(i)} - a^{(i)}(\beta)\mathbf{q}^{(i)}(\beta) - b^{(i)}(\beta)$$

  . where $a^{(i)}(\beta), b^{(i)}(\beta), \mathbf{q}^{(i)}(\beta)$ are as defined in Equation (2).

  end for

end for

Initialize clipping strengths for each group $\gamma^{(0)}, \ldots \gamma^{(d_{\text{in}}/g-1)}$

Compute Hessian H as: $H \leftarrow X^T X$

$\mathbf{v}_i \leftarrow -2H_i(a\mathbf{q} + b - w)$ where H_i is the sub-matrix of H corresponding to the columns of group i.

for $t \in [1 : T]$ **do**

  Find the group that leads to the largest reduction in loss

$$i^*, \beta^* = \underset{\substack{i\in\{0,1,\ldots d_{\text{in}}/g-1\},\\ \beta\in\Gamma}}{\arg\min} (\Delta(i, \gamma^{(i)}) - \Delta(i, \beta))^T H_{i,i}(\Delta(i, \gamma^{(i)}) - \Delta(i, \beta)) + \mathbf{v}_i^T(\Delta(i, \gamma^{(i)}) - \Delta(i, \beta))$$

  where $H_{i,i}$ is the block diagonal element of H corresponding to group i.

  Update $\mathbf{v} \leftarrow \mathbf{v} + 2(\Delta(i^*, \gamma^{(i^*)}) - \Delta(i^*, \beta^*))^T H_{i^*}$

  Update $\gamma^{(i^*)} \leftarrow \beta^*$

end for

H Additional Experimental Results

This section presents additional experimental results:

1. In Appendix H.1, we present results for 3,4 bit quantization with comparisons of various techniques on evaluation perplexity .

2. In Appendix H.2, we present downstream evaluation results on various datasets. In particular, we use TriviaQA [Joshi et al., 2017], SQuAD [Rajpurkar et al., 2018], NaturalQuestions [Kwiatkowski et al., 2019] and WebQuestions [Berant et al., 2013] to evaluate *generation* capabilities of the quantized models, and to evaluate their *reasoning* capabilities, we test on ARC-c, ARC-e [Clark et al., 2018], HellaSwag [Zellers et al., 2019], BoolQ [Clark et al., 2019], PIQA [Bisk et al., 2020] and WinoGrande [Sakaguchi et al., 2020].
3. In Appendix H.3, we present results for the setting where we quantize both attention and FFN layers.
4. In Appendix H.4, we present runtime numbers for CD and BCD algorithms

H.1 $3, 4$ bit FFN Quantization

We use the same experimental setup as described in Section 4. Table 6 present the eval perplexity numbers of various techniques for $3, 4$ bit quantization of FFN weights. It can be seen that both our CD and BCD methods have a clear advantage over GPTQ, leading to lower perplexity scores for all models and quantization levels. In particular, we achieve significant quality improvements for quantization of smaller models such as Gemma-1 7B, and PaLM2 Gecko.

Table 6: Table presents the perplexity evaluations for GPTQ, CD, BCD for INT3, INT4 quantization of FFN weights.

Config	Method	Gemma-1 7B	Gemma-2 9B	Gemma-2 27B	PaLM2 Gecko	PaLM2 Otter	PaLM2 Bison
w16a16	-	10.348	10.683	8.682	7.948	5.984	5.298
w3a16	OWC	$2.885e4$	11.666	11.823	12.570	17.928	6.169
	GPTQ	48.157	11.382	9.681	11.347	7.176	5.774
	CD	19.614	11.301	9.551	10.920	7.002	5.739
	BCD(k=2)	**18.552**	**11.284**	**9.526**	**10.898**	**6.979**	**5.733**
w3a16g128	OWC	21.815	11.338	9.655	11.597	8.342	5.847
	GPTQ	15.561	11.193	9.260	10.414	6.635	5.677
	CD	13.496	11.182	9.237	10.273	6.655	5.656
	BCD(k=2)	13.501	11.180	9.214	10.259	6.545	5.654
	OWC-CD	14.827	11.220	9.290	10.706	6.635	5.686
	OWC-CD + CD	13.042	11.131	**9.194**	10.143	6.528	5.650
	OWC-CD + BCD(k=2)	**13.004**	**11.131**	9.199	**10.138**	**6.527**	**5.647**
w4a16	OWC	26.438	10.929	9.252	8.946	6.693	5.475
	GPTQ	13.355	10.896	8.923	8.764	6.249	5.417
	CD	12.142	**10.860**	8.910	8.694	6.195	5.407
	BCD(k=2)	**12.048**	10.863	**8.899**	**8.691**	**6.192**	**5.405**
w4a16g128	OWC	11.598	10.82	8.870	8.613	6.264	5.401
	GPTQ	11.086	10.784	8.786	8.498	6.112	5.377
	CD	10.838	10.777	8.781	8.456	6.097	5.373
	BCD(k=2)	10.797	**10.775**	8.786	8.454	6.097	5.372
	OWC-CD	10.981	10.786	8.802	8.519	6.106	5.377
	OWC-CD + CD	10.766	10.776	8.778	8.436	6.092	5.371
	OWC-CD + BCD(k=2)	**10.760**	10.780	**8.774**	**8.434**	**6.091**	**5.371**

H.2 Detailed Downstream Evaluation Results

In this section, we present the downstream evaluation numbers for various quantization techniques. For this evaluation, we use TriviaQA [Joshi et al., 2017], SQuAD [Rajpurkar et al., 2018], NaturalQuestions [Kwiatkowski et al., 2019] and WebQuestions [Berant et al., 2013] to evaluate *generation* capabilities of the quantized models, and to evaluate their *reasoning* capabilities, we test on ARC-c, ARC-e [Clark et al., 2018], HellaSwag [Zellers et al., 2019], BoolQ [Clark et al., 2019], PIQA [Bisk et al., 2020] and WinoGrande [Sakaguchi et al., 2020]. We run the aforementioned evaluations in the zero-shot setting.

Results for the PaLM 2 and Gemma model families are presented in Tables 7 and 8, respectively. These tables report the average performance across generation and ranking tasks. For a detailed

per-task breakdown of the results please refer to Tables 9-14. These results demonstrate that our models achieve comparable, if not superior, performance to GPTQ across all tasks. Notably, our techniques exhibit significant improvements over GPTQ for smaller models like PaLM2-Gecko and Gemma-1 7B.

Table 7: Table presents *downstream evaluation* (zero-shot) numbers for GPTQ, CD, BCD for INT3, INT4 quantization of FFN weights for the PaLM2 family of models. *Gen*, *Rank* columns correspond to generation and ranking tasks.

Config	Method	PaLM2-Gecko			PaLM2-Otter			PaLM2-Bison		
		Gen.	Rank	Avg.	Gen.	Rank	Avg.	Gen.	Rank	Avg.
w16a16	-	20.1	63.02	43.84	36.23	79.51	58.57	44.29	85.44	64.55
w3a16	OWC	15.45	55.73	39.62	15.06	59.29	41.60	39.58	75.45	61.10
	GPTQ	13.08	56.92	39.39	31.80	71.94	55.88	41.42	76.70	62.58
	CD	15.82	58.19	41.24	32.34	72.03	56.16	41.66	76.71	62.69
	BCD(k=2)	16.16	57.85	41.17	32.04	72.22	56.15	41.37	76.78	62.62
w3a16g128	OWC	18.54	56.95	41.59	26.62	68.09	51.50	41.54	76.51	62.52
	GPTQ	17.96	58.30	42.16	33.43	72.98	57.16	43.47	76.65	63.38
	CD	16.81	58.10	41.58	32.73	72.62	56.66	42.95	77.03	63.4
	BCD(k=2)	16.33	57.81	41.22	32.13	72.67	56.46	42.67	76.85	63.18
	OWC-CD	16.73	57.74	41.34	32.90	72.40	56.60	42.23	77.02	63.11
	OWC-CD + CD	16.37	58.22	41.48	33.53	72.64	56.99	43.09	76.91	63.38
	OWC-CD + BCD(k=2)	16.47	58.45	41.66	33.71	72.73	57.12	43.07	76.70	63.25
w4a16	OWC	16.07	59.42	42.08	33.76	72.22	56.83	43.59	77.13	63.71
	GPTQ	18.77	59.5	43.21	34.03	73.53	57.73	44.16	77.54	64.19
	CD	18.20	59.95	43.25	35.01	73.42	58.06	43.97	77.55	64.12
	BCD(k=2)	17.89	59.64	42.94	35.10	73.31	58.02	43.95	77.77	64.24
w4a16g128	OWC	19.00	60.42	43.85	34.52	72.83	57.5	44.45	77.71	64.41
	GPTQ	19.07	60.40	43.87	35.34	73.60	58.29	44.25	77.65	64.29
	CD	20.35	60.94	44.70	36.20	73.67	58.68	44.21	77.72	64.31
	BCD(k=2)	20.44	60.69	44.59	36.11	73.54	58.57	44.17	77.56	64.2
	OWC-CD	19.80	60.39	44.15	36.33	73.62	58.71	44.75	77.70	64.52
	OWC-CD + CD	20.47	60.85	44.70	35.98	73.59	58.54	44.77	77.52	64.42
	OWC-CD + BCD(k=2)	20.15	60.49	44.36	36.00	73.62	58.57	44.70	77.70	64.50

Table 8: Table presents *downstream evaluation* (zero-shot) numbers for GPTQ, CD, BCD for INT3, INT4 quantization of FFN weights for both, Gemma-1 and Gemma-2 models. *Gen*, *Rank* columns correspond to generation and ranking tasks.

Config	Method	Gemma-1 7B			Gemma-2 9B			Gemma-2 27B		
		Gen.	Rank	Avg.	Gen.	Rank	Avg.	Gen.	Rank	Avg.
w16a16		40.6	76.96	58.35	49.34	82.6	64.36	52.57	77.53	67.55
w316	OWC	0.76	43.95	26.6	45.84	73.51	62.44	47.29	75.21	64.04
	GPTQ	19.58	58.33	40.87	45.82	73.11	62.19	49.91	75.53	65.28
	CD	27.83	69.75	50.2	46.48	73.4	62.63	50.09	75.98	65.62
	BCD(k=2)	28.21	70.08	50.51	46.23	73.34	62.49	50.34	76.03	65.75
w3a16g128	OWC	28.47	66.85	48.65	46.9	73.35	62.77	50.77	76.81	66.39
	GPTQ	32.94	73.23	53.82	46.62	73.52	62.76	51.26	76.49	66.4
	CD	34.22	73.03	54.08	46.59	73.34	62.64	51.32	77.1	66.78
	BCD(k=2)	34.52	73.82	54.65	46.65	73.35	62.67	51.22	77.22	66.82
	OWC-CD	32.76	72.61	53.39	46.64	73.03	62.48	50.99	76.97	66.57
	OWC-CD + CD	34.3	73.8	54.57	46.7	73.46	62.75	51.55	77.07	66.86
	OWC-CD + BCD(k=2)	34.59	74.21	54.9	46.7	73.47	62.76	51.49	77.12	66.87
w4a16	OWC	30.69	68.82	50.5	48.49	74.1	63.85	51.63	76.9	66.79
	GPTQ	35.95	74.94	55.75	48.5	74.39	64.03	52.12	76.98	67.04
	CD	37.95	75.6	56.74	48.74	74.27	64.05	51.85	77.2	67.06
	BCD(k=2)	38.11	75.95	57	48.72	74.3	64.06	51.9	77.11	67.03
w4a16g128	OWC	38.51	75.35	56.76	49.22	73.98	64.08	52.05	77.6	67.38
	GPTQ	39.85	76.68	57.96	48.9	73.98	63.95	52.12	77.63	67.43
	CD	40.01	76.79	58.08	48.9	74.05	63.99	52.29	77.6	67.47
	BCD(k=2)	40.14	76.61	58.01	49.02	73.98	64	52.22	77.65	67.48
	OWC-CD	39.44	75.99	57.43	49.26	74.21	64.23	52.15	77.75	67.51
	OWC-CD + CD	40.19	76.89	58.19	49.06	74.13	64.1	52.24	77.71	67.52
	OWC-CD + BCD(k=2)	39.94	76.52	57.89	49.1	74.17	64.14	52.14	77.69	67.47

Table 9: Table presents *downstream evaluation* (zero-shot) numbers of PaLM2-Gecko - for GPTQ, CD, BCD for INT3, INT4 quantization of FFN weights.

Config	Method	PaLM2-Gecko									
		NatualQ.	SQuAD	TriviaQA	WebQ	ARC-c	ARC-e	BoolQ	HellaSwag	PIQA	WinoGrande
w16a16	-	4.68	43.17	27.46	5.07	35.24	65.66	60.24	60.69	74.7	61.48
w3a16	OWC	2.55	47.57	9.81	1.87	29.69	56.73	64.8	53.28	69.59	60.3
	GPTQ	3.35	44.13	3.97	0.89	30.2	62.12	63.79	56.33	71.22	57.85
	CD	3.55	45.71	11.32	2.71	30.63	60.56	66.45	57.2	71.87	62.43
	BCD(k=2)	3.63	47.04	11.22	2.76	30.29	60.52	65.32	57.33	71.82	61.8
w3a16g128	OWC	3.68	55.38	12.5	2.61	31.66	60.14	66.12	53.19	71.71	58.88
	GPTQ	4.02	50.04	14.64	3.15	32.42	61.91	66.64	56.05	73.01	59.75
	CD	3.38	52.01	9.62	2.21	31.23	63.13	65.9	55.98	72.47	59.91
	BCD(k=2)	3.1	50.95	9.36	1.92	31.14	62.25	65.41	56.16	72.91	58.96
	OWC-CD	3.6	51.69	9.33	2.31	31.83	61.74	65.02	56.23	72.2	59.43
	OWC-CD + CD	3.82	50.39	9.5	1.77	32.51	62.25	66.33	56.91	71.27	60.06
	OWC-CD + BCD(k=2)	3.63	49.87	10.26	2.12	32.76	62.88	66.24	56.79	71.82	60.22
w4a16	OWC	4.02	46.96	10.63	2.66	33.19	61.99	66.7	59.26	73.67	61.72
	GPTQ	4.46	48.69	18.84	3.1	34.47	62.96	64.77	60.09	73.61	61.09
	CD	4.13	48.88	16.64	3.15	33.53	63.89	66.79	60.31	73.78	61.4
	BCD(k=2)	4.24	48.31	15.73	3.3	33.28	63.34	66.39	60.26	73.56	61.01
w4a16g128	OWC	4.99	46.1	20.62	4.28	34.73	64.48	67.83	59.91	74.32	61.25
	GPTQ	5.18	49.79	18.17	3.15	35.15	64.6	66.24	60.21	74.32	61.88
	CD	5.73	46.96	24.17	4.53	36.18	66.04	67.8	60.7	73.67	61.25
	BCD(k=2)	5.79	47.28	24.11	4.58	36.01	65.7	67.06	60.73	73.61	61.01
	OWC-CD	5.54	47.24	21.71	4.72	34.3	64.73	68.01	60.72	73.56	61.01
	OWC-CD + CD	5.62	47.6	24	4.68	35.49	66.16	67.06	60.78	74.1	61.48
	OWC-CD + BCD(k=2)	5.57	47.17	23.05	4.82	35.24	65.82	66.02	60.9	73.23	61.72

Table 10: Table presents *downstream evaluation* (zero-shot) numbers of PaLM2-Otter - for GPTQ, CD, BCD for INT3, INT4 quantization of FFN weights.

Config	Method	PaLM2-Otter									
		NatualQ.	SQuAD	TriviaQA	WebQ	ARC-c	ARC-e	BoolQ	HellaSwag	PIQA	WinoGrande
w16a16	-	12.85	65.27	58.34	8.46	51.79	76.09	81.8	79.3	79.6	72.22
w3a16	OWC	2.27	49.45	8.02	0.49	41.98	60.14	48.78	66.91	71.87	66.06
	GPTQ	9.09	64.53	46.38	7.19	49.83	73.99	81.96	76.87	78.89	70.09
	CD	10.72	64.01	48.03	6.59	50.77	76.6	81.01	75.77	77.97	70.09
	BCD(k=2)	10.55	64.31	47.25	6.05	50.85	76.43	81.41	75.91	78.45	70.24
w3a16g128	OWC	6.18	66.37	30.48	3.44	47.44	70.16	69.63	74.61	76.88	69.85
	GPTQ	10.17	66.53	49.29	7.73	51.37	75.93	82.35	77	79.11	72.14
	CD	7.17	68.73	49.42	5.61	50.26	76.18	82.39	76.79	78.13	71.98
	BCD(k=2)	6.68	68.18	48.39	5.27	50.26	75.93	81.41	76.84	79	72.61
	OWC-CD	8.98	68.37	48.33	5.91	50.51	75.46	81.35	76.83	78.18	72.06
	OWC-CD + CD	9.72	67.87	49.87	6.64	51.37	76.68	81.59	76.78	77.91	71.51
	OWC-CD + BCD(k=2)	9.94	68.26	49.91	6.74	50.85	76.22	81.68	76.99	78.89	71.74
w4a16	OWC	11.19	66.67	49.56	7.63	50	73.91	80	77.36	79.11	72.93
	GPTQ	12.27	66.65	48.72	8.46	51.71	75.84	82.02	78.2	80.09	73.32
	CD	12.52	66.85	53.16	7.53	52.3	76.43	81.28	78.3	79.6	72.61
	BCD(k=2)	12.33	67	53.45	7.63	51.96	75.97	80.92	78.47	80.47	72.06
w4a16g128	OWC	11.33	65.98	53.57	7.19	49.66	74.92	82.17	78.22	79.54	72.45
	GPTQ	11.5	65.81	56.36	7.68	51.71	76.52	82.08	78.86	79.27	73.16
	CD	11.72	66.85	57.63	8.61	51.19	76.6	82.51	78.89	79.49	73.32
	BCD(k=2)	11.66	66.73	57.5	8.56	51.19	76.3	82.29	78.88	79.27	73.32
	OWC-CD	12.27	66.28	58.21	8.56	51.62	76.85	82.32	78.85	79.49	72.61
	OWC-CD + CD	11.63	66.28	58.06	7.92	51.62	76.94	82.45	78.72	79.11	72.69
	OWC-CD + BCD(k=2)	11.72	65.96	58.29	8.02	51.79	76.77	82.17	78.8	79.49	72.69

Table 11: Table presents *downstream evaluation* (zero-shot) numbers of PaLM2-Bison - for GPTQ, CD, BCD for INT3, INT4 quantization of FFN weights.

Config	Method	PaLM2-Bison									
		NatualQ.	SQuAD	TriviaQA	WebQ	ARC-c	ARC-e	BoolQ	HellaSwag	PIQA	WinoGrande
w16a16	-	21.36	72.92	70	12.89	55.38	81.27	87.86	82.72	82.97	78.14
w3a16	OWC	15.87	73.01	58.86	10.58	52.47	76.43	86.91	79.76	81.01	76.09
	GPTQ	17.31	71.25	65.53	11.56	54.44	79.25	87.22	80.78	81.61	76.87
	CD	17.84	71.24	64.76	12.8	54.44	79.17	87.58	80.79	82.1	76.16
	BCD(k=2)	17.81	71.3	64.29	12.06	53.5	79.08	87.65	80.91	82.21	77.35
w3a16g128	OWC	15.98	71.28	68.02	10.88	53.58	79.55	86.73	80.9	81.61	76.72
	GPTQ	18.95	71.49	69.67	13.78	53.24	79.84	86.79	81.05	82.05	76.95
	CD	18.31	71.81	68.56	13.14	53.41	79.8	87.55	81.31	82.43	77.66
	BCD(k=2)	18.53	71.55	67.62	12.99	53.58	79.59	87.49	81.14	82.81	76.48
	OWC-CD	17.26	71.79	67.53	12.35	54.69	80.85	85.72	81.02	81.88	77.98
	OWC-CD + CD	18.75	72.05	68.35	13.19	54.1	80.18	86.36	80.86	82.48	77.51
	OWC-CD + BCD(k=2)	19.36	71.47	68.45	12.99	53.58	80.47	86.02	81.15	82.26	76.72
w4a16	OWC	20	72.51	69.34	12.5	54.61	80.39	87.06	81.5	82.05	77.19
	GPTQ	21.25	72.39	70.6	12.4	55.2	80.64	87.58	81.98	82.26	77.58
	CD	21.5	71.57	70.04	12.8	55.29	80.56	88.07	81.78	82.59	77.03
	BCD(k=2)	21.16	71.92	70.01	12.7	55.63	81.19	87.77	81.82	82.81	77.43
w4a16g128	OWC	21.22	73.06	70.07	13.44	55.8	80.47	87.77	82.08	82.64	77.51
	GPTQ	20.72	73.17	70.12	12.99	55.38	80.89	87.55	82.14	82.59	77.35
	CD	21.8	73.37	69.06	12.6	54.78	80.72	87.25	82.16	83.19	78.22
	BCD(k=2)	21.5	73.06	69.34	12.8	55.29	80.98	87.31	82.14	82.54	77.11
	OWC-CD	21.36	73.85	70.07	13.73	55.12	81.23	87.71	82.27	82.59	77.27
	OWC-CD + CD	21.41	73.32	71.46	12.89	54.18	81.4	87.28	82.17	82.37	77.74
	OWC-CD + BCD(k=2)	21.19	73.24	71.64	12.75	55.03	81.52	87.28	82.06	82.64	77.66

Table 12: Table presents *downstream evaluation* (zero-shot) numbers of Gemma-1 7B - for GPTQ, CD, BCD for INT3, INT4 quantization of FFN weights.

Config	Method	Gemma-1 7B									
		NaturalQ	SQuAD	TriviaQA	WebQ	ARC-c	ARC-e	BoolQ	HellaSwag	PIQA	WinoGrande
w16a16		15.04	71.94	62.38	13.04	43.69	65.53	80.43	78.55	80.79	72.14
w316	OWC	0.03	2.87	0.15	0	30.55	38.8	63.09	29	52.61	48.86
	GPTQ	2.24	58.68	14.35	3.05	35.15	53.16	63.76	53.84	64.58	59.91
	CD	6.04	68.77	32.22	4.28	41.04	62.37	76.51	68.95	74.76	67.01
	BCD(k=2)	6.76	69.03	33.32	3.74	40.7	62.21	76.91	69.1	75.24	68.11
w3a16g128	OWC	6.51	71.04	30.86	5.46	38.14	60.1	70.7	64.51	75.3	63.85
	GPTQ	10.03	71.66	42.25	7.82	41.64	64.06	80.83	72.37	78.18	69.38
	CD	10.08	71.94	46.39	8.46	40.7	62.75	80.43	73.88	78.56	67.64
	BCD(k=2)	10.78	71.99	46.65	8.66	42.32	63.01	81.25	74.17	78.18	69.46
	OWC-CD	9.7	71.62	41.44	8.27	41.47	64.44	79.94	72.19	77.53	67.32
	OWC-CD + CD	10.66	71.78	46.73	8.02	42.41	64.73	80.46	74.29	78.94	67.64
	OWC-CD + BCD(k=2)	10.69	71.84	47.05	8.76	42.41	65.36	80.03	74.47	79.49	68.9
w4a16	OWC	7.09	63.51	44.98	7.19	41.38	59.34	77.06	67.29	73.29	63.85
	GPTQ	10.66	70.95	52.63	9.55	44.11	64.56	79.39	75.9	79.16	70.56
	CD	12.71	70.41	57.39	11.27	43.69	65.11	79.66	77.02	79.98	70.17
	BCD(k=2)	12.44	70.5	57.55	11.96	45.14	65.4	79.79	76.66	79.54	71.03
w4a16g128	OWC	13.55	71.61	56.09	12.8	42.49	63.85	80.92	76.77	79.22	70.32
	GPTQ	13.88	71.94	59.45	14.12	43.52	66.12	81.96	77.54	79.98	71.11
	CD	15.26	71.73	59.81	13.24	45.05	65.45	81.25	77.52	80.58	70.88
	BCD(k=2)	15.54	71.94	59.83	13.24	43.69	64.6	80.89	77.68	81.23	71.43
	OWC-CD	14.43	72.08	58.69	12.55	43.52	64.94	80.46	77.46	79.49	70.64
	OWC-CD + CD	14.82	72.32	59.98	13.63	45.05	65.82	80.43	77.46	80.96	71.43
	OWC-CD + BCD(k=2)	14.93	72.39	59.49	12.94	44.45	66.08	80.21	77.6	80.03	70.8

Table 13: Table presents *downstream evaluation* (zero-shot) numbers of Gemma-2 9B - for GPTQ, CD, BCD for INT3, INT4 quantization of FFN weights.

Config	Method	Gemma-2 9B									
		NaturalQ	SQuAD	TriviaQA	WebQ	ARC-c	ARC-e	BoolQ	HellaSwag	PIQA	WinoGrande
w16a16		26.76	72.06	76.83	21.7	58.96	77.57	83.33	77.31	81.12	67.96
w316	OWC	21.02	74.18	69.76	18.41	56.23	76.52	84.37	75.24	80.79	67.88
	GPTQ	21.22	74	69.99	18.06	57	77.19	82.35	74.96	80.69	66.46
	CD	22.24	73.41	70.67	19.59	56.91	77.74	84.13	75.21	79.82	66.61
	BCD(k=2)	21.77	73.33	71	18.8	57.08	77.69	83.18	75.04	80.03	67.01
w3a16g128	OWC	23.1	73.42	71.79	19.29	56.31	76.6	83.94	75.28	80.74	67.25
	GPTQ	22.33	72.77	71.68	19.69	56.66	77.44	83.46	75.19	80.74	67.64
	CD	22.8	71.94	72.24	19.39	56.74	77.36	84.13	75.27	80.14	66.38
	BCD(k=2)	23.02	72.3	71.99	19.29	56.57	77.06	84.37	75.36	80.3	66.46
	OWC-CD	22.55	72.69	72.13	19.19	55.46	76.85	82.81	75.36	80.79	66.93
	OWC-CD + CD	22.99	72.42	72.28	19.09	56.91	77.36	82.75	75.48	80.52	67.72
	OWC-CD + BCD(k=2)	23.27	72.39	72.23	18.9	57	77.31	83.27	75.49	80.41	67.32
w4a16	OWC	25.48	73.21	74.93	20.32	58.45	76.81	83.94	76.58	80.69	68.11
	GPTQ	25.29	72.9	74.65	21.16	58.7	77.27	83.55	76.6	81.23	68.98
	CD	25.57	73.03	75.42	20.92	58.36	77.61	83.79	76.75	80.41	68.67
	BCD(k=2)	25.6	72.86	75.35	21.06	58.62	77.4	83.73	76.74	80.69	68.59
w4a16g128	OWC	26.26	73.33	75.73	21.56	58.7	77.4	83.33	76.76	80.85	66.85
	GPTQ	26.12	72.53	75.75	21.21	58.53	77.65	83.06	76.72	81.28	66.61
	CD	26.23	72.57	75.83	20.96	58.28	77.53	83.15	76.83	81.12	67.4
	BCD(k=2)	26.4	72.62	75.94	21.11	58.36	77.4	83.09	76.72	80.85	67.48
	OWC-CD	26.15	73.46	75.88	21.56	59.3	77.74	82.81	76.72	80.96	67.72
	OWC-CD + CD	26.15	72.7	75.98	21.41	58.62	77.78	82.94	76.86	81.07	67.48
	OWC-CD + BCD(k=2)	26.32	72.75	76.13	21.21	59.13	77.48	83	76.92	81.01	67.48

Table 14: Table presents *downstream evaluation* (zero-shot) numbers of Gemma-2 27B - for GPTQ, CD, BCD for INT3, INT4 quantization of FFN weights.

Config	Method	Gemma-2 27B									
		NaturalQ	SQuAD	TriviaQA	WebQ	ARC-c	ARC-e	BoolQ	HellaSwag	PIQA	WinoGrande
w16a16		30.42	75.04	81.5	23.33	60.49	78.96	83.18	82.24	84.6	75.69
w316	OWC	23.3	73.28	73.2	19.39	56.66	77.53	81.01	79.82	83.24	73.01
	GPTQ	26.09	75.08	76.62	21.85	57.51	78.96	77.4	80.57	83.73	74.98
	CD	26.51	75.57	77.21	21.06	59.13	79.76	78.29	80.1	83.95	74.66
	BCD(k=2)	26.7	75.67	77.34	21.65	58.45	79.34	78.47	79.93	83.41	76.56
w3a16g128	OWC	26.7	75.95	78.61	21.8	58.45	79.63	83.55	80.49	82.75	76.01
	GPTQ	27.48	75.97	78.72	22.88	59.56	80.05	80.28	81.03	83.68	74.35
	CD	27.98	76.15	78.59	22.54	59.73	79.88	81.77	80.99	84.11	76.09
	BCD(k=2)	27.78	75.99	78.62	22.49	59.56	80.01	81.99	81.08	84.11	76.56
	OWC-CD	26.87	76.03	79	22.05	58.96	80.18	81.28	80.94	83.79	76.64
	OWC-CD + CD	27.87	76.36	79.62	22.34	59.3	80.05	81.59	80.97	83.79	76.72
	OWC-CD + BCD(k=2)	27.7	76.45	79.38	22.44	59.73	80.05	81.83	80.98	83.57	76.56
w4a16	OWC	29	75.51	80.05	21.95	59.39	79.04	81.13	81.93	84.39	75.53
	GPTQ	29.5	75.7	80.41	22.88	59.47	78.7	81.62	81.91	84.33	75.85
	CD	29.36	75.23	80.23	22.59	59.9	79.67	81.53	81.84	84.49	75.77
	BCD(k=2)	29.42	75.34	80.31	22.54	59.3	79.46	81.83	81.72	84.44	75.93
w4a16g128	OWC	29.25	75.41	80.91	22.64	59.98	79.46	83.76	82.14	84.22	76.01
	GPTQ	29.53	75.61	80.85	22.49	60.41	79.59	83.61	82.14	84.66	75.37
	CD	29.58	75.38	81.05	23.13	59.98	79.42	83.73	82.08	84.44	75.93
	BCD(k=2)	29.61	75.44	80.91	22.93	60.32	79.71	83.85	82.06	84.44	75.53
	OWC-CD	29.11	75.93	80.85	22.69	60.75	79.67	83.33	82.2	84.87	75.69
	OWC-CD + CD	29.36	75.66	80.96	22.98	59.9	79.59	83.82	82.05	85.15	75.77
	OWC-CD + BCD(k=2)	29.34	75.41	80.86	22.93	59.9	79.84	83.94	82.14	84.93	75.37

H.3 Attention + FFN Quantization

Before presenting the results for quantization of both attention and FFN layers, we first explain why quantizing the attention weights does not yield inference latency gains.

H.3.1 Attention Quantization doesn't yield Latency Gains

In Table 15, we present the end-to-end prefix and decode latency for Gemma-2 27B with three different prefix lengths. The latencies are obtained by running Gemma-2 27B on 4 TPUv5 chips with a per TPU batch size of 4 and no model parallelism.

Prefix phase. During the prefix phase, neither FFN nor attention quantization yielded significant latency improvements, likely due to the compute-bound nature of the prefix.

Decode phase. In the decode phase, quantizing attention weights from 16 bits to 8 bits provided marginal gains, as attention compute is primarily memory-bound by the KV cache size, which typically exceeds attention weight size for moderate-to-large context lengths. Further reduction to 4-bit quantization offered negligible latency improvement while incurring a non-trivial drop in quality (Table 16 and Table 6), thus questioning the utility of attention weight quantization. Conversely, FFN layers, processing only a single token activation during decoding, are memory-bound by the significantly larger FFN weights (9x the size of attention weights in Gemma-2 27B). Consequently, FFN weight quantization effectively reduces memory transfers and improves latency. We observed a $32\% - 35\%$ latency reduction when quantizing from 16 bits to 8 bits, with an additional 10% reduction when using 4-bit quantization.

These results clearly highlight that quantization of attention weights doesn't yield any inference latency gains.

Table 15: Table presents the prefix and decode time inference latency of Gemma-2 27B with three different prefix lengths. In the Attention-only setting, only the attention parameters are quantized and rest of the model is in 16 bits. Similarly, in the FFN-only setting, only the FFN parameters are quantized.

Quantization Setting	Prefix Length	Config	Prefix time (ms)	Decode per step time (ms)
Attention-only	1024	w16a16	299.958	26.178
		w8a16	302.199	25.710
		w4a16	301.899	25.595
	2048	w16a16	616.460	26.532
		w8a16	616.080	26.117
		w4a16	615.601	25.964
	4096	w16a16	1296.000	27.254
		w8a16	1290.000	26.685
		w4a16	1286.000	26.653
FFN-only	1024	w16a16	299.958	26.178
		w8a16	292.630	17.410
		w4a16	284.680	15.635
	2048	w16a16	616.460	26.532
		w8a16	604.941	17.868
		w4a16	602.306	16.006
	4096	w16a16	1296.000	27.254
		w8a16	1276.000	18.521
		w4a16	1273.000	16.626

H.3.2 Results for Attention + FFN quantization

We now perform FFN + Attention weight quantization, results for which are presented in Table 16. We use the same experimental setting as the one described in Section 4. In our experiments we noticed that the inputs to the attention layer are often aligned in a handful of directions. Consequently, performing quantization using such a data leads to a huge drop in performance, as the algorithms would primarily focus on a few directions and ignore the rest. To mitigate this, we clip the largest eigenvalues of the Hessian to ensure a more balanced Hessian. This technique, reminiscent of the weight clipping in OmniQuant [Shao et al., 2023], improves the performance of both GPTQ and CDQuant[1]. For instance, for PaLM2-Gecko, the perplexity for w3a16-GPTQ improves from 34.872 to 24.054 with clipping, and for w4a16-GPTQ, it improves from 10.966 to 10.005. Table 16 presents the results from this experiment, where we run all the algorithms on the clipped Hessian. We once again notice that in almost all the settings, our coordinate descent algorithms outperform GPTQ. For instance, we see $5 - 10\%$ improvement in perplexity for 3-bit per-channel quantization of Gemma-2, and PaLM2 Gecko models.

[1]One could also rely on existing techniques such as AWQ and SmoothQuant to reduce the effect of outliers. But in our experiments we noticed that both AWQ and SmoothQuant performed poorly compared to the simple eigenvalue clipping technique.

Table 16: Table presents the perplexity evaluations for GPTQ, CD, BCD for INT3, INT4 quantization of both FFN and attention weights.

Config	Method	Gemma-1 7B	Gemma-2 9B	Gemma-2 27B	PaLM2 Gecko	PaLM2 Otter	PaLM2 Bison
w16a16	-	10.348	10.683	8.682	7.948	5.984	5.298
w3a16	OWC	$1.89e6$	12.558	15.736	41.751	54.499	6.446
	GPTQ	91.586	11.849	11.423	24.054	**11.384**	5.801
	CD	36.102	11.808	10.941	19.692	11.542	5.770
	BCD(k=2)	**32.850**	**11.784**	**10.926**	**19.312**	11.392	**5.762**
w3a16g128	OWC	33.710	11.792	10.486	20.301	9.785	5.973
	GPTQ	20.045	11.491	9.766	15.042	7.501	5.698
	CD	15.850	11.500	9.689	14.757	7.454	5.679
	BCD(k=2)	15.466	11.487	9.657	14.691	**7.438**	5.673
	OWC-CD	18.599	11.583	9.820	16.268	7.617	5.746
	OWC-CD + CD	15.792	11.465	**9.559**	14.285	7.466	5.675
	OWC-CD + BCD(k=2)	**15.777**	**11.463**	9.564	**14.242**	7.458	**5.670**
w4a16	OWC	40.563	11.100	9.706	10.683	7.194	5.508
	GPTQ	15.805	11.021	9.217	10.005	6.645	5.425
	CD	14.379	**10.979**	9.177	9.991	6.627	5.415
	BCD(k=2)	**14.056**	10.982	**9.167**	**9.965**	**6.621**	**5.413**
w4a16g128	OWC	12.222	10.919	9.027	9.402	6.415	5.420
	GPTQ	11.548	10.863	8.888	9.160	6.241	5.381
	CD	11.306	10.856	8.890	9.124	6.226	5.377
	BCD(k=2)	11.105	**10.853**	8.892	9.120	**6.225**	5.376
	OWC-CD	11.342	10.876	8.924	9.241	6.261	5.387
	OWC-CD + CD	11.149	10.863	8.873	9.094	6.244	5.375
	OWC-CD + BCD(k=2)	**11.140**	10.866	**8.873**	**9.093**	6.242	**5.374**

H.4 Runtime Analysis

In Tables 17 18, we present the quantization runtime of GPTQ, CD and BCD on Gemma models using H100s (in one setting we used 1 H100 and in another we used 8H100s). In particular, in Table 17, we present runtime for quantizing all the attention and FFN layers (which includes FFN1, FFN1-gate, FFN2) in the model. Table 18 presents a more detailed breakdown; we present the per-layer runtime for quantizing FFN1 and FFN2 layers.

Runtime on 8 **H100's.** With 8 H100s, CD is as fast as GPTQ for quantizing attention weights, whereas BCD is $2\times$ slower. However, for FFN1 (FFN2) quantization, CD is $5\times$ ($2\times$) slower than GPTQ, whereas BCD is an order of magnitude slower than GPTQ. For quantization of the entire model, CD is $2\times$ slower than GPTQ (because quantizing FFN2 takes up most of the time). We attribute the slowness of CD and BCD to the numerous scatters and gathers involved in finding the most optimal coodrinate/block of coordinate to update, and then eventually updating it. Since ML accelerators are slow with gather and scatter operations, we are bound by the time it takes to do them. Performing these operations on the CPU should help speed up CD and BCD.

Runtime on 1 **H100.** With a single H100, GPTQ is 2x-3.8x faster than CD for the smaller, attention weights, and is an order of magnitude faster than BCD. For the larger, FFN weights, GPTQ is an order of magnitude faster than CD, and in most cases is two orders of magnitude faster than BCD. As stated previously, both CD and BCD are bound by the time it takes to do gathers and scatters.

Speeding up BCD. To algorithmically speed up BCD, we propose to use CD for FFN2 quantization, and use BCD for the rest of the layers. We find that BCD spends most of its time quantizing the FFN2 weight matrix (as can be seen in Table 18) where the quantization axis is of size d_{hidden}. In most cases, the time it takes to FFN2 is 10x the time it takes to quantize FFN1. Thus, to speed up BCD, we quantize FFN2 with CD, and FFN1 and FFN1-gate with BCD (where the quantization axis is of size d_{model}. Results for this setting can be found in Table 19. It can be seen that quantizing FFN2 with CD leads to negligible to no drop in performance especially for the larger, and better Gemma-2 9B. In fact, it improves the performance of BCD for w3a16g128. With this setup, BCD's runtime is reduced significantly and is comparable to that of CD.

Speeding up CD. We find that CD converges to an optimal solution significantly before d_{in} iterations. Based on this finding, we run CD for lesser number of iterations. In Table 20, one can see that there is negligible quality drop for CD, and even with $d_{\text{in}}/8$ iterations, CD outperforms GPTQ. With lesser number of iterations, CD's runtime can be made faster than/comparable to that of GPTQ.

Table 17: Table presents the per-channel quantization runtime (in minutes) of GPTQ, CD, BCD (where each run is for d_{in} iterations). FFN includes the combined runtime of quantizing FFN1, FFN1-gate and FFN2 layers.

Config	Method	8 H100s						1 H100					
		Gemma-1 7B		Gemma-2 9B		Gemma-2 27B		Gemma-1 7B		Gemma-2 9B		Gemma-2 27B	
		FFN (mins.)	Attn. (mins.)	FFN (mins.)	Attn. (mins.)	FFN (mins.)	Attn. (mins.)	FFN (mins.)	Attn. (mins.)	FFN (mins.)	Attn. (mins.)	FFN (mins.)	Attn. (mins.)
w3a16	GPTQ	1.68	0.67	1.15	1.05	7.98	1.18	2.13	0.65	1.36	1.04	10.31	1.32
	CD	3.03	0.19	2	0.31	16.55	0.51	22.3	1.2	13.31	1.87	122.35	3.2
	BCD(k=2)	8.52	0.71	5.95	1.18	66.97	2.11	88.14	3.85	61.5	7.24	541.03	13.62
w4a16	GPTQ	1.69	0.77	1.15	1.06	7.95	1.18	2.1	0.63	1.36	0.98	10.16	1.12
	CD	3.9	0.22	2.53	0.36	21.16	0.64	29.27	1.47	17.66	2.25	163.78	4.27
	BCD(k=2)	26.2	1.76	18.51	2.66	162.43	4.64	218.25	12.89	163.64	20.12	1296.36	36.68

Table 18: Table presents the *per-layer*, per-channel quantization runtime (in seconds) of GPTQ, CD, BCD (where each run is for d_{in} iterations) for the FFN layer. Note that FFN1 and FFN1-Gate have the exact same runtimes for all compared methods, hence we report only FFN1's runtime.

Config	Method	8 H100s						1 H100					
		Gemma-1 7B		Gemma-2 9B		Gemma-2 27B		Gemma-1 7B		Gemma-2 9B		Gemma-2 27B	
		FFN1 (sec.)	FFN2 (sec.)	FFN1 (sec.)	FFN2 (sec.)	FFN1 (sec.)	FFN2 (sec.)	FFN1 (sec.)	FFN2. (sec.)	FFN1 (sec.)	FFN2 (sec.)	FFN1 (sec.)	FFN2 (sec.)
w3a16	GPTQ	0.34	2.93	0.35	0.94	0.39	9.61	0.43	3.69	0.39	1.16	0.81	11.64
	CD	0.46	5.57	0.41	2.04	1.6	18.38	3.34	41.11	2.86	13.31	12.24	135.11
	BCD(k=2)	1.4	15.46	1.27	5.96	7.86	71.67	12.55	163.74	11.95	63.95	57.38	590.89
w4a16	GPTQ	0.35	2.93	0.36	0.94	0.4	9.58	0.42	3.68	0.39	1.14	0.81	11.63
	CD	0.53	7.28	0.46	2.71	2.15	23.31	3.94	54.85	3.22	18.81	17.24	179.15
	BCD(k=2)	4.54	47.07	3.71	19.02	18.97	173.92	35.95	395.78	30.34	173.09	151.36	1388.19

Table 19: Table presents the perplexity evaluations for INT3 quantization with BCD wherein the FFN2 may be quantized either with CD or BCD, while rest of the FFN parameters are quantized with BCD.

Config	Method	Gemma-1 7B	Gemma-2 9B
w16a16		10.384	10.683
w3a16	CD, FFN2	19.709	11.288
	BCD(k=2), FFN2	18.552	11.284
w3a16g128	CD, FFN2	13.512	11.179
	BCD(k=2), FFN2	13.501	11.180

Table 20: Table presents the perplexity evaluations for INT3 quantization with CD, wherein instead of running CD for d_{in} iterations, we run it for $d_{\text{in}}/2$, $d_{\text{in}}/4$, and $d_{\text{in}}/8$ iterations.

Config	Method	Epochs	Gemma-1 7B	Gemma-2 9B
	w16a16		10.384	10.683
w3a16	GPTQ	-	48.157	11.382
	CD	1	19.614	11.301
		1/2	19.614	11.301
		1/4	19.621	11.301
		1/8	19.597	11.305
w3a16g128	GPTQ	-	15.561	11.193
	CD	1	13.496	11.182
		1/2	13.488	11.189
		1/4	13.480	11.189
		1/8	13.560	11.188