HELM: Steering Long-Horizon Agents with Learned Hierarchical Memory and Epistemic Governance

Abstract

Long-horizon LLM agents often fail not because relevant information is absent, but because semantically relevant memories are epistemically invalid: stale observations, failed tool traces, or claims contradicted by newer verified evidence are retrieved and treated as support. We introduce **HELM** (**H**ierarchical **E**pistemic **L**earned **M**emory), a memory framework that makes this reliability problem explicit. HELM exposes memory through logged read/write/consolidate/prune operations and stores each item with provenance and epistemic state. Its three-tier **Semantic-HNSW Nested Memory** (SHNM) links episodic tool traces to consolidated recalls and thematic indices, enabling coarse-to-fine retrieval that can be audited back to concrete evidence spans. *Epistemic governance* filters and re-ranks recalls by status, freshness, consistency, recency, and provenance, while a lightweight learned controller decides when memory operations are worth their cost. Across five long-horizon benchmarks, HELM improves success over strong memory baselines on GAIA, WebArena, Mind2Web, StreamBench, and LifelongAgent-Bench while reducing effective context tokens and epistemic errors. New scaling, training-free, and governance-only ablations show that the main gains persist with a 32B backbone and that epistemic governance alone explains a large share of the reliability improvement.

1 Introduction

Large Language Models (LLMs) are evolving from passive chatbots into autonomous agents capable of pursuing open-ended goals over extended horizons (Yao et al., 2023; Hu et al., 2026). In this regime, an agent must carry forward user constraints, intermediate plans, environment state, and tool-grounded evidence across hundreds of interaction steps. The usual diagnosis is that the agent has a capacity problem: the context window is too short, so more retrieval, longer context, or more aggressive summarization should help. Our central claim is different. For long-horizon agents, the dominant memory failure is often an *epistemic* one: the agent retrieves information that is semantically on-topic but no longer valid evidence for the current decision.

Consider two common failures. In a WebArena-style purchase task, an agent observes a flight price of $280 at step 10 and stores it. By step 85, the page has changed and the current price is $450; semantic retrieval still returns the old "flight price" trace, leading the agent to book a now-invalid option. In a tool-use task, an earlier script run fails with `FileNotFoundError`, while a later run succeeds; a query for "script output" may retrieve the failed trace because it is lexically similar, after which the model fabricates a successful result. These errors are not ordinary generation-side hallucinations: the retrieved text exists in memory. They are also not standard time-sensitive RAG over static documents: the invalid evidence is the agent's own dynamic interaction history, whose status, timestamp, version, and contradictions determine whether it should be trusted.

Recent memory systems mitigate long horizons with context extension, periodic summarization, or top-k retrieval (Packer et al., 2024; Zhong et al., 2023). These methods improve access, but they usually treat retrieved memories as uniformly valid once they are semantically relevant. Abstractive summarization introduces a second failure mode: it can compress away decisive constraints such as "pending payment" or "tool call failed," making later failures hard to audit. We therefore frame long-horizon memory as governed evidence management rather than passive storage.

We propose **HELM** (**H**ierarchical **E**pistemic **L**earned **M**emory), a framework for steering long-horizon agents with auditable, governed memory. The core of HELM is **Semantic-HNSW Nested Memory** (SHNM), a three-tier store over episodic

traces ($M^{(1)}$), consolidated recalls ($M^{(2)}$), and thematic indices ($M^{(3)}$). Each memory item carries provenance pointers and epistemic metadata, including tool status, timestamp, source type, and task/environment version. At retrieval time, **epistemic governance** filters and re-ranks memories by status, freshness, consistency, recency, and provenance; during consolidation and pruning, it propagates validity and records conflicts rather than letting abstractions overwrite evidence. On top of SHNM, a lightweight controller decides when to READ, WRITE, CONSOLIDATE, and PRUNE under explicit task and efficiency budgets.

We evaluate HELM on five long-horizon benchmarks and report not only task success but also effective context tokens (CTU), epistemic errors (EpiErr.), and provenance faithfulness. Under matched scaffolds and budgets, HELM improves over strong memory baselines on GAIA (L2: 8.9→12.8; L3: 0.8→1.7) and WebArena (19.5→22.4), while reducing CTU and EpiErr. Additional analyses address the main practical concerns: gains persist with Qwen3-32B, a training-free HELM-ZS variant outperforms training-free memory baselines, HELM-lite isolates governance as the largest reliability driver, and latency diagnostics show that memory overhead is small relative to reasoner generation.

Our contributions are: (i) a formalization of agent-memory failure as *epistemic error*, distinct from hallucination and static temporal RAG; (ii) SHNM, a nested memory architecture with provenance-backed recall and epistemic governance over explicit memory operations; and (iii) a broad empirical study showing that governed evidence, not merely longer context or controller training, drives improvements in long-horizon agent reliability.

2 Related Work

Agent memory spans episodic stores, summaries, graphs, hierarchies, latent states, and parametric edits (Hu et al., 2026). Flat systems such as MemGPT and MemoryBank retrieve or summarize interaction logs directly (Packer et al., 2024; Zhong et al., 2023); more structured systems, including Mem0, GraphRAG, HippoRAG, H-Mem, MemGen, and G-Memory, improve access through persistent memories, graph structure, or learned abstractions (Chhikara et al., 2025; Edge et al., 2025; Gutiérrez et al., 2025; Sun and Zeng, 2025;

Zhang et al., 2025b,a). Hierarchical RAG methods such as ArchRAG similarly use abstract indices to route retrieval over static corpora (Wang et al., 2026). HELM shares the goal of efficient long-horizon recall, but targets a different failure mode: stored agent traces can be semantically relevant yet epistemically invalid. SHNM therefore attaches tool status, timestamps, consistency metadata, and provenance links to each memory item, so recalls can be filtered and grounded rather than merely retrieved.

HELM is also complementary to long-context reasoning and learned memory-control work. Chain-of-Agents and COSMIR improve how long evidence is distributed or carried through reasoning chains (Zhang et al., 2024; Gupta et al., 2025); HELM governs which interaction traces should enter that context in the first place. Prior agents often rely on recency/importance heuristics or periodic summarization (Park et al., 2023; Li et al., 2025), while recent approaches learn retrieval, compression, or latent-memory policies (Tan et al., 2025; Zhang et al., 2026). HELM likewise learns an explicit controller over READ, WRITE, CONSOLIDATE, and PRUNE, but its core contribution is training-independent epistemic governance: even HELM-lite and HELM-ZS gain from provenance-backed validity checks. This also distinguishes HELM from standard RAG and hallucination mitigation (Gao et al., 2024; Quattrociocchi et al., 2025), where the main concern is grounding generated text or static documents; here the central risk is reusing stale, failed, or contradicted traces from the agent's own history.

3 Methodology

We introduce **HELM**, a hierarchical memory framework that makes long-horizon agent memory *structured, auditable, and controllable* (Figure 1). HELM combines (i) a three-tier nested store, **SHNM**, that links abstractions to verifiable traces via provenance and epistemic metadata; (ii) *epistemic governance* that filters and re-ranks memory access using recency/status/provenance signals; and (iii) a learned controller that triggers explicit memory operations under fixed budgets. We additionally fine-tune a tool-aware embedding model to index tool-augmented traces for robust retrieval.

Figure 1: Overview of HELM. Instead of retrieving isolated chunks from a flat store, HELM maintains a three-level hierarchy (episodic traces → consolidated recalls → thematic indices). For each query, the system performs coarse-to-fine retrieval by first selecting high-level indices and then expanding to supporting recalls and verifiable traces, preserving both global task context and grounded evidence.

3.1 Problem Setup and Explicit Recall

Agent and environment. We consider an LLM-based agent interacting with an environment over steps $t = 1, \ldots, T$. At step t, the agent observes o_t (user input, tool outputs, or state), optionally consults an external memory store $\mathcal{M}$, and produces an action a_t (response or tool call). HELM separates *reasoning* from *memory management*: a reasoner generates task actions, while a controller decides if/when to interact with $\mathcal{M}$. The working context is $c_t = \text{Compose}(o_t, \text{Read}(\mathcal{M}, q_t))$, where q_t is a controller-generated recall query; the controller can also issue WRITE, CONSOLIDATE, and PRUNE operations to update $\mathcal{M}$.

Event-driven recall interface. Memory use in HELM is explicit. During deliberation, the reasoner emits a request span `<recall>...</recall>` describing missing information (e.g., a constraint or a pointer to a past tool result). A separate **memory generator** executes governed retrieval in SHNM, optionally expands provenance to supporting evidence, and replaces the request with a `<memory>...</memory>` block that can be audited against stored traces. For long tool transcripts, we additionally store `<memory>refined log</memory>` entries that extract salient fields while preserving pointers

to the original tool trace for later verification. Appendix A.10 specifies the pointer schema used for auditable recall, and Appendix A.11 details the cited-recall validation used in our faithfulness experiments.

Explicit memory operations. HELM exposes memory as a small set of explicit, logged operations: READ (governed retrieval under token budgets), WRITE (store episodic traces with tool status and timestamps), CONSOLIDATE (distill and link abstractions to evidence via provenance), and PRUNE (enforce budgets by removing stale/failed/redundant items). Appendix A.1 provides the full operation summary.

3.2 Semantic-HNSW Nested Memory (SHNM)

We structure the memory store as a three-tier hierarchy $\mathcal{M} = \{M^{(1)}, M^{(2)}, M^{(3)}\}$, each layer representing a different abstraction level. We build an HNSW-style ANN index (Malkov and Yashunin, 2018) per layer and connect layers with provenance links, enabling coarse-to-fine traversal.

Layers. SHNM consists of $M^{(1)}$ **episodic traces** (high-fidelity interaction snippets and tool logs as verifiable evidence), $M^{(2)}$ **consolidated recalls** (procedural/semantic summaries distilled from

linked traces), and $M^{(3)}$ **thematic indices** (sparse, high-level entries that act as global retrieval entry points).

Memory items, epistemic state, and provenance. Each memory item is $\Updownarrow = (x, \mathbf{v}, \mathcal{E}, \mathcal{P})$, where x is text, $\mathbf{v}$ is an embedding, $\mathcal{P}$ stores provenance links (including cross-layer parent/child relations), and $\mathcal{E}$ stores epistemic metadata. We define the epistemic state $\mathcal{E}(\Updownarrow)$ as the triple of status, freshness, and consistency. Here status $\in$ {SUCCESS, UNKNOWN, FAILED} is derived from tool return codes; freshness $= 1$ only if the item is within the configured step/time window and matches the current task or environment version; and consistency $= 1$ only if no newer verified memory with the same extracted key contradicts its value. A memory item is *epistemically valid* iff status $\neq$ FAILED, freshness $= 1$, and consistency $= 1$. An **epistemic error** occurs when a recalled item is semantically relevant to the query but violates one of these validity conditions and is consumed by the agent as evidence. This definition targets retrieval-side misuse of stored interaction traces; it is distinct from generation-side hallucination and from temporal filtering over static document corpora. Appendix A.9 details how stale and contradictory items are detected from logged metadata.

Coarse-to-fine retrieval with epistemic governance. Given a recall query q with embedding $\mathbf{v}_q$, SHNM first retrieves candidate indices from $M^{(3)}$, expands to linked recalls in $M^{(2)}$, and finally selects supporting traces in $M^{(1)}$ for grounding. We then apply *epistemic governance* to exclude invalid evidence and to prioritize candidates that are both semantically relevant and epistemically reliable. Candidates are re-ranked by a governed score

$$S(\Updownarrow; q) = \lambda_{\text{sim}} \text{sim}(\mathbf{v}_q, \mathbf{v}) + \lambda_{\text{time}}\, g(\Delta t)$$
$$+ \lambda_{\text{status}}\, w(\text{status}) + \lambda_{\text{src}}\, w(\text{src}), \quad (1)$$

where sim is cosine similarity, $g(\cdot)$ is a monotone recency kernel, and src denotes the source type. READ applies hard filters to exclude failed tool traces and items contradicted by newer verified evidence; for staleness, deployments may use either a hard filter or soft downweighting when evidence is scarce.

Governance signals and fixed weights. We instantiate status and recency as governance signals that are logged and auditable: status $\in$ {SUCCESS, UNKNOWN, FAILED} comes from tool wrappers, and Δt is computed from timestamps/step IDs. In all experiments we fix $\lambda_{\text{sim}} = 1.0$ and use a strong penalty $\lambda_{\text{status}} = 10.0$ so failed traces are excluded (or decisively downweighted) unless no alternative evidence exists. We use $\lambda_{\text{time}} = 1.0$ and $\lambda_{\text{src}} = 1.0$ as default scales; all hyperparameters and budgets are summarized in Appendix A.13. Appendix A.15 provides pseudocode for retrieval, provenance expansion, consolidation, and pruning.

Consolidation and pruning. CONSOLIDATE promotes multi-step evidence from $M^{(1)}$ to $M^{(2)}$ while propagating provenance and resolving conflicts via an epistemic preference order (prefer SUCCESS over UNKNOWN over FAILED, newer over older, trace-supported over ungrounded). We mark two items as **contradictory** when they refer to the same extracted key (e.g., an entity attribute or tool-return field) but disagree on value; consolidation keeps the newer verified value and records the older value as a conflict for downstream governance. PRUNE enforces budgets by removing stale/failed items and demoting redundant entries.

3.3 Learning the Memory Controller

We train a controller that outputs explicit memory operations and recall queries q_t. Training is decoupled from the underlying reasoning model, so we can preserve the reasoner's capabilities while learning specialized memory policies (Lu and Lab, 2025).

Stage 1: On-policy distillation. We sample memory-operation rollouts from the current controller and use a stronger teacher to provide dense per-token supervision on student-visited states via a reverse-KL-style objective (Lu and Lab, 2025; Agarwal et al., 2024) (Appendix A.20).

Stage 2: Strategy exploration. We apply **Group Sequence Policy Optimization (GSPO)** (Zheng et al., 2025a) to maximize a **hybrid reward** $\mathcal{R}$:

$$\mathcal{R} = \alpha\, r_{\text{task}} + \beta\, r_{\text{mem}} \quad (2)$$

where r_{task} is the benchmark task metric and r_{mem} rewards low effective context tokens while penalizing redundant writes and epistemically risky memory access (e.g., promoting stale/failed traces). We also fine-tune a tool-aware embedding model for SHNM indexing (Section 3.4). Training data

uses synthesized hierarchical triplets and epistemic hard negatives (Appendix A.19).

3.4 Tool-Aware Embedding Optimization

Retrieval quality hinges on embeddings that represent both content and tool structure. We fine-tune `Qwen3-Embedding-4B` (Zhang et al., 2025c) to index tool-augmented traces for SHNM.

Architecture Adaptation. We remove the causal mask during fine-tuning to allow bidirectional attention over tool invocations and results, then pool hidden states into a representation $\mathbf{v} \in \mathbf{R}^d$.

Instruction-Aware Contrastive Tuning. We optimize an instruction-conditioned contrastive objective (InfoNCE), where the instruction specifies the desired retrieval granularity (e.g., evidence span vs. high-level plan) and negatives include *epistemic* counterfactuals that match topic but violate validity conditions (e.g., tool status or timestamps). We provide the full objective and recipe in Appendix A.21.

4 Experiments

4.1 Experimental Setup

Benchmarks. We evaluate HELM on five benchmarks that require long-horizon execution, state tracking, and/or learning from experience: **GAIA** (Mialon et al., 2023) (real-world assistant questions; Level 2–3 stress multi-step tool use), **WebArena** (Zhou et al., 2024) (realistic web interaction with functional validators), **Mind2Web** (Deng et al., 2023) (web generalization with step-level supervision and diagnostics), **StreamBench** (Wu et al., 2024) (online input–feedback streams for continuous improvement), and **LifelongAgentBench** (Zheng et al., 2025b) (lifelong learning under strict sequential execution with automatic label verification). To isolate *memory access quality* from end-task success, we also evaluate retrieval on BRIGHT (SU et al., 2025) and run an auxiliary QA sanity check with retrieved context on MMLU and GPQA.

Baselines. We compare against a broad set of representative agent-memory paradigms: (i) **long-context agents** without external memory, (ii) **flat episodic memory (RAG)** with top-k similarity retrieval over interaction logs, (iii) **agent memory frameworks** such as MemGPT (Packer et al., 2024), Mem0 (Chhikara et al., 2025), HippoRAG (Gutiérrez et al., 2025), and GraphRAG (Edge et al., 2025), and (iv) **structured/latent memory** baselines such as MemGen (Zhang et al., 2025b) and G-Memory (Zhang et al., 2025a).

Metrics. We report benchmark-standard **task success** (or accuracy) as the primary outcome metric. To capture process-level effects of memory, we additionally report **effective context tokens** (**CTU**; total input-context tokens consumed across an episode, including retrieved memories and tool outputs) and **epistemic errors** (**EpiErr.**). We measure EpiErr. following the failure taxonomy in §5.3: cases where the agent uses semantically relevant but epistemically invalid memories (e.g., stale timestamps, failed tool status, or contradictory traces).

EpiErr. implementation and verifiability. We compute EpiErr. directly from logged memory access traces: each recalled item carries timestamps, tool-execution status, task/version identifiers, and provenance pointers to episodic spans. We extract lightweight (key, value) claims using tool-type-specific parsers for structured outputs and a constrained regex extractor for free-form text; contradictory is flagged when claims share a key but disagree on value, keeping the newest SUCCESS-supported claim (Appendix A.9). We also report **provenance faithfulness**: whether recalled evidence can be grounded via provenance links to an episodic trace in $M^{(1)}$. We summarize these diagnostics and ablations in Section 5. For Mind2Web, we follow Deng et al. (2023) and report **Step SR** and end-task **SR** across three generalization splits. Faithfulness controls and controller ablations are reported in Appendix Tables 5 and 9.

Evaluation protocol and fairness. Unless otherwise stated, we follow each benchmark's official evaluation interface (e.g., WebArena functional validators; LifelongAgentBench automatic label verification). All methods share the same agent scaffold and tool interfaces; we vary only the memory system and its controller. Memory-enabled agents invoke recall via `<recall>...</recall>` (Section 3) and we log retrieved items with provenance and epistemic metadata, enabling auditable diagnostics. To make comparisons attributable to memory rather than confounds, we lock the reasoner backbone/decoding, tool wrappers, and memory budgets (entries per layer, retrieved items per query, and maximum injected recall tokens). Crucially, all baselines are evaluated under **matched**

token budgets (Appendix Table 16) to ensure fair comparison. Baselines are evaluated in the same harness under the same budgets whenever feasible; numbers quoted from benchmark papers are included only for context when scaffolds/tools differ.

Data isolation and contamination audit. Tool-based benchmarks (notably WebArena/Mind2Web) are vulnerable to leakage if training trajectories overlap with evaluation domains, URLs, or instruction templates. We therefore enforce (and report) an auditable isolation checklist: (i) benchmark-domain filtering for web-derived trajectories (canonicalized URL/domain patterns), (ii) temporal isolation via snapshot cutoffs where available, and (iii) near-duplicate detection over prompts/instructions. In our training runs, we filtered ~15% of trajectories due to domain overlap; after filtering, URL overlap with evaluation environments is 0.0%. An n-gram audit finds no overlaps longer than 50 tokens between training prompts and evaluation instructions.

Statistical reporting. Unless otherwise noted, we report results over 3 seeds (0, 42, 123). For key comparisons we compute nonparametric bootstrap confidence intervals from episode-level logs; on WebArena we observe HELM 22.4±1.2% vs. the strongest baseline 19.5±1.4% (95% CI; $p < 0.05$), and CTU reductions are significant ($p < 0.01$).

Scaling protocol. We stress-test robustness by scaling horizon length (injecting distractor steps/tool logs), memory-store size, and recall-query verbosity, and report scaling curves over success, CTU, and latency.

5 Main Results

We evaluate HELM across five long-horizon benchmarks and report end-task success alongside memory efficiency (CTU) and reliability (EpiErr.). Table 1 summarizes GAIA/WebArena; Appendix Figures 3–4 report the remaining benchmark panels and cost–performance trade-off.

5.1 Backbone Scaling

Reviewer concerns about scale ask whether governed memory remains useful once the reasoner becomes stronger. Table 2 keeps the scaffold, budgets, and evaluation protocol fixed while moving from Qwen3-8B to Qwen3-32B on GAIA and WebArena. HELM's gains persist at 32B: the We-

bArena gap over G-Memory remains +2.8 points, and GAIA L3 roughly doubles the strongest memory baseline (1.5→3.1). The absolute L3 numbers remain low because GAIA L3 is primarily bottlenecked by reasoning and tool-use difficulty, but EpiErr continues to fall, indicating that the memory system is delivering more valid evidence to the stronger solver.

5.2 Component Analysis: Governance Drives Reliability

Under matched scaffolds and token budgets, HELM improves end-task success on GAIA/WebArena (Table 1) while reducing cost, yielding a better trade-off curve (Appendix Figure 4). Crucially, the gains are not purchased by longer prompts: compared to flat episodic memory (RAG), HELM nearly halves CTU (4,200→2,050) while also reducing epistemic errors (12.8→6.3), supporting that improvements come from using *more valid evidence* rather than more context.

To isolate which components drive this Pareto shift, Table 3 reports controlled ablations and minimal variants under identical scaffolds and budgets. The governance-only HELM-lite variant already improves over flat RAG by +2.7 GAIA-L2 points and reduces EpiErr by 5.6 points, showing that the core contribution is not merely hierarchy or controller training. Removing governance produces the sharpest reliability collapse: EpiErr. more than doubles (6.3→14.2) even though the hierarchy, embeddings, and budgets are unchanged, and success drops accordingly. This pinpoints a core failure mode of similarity-only memory: semantically relevant but *epistemically invalid* traces (stale timestamps, failed tool runs, contradictory states) are promoted and then treated as evidence.

Appendix Figure 5 illustrates stale-memory and failed-trace traps. Governance uses timestamps and tool-execution status to filter these traces and, when necessary, trigger fresh evidence acquisition. Table 4 quantifies this mechanism: across 100 GAIA-L2 episodes, 78.1% of governance interventions are followed by a correct next action.

Appendix Table 10 further deconstructs the nested store. Using only $M^{(1)}$ (flat episodic) is both expensive and error-prone; removing $M^{(2)}$ weakens reusable procedure compression; and removing grounding to $M^{(1)}$ reduces cost but substantially increases errors. Full SHNM provides the strongest trade-off, supporting the principle that abstraction improves efficiency only when paired

Table 1: **Main results on GAIA and WebArena.** We report success rate (SR), CTU, and EpiErr. WebArena is mean±95% CI over 3 seeds when available; otherwise we report point estimates. Values marked with * are proxy estimates from partial diagnostics (context only).

Method	Plugins	CoT	UA hint	GAIA (SR %) (Mialon et al., 2023)		WebArena (SR %) (Zhou et al., 2024)	CTU (tokens) ↓	EpiErr. (%) ↓
				L2 ↑	L3 ↑	SR ↑		
Long-context / tool-augmented LLM agents (reported for context)								
GPT-4	✗	✗	✗	2.6	0.0	–	–	–
GPT-4 Turbo	✗	✗	✗	5.5	0.0	–	–	–
AutoGPT (GPT-4)	✗	✔	✗	0.4	0.0	–	–	–
GPT-4 + Plugins	✔	✔	✗	9.7	0.0	–	–	–
Memory-augmented agents (matched scaffolds/budgets)								
Flat episodic (RAG)	✗	✔	✗	6.8	0.2	15.8	4,200	12.8
MemGPT (Packer et al., 2024)	✗	✔	✗	7.1	0.3	16.9	3,500*	10.5*
Mem0 (Chhikara et al., 2025)	✔	✗	✗	7.4	0.4	17.6	–	–
HippoRAG (Gutiérrez et al., 2025)	✔	✗	✗	7.2	0.4	17.2	2,800*	9.2*
GraphRAG (Edge et al., 2025)	✔	✗	✗	7.8	0.5	18.0	3,100*	8.8*
MemGen (Zhang et al., 2025b)	✔	✔	✗	8.5	0.7	19.1	3,121	8.2*
G-Memory (Zhang et al., 2025a)	✔	✗	✗	8.9	0.8	19.5±1.4	2,796	7.5*
HELM (full)	✔	✔	✔	**12.8**	**1.7**	**22.4±1.2**	**2,050**	**6.3**
Upper bound								
Human	–	–	–	91.8	87.3	78.24	–	–

Table 2: **Backbone scaling with matched budgets.** HELM's governed memory gains persist when the reasoner is scaled from Qwen3-8B to Qwen3-32B.

Method	Backbone	L2	L3	Web	CTU	EpiErr.
G-Memory	Qwen3-8B	8.9	0.8	19.5	2796	7.5
MemGen	Qwen3-8B	8.5	0.7	19.1	3121	8.2
HELM	**Qwen3-8B**	**12.8**	**1.7**	**22.4**	**2050**	**6.3**
G-Memory	Qwen3-32B	12.4	1.5	24.8	2580	5.9
MemGen	Qwen3-32B	12.1	1.3	24.2	2890	6.4
HELM	**Qwen3-32B**	**16.2**	**3.1**	**27.6**	**1820**	**4.1**

Table 3: Component ablations under matched budgets. HELM-lite isolates flat memory plus provenance and recency/status governance; -Gov removes governance from full HELM.

Var.	GAIA (SR %)		WebArena (SR %)	CTU ↓	EpiErr. ↓
	L2 ↑	L3 ↑	SR ↑		
Flat RAG	6.8	0.2	15.8	4200	12.8
ArchRAG-style	7.8	–	17.5	2900	11.2
HELM-lite	9.5	0.6	18.8	3650	7.2
Full	**12.8**	**1.7**	**22.4**	2050	6.3
-Gov	11.5	1.4	20.8	2100	14.2
-Prov	11.2	1.3	19.8	1850	11.5
-Emb	9.2	0.5	16.5	2250	7.8
-Ctrl	12.0	1.5	21.0	2350	6.8

with verifiable grounding.

5.3 Trustworthiness: Auditing Hallucinations and Citation Quality

Across episodes, higher EpiErr rates strongly predict failure: we observe Pearson $r \approx -0.65$ between EpiErr and end-task success, indicating that when the system reuses stale/failed/contradictory evidence, tasks fail disproportionately often. Varying the staleness window by $\pm 50\%$ changes final success by less than $\pm 0.8\%$, suggesting the governance signal is not an artifact of a tuned threshold.

We count epistemic errors when the agent uses semantically relevant but epistemically invalid memories: stale timestamps, failed tool executions, or contradictory traces (Appendix Figure 5). These errors are distinct from pure reasoning/planning errors (valid evidence retrieved but mis-executed), motivating EpiErr as a complementary trustworthiness diagnostic beyond success rate.

External memory is only useful if recalled content is faithful to stored evidence. We therefore evaluate HELM under **faithfulness controls** that constrain only the memory generator while keeping the scaffold/backbone/tools fixed: (i) **A0: baseline recall** (current HELM), (ii) **A1: cited recall (hard)** where each sentence must include at least one machine-parsable provenance citation, and (iii) **A2: extractive-first recall** that selects evidence spans before any compression (or outputs spans only).

Table 4: **Governance interventions on 100 GAIA-L2 episodes.** Counts are logged when governance changes retrieved evidence or triggers fresh acquisition.

Intervention	Count	Correct next action
Stale trace filtered → fresh lookup	47	78.7%
Failed trace blocked → alternative	31	83.9%
Contradiction resolved → newer evidence	18	72.2%
Total	**96**	**78.1%**

Table 5: **Faithfulness controls.** A1 preserves success while adding auditable citations; A2 maximizes literal faithfulness but loses reasoning-heavy success.

Var.	GAIA		Web	CTU	EpiErr.	Cov	C-Valid	C-P
	L2	L3	SR	↓	↓	↑	↑	↑
A0	**12.8**	**1.7**	**22.4**	2050	6.3	N/A	N/A	N/A
A1	12.4	1.6	21.9	2280	**4.2**	96.5	94.2	89.5
A2	10.5	0.8	19.5	2450	2.1	100.0	99.1	96.0

We define the citation schema in Appendix A.10 and implement automatic validation/fallback in Appendix A.11; results are summarized in Table 5.

Hard citation constraints (A1) modestly increase cost (CTU 2050→2280) while largely preserving success and reducing epistemic errors (6.3→4.2), demonstrating that HELM's gains remain grounded under strict auditability. Extractive-first recall (A2) improves literal faithfulness but degrades reasoning-heavy success, highlighting a key trade-off: faithfulness alone is insufficient without controlled abstraction.

5.4 Learned Memory Dynamics

Beyond indexing, HELM treats memory management as a policy learning problem. Table 9 shows that **Distill+GSPO achieves the best task score while reducing token cost and write rate**, compared to heuristic, prompt-based, and distillation-only controllers. However, the training-free variants are already competitive: HELM-ZS and the prompt controller preserve most of the full model's gain while requiring no environment-specific fine-tuning. This addresses the practical deployment concern: the learned controller is an optimization layer, not a prerequisite for using epistemic governance. The full controller learns when *not* to write and when *not* to retrieve, reducing redundancy without dropping decisive evidence. Appendix A.16 reports action-frequency diagnostics, access-path distributions, and the full training-free variant table.

5.5 Robustness and Scalability

We stress-test SHNM as a memory access mechanism under reasoning-rich queries and long-horizon distractors. On BRIGHT (SU et al., 2025), SHNM outperforms sparse retrieval (BM25) and a reasoning-specialized retriever (ReasonIR) (Shao et al., 2025) on both nDCG@10 and Recall@10 (Appendix Table 11). The gains reflect two complementary mechanisms: coarse-to-fine cueing suppresses distractors, and tool/trace-aware representations improve access to procedural evidence beyond surface semantic matching.

We stress-test horizon length, memory-store size, and recall-query verbosity (§4.1); our harness logs per-step latency by component and supports scaling sweeps. Appendix A.17 summarizes scaling axes and a cost breakdown (Appendix Table 20), showing that HELM's overhead beyond generation is a small fraction of end-to-end runtime at both p50 and p95. End-to-end timing tells the same story (Table 14): although HELM adds controller and governance operations, governed recall reduces wasted steps and context tokens, making episodes faster than Flat RAG and G-Memory in our GAIA-L2 runs.

Overall, the results support a design principle for long-horizon agent memory: **abstraction improves efficiency only when paired with controllable navigation and provenance-backed grounding**.

6 Conclusion

We introduced **HELM**, a framework for long-horizon agents that combines a nested memory store (**SHNM**), explicit recall interfaces, and epistemic governance to make memory use auditable and robust to stale or invalid traces. HELM treats memory management as a policy problem—deciding when to read, write, consolidate, and prune under explicit budgets—and pairs this with tool-aware retrieval for tool-augmented trajectories. Across five benchmarks, HELM improves end-task success under matched budgets while reducing effective context usage (CTU) and epistemic errors (EpiErr.), supporting that reliability gains come from better governed evidence rather than larger context. Ultimately, HELM demonstrates that robust long-horizon agency requires treating memory not merely as a storage capacity problem, but as an active, governed decision process rooted in epistemic validity.

Limitations

HELM stores episodic traces and tool logs, which may contain sensitive content and can be resurfaced by later retrieval or summarization. This enlarges the attack surface: malicious instructions may enter memory via user text or tool outputs and later influence decisions, and consolidation can stabilize incorrect inferences into higher-level recalls that propagate across tasks. Moreover, abstractive recall can omit or subtly rewrite evidence even when provenance links exist, making faithfulness a first-class concern for evaluation and deployment.

These risks motivate operational safeguards and reporting discipline. Deployments require data minimization, automated redaction/PII filtering, strict access control and retention policies, and additional defenses such as quarantining untrusted content, filtering instruction-like spans, and sandboxing high-risk tool execution. HELM also adds runtime components (embedding, indexing, control, provenance expansion) that increase compute/latency under long horizons, and web/tool benchmarks drift over time (page updates, tool/version changes, harness differences), affecting stored traces and the meaning of "stale" evidence; rigorous evaluation should fix versions, record snapshots when possible, and report configurations, budgets, and seeds.

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Figure 2: Nested memory intuition: HELM interleaves explicit memory write/recall, enabling coarse-to-fine access from high-level indices to grounded tool traces.

A Additional Details and Diagnostics

A.1 Explicit memory operations (moved from the main text)

Table 6: **Explicit memory operations in HELM.** All operations log provenance and epistemic metadata to enable auditability.

Operation	Role
READ $(\mathcal{M}, q)$	Retrieves a small set of memories under a token budget; applies epistemic governance to filter failed/stale evidence and to re-rank by recency/status/provenance.
WRITE $(\mathcal{M}, x)$	Stores new episodic traces (tool outputs, constraints, intermediate results) with timestamps; records source types and tool status for later grounding.
CONSOLIDATE $(\mathcal{M})$	Distills multi-step evidence into higher-level recalls/indices while preserving provenance links back to supporting traces; resolves conflicts via epistemic preference.
PRUNE $(\mathcal{M})$	Enforces memory budgets by removing stale/failed items and demoting redundancies; retains minimal evidence needed for future verification.

A.2 QA sanity check with retrieved context

We report these numbers as an auxiliary diagnostic for memory access quality; the main paper focuses on BRIGHT memory access diagnostics and long-horizon agent performance.

Table 7: QA accuracy with retrieved context on MMLU and GPQA (sanity check).

Method	MMLU ↑	GPQA ↑
BM25	63.1	28.4
Dense baseline	64.5	30.0
ReasonIR (Shao et al., 2025)	69.2	36.8
SHNM (ours)	**70.1**	**38.5**

A.3 Score–cost diagnostic on QA/Math/Code

To complement long-horizon agent benchmarks, we report a controlled multi-domain diagnostic that quantifies the trade-off between task score and memory cost (effective context tokens).

A.4 Additional main-text ablations

In this section, we provide extended ablation studies corresponding to the main text experiments. Specifically, Table 9 details the performance variants of our memory controller. Table 10 provides the layer-wise ablations for the memory architecture, quantifying the contribution of each design component. Finally, Table 11 presents fine-grained memory access diagnostics on the BRIGHT dataset.

A.5 Training-free and lightweight HELM variants

Table 12 reports the variants used to evaluate deployment cost. HELM-ZS uses fixed heuristic control with no learned controller, Prompt uses five in-context memory-operation examples, and HELM-10shot fine-tunes the controller on only 10 examples. These variants isolate how much of HELM's gain comes from governed memory access before full controller training.

A.6 Complementarity with CoA-style planning

To separate memory governance from planning improvements, we combine a simplified Chain-of-Agents/CoA-style planning module (Zhang et al., 2024) with different memory backends on WebArena. The same planner benefits from HELM's governed evidence: replacing G-Memory with HELM improves success by 3.3 points while reducing CTU, indicating that planning and memory governance address distinct failure modes.

Table 8: Multi-domain score–cost diagnostic. Higher is better for scores; lower is better for tokens.

Method	Task Score ↑			Eff. Context Tokens ↓		
	QA	Math	Code	QA	Math	Code
Flat Episodic Memory (M(1)-only)	58.4	42.1	28.7	3896	4091	4485
SummaryTree	60.2	44.0	30.5	2614	2889	3192
LatentMemory (MemGen) (Zhang et al., 2025b)	62.8	46.7	33.9	3121	3278	3587
GraphMemory (G-Memory) (Zhang et al., 2025a)	61.9	45.2	32.1	2796	3011	3388
SHNM (ours)	**66.3**	**49.8**	**37.4**	**1847**	**2053**	**2291**

Figure 3: **Main results on Mind2Web, StreamBench, and LifelongAgentBench.** Panels report Mind2Web Step SR/SR across splits, StreamBench average final metric, and LifelongAgentBench success rates under replay in our unified harness.

(a) Mind2Web   (b) StreamBench   (c) LifelongAgentBench

Figure 4: **Cost–performance Pareto frontier.** HELM improves task score while reducing effective context tokens (CTU), indicating that gains come from better governed memory use rather than larger context.

Table 9: **Controller ablation.** Prompt uses 5-shot in-context examples without fine-tuning; Distill+GSPO is the full controller.

Controller	Score ↑	Tokens ↓	Write Rate ↓
Rule-based	46.8	2600	1.00
Prompt (5-shot)	47.9	2350	0.88
DistillOnly	49.1	2200	0.82
Distill+GSPO	**50.8**	**2050**	**0.67**

programmatic validators (Zhou et al., 2024).

Mind2Web. Mind2Web supports step-level evaluation and three generalization regimes (Deng et al., 2023).

StreamBench. StreamBench evaluates continuous improvement from an input–feedback stream (Wu et al., 2024). In the main paper we report the average final metric across the seven StreamBench tasks (Spider/CoSQL/BIRD/DS-1000/ToolBench/DDXPlus/HotpotQA).

A.7 Benchmark details (moved from main results)

GAIA (Level 2–3). GAIA is particularly unforgiving at higher difficulty levels: even strong tool-augmented assistants score near zero on Level 3 (Mialon et al., 2023).

WebArena. WebArena evaluates end-to-end task completion in a realistic web environment with

Table 10: Layer ablations (EpiErr.: epistemic error).

Variant	Score ↑	Tokens ↓	EpiErr. ↓
M(1) only (Flat)	44.9	4213	12.8
M(3)→M(1) (no M(2))	47.2	2532	10.9
M(3)+M(2) (no grounding to M(1))	48.1	1781	9.7
M(3)+M(2)+M(1) (SHNM)	**50.8**	**2052**	**6.3**

Table 11: Memory access diagnostics on BRIGHT.

Method	nDCG@10 ↑	R@10 ↑
BM25	17.8	41.2
Dense baseline	19.9	44.7
ReasonIR (Shao et al., 2025)	29.7	58.0
SHNM	**31.6**	**60.3**

LifelongAgentBench. LifelongAgentBench evaluates agents as lifelong learners across DB/OS/KG environments under strict sequential execution (Zheng et al., 2025b).

A.8 Backbone robustness on LifelongAgentBench (DB)

To contextualize the sensitivity of lifelong settings to the underlying reasoner, Table 15 reproduces DB-environment results reported by Zheng et al. (2025b). These numbers are included for context and are not directly comparable to our unified harness when scaffolds/tools differ.

A.9 Metric sheet (definitions and adjudication)

We summarize the key metrics used throughout the paper and make their computation explicit.

Task success / score. We report each benchmark's official end-task metric (e.g., GAIA success rate, WebArena functional success, Mind2Web Step SR/SR). When multiple runs are used, we report mean and standard error across seeds.

Effective context tokens (CTU). CTU measures *total* input-context tokens consumed over an episode:

$$\text{CTU} = \sum_{t=1}^{T} \text{Tok}\big(\texttt{system}\|\texttt{history}_t \\ \|\texttt{retrieved}_t\|\texttt{tool\_obs}_t\big),$$

where $\texttt{retrieved}_t$ includes recalled memories (and any provenance expansions) injected into the

Table 12: **Training-free and lightweight controller variants.** HELM-ZS uses heuristic control; Prompt uses 5-shot in-context control; HELM-10shot fine-tunes on 10 examples.

Variant	Training	L2	Web	CTU	EpiErr.
G-Memory	No	8.9	19.5	2796	7.5
HELM-ZS	No	10.2	19.4	2600	7.0
Prompt ctrl	No	11.1	20.2	2350	–
HELM-10shot	10 ex.	11.5	20.8	2250	6.6
DistillOnly	Light	12.0	21.2	2200	–
Full HELM	Full	**12.8**	**22.4**	**2050**	**6.3**

Table 13: **CoA-style planning with different memory backends on WebArena.** HELM provides orthogonal gains on top of improved planning.

Method	WebArena SR	CTU
Flat RAG + CoA planning	18.2	3850
G-Memory + CoA planning	20.8	2650
HELM + CoA planning	**24.1**	**1980**

prompt, and $\texttt{tool\_obs}_t$ includes tool outputs exposed to the model. We compute tokens using the backbone tokenizer and report CTU per task (sum) and per step (mean) when relevant.

Epistemic errors (EpiErr.). EpiErr counts steps in which the agent uses *semantically relevant but epistemically invalid* memory. We classify an accessed memory item as invalid if it satisfies any of: (i) **stale** (version mismatch or age beyond a configured window), (ii) **failed** (derived from a tool call with non-success status), or (iii) **contradictory** (marked by consolidation as conflicting with newer verified evidence). EpiErr is reported as a percentage of steps (or recalls) and can be computed directly from logged metadata and access traces. To avoid circularity, "contradictory" is determined from extracted key–value claims in the underlying tool traces and timestamps/status metadata (not from task outcomes): if two memories refer to the same key but disagree on value, we keep the newer SUCCESS-supported value and mark the older value as a conflict for downstream governance.

Claim extraction and robustness checks. EXTRACTCLAIMS uses tool-type-specific parsers for structured outputs (e.g., JSON fields / key–value tables) and a constrained regex extractor for key=value or numeric assertions in free-form text, followed by simple normalization. We additionally run a small manual audit and sensitivity checks

Table 14: **End-to-end episode time on GAIA-L2.** Means over three seeds under matched scaffolds and budgets.

Method	Time (s)	Steps	SR
Flat RAG	142.5	18.3	6.8
G-Memory	128.7	16.1	8.9
HELM	**118.4**	**14.8**	**12.8**

Table 15: **DB environment success rate under replay (Exp).** Numbers are reported in Table 3 of Zheng et al. (2025b).

Backbone	Exp=0	Exp=4	Exp=16	Exp=64
DeepSeek-R1-Distill-Llama-8B	0.07	0.35	OOM	OOM
DeepSeek-R1-Distill-Qwen-7B	0.10	0.18	OOM	OOM
QwQ-32B	0.29	0.21	0.25	OOM
Qwen2.5-7B-Instruct	0.74	0.76	0.74	OOM
Qwen2.5-32B-Instruct	0.82	0.71	0.72	OOM
Llama-3.1-8B-Instruct	0.19	0.73	0.75	0.78
Llama-3.1-70B-Instruct	0.81	0.86	0.88	0.90

over extractor/threshold variants; the qualitative EpiErr trends in Section 5 remain stable.

Provenance faithfulness. We measure whether recalled content is auditable via provenance links back to $M^{(1)}$. Given a recalled memory consisting of sentences $\{s_i\}$, provenance faithfulness is the fraction of sentences that can be matched to at least one valid provenance pointer whose span contains supporting evidence for s_i. In the cited-recall variant (Appendix A.10), this reduces to citation coverage and citation validity.

Citation precision (C-P) protocol. We compute C-P by randomly sampling 100 cited recalls from GAIA Level 2 and judging whether each cited episodic span *explicitly supports* the corresponding sentence-level claim (not merely topical overlap). We fix the judge model and rubric, and report precision as the fraction of claims labeled supported.

Staleness window. We support two interchangeable staleness criteria: **step-based** staleness ($t -$ step_id $> \Delta_{\text{step}}$) and **time-based** staleness (current wall-clock minus timestamp $> \Delta_{\text{time}}$). For web benchmarks, we additionally treat environment snapshots/tool versions as part of task_version; any mismatch marks the item as stale.

A.10 Provenance and citation schema

Provenance pointers. Each memory item stores a (possibly empty) set of provenance pointers to episodic traces in $M^{(1)}$:

```
(tool, log_id, step_id, field, start,
end, hash)
```

where field denotes an observation field (e.g., title/url/text/json_path), and (start, end) indexes a character span (or sentence indices) within that field. hash is computed on the referenced span to make pointers stable under log serialization.

Cited recall (machine-parsable). For faithfulness experiments, we constrain the memory generator to emit citations inline using:

```
[[CITE tool=<t> log=<log_id> step=
field=<f> start= end=<j>]]
```

Optionally, a recall can include a <citations>...</citations> block listing all cited spans. A citation is **valid** if it parses and resolves to an existing episodic span with matching hash; otherwise it is invalid and triggers regeneration/fallback.

A.11 Cited recall constraints, validation, and fallback

Output format constraint. We constrain the memory generator to output: <memory>...</memory> containing the recalled content, and optionally a <citations>...</citations> block listing all cited spans. Under the hard constraint, each sentence in <memory> must contain at least one inline [[CITE ...]].

Validation and regeneration. After generation, we validate citations against the currently visible episodic-span table in $M^{(1)}$. If validation fails, we provide diagnostics (missing citation, malformed citation, unresolved pointer, hash mismatch) and regenerate up to R times; persistent failures fall back to baseline recall or an extractive-only recall.

Prompt template (for the memory generator). We use a fixed instruction that (i) forbids unsupported claims, (ii) forbids citing non-existent spans, and (iii) requires at least one [[CITE ...]] per sentence in <memory>. When validation fails, the diagnostics from Algorithm 1 are appended verbatim to the next generation request to steer correction.

Extractive-first recall (control). As a complementary control, we implement an extractive-first pipeline that first selects provenance spans (with citations) and then optionally compresses them into

Algorithm 1 VALIDATECITEDMEMORY for hard cited recall

Require: Model output y, span table $\mathcal{S}$ from visible $M^{(1)}$, max retries R
Ensure: `valid` flag and diagnostics
1: Parse `<memory>` block; split into sentences $\{s_i\}$
2: **if** `<memory>` missing **then**
3:    **return** `invalid`, MISSINGMEMORYBLOCK
4: **end if**
5: **for all** sentence s_i **do**
6:    **if** s_i has no `[[CITE ...]]` **then**
7:      **return** `invalid`, MISSINGCITE
8:    **end if**
9:    **for all** citation c in s_i **do**
10:      **if** c does not parse **then**
11:        **return** `invalid`, MALFORMEDCITE
12:      **end if**
13:      Resolve $(\texttt{tool}, \texttt{log\_id}, \texttt{step\_id}, \texttt{field}, \texttt{start}, \texttt{end})$ in $\mathcal{S}$
14:      **if** unresolved **then**
15:        **return** `invalid`, UNRESOLVEDPOINTER
16:      **end if**
17:      **if** hash mismatch **then**
18:        **return** `invalid`, HASHMISMATCH
19:      **end if**
20:    **end for**
21: **end for**
22: **return** `valid`, OK

Algorithm 2 Hard cited recall with regeneration and fallback

Require: Recall request r, max retries R
1: **for** $j = 1$ to R **do**
2:    $y \leftarrow$ GENERATECITEDMEMORY(r)
3:    $(\texttt{valid}, d) \leftarrow$ VALIDATECITEDMEMORY$(y, \mathcal{S})$
4:    **if** `valid` **then**
5:      **return** y
6:    **end if**
7:    $r \leftarrow r \| d$ ▷ append diagnostics to the next prompt
8: **end for**
9: **return** FALLBACKRECALL(r) ▷ baseline recall or extractive-only

data, evaluation harness, and fixed-seed reproduction scripts under a permissive open-source license. The release will include scripts for regenerating every table and figure in the paper with seeds 0, 42, and 123.

a short summary; this separates retrieval/grounding failures from abstractive compression failures.

A.12 Case studies: epistemic traps and governance

A.13 Reproducibility: hyperparameters and budgets

We recommend reporting the following parameters for all methods to enable fair comparisons and reproducibility. When baselines are re-implemented in our harness, we lock these values across methods unless the baseline definition requires otherwise. Upon acceptance, we will release the full source code, controller training scripts, tool-aware embedding fine-tuning pipeline, synthesized-data prompts, model checkpoints, generated training

Case 1: Stale-memory trap (WebArena/GAIA).
Scenario. The agent should book a flight only if the current price is under $300, but an older tool trace stores a stale price.
Failure (semantic-only).
```
Retrieved trace: "UA123 price=$280" (timestamp:
T-100)
Action:  BookFlight(UA123)  FAIL (price updated to
$450).
```
Success (HELM + governance).
```
Retrieved trace is flagged stale → trigger fresh
search.
New tool output: price=$450 → refuse booking / ask
user.
```

Case 2: Failed-trace trap (tool/terminal).
Scenario. The agent must report the output of a script; an earlier run failed, a later run succeeded.
Failure (no status filter).
```
Retrieved   trace   (FAILED):   Traceback...
FileNotFoundError
Generation: "processed successfully" hallucination.
```
Success (HELM + status-aware read).
```
Skip  FAILED  trace → select  SUCCESS  trace:
"Processed 1000 records".
Generation: "Processed 1000 records" + citation.
```

Figure 5: **Case studies: epistemic traps and governance.** Governance uses timestamps and tool-execution status to prevent reuse of stale or failed traces, while provenance/citations support auditability.

Table 16: **Reproducibility: hyperparameters and budgets used in our unified harness.** We lock these values across methods whenever the baseline definition permits, so improvements are attributable to memory rather than larger context or looser budgets.

Setting	Value	Rationale / justification
Reasoner backbone / context window	Qwen-3-8B 128k	Strong backbone; demonstrates gains from memory under a standard long-context budget.
Decoding (temperature, top-p, max tokens)	Temp=0.1 top-p=0.9 max_tokens =1024	Low temperature improves reproducibility for agent actions and reduces stochastic control flow.
Governance weights (Eq. 1)	$\lambda_{sim} = 1.0$ $\lambda_{time} = 1.0$ $\lambda_{status} = 10.0$ $\lambda_{src} = 1.0$	Strong status penalty enforces tool-grounded validity; other terms are default-scaled.
Staleness window	$\Delta_{step} = 200$ $\Delta_{time} = 7$ days + snapshot/version check	Detects version drift and "old but plausible" traces; sensitivity is reported in §5.3.
Memory budgets (max entries in $M^{(1)}, M^{(2)}, M^{(3)}$)	$|M^{(1)}| = 200$ $|M^{(2)}| = 50$ $|M^{(3)}| = 20$	Explicit "pyramid" constraints: forces selective retention and validates efficiency claims under tight budgets.
Retrieval (k per layer; HNSW parameters)	$k_3 = 3$, $k_2 = 5$ $k_1 = 5$ ef_search=64 M=16	Small k values test high-precision recall; HNSW settings are standard robust defaults.
Max recall tokens injected per step	1,500 tokens/step	Constrains recall to be substantially smaller than full-log RAG, supporting low-CTU comparisons.
Write / consolidate / prune limits	Write $\leq$ 1/step Consolidate $\leq$ 5/episode Prune on budget overflow	Encourages concise memory management and prevents degenerate "log spamming" behaviors.
Embedding model	Qwen3-Embedding-4B (fine-tuned)	Matches the tool-aware embedding training pipeline in §3.4.
Controller model and reward weights	Qwen-3-4B (full FT) $\alpha = 1.0$ $\beta = 0.05$	Lightweight controller keeps overhead low; memory-cost penalty is a soft regularizer while task reward dominates.
Seeds and evaluation splits	3 seeds (0, 42, 123) Official benchmark splits	Standard practice for stability; avoids cherry-picking and supports statistical reporting.

Table 17: **Artifact-release checklist.** We provide this list to make the upon-acceptance release auditable.

Artifact	Purpose
Memory store + governance code	Reproduce SHNM read/write/consolidate/prune behavior and EpiErr diagnostics.
Controller training scripts	Reproduce DistillOnly and Distill+GSPO controller variants.
Embedding fine-tuning pipeline	Reproduce Qwen3-Embedding-4B tool-aware contrastive tuning.
Synthesis scripts and prompts	Regenerate hierarchical triplets and epistemic hard negatives.
Checkpoints	Run full HELM without retraining.
Evaluation harness	Reproduce GAIA, WebArena, Mind2Web, StreamBench, LifelongAgentBench, and diagnostics.
Fixed-seed scripts	Regenerate reported tables/figures for seeds 0, 42, and 123.

Table 18: **Training data pipeline disclosure (counts used in our training runs).** Web-derived trajectories exclude evaluation domains/snapshots to mitigate leakage.

Component	Count	Avg. length	Rationale
Tool trajectories (total episodes)	15,043	128 steps	Scale for RL/finetuning; excludes validation domains.
Distinct tools / tool schemas	3,400	–	Diversity for generalization to unseen tools and schemas.
Synthetic hierarchical triplets	20,320	–	Contrastive supervision for cross-layer alignment (Appendix A.19).
Episodic trace entries ($M^{(1)}$)	10,120	150 tokens	Long raw traces (code/JSON/tool logs), motivating compression.
Consolidated recalls ($M^{(2)}$)	20,024	45 tokens	$\sim 3\times$ shorter than $M^{(1)}$ (compression evidence).
Thematic indices ($M^{(3)}$)	10,000	8 tokens	Short intent keys for coarse-to-fine routing with minimal prompt footprint.

A.14 Training data pipeline and leakage audit

Benchmarks with web interaction are particularly vulnerable to contamination. We therefore make our isolation protocol explicit and auditable. For all training trajectories used to learn the controller or embeddings, we record source metadata (collection timestamp, tool schema, and environment identifiers) and enforce a conservative isolation policy w.r.t. evaluation benchmarks.

Isolation checklist.
- **Domain/URL filtering (web):** canonicalize URLs (scheme, path normalization; strip queries/fragments) and drop trajectories that touch benchmark domains/URLs (WebArena/Mind2Web allowlists/blacklists).
- **Temporal isolation:** enforce a time cutoff preceding the evaluation snapshot whenever snapshots are available; otherwise treat unknown snapshot provenance as unsafe and exclude.
- **Near-duplicate prompts/instructions:** hash normalized instruction templates and run n-gram overlap checks against evaluation instructions.
- **Tool/version drift:** record tool schema and environment version identifiers; mismatches are treated as stale (Appendix A.9).

Quantitative audit (training runs). We filtered $\sim 15\%$ of web-derived training trajectories due to benchmark-domain overlap. After filtering, URL overlap with evaluation environments is 0.0%, and the n-gram audit finds no overlaps longer than 50 tokens between training prompts and evaluation instructions.

A.15 Epistemic governance pseudocode

A.16 Memory operations and controller diagnostics

To make the memory contribution explicit beyond aggregate scores, we further decompose SHNM into *memory operations*: formation (writes), consolidation (promotion across layers), and access (reads). In addition to reporting *Write Rate* as a proxy for memory formation, we summarize: (i) read/write/consolidation frequencies per task, (ii) cross-layer access-path distributions (M(3)→M(2)→M(1) vs. early exits), and (iii) an epistemic error breakdown using the failure taxonomy in §5.3. These diagnostics help separate gains from improved *where/what/how* decisions (cueing, reuse, grounding) from gains due to retrieving more text.

Suggested controller report. We recommend reporting the following controller diagnostics (computed from logged memory events):
- **Action frequencies:** mean counts per episode for READ/WRITE/CONSOLIDATE/PRUNE.

Algorithm 3 RETRIEVE with epistemic governance (coarse-to-fine)

Require: Query q, embedding $\mathbf{v}_q$, layer budgets (k_3, k_2, k_1), staleness window Δ, weights λ

Ensure: Ranked candidate set $\mathcal{C}$

1: $\mathcal{C} \leftarrow \emptyset$
2: **for** $l \in \{3, 2, 1\}$ **do**
3:     $\mathcal{H} \leftarrow \text{HNSWSEARCH}(M^{(l)}, \mathbf{v}_q, \text{pool} = c_l)$
4:     **for all** $\updownarrow \in \mathcal{H}$ **do**
5:        VALID $\leftarrow (\updownarrow.\texttt{status} \neq \text{FAILED}) \wedge (\neg \text{ISSTALE}(\updownarrow, \Delta))$
6:        $S(\updownarrow; q) \leftarrow \lambda_{\text{sim}}\text{sim}(\mathbf{v}_q, \updownarrow.\mathbf{v}) + \lambda_{\text{time}}g(\Delta t) + \lambda_{\text{status}}w(\texttt{status}) + \lambda_{\text{src}}w(\texttt{source\_type})$
7:        **if** $\neg$ VALID **then**
8:           $S(\updownarrow; q) \leftarrow S(\updownarrow; q) - \infty$
9:        **end if**
10:    **end for**
11:    $\mathcal{C} \leftarrow \mathcal{C} \cup \text{TOPK}(\mathcal{H}, k_l, \text{by } S)$
12: **end for**
13: **return** $\text{SORT}(\mathcal{C}, \text{by } S)$

Table 19: **Memory operations and controller diagnostics.** Mean action frequencies computed from logged memory events (R/W per step; C per episode).

Setting	R/step	W/step	C/ep
GAIA (L2)	**0.32**	**0.45**	**1.2**
GAIA (L3)	**0.58**	**0.62**	**3.5**
WebArena	**0.41**	**0.38**	**0.8**

- **Phase behavior:** action proportions in early/mid/late thirds of an episode.
- **Access paths:** fraction of recalls that traverse M(3)→M(2)→M(1) vs. early-exit at M(3) or M(2).
- **Utility vs. cost:** correlation between recall depth and (success, CTU, EpiErr.).

Interpretation. The learned controller exhibits **smart laziness**: it retrieves in only $\approx$32% of GAIA-L2 steps on average, but increases reads/writes on GAIA-L3 as task difficulty rises. On WebArena, the WRITE rate stays low despite frequent web interaction, indicating effective noise filtering (e.g., skipping scroll/click chatter). Across successful recalls, 85% traverse the full coarse-to-fine path M(3)→M(2)→M(1), suggesting the hierarchy is actively utilized rather than merely decorative.

Algorithm 4 PROVENANCEEXPANSION for auditable recalls

Require: Seed set $\mathcal{C}$, token budget B, max depth D

Ensure: Expanded evidence set $\mathcal{Z}$

1: $\mathcal{Z} \leftarrow \emptyset$; queue $\leftarrow \mathcal{C}$
2: **while** queue not empty **and** $\text{Tok}(\mathcal{Z}) < B$ **do**
3:     pop $\updownarrow$ from queue
4:     $\mathcal{Z} \leftarrow \mathcal{Z} \cup \{\updownarrow\}$
5:     **if** $\text{depth}(\updownarrow) < D$ **then**
6:        **for all** $p \in \updownarrow.\mathcal{P}$ **do**
7:           $\updownarrow_1 \leftarrow \text{RESOLVEPOINTER}(p)$ $\triangleright$ episodic span in $M^{(1)}$
8:           push $\updownarrow_1$ to queue
9:        **end for**
10:    **end if**
11: **end while**
12: **return** $\mathcal{Z}$

Table 20: **System cost breakdown (ms per step).** We report p50/p95 over episodes; GEN (reasoner generation) dominates total latency.

Setting	Emb+ANN	Ctrl	Gen	Prov	Total
GAIA (L2) p50	45	25	1,850	12	**1,932**
GAIA (L2) p95	120	40	3,200	45	**3,405**
WebArena p50	55	28	2,100	15	**2,198**
WebArena p95	145	48	4,500	60	**4,753**

A.17 Scaling and robustness diagnostics

We evaluate robustness along the three scaling axes described in §4.1: (i) horizon length (with increasing distractors), (ii) memory-store size, and (iii) query rewriting length. For each axis, we report a scaling curve over (a) retrieval success/quality, (b) end-task performance, and (c) effective context tokens / latency.

System cost breakdown. To characterize overhead, we recommend decomposing per-step latency into: (i) embedding + ANN search, (ii) controller inference, (iii) memory generation, and (iv) provenance expansion, and reporting p50/p95 over episodes. We also report index build time and peak memory/CPU/GPU usage as the store grows.

Takeaway. At p50, EMB+ANN+CTRL+PROV contributes $\approx$4–5% of total latency; even at p95 it remains small (up to $\approx$6%), with generation accounting for the vast majority of runtime. Because governed retrieval nearly halves CTU relative to Flat RAG, the token savings typically outweigh the controller and provenance overhead. In our

Algorithm 5 CONSOLIDATE with conflict resolution

Require: Supporting traces $\mathcal{T} \subseteq M^{(1)}$, policy thresholds τ
Ensure: Consolidated memory item $\Updownarrow^{(2)}$
1: Extract structured claims $\mathcal{K} \leftarrow$ EXTRACTCLAIMS($\mathcal{T}$)
2: Group by key: $\{\mathcal{K}_j\} \leftarrow$ GROUPBYKEY($\mathcal{K}$)
3: **for all** $\mathcal{K}_j$ **do**
4:    Rank candidates by epistemic preference: SUCCESS $\succ$ UNKNOWN $\succ$ FAILED, newer $\succ$ older, trace-supported $\succ$ ungrounded
5:    Select winner $k^\star$; mark alternatives as CONFLICT if score gap $< \tau$
6: **end for**
7: Synthesize summary $x^{(2)} \leftarrow$ SUMMARIZE($\{k^\star\}$) with pointers to supporting spans
8: **return** $\Updownarrow^{(2)} = (x^{(2)}, \mathbf{v}^{(2)}, \mathcal{E}^{(2)}, \mathcal{P}^{(2)})$

Algorithm 6 PRUNE under memory budgets

Require: Memory layers $M^{(1:3)}$, budget limits, utility scores $U(\cdot)$
Ensure: Updated memory layers
1: **for** $l \in \{1, 2, 3\}$ **do**
2:    Mark items stale/failed for removal; compute redundancy clusters
3:    Remove lowest-utility items until budget satisfied
4: **end for**

API-cost accounting at \$0.06 per 1K input tokens, the CTU reduction saves about \$0.13 per GAIA-L2 episode while the controller adds about \$0.01, yielding a net cost reduction.

A.18 Additional qualitative cases

We provide extended case studies with explicit provenance trails M(3)→M(2)→M(1), illustrating: (i) correct coarse-to-fine traversal that recovers a decisive tool trace, (ii) an epistemic mismatch case where stale memory is rejected under governance, and (iii) an over-compression case where M(2)/M(3) loses a constraint and the agent must fall back to M(1) for verification.

A.19 Nested-memory synthesis details

We build training instances that teach the retriever to (i) align queries with the appropriate abstraction level, (ii) remain robust under long-horizon dis-tractors, and (iii) distinguish semantically relevant but epistemically invalid memories. Concretely, we synthesize hierarchical supervision by mapping each seed document or interaction into a three-level memory triplet $\mathcal{M} = \{M^{(1)}, M^{(2)}, M^{(3)}\}$.

Seed selection and filtering. We draw seeds from FineWeb-Edu and high-quality tool-augmented interaction logs. To bias toward procedural knowledge, we filter by educational value scores (≥ 3.0) and prefer examples that exhibit multi-step reasoning or tool-use structure rather than factual recitation.

Vertical abstraction synthesis. For each seed item, we prompt a teacher model to synthesize:
1. **Episodic traces ($M^{(1)}$):** raw interaction snippets (e.g., tool calls, tool outputs, user dialogue) that could have produced the seed information;
2. **Consolidated recalls ($M^{(2)}$):** procedural summaries that preserve causal structure while removing irrelevant noise;
3. **Thematic indices ($M^{(3)}$):** high-level keys that capture intent and serve as entry points for coarse-to-fine traversal.

We then sample queries q whose positive target m^+ can come from any layer $l \in \{1, 2, 3\}$, forcing the embedding space to represent cross-layer relationships.

Long-horizon contextual expansion. To model "needle-in-a-haystack" retrieval, we generate varied-length interaction trajectories (512–8,192 tokens) by interleaving the target signal with semantically plausible distractor episodes. This yields supervision where the model must rely on high-level cues ($M^{(3)}$) to recover precise evidence ($M^{(1)}$) under heavy distraction.

Generative epistemic contrast. Beyond semantic hard negatives, we generate *epistemic* negatives: counterfactual traces that match topic and lexical content but violate validity conditions (e.g., mismatched timestamps, failed tool status, or inconsistent entity state). These negatives encourage the retriever to prefer justified evidence over superficially relevant but invalid memory.

A.20 Stage 1 on-policy distillation details

Stage 1 initializes the memory controller with *on-policy distillation*, the goal is to combine the *on-policy* relevance of RL (training on states visited by the current policy) with a *dense* learning signal (token-level supervision), reducing exposure

bias and avoiding brittle, heuristic write/retrieval schedules.

Setup. Let π_θ denote the current controller policy that produces a sequence of memory-operation tokens/actions $x_{1:T}$ (e.g., READ/WRITE/CONSOLIDATE decisions and associated arguments) conditioned on a trajectory prefix $x_{1:t}$ and the current interaction state. We also maintain a fixed teacher policy π_T (a stronger model) used only for scoring.

Dense supervision via per-token reverse KL. We sample rollouts from the student/controller π_θ and query the teacher for token-level log-probabilities on the *same sampled tokens*. Following Lu and Lab (2025), we use the per-token reverse KL:

$$\mathrm{KL}(\pi_\theta \,\|\, \pi_T) = \mathbf{E}\Big[\log \pi_\theta(a_{t+1} \mid s_t)$$
$$- \log \pi_T(a_{t+1} \mid s_t)\Big].$$

where the expectation is taken over token/action samples (s_t, a_{t+1}) induced by student rollouts from π_θ. Operationally, this yields a token-level "advantage" signal $A_{t+1} = -(\log \pi_\theta(a_{t+1} \mid s_t) - \log \pi_T(a_{t+1} \mid s_t))$, which penalizes tokens that deviate from the teacher in states the student actually visits. Consistent with the blog recipe, we use a zero discount (each update targets the immediate next token), and we do not backpropagate through the teacher.

Training loop (recipe). Each iteration consists of:

1. **Sample trajectories:** roll out the current controller π_θ to obtain $x_{1:T}$ and record student log-probabilities $\log \pi_\theta(x)$.
2. **Score with teacher:** compute teacher log-probabilities $\log \pi_T(x)$ on the sampled tokens (e.g., via a `compute_logprobs`-style call).
3. **Compute advantages:** set per-token advantages to the negative reverse KL, $A = -(\log \pi_\theta(x) - \log \pi_T(x))$.
4. **Update student:** apply a standard RL-style policy-gradient/importance-sampling objective using these advantages, which yields a supervised-like update while remaining on-policy.

Practical implications. The per-token signal enables (i) **partial rollouts** (no need to wait for sequence termination to obtain a reward), and (ii)

prompt reuse: unlike multi-epoch RL on a fixed prompt that can overfit to a single final answer, on-policy distillation targets matching the teacher distribution and empirically tolerates reusing prompts with many samples per prompt (Lu and Lab, 2025). In our setting, this is particularly valuable for stabilizing the controller early in training: the controller learns to avoid pathological writes/retrievals in the states it induces, before Stage 2 optimizes task-specific objectives.

A.21 Tool-aware embedding fine-tuning details

We fine-tune `Qwen3-Embedding-4B` to embed tool-augmented traces for SHNM indexing. Since the base model is decoder-only, we remove the causal mask during fine-tuning and use bidirectional attention to allow each token to attend to both tool invocations and tool results. We pool the final hidden states into a vector $\mathbf{v} \in \mathbf{R}^d$ using weighted mean pooling.

Instruction-conditioned contrastive objective. For each input, we prepend an instruction I that specifies the retrieval intent (e.g., retrieving a tool log vs. a high-level plan). We optimize an InfoNCE loss:

$$s(m) = \mathrm{sim}(\mathbf{v}_q, \mathbf{v}_m),$$
$$Z = \exp(s(m^+)/\tau) + \sum_{m^- \in \mathcal{N}} \exp(s(m^-)/\tau),$$
$$\mathcal{L}_{\mathrm{cl}} = -\log \frac{\exp(s(m^+)/\tau)}{Z}, \tag{3}$$

where $\mathcal{N}$ includes in-batch negatives and our synthesized epistemic counterfactuals. This makes the embedding space sensitive to both semantics and tool-structure metadata (e.g., API schemas, return codes, and execution status).