Expected Free Energy-based Planning as Variational Inference

Anonymous authors
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Abstract

Planning under uncertainty requires agents to balance goal achievement with information gathering. Active inference addresses this through the Expected Free Energy (EFE), a cost function that unifies instrumental and epistemic objectives. However, existing EFE-based methods typically employ specialized optimization procedures that are difficult to extend or analyze. In this paper, we show that EFE-based planning can be formulated as standard variational inference on a generative model augmented with epistemic priors. Our main result demonstrates that minimizing a Variational Free Energy functional with appropriately chosen priors yields a decomposition into expected plan costs (the EFE) plus a complexity term. This formulation reinforces theoretical consistency with the Free Energy Principle by casting planning as the same inferential process that governs perception and learning. We validate our approach on a T-maze task, demonstrating that the epistemic priors are sufficient for inducing information-seeking behavior.

1 Introduction

Agents operating in uncertain environments face a fundamental tension: should they exploit current knowledge to achieve immediate goals, or explore to gather information that may improve future decisions? Classical approaches in reinforcement learning and optimal control typically address this through value function estimation or policy learning (Sutton & Barto, 2018; Bertsekas, 2012), but often treat reward maximization and uncertainty reduction as separate objectives, relying on heuristics to balance exploration and exploitation.

Active inference offers an alternative, grounded in the Free Energy Principle (FEP), which casts perception, learning, and action selection as inference processes that minimize a variational bound on surprise (Friston, 2010; Parr et al., 2022). Central to this framework is the Expected Free Energy (EFE), an objective that combines instrumental (goal-directed) and epistemic (information-seeking) components (Friston et al., 2015). Minimizing EFE yields behavior that simultaneously pursues preferred outcomes and resolves uncertainty.

However, existing implementations of EFE-based planning typically employ specialized optimization procedures, such as tree search or explicit enumeration over policies, that are difficult to extend or analyze using standard tools (Friston et al., 2021; Paul et al., 2024a). This has limited both the theoretical understanding of EFE minimization and its practical scalability.

In this work, we show that EFE-based planning can be formulated as standard variational inference. Our central result (Theorem 1) demonstrates that minimizing a Variational Free Energy (VFE) functional on a generative model augmented with epistemic priors yields a decomposition into expected plan costs (the EFE) plus a complexity term. This formulation achieves theoretical alignment with the FEP: planning emerges from the same variational principle that governs perception and learning. Our contributions are as follows:

- We derive the *EFE Theorem*, which shows that minimizing a Variational Free Energy functional with appropriately chosen epistemic priors yields EFE-based planning as a natural consequence.
- We identify the specific epistemic priors that induce risk-minimizing, ambiguity-reducing, and novelty-seeking behavior.

- We provide empirical validation demonstrating that these epistemic priors are sufficient for information-seeking behavior in a canonical T-maze benchmark task.

The results demonstrate that the epistemic priors from our theorem are necessary and sufficient for inducing the information-gathering behavior that distinguishes active inference from standard planning-as-inference methods.

The remainder of this paper is organized as follows. Section 2 defines the planning problem and introduces the Expected Free Energy as a cost function for plan evaluation. Section 3 reviews prior work on EFE minimization and planning as inference. Our central contribution is presented in Section 4. Section 5 provides empirical validation on the T-maze task, and Section 6 discusses implications for policy inference and future directions.

2 The Expected Free Energy Cost Function

2.1 The Planning Problem

In the context of sequential decision-making, planning is the process of determining an optimal course of action to achieve specific objectives. It is important to distinguish between two related but distinct concepts (Bertsekas, 2012; LaValle, 2006):

- A policy $\pi(u_t|x_{t-1})$ is a distribution over actions conditioned on states. Under the Markov assumption, the current state is a sufficient statistic for action selection.
- A plan $\boldsymbol{u} = (u_1, \ldots, u_T)$ is a fixed sequence of actions determined in advance.

Formally, sequential decision problems are often described within the framework of a Markov Decision Process (MDP), defined by the tuple

$$(\mathcal{X}, \mathcal{A}, p(x_0), p(x_{t+1}|x_t, u_t), R(x_t, u_t), T)\,,$$

where $\mathcal{X}$ is the state space, $\mathcal{A}$ is the action space, $p(x_0)$ is the initial state distribution, $p(x_{t+1}|x_t, u_t)$ represents the transition dynamics, $R(x_t, u_t)$ is the reward function, and T is the planning horizon.

The classical planning objective is to find a policy π that maximizes expected cumulative reward:

$$\pi^* = \arg\max_{\pi} \mathbb{E}_{\pi}\left[\sum_{t=1}^{T} R(x_t, u_t)\right]\,. \tag{1}$$

While this formulation is standard in reinforcement learning and optimal control (Sutton & Barto, 2018, Equation 3.13), it does not naturally accommodate epistemic objectives such as uncertainty reduction or information-seeking behavior.

2.2 The Generative Model

Consider an agent described by a generative model $p(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta)$.[1] In this paper, we are only concerned with planning, so we will assume that the model predicts a sequence of future observations. A typical example of this model would be a rollout of a state space model, for instance

$$p(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta) = p(x_0)p(\theta)\underbrace{\prod_{t=1}^{T} p(y_t|x_t, \theta)p(x_t|x_{t-1}, u_t, \theta)p(u_t)}_{\text{rollout to the future}}\,, \tag{2}$$

where $t = 0$ denotes the current time and $p(x_0)$ is the prior over the current state. In this model, $\boldsymbol{y}$ denotes the sequence of future observations, $\boldsymbol{x}$ represents the (latent) states, θ contains the model parameters, and

[1] We use boldface to denote sequences of random variables, e.g., $\boldsymbol{x} = x_{1:T}$, while individual variables such as x_t are not boldface.

$\boldsymbol{u}$ refers to the plan, i.e., a sequence of future actions (controls). Because all of these variables are defined as part of a model rollout into the future, they are all treated as unobserved variables. Since (2) is designed to predict how the future is expected to unfold, we refer to it as the generative model.

In model (2), the prior distribution $p(\boldsymbol{u})$ constrains allowable plans.

2.3 Planning-as-Inference

An alternative to reward maximization replaces the reward function with a *preference distribution* $\hat{p}(\boldsymbol{x})$ over desired future states (Levine, 2018). The preference distribution encodes the desired states, and can be understood as proportional to an exponentiated reward. Given the generative model (2) augmented with a preference distribution $\hat{p}(\boldsymbol{x})$, the planning objective becomes inferring $q(\boldsymbol{u})$, a posterior over plans encoding beliefs about which action sequences would efficiently guide the agent to these preferred states.[2]

This perspective is called Planning-as-Inference (PAI) (Attias, 2003; Toussaint & Storkey, 2006): any procedure that employs (variational) Bayesian inference on the generative model to obtain $q(\boldsymbol{u})$, or a joint posterior from which plans or policies can be derived. The result is a distribution rather than a point estimate, naturally accommodating uncertainty in the planning process.

2.4 The Expected Free Energy

Having established the generative model and the planning-as-inference formulation, we now turn to how candidate plans are evaluated. In the active inference literature, plans are evaluated by a cost function $G(\boldsymbol{u})$, known as the Expected Free Energy (EFE) (Da Costa et al., 2020), which is defined as

$$G(\boldsymbol{u}) = \underbrace{\mathbb{E}_q\left[\log \frac{q(\boldsymbol{x}|\boldsymbol{u})}{\hat{p}(\boldsymbol{x})}\right]}_{\text{risk}} + \underbrace{\underbrace{\mathbb{E}_q\left[\log \frac{1}{q(\boldsymbol{y}|\boldsymbol{x})}\right]}_{\text{ambiguity}} - \underbrace{\mathbb{E}_q\left[\log \frac{q(\theta|\boldsymbol{y},\boldsymbol{x})}{q(\theta|\boldsymbol{x})}\right]}_{\text{novelty}}}_{\text{epistemic costs}}, \tag{3}$$

where the expectations are with respect to $q = q(\boldsymbol{y}, \boldsymbol{x}, \theta|\boldsymbol{u})$. Plans with a lower $G(\boldsymbol{u})$ value are regarded as more favorable, i.e., a priori more likely to be selected. Active inference processes are based on the Free Energy Principle, a theory that accounts for the behavior of living systems as if $G(\boldsymbol{u})$ were their planning cost function (Parr et al., 2022; Friston, 2010; 2019). On a more practical level, we summarize this theory by discussing the incentives behind the three components of $G(\boldsymbol{u})$.

- **Risk** refers to the KL-divergence between $q(\boldsymbol{x}|\boldsymbol{u})$, which is the distribution over the state trajectory that we expect to reach under plan $\boldsymbol{u}$, and the target state $\hat{p}(\boldsymbol{x})$. EFE minimization aligns with risk minimization.

- **Ambiguity** is the expected entropy $\mathbb{E}_{q(\boldsymbol{x}|\boldsymbol{u})}\left[\mathbb{H}\left[q(\boldsymbol{y}|\boldsymbol{x})\right]\right]$ of future observations $\boldsymbol{y}$ under plan $\boldsymbol{u}$. Minimizing EFE biases inference toward state trajectories that predict a low-entropy distribution over future observations, which can be interpreted as a preference for states that lead to well-predicted observations.

- **Novelty** extends information-seeking behavior to include active parameter learning. Minimizing EFE results in plans that maximize the mutual information between observations $\boldsymbol{y}$ and parameters θ (averaged over states $\boldsymbol{x}$).

EFE minimization can be viewed as a unifying framework for planning under uncertainty, integrating principles from both decision theory and optimal control. As an aside, if preferences were instead described by

[2]Since $q(\boldsymbol{u})$ is conditioned solely on priors, namely the generative model $p(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta)$ and a preference prior $\hat{p}(\boldsymbol{x})$, it would be more appropriate to speak about an updated prior rather than a posterior. For simplicity, in this paper, all distributions that result from inference are denoted by $q(\cdot)$ and termed posteriors.

a distribution $\hat{p}(\boldsymbol{y})$ over desired future observations, it is common to define an alternative EFE as

$$G'(\boldsymbol{u}) = \underbrace{\mathbb{E}_q\left[\log \frac{1}{\hat{p}(\boldsymbol{y})}\right]}_{\substack{\text{pragmatic}\\ \text{costs}}} \underbrace{- \underbrace{\mathbb{E}_q\left[\log \frac{q(\boldsymbol{x}|\boldsymbol{y})}{q(\boldsymbol{x}|\boldsymbol{u})}\right]}_{\text{salience}} + \underbrace{\mathbb{E}_q\left[\log \frac{q(\theta|\boldsymbol{y},\boldsymbol{x})}{q(\theta|\boldsymbol{x})}\right]}_{\text{novelty}}}_{\text{epistemic costs}}, \tag{4}$$

where, as before, the expectations are with respect to $q = q(\boldsymbol{y}, \boldsymbol{x}, \theta|\boldsymbol{u})$. Although $G(\boldsymbol{u})$ and $G'(\boldsymbol{u})$ are distinct cost functions with different definitions of epistemic costs, they are centered around the same criteria. They can be regarded as specific instances of a broader policy cost function template given by

$$\mathbb{E}_{q(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})}\left[\log \frac{q(\boldsymbol{x},\theta|\boldsymbol{u})}{\bar{p}(\boldsymbol{y},\boldsymbol{x},\theta)}\right], \tag{5}$$

where the denominator features a "biased" model $\bar{p}(\boldsymbol{y}, \boldsymbol{x}, \theta)$, incorporating both preference incentives $\hat{p}(\cdot)$ and variational posteriors $q(\cdot)$. Specifically, $G(\boldsymbol{u})$ in Equation 3 uses $\bar{p}(\boldsymbol{y}, \boldsymbol{x}, \theta) = q(\theta|\boldsymbol{y}, \boldsymbol{x})q(\boldsymbol{y}|\boldsymbol{x})\hat{p}(\boldsymbol{x})$, and $\bar{p}(\boldsymbol{y}, \boldsymbol{x}, \theta)$ in Equation 4 factorizes as $\bar{p}(\boldsymbol{y}, \boldsymbol{x}, \theta) = q(\theta|\boldsymbol{y}, \boldsymbol{x})q(\boldsymbol{x}|\boldsymbol{y})\hat{p}(\boldsymbol{y})$. In the following discussion, we focus on $G(\boldsymbol{u})$, noting that similar derivations and arguments apply to $G'(\boldsymbol{u})$.

Having established the EFE as a cost function for plan evaluation, we now review prior approaches to EFE minimization and planning as inference before presenting our main theoretical contribution.

3 Related Work

3.1 Planning as Inference

The formulation of planning as probabilistic inference has a rich history. Classical optimal control (Bellman, 1954; 1966; Pontryagin, 2018) provides a mathematical framework for determining control inputs that minimize a predefined cost function, while Model Predictive Control extends this to real-time settings through receding horizon strategies (Bertsekas, 2012; Richalet et al., 1978; Cutler & Ramaker, 1979).

A significant paradigm shift involves recasting these problems as inference. Todorov (2006) demonstrated that a class of optimal control problems can be solved via inference in exponential family models, giving rise to KL control. Rawlik et al. (2012) extended this perspective to stochastic optimal control, showing that the Bellman equation can be derived from variational inference principles. When dealing with stochastic dynamics or state estimation under uncertainty, stochastic optimal control methods can be reformulated using variational inference (Kappen et al., 2012; Ito & Kashima, 2022), where intractable posteriors over states and controls are approximated by tractable variational distributions. The maximum causal entropy framework (Ziebart, 2010) provides a principled foundation for entropy-regularized control, connecting optimal control to probabilistic inference through the principle of maximum entropy. Under these formulations, the backward propagation of value functions corresponds to message passing on a factor graph (Levine, 2018).

The Planning-as-Inference (PAI) framework, introduced by Attias (2003) and extended by Toussaint & Storkey (2006), Toussaint (2009), and Solway & Botvinick (2012), formalizes this perspective: the objective is to infer action trajectories consistent with prior preferences over outcomes, enabling the use of approximate inference techniques such as variational inference and message passing. However, these PAI formulations focus on maximizing expected utility and do not explicitly incorporate epistemic value, i.e., the drive to reduce uncertainty. As a result, their applicability in partially observable environments is limited, as they lack a principled mechanism for information-seeking behavior.

A known challenge in PAI formulations is the phenomenon of optimistic inference (Levine, 2018). When the goal is encoded as a conditioning event, standard inference procedures can yield posteriors that implicitly assume favorable outcomes will occur regardless of the chosen actions. This arises because conditioning on goal achievement biases the posterior toward trajectories in which stochastic transitions occur to realize the goal, even when such realizations are unlikely under the agent's actual control. Lázaro-Gredilla et al. (2024) provided a comprehensive analysis of this issue and proposed a correction based on the observation

that proper sequential decision-making should account for the fact that the environment noise can not be manipulated. Specifically, their formulation augments the variational objective with an entropy correction, which turns the variational objective into a negative expectation of a reward function, essentially turning the inference procedure into expected reward maximization.

3.2 Active Inference

Active inference (Da Costa et al., 2020; 2024; Parr et al., 2022) offers a complementary perspective grounded in the Free Energy Principle. Rather than treating reward maximization and uncertainty reduction as separate objectives, active inference proposes that information gain constitutes an intrinsic objective. The framework balances exploration and exploitation through the Expected Free Energy (EFE) (Friston et al., 2015), which combines epistemic value (the drive to reduce uncertainty) with instrumental value (the drive to achieve preferred outcomes).

Existing methods for EFE minimization typically employ specialized optimization procedures rather than inference. Sophisticated Inference (Friston et al., 2021) evaluates policies through explicit tree search with recursive belief modeling; Branching Time Active Inference (Champion et al., 2022) extends this to handle counterfactual branches; and Dynamic Programming EFE (DPEFE) (Paul et al., 2024a;b) computes EFE recursively using dynamic programming principles. While these approaches offer computational strategies for EFE minimization, they rely on procedural algorithms for policy selection rather than deriving policy selection from inference. From the perspective of the Free Energy Principle, which posits that cognitive and behavioral processes emerge solely from Variational Free Energy minimization (Friston, 2010), policy selection should ideally arise through inference rather than through engineered algorithmic procedures.

3.3 Unifying EFE with Variational Inference

Several recent works have sought to reconcile EFE minimization with standard inference procedures. Palmieri et al. (2022) introduced a framework that unifies estimation and control via belief propagation on factor graphs. Building on this, Koudahl et al. (2023); van de Laar et al. (2024) proposed modifying the Variational Free Energy by subtracting a mutual information term when inference is performed over future segments of the factor graph, based on the Generalized Free Energy (Parr & Friston, 2019). This modification successfully allows message passing to account for both instrumental and epistemic value, yielding an interruptible inference procedure for policy evaluation.

A natural question is whether the same result can be achieved without requiring different objectives for different parts of the factor graph. This motivates the present work, which seeks a formulation where EFE-based planning emerges from a single, unified variational objective.

4 EFE-based Planning as Variational Inference

The main contribution of this paper is the following theorem, which demonstrates that EFE-based planning can be formulated as standard Variational Free Energy minimization without requiring modified cost functions or procedural algorithms. By augmenting the generative model with specific epistemic priors, the variational objective decomposes into expected plan costs (the EFE) plus a complexity term. The epistemic priors account for entropic contributions from conditional state, observation, and parameter beliefs, corresponding to the risk, ambiguity, and novelty terms, respectively.

Theorem 1 (Expected Free Energy Theorem)**.** *Consider an agent with generative (predictive) model $p(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta)$ and prior beliefs $\hat{p}(\boldsymbol{x})$ about future desired states. We define the Variational Free Energy functional $F[q]$ as*

$$F[q] \triangleq \mathbb{E}_{q(\boldsymbol{y},\boldsymbol{x},\boldsymbol{u},\theta)}\left[\log \frac{\overbrace{q(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta)}^{posterior}}{\underbrace{p(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta)}_{\substack{generative\\model}} \; \underbrace{\hat{p}(\boldsymbol{x})}_{\substack{preference\\prior}} \; \underbrace{\tilde{p}(\boldsymbol{u})\tilde{p}(\boldsymbol{x})\tilde{p}(\boldsymbol{y}, \boldsymbol{x})}_{epistemic\ priors}}\right], \tag{6}$$

where the generative model in the denominator is augmented by both a preference prior $\hat{p}(\cdot)$ and epistemic priors $\tilde{p}(\cdot)$. Assume that the variational posterior $q(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta)$ factorizes as

$$q(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta) = q(\boldsymbol{y}, \theta|\boldsymbol{x})q(\boldsymbol{x}|\boldsymbol{u})q(\boldsymbol{u})\,. \tag{7}$$

If the epistemic priors are chosen as

$$\tilde{p}(\boldsymbol{u}) \propto \exp(\mathbb{H}\left[q(\boldsymbol{x}|\boldsymbol{u})\right]) \tag{8a}$$

$$\tilde{p}(\boldsymbol{x}) \propto \exp(-\mathbb{H}\left[q(\boldsymbol{y}|\boldsymbol{x})\right]) \tag{8b}$$

$$\tilde{p}(\boldsymbol{y}, \boldsymbol{x}) \propto \exp(\mathbb{D}_{\mathrm{KL}}\left[q(\theta|\boldsymbol{y}, \boldsymbol{x})\|q(\theta|\boldsymbol{x})\right])\,. \tag{8c}$$

then $F[q]$ decomposes as

$$F[q] = \underbrace{\mathbb{E}_{q(\boldsymbol{u})}\left[G(\boldsymbol{u})\right]}_{\substack{\textit{expected plan}\\ \textit{costs}}} + \underbrace{\mathbb{E}_{q(\boldsymbol{y},\boldsymbol{x},\boldsymbol{u},\theta)}\left[\log \frac{q(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta)}{p(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta)}\right]}_{\textit{complexity}} + \text{constant}\,, \tag{9}$$

where $G(\boldsymbol{u})$ is the Expected Free Energy as defined in (3)*.*

In (8), $\mathbb{H}\left[q(\boldsymbol{x}|\boldsymbol{u})\right] = -\sum_{\boldsymbol{x}} q(\boldsymbol{x}|\boldsymbol{u}) \log q(\boldsymbol{x}|\boldsymbol{u})$ denotes the entropy functional, and $\mathbb{D}_{\mathrm{KL}}\left[q(\theta|\boldsymbol{y}, \boldsymbol{x})\|q(\theta|\boldsymbol{x})\right] = \sum_{\theta} q(\theta|\boldsymbol{y}, \boldsymbol{x}) \log \frac{q(\theta|\boldsymbol{y},\boldsymbol{x})}{q(\theta|\boldsymbol{x})}$ is the Kullback-Leibler divergence.

Proof. The proof of (9) is given in Appendix A. □

A key consequence of (9) is that minimizing $F[q]$ leads to reducing the expected plan costs $\mathbb{E}_{q(\boldsymbol{u})}\left[G(\boldsymbol{u})\right]$, while also balancing this with the drive to reduce complexity, which are the costs associated with changing beliefs.

4.1 Optimal Posterior over Plans

Starting from (18c) (see proof in Appendix A), we can compute the optimal $q^*(\boldsymbol{u})$ through

$$\begin{aligned} F[q] &= \mathbb{E}_{q(\boldsymbol{u})}\left[\log \frac{q(\boldsymbol{u})}{p(\boldsymbol{u})} + G(\boldsymbol{u}) + \underbrace{\mathbb{E}_{q(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})}\left[\log \frac{q(\boldsymbol{y}, \boldsymbol{x}, \theta|\boldsymbol{u})}{p(\boldsymbol{y}, \boldsymbol{x}, \theta|\boldsymbol{u})}\right]}_{=C(\boldsymbol{u})\ \text{(complexity)}}\right] \\ &= \mathbb{E}_{q(\boldsymbol{u})}\left[\log \frac{q(\boldsymbol{u})}{\exp\big(-P(\boldsymbol{u}) - G(\boldsymbol{u}) - C(\boldsymbol{u})\big)}\right]\,, \end{aligned} \tag{10}$$

where $P(\boldsymbol{u}) = -\log p(\boldsymbol{u})$ denotes the plan prior expressed as a cost function. Equation (10) is a Kullback-Leibler divergence that is minimized for

$$\begin{aligned} q^*(\boldsymbol{u}) &= \arg\min_q F[q] \\ &= \sigma\big(-P(\boldsymbol{u}) - G(\boldsymbol{u}) - C(\boldsymbol{u})\big)\,, \end{aligned} \tag{11}$$

where

$$\sigma(a)_k = \frac{\exp(a_k)}{\sum_{k'} \exp(a_{k'})} \tag{12}$$

is the softmax function.

Equation (11) is not new. A comparable formula for $q^*(\boldsymbol{u})$ can be found in Equation 2.1 of (Friston et al., 2021). A main contribution of this paper is to demonstrate that (11) can be obtained through standard variational minimization of an appropriately defined free energy functional $F[q]$. Importantly, while $q^*(\boldsymbol{u})$ is a distribution over plans (action sequences), the full joint posterior $q(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta)$ can be marginalized to obtain state-conditioned action beliefs $q(u_t|x_{t-1})$, effectively yielding a policy. This distinction is elaborated in Section 6.

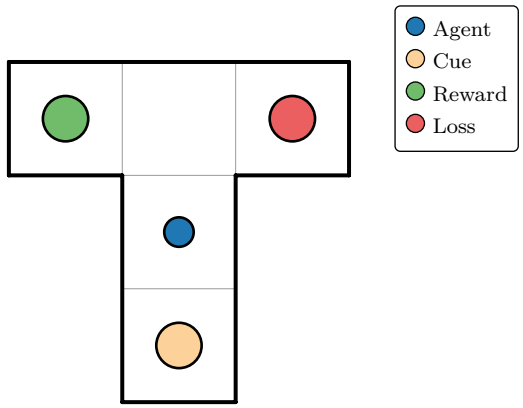


Figure 1: T-maze environment. The agent starts at the center junction and must reach the rewarding arm. Visiting the cue location (orange) provides an observation that reveals the reward location. The two arms (green and red) represent potential goal states.

5 Validation

This section validates the influence of epistemic priors on planning by comparing three alternative objectives on a T-maze task.

5.1 Environment

The T-maze (Figure 1) is a canonical benchmark for information-seeking behavior in active inference (Friston et al., 2015). Our variant consists of five locations: a starting position, a center junction, two goal arms (left and right), and a cue location below the junction. Unlike traditional formulations where agents can transition directly between any locations, we require deterministic movement between adjacent locations only, which increases the planning horizon required to gather information. Figure 1 shows the initial state of the environment, with the reward location (green) on the left arm.

One of the two arms contains a reward; the rewarding arm is determined by a hidden context variable that remains fixed within each episode. At the cue location, the agent receives an observation that unambiguously reveals which arm contains the reward. At all other locations, observations are uninformative.

5.2 Generative Model

The agent maintains a generative model of the form (2) for planning over a horizon of $T = 4$ timesteps. The model components are specified as follows.

- **States** $x_t \in \mathcal{X}$: The five locations, with $|\mathcal{X}| = 5$.
- **Actions** $u_t \in \mathcal{U}$: Four movement directions (up, down, left, right), with $|\mathcal{U}| = 4$. The transition model $p(x_t|x_{t-1}, u_t)$ encodes deterministic adjacency-respecting movement.
- **Observations** $y_t \in \mathcal{Y}$: Binary observations with $|\mathcal{Y}| = 2$. The observation model $p(y_t|x_t, \theta)$ yields an informative observation (deterministically revealing θ) when x_t is the cue location, and a uniform distribution over observation values otherwise.
- **Context parameter** $\theta \in \{\text{left}, \text{right}\}$: Indicates which arm contains the reward. The prior $p(\theta)$ reflects the agent's current belief, initialized to uniform and updated through state inference.
- **Preference prior**: The preference distribution factorizes as $\hat{p}(\boldsymbol{x}) = \hat{p}(x_T) \prod_{t=1}^{T-1} \hat{p}(x_t)$, where intermediate states have uniform preferences $\hat{p}(x_t) \propto 1$, and the terminal preference is $\hat{p}(x_T|\theta) =$

$\sigma(\gamma \cdot \mathbb{1}[x_T = \theta])$ with $\gamma = 2$, placing high probability on reaching the arm matching the reward location.

5.3 Comparison of Objectives

To isolate the contribution of epistemic priors, we define three objectives for comparison, all formulated as VFE minimization over a joint posterior $q(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta)$:

$$F_{\text{marginal}}[q] = \mathbb{E}_{q(\boldsymbol{y},\boldsymbol{x},\boldsymbol{u},\theta)} \left[\log \frac{q(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta)}{p(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta)\hat{p}(\boldsymbol{x})} \right] , \tag{13}$$

$$F_{\text{planning}}[q] = F_{\text{marginal}}[q] + \sum_{t=1}^{T} \left(\mathbb{H}\left[q(x_{t-1}, u_t)\right] - \mathbb{H}\left[q(x_{t-1})\right] \right) , \tag{14}$$

$$F_{\text{active}}[q] = \mathbb{E}_{q(\boldsymbol{y},\boldsymbol{x},\boldsymbol{u},\theta)} \left[\log \frac{q(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta)}{p(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta)\hat{p}(\boldsymbol{x})\tilde{p}(\boldsymbol{x})\tilde{p}(\boldsymbol{u})\tilde{p}(\boldsymbol{y}, \boldsymbol{x})} \right] . \tag{15}$$

Equation (13) is standard VFE with preference prior, consistent with KL control (Todorov, 2006; Rawlik et al., 2012, Equation 3). Equation (14) applies the entropy correction from Lázaro-Gredilla et al. (2024, Equation 4) to account for sequential decision structure and addresses the optimistic planning tendency of KL control by accounting for aleatoric uncertainty in state transitions (Levine, 2018). Equation (15) augments VFE with the full set of epistemic priors from Theorem 1. The main question is whether these epistemic priors provide additional value by inducing information-seeking behavior that anticipates the reduction of epistemic uncertainty.

5.4 Inference Procedure

Planning is performed by minimizing the Variational Free Energy over a joint posterior $q(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta) = q(\boldsymbol{y}, \theta|\boldsymbol{x})q(\boldsymbol{x}|\boldsymbol{u})q(\boldsymbol{u})$, respecting the factorization assumption in Theorem 1, using the Adam optimizer (Kingma & Ba, 2015), implemented in JAX (Bradbury et al., 2018). The variational distribution is represented as a tabular categorical distribution, with scalability implications discussed in Section 6.3. Optimization runs for 5000 iterations per planning step (see Appendix B for convergence diagnostics).

At each timestep, the agent receives an observation from the environment and updates its belief over the current state and context parameter θ. Since the cue observation deterministically reveals θ and observations elsewhere reveal the current location, state inference reduces to direct belief updates. The agent then performs planning by minimizing the VFE objective over horizon T, and executes the action with highest marginal probability:

$$\hat{u}_{t+1} = \arg\max_{u_{t+1}} q(u_{t+1}) . \tag{16}$$

This receding-horizon scheme replans at each timestep, incorporating new observations. Episodes terminate upon reaching a goal arm, with a maximum of T steps.[3]

5.5 Results

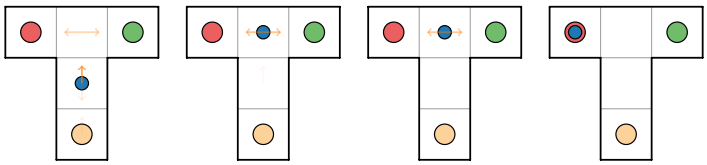
We evaluate each objective over 100 episodes with the reward location θ sampled uniformly. Table 1 summarizes the results.

Both F_{marginal} and F_{planning} achieve complementary success rates that sum to 100%, reflecting that each consistently commits to a different arm without gathering information. This asymmetry is a numerical artifact of the optimization; the key observation is that neither objective visits the cue. Without epistemic priors, both objectives yield posterior action distributions that are approximately uniform over the non-downward actions, providing no mechanism to anticipate that visiting the cue would resolve uncertainty about θ.

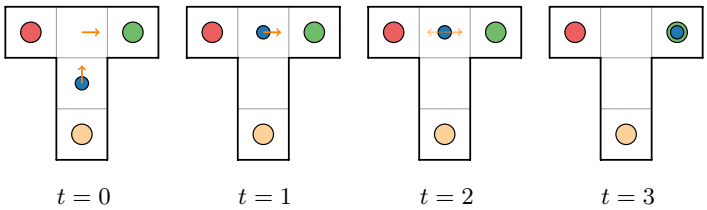
[3]Code is available at `https://anonymous.4open.science/r/EpistemicPriorsTMazeJAX-053C`.

Table 1: Performance comparison on the T-maze (100 episodes).

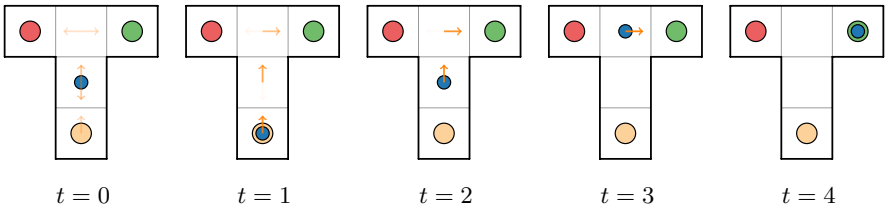
Objective	Success Rate	Mean Reward	Cue Visit Rate	Avg. Steps
F_{marginal}	44%	-0.12	0%	3.0
F_{planning}	56%	$+0.12$	0%	3.0
F_{active}	**100%**	**+1.00**	**100%**	4.0



(a) F_{marginal}: The agent proceeds directly toward an arm. The plan shows no preference for visiting the cue; without epistemic priors, the agent cannot anticipate the value of information.



(b) F_{planning}: Similar to marginal inference, the agent proceeds directly to an arm. The entropy correction prevents optimistic inference but does not induce information-seeking behavior.



(c) F_{active}: At $t = 0$, the plan reflects intention to visit the cue, with probability mass on the downward action. After observing the cue ($t = 1$), the agent forms a strong preference for the correct arm, visible in the concentrated arrow at subsequent times.

Figure 2: Representative trajectories with planned actions (reward in left arm). The blue circle indicates agent position. Arrows visualize the complete plan: at each location, opacity encodes the marginal probability $q(u_t|x_{t-1})$ assigned to that action at future timestep t. Only F_{active} visits the cue (orange) before committing to a goal.

Figure 2 illustrates representative trajectories. Each frame shows the agent's current position (blue circle) along with its complete plan: arrows at all locations indicate the marginal action probabilities $q(u_t|x_{t-1})$ the agent assigns to future timesteps, with opacity proportional to probability. This visualization reveals what the agent intends to do not only at its current location, but also at states it anticipates visiting later in the episode.

The planned actions reveal a key distinction. Under F_{marginal} and F_{planning}, the agent is ambivalent at the junction: with no mechanism to value information, it proceeds directly toward an arm with roughly equal probability for left and right. In contrast, under F_{active}, the agent visits the cue location first. After observing

the cue at $t = 2$, it exhibits a strong preference for the correct arm. These results demonstrate that the epistemic priors from Theorem 1 are sufficient to induce information-seeking behavior within the variational framework.

6 Discussion

6.1 From Plans to Policies

The Expected Free Energy $G(\boldsymbol{u})$ is traditionally defined as a cost function for evaluating plans, where a plan $\boldsymbol{u} = (u_1, \ldots, u_T)$ is a fixed sequence of actions. Traditional EFE-based methods compute $G(\boldsymbol{u})$ for each candidate plan independently. This plan-based approach does not maintain a joint distribution over states and actions, and therefore cannot represent contingent action selection.

When the transition dynamics $p(x_t|x_{t-1}, u_t)$ are stochastic (i.e., not a delta function), plan-based evaluation becomes fundamentally limited (Lázaro-Gredilla et al., 2024, Appendix I). Suppose an agent faces a goal that can be reached via two different routes, both requiring an initial action u_1. After taking u_1, a stochastic state transition may lead to one of two intermediate states. From each intermediate state, a different second action $u_2^{(1)}$ or $u_2^{(2)}$ leads efficiently to the goal. The optimal course of action would be to take u_1 first, then adapt the second action to whichever state is realized.

However, plan-based EFE evaluation considers each complete action sequence independently. The plan $(u_1, u_2^{(1)})$ may have high probability of failure if the state transition leads to the other branch. Similarly, $(u_1, u_2^{(2)})$ fails if the transition goes the other way. Consequently, both plans involving u_1 receive high (unfavorable) EFE costs. Plan-based methods cannot represent the strategy of taking u_1 and then adapting, because they evaluate fixed action sequences that commit to specific future actions regardless of what state is reached. The value of the initial action u_1 becomes underestimated because all plans containing it appear poor when evaluated independently.

A policy-based approach represents action selection through state-conditioned distributions $\pi(u_t|x_{t-1})$, which specify what action to take for each possible state. This enables contingent action selection: different actions can be taken depending on which state is realized. Our key insight is that by performing inference over the joint posterior $q(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta)$, we implicitly optimize these policy distributions through marginalization. When we marginalize the joint posterior to obtain:

$$q(u_t|x_{t-1}) = \frac{q(u_t, x_{t-1})}{q(x_{t-1})}\,, \quad \text{where} \quad q(u_t, x_{t-1}) = \sum_{\boldsymbol{y}, \boldsymbol{x}_{\neq t-1}, \boldsymbol{u}_{\neq t}, \theta} q(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta)\,, \tag{17}$$

the resulting distribution accounts for all possible future state transitions and the agent's planned responses to each. Crucially, when computing the value of the initial action u_1, marginalization accounts for all possible future trajectories. Even though every specific plan starting with u_1 may individually have low probability of success, the marginal distribution $q(u_1|x_0)$ can correctly assign high probability to u_1 precisely because it factors in the joint value of the initial action paired with all possible adaptive responses to the stochastic outcomes that follow. The policy representation thus captures future adaptability within a single planning step before any actions are executed.

This time-dependent policy emerges naturally from the factorization structure of the variational posterior without requiring explicit recursive modeling or procedural algorithms. The within-horizon adaptability is implicit in the joint inference: marginalizing over all future trajectories automatically accounts for all possible outcomes and the agent's anticipated responses.

The Sophisticated Inference framework (Friston et al., 2021) achieves a similar effect through recursive modeling of belief evolution. By explicitly representing how beliefs update at future timesteps, Sophisticated Inference implicitly represents state-contingent action selection. Our variational formulation provides an alternative route to the same policy-based objective: through marginalization of a joint posterior rather than recursive belief modeling.

It is important to note that our T-maze experiments in Section 5 test the epistemic value contribution (novelty and ambiguity terms), not the plan-versus-policy distinction. The T-maze has deterministic transitions, so the advantage of policy-based evaluation does not manifest in those results. The superiority in stochastic domains remains a theoretical property of the variational approach.

6.2 Interpretation of the Epistemic Priors

The epistemic priors in Theorem 1 are not chosen to encode intuitive a priori preferences, but rather are constructed so that the VFE decomposes into expected plan costs (the EFE) plus complexity. Each prior corresponds to a specific term in the EFE decomposition:

- The prior $\tilde{p}(\boldsymbol{u}) \propto \exp(\mathbb{H}[q(\boldsymbol{x}|\boldsymbol{u})])$ biases action selection toward plans that maintain high entropy over future states, reflecting a preference for maintaining flexibility. When $\mathbb{H}[q(\boldsymbol{x}|\boldsymbol{u})]$ is high, the agent keeps multiple future states open rather than committing to a narrow set of outcomes, which can be interpreted as a preference for adaptability in the face of uncertain dynamics.
- The prior $\tilde{p}(\boldsymbol{x}) \propto \exp(-\mathbb{H}[q(\boldsymbol{y}|\boldsymbol{x})])$ introduces the ambiguity term, biasing inference toward state trajectories that predict a low-entropy distribution over future observations, which can be interpreted as a preference for states that lead to well-predicted observations.
- The prior $\tilde{p}(\boldsymbol{y}, \boldsymbol{x}) \propto \exp(\mathbb{D}_{\mathrm{KL}}[q(\theta|\boldsymbol{y}, \boldsymbol{x}) \| q(\theta|\boldsymbol{x})])$ introduces the novelty term, favoring observation-state pairs that maximize information gain about parameters.

A notable feature of these priors is that they depend on the variational posterior q, which is unusual from a Bayesian perspective where priors are typically specified independently of the posterior. This self-referential structure reflects the fact that epistemic value is inherently defined in terms of how observations would update beliefs.

The priors can be understood as implicit constraints that couple the generative model to the agent's current belief state, rather than as conventional prior distributions expressing uncertainty before observing data.

6.3 Scalability

The illustrative example in Section 5 performs inference over a joint posterior $q(\boldsymbol{y}, \boldsymbol{x}, \boldsymbol{u}, \theta) = q(\boldsymbol{y}, \theta|\boldsymbol{x})q(\boldsymbol{x}|\boldsymbol{u})q(\boldsymbol{u})$ using a tabular representation. In this case, the dimensionality grows exponentially with the planning horizon T. For the T-maze with $|\mathcal{Y}| = 2$ observation values, $|\mathcal{X}| = 5$ states, $|\mathcal{U}| = 4$ actions, $|\Theta| = 2$ context values, and $T = 4$, the joint posterior requires $(|\mathcal{Y}|^T \times |\Theta| \times |\mathcal{X}|^T) + (|\mathcal{X}|^T \times |\mathcal{U}|^T) + |\mathcal{U}|^T = 180.256$ parameters. This computational burden is specific to the tabular representation, not to the variational formulation itself. By recasting EFE-based planning as variational inference, the problem becomes amenable to the same approximation techniques that have enabled variational inference to scale in other domains. Exploiting the factorization structure of the generative model through message passing offers a natural starting point for future work.

7 Conclusion

We have presented a variational formulation of Expected Free Energy-based planning. Our main result, the EFE Theorem, demonstrates that minimizing a Variational Free Energy functional over a generative model augmented with specific epistemic priors yields a natural decomposition into expected plan costs and complexity. This establishes that EFE-based planning can be understood as standard variational inference, achieving theoretical alignment with the Free Energy Principle.

The epistemic priors introduced in our formulation encode preferences for ambiguity-minimizing and novelty-seeking behavior. Empirical validation on the T-maze task confirms that these priors are sufficient to induce the information-gathering behavior characteristic of active inference agents within our framework. Without them, planning reduces to standard risk-minimizing control; with them, agents actively seek observations that resolve uncertainty about hidden states and parameters.

A key advantage of the variational formulation is that the full joint posterior can be marginalized to obtain state-conditioned action beliefs, effectively yielding a policy rather than a fixed plan. This policy-coordinate posterior implicitly accounts for the agent's ability to adapt future actions to whatever state actually occurs, providing robustness to stochastic dynamics that plan-based evaluation lacks. Moreover, by recasting EFE minimization as variational inference, we unlock access to the extensive toolkit of scalable variational inference methods, including message passing on factor graphs, amortized inference, and stochastic optimization, providing a principled path toward active inference agents that can operate in complex, high-dimensional environments.

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A Proof of Theorem 1

Proof of Theorem 1.

$$F[q] = \mathbb{E}_{q(\boldsymbol{y},\boldsymbol{x},\boldsymbol{u},\theta)}\left[\log \frac{q(\boldsymbol{y},\boldsymbol{x},\boldsymbol{u},\theta)}{p(\boldsymbol{y},\boldsymbol{x},\boldsymbol{u},\theta)\hat{p}(\boldsymbol{x})\tilde{p}(\boldsymbol{u})\tilde{p}(\boldsymbol{x})\tilde{p}(\boldsymbol{y},\boldsymbol{x})}\right] \tag{18a}$$

$$= \mathbb{E}_{q(\boldsymbol{u})}\left[\log \frac{q(\boldsymbol{u})}{p(\boldsymbol{u})} + \underbrace{\mathbb{E}_{q(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})}\left[\log \frac{q(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})}{p(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})\hat{p}(\boldsymbol{x})\tilde{p}(\boldsymbol{u})\tilde{p}(\boldsymbol{x})\tilde{p}(\boldsymbol{y},\boldsymbol{x})}\right]}_{C(\boldsymbol{u})}\right] \tag{18b}$$

$$= \mathbb{E}_{q(\boldsymbol{u})}\left[\log \frac{q(\boldsymbol{u})}{p(\boldsymbol{u})} + \underbrace{G(\boldsymbol{u}) + \mathbb{E}_{q(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})}\left[\log \frac{q(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})}{p(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})}\right] + \text{constant}}_{C(\boldsymbol{u}) \text{ if (8) holds}}\right] \tag{18c}$$

$$= \mathbb{E}_{q(\boldsymbol{u})}\left[G(\boldsymbol{u})\right] + \mathbb{E}_{q(\boldsymbol{y},\boldsymbol{x},\boldsymbol{u},\theta)}\left[\log \frac{q(\boldsymbol{y},\boldsymbol{x},\boldsymbol{u},\theta)}{p(\boldsymbol{y},\boldsymbol{x},\boldsymbol{u},\theta)}\right] + \text{constant} \quad \text{if (8) holds} \tag{18d}$$

□

In the above derivation, we still need to prove the transition for $C(\boldsymbol{u})$ from (18b) to (18c), which we address next. In the following, all expectations are with respect to $q(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})$ unless otherwise indicated.

Lemma 1 (Proof of equivalence $C(\boldsymbol{u})$ in (18b) and (18c))**.**

Proof.

$$C(\boldsymbol{u}) = \mathbb{E}\left[\log \frac{\overbrace{q(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})}^{\text{posterior}}}{\underbrace{p(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})}_{\text{predictive}}\underbrace{\hat{p}(\boldsymbol{x})}_{\text{utility}}\underbrace{\tilde{p}(\boldsymbol{u})\tilde{p}(\boldsymbol{x})\tilde{p}(\boldsymbol{y},\boldsymbol{x})}_{\text{epistemic priors}}}\right] \tag{19a}$$

$$= \underbrace{\mathbb{E}\left[\log\left(\underbrace{\frac{q(\boldsymbol{x}|\boldsymbol{u})}{\hat{p}(\boldsymbol{x})}}_{\text{risk}} \cdot \underbrace{\frac{1}{q(\boldsymbol{y}|\boldsymbol{x})}}_{\text{ambiguity}} \cdot \underbrace{\frac{q(\theta|\boldsymbol{x})}{q(\theta|\boldsymbol{y},\boldsymbol{x})}}_{-\text{novelty}}\right)\right]}_{G(\boldsymbol{u})=\text{Expected Free Energy}} + \tag{19b}$$

$$+ \mathbb{E}\left[\log\left(\underbrace{\frac{\hat{p}(\boldsymbol{x})q(\boldsymbol{y}|\boldsymbol{x})q(\theta|\boldsymbol{y},\boldsymbol{x})}{q(\boldsymbol{x}|\boldsymbol{u})q(\theta|\boldsymbol{x})}}_{\text{inverse factors from } G(\boldsymbol{u})} \cdot \underbrace{\frac{q(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})}{p(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})\hat{p}(\boldsymbol{x})\tilde{p}(\boldsymbol{u})\tilde{p}(\boldsymbol{x})\tilde{p}(\boldsymbol{y},\boldsymbol{x})}}_{\text{factors from (19a)}}\right)\right]$$

$$= G(\boldsymbol{u}) + \underbrace{\mathbb{E}\left[\log \frac{q(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})}{p(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})}\right]}_{=B(\boldsymbol{u})} + \underbrace{\mathbb{E}\left[\log \frac{q(\boldsymbol{y}|\boldsymbol{x})q(\theta|\boldsymbol{y},\boldsymbol{x})}{q(\boldsymbol{x}|\boldsymbol{u})q(\theta|\boldsymbol{x})\tilde{p}(\boldsymbol{u})\tilde{p}(\boldsymbol{x})\tilde{p}(\boldsymbol{y},\boldsymbol{x})}\right]}_{\text{choose epistemic priors to let this be constant}} \tag{19c}$$

$$= G(\boldsymbol{u}) + B(\boldsymbol{u}) + \tag{19d}$$

$$+ \mathbb{E}\left[\log \frac{1}{q(\boldsymbol{x}|\boldsymbol{u})\tilde{p}(\boldsymbol{u})}\right] + \mathbb{E}\left[\log \frac{q(\boldsymbol{y}|\boldsymbol{x})}{\tilde{p}(\boldsymbol{x})}\right] + \mathbb{E}\left[\log \frac{q(\theta|\boldsymbol{y},\boldsymbol{x})}{q(\theta|\boldsymbol{x})\tilde{p}(\boldsymbol{y},\boldsymbol{x})}\right]$$

$$
\begin{aligned}
&= G(\boldsymbol{u}) + B(\boldsymbol{u}) + \quad \text{(expand by using factorization (7))} && \text{(19e)}\\
&\quad + \sum_{\boldsymbol{x}} q(\boldsymbol{x}|\boldsymbol{u}) \Big(-\log q(\boldsymbol{x}|\boldsymbol{u}) - \log \tilde{p}(\boldsymbol{u}) \Big)\\
&\quad + \sum_{\boldsymbol{x}} q(\boldsymbol{x}|\boldsymbol{u}) \sum_{\boldsymbol{y}} q(\boldsymbol{y}|\boldsymbol{x}) \Big(\log q(\boldsymbol{y}|\boldsymbol{x}) - \log \tilde{p}(\boldsymbol{x}) \Big)\\
&\quad + \sum_{\boldsymbol{x}} q(\boldsymbol{x}|\boldsymbol{u}) \sum_{\boldsymbol{y}} q(\boldsymbol{y}|\boldsymbol{x}) \sum_{\theta} q(\theta|\boldsymbol{y},\boldsymbol{x}) \Big(\log \frac{q(\theta|\boldsymbol{y},\boldsymbol{x})}{q(\theta|\boldsymbol{x})} - \log \tilde{p}(\boldsymbol{y},\boldsymbol{x}) \Big)\\
&= G(\boldsymbol{u}) + B(\boldsymbol{u}) + \quad \text{(move epistemic priors out of sums)} && \text{(19f)}\\
&\quad - \log \tilde{p}(\boldsymbol{u}) \underbrace{- \sum_{\boldsymbol{x}} q(\boldsymbol{x}|\boldsymbol{u}) \log q(\boldsymbol{x}|\boldsymbol{u})}_{=\mathbb{H}[q(\boldsymbol{x}|\boldsymbol{u})]}\\
&\quad + \sum_{\boldsymbol{x}} q(\boldsymbol{x}|\boldsymbol{u}) \Big(-\log \tilde{p}(\boldsymbol{x}) + \underbrace{\sum_{\boldsymbol{y}} q(\boldsymbol{y}|\boldsymbol{x}) \log q(\boldsymbol{y}|\boldsymbol{x})}_{=-\mathbb{H}[q(\boldsymbol{y}|\boldsymbol{x})]} \Big)\\
&\quad + \sum_{\boldsymbol{x}} q(\boldsymbol{x}|\boldsymbol{u}) \sum_{\boldsymbol{y}} q(\boldsymbol{y}|\boldsymbol{x}) \Bigg(-\log \tilde{p}(\boldsymbol{y},\boldsymbol{x}) + \underbrace{\sum_{\theta} q(\theta|\boldsymbol{y},\boldsymbol{x}) \Big(\log \frac{q(\theta|\boldsymbol{y},\boldsymbol{x})}{q(\theta|\boldsymbol{x})} \Big)}_{=\mathbb{D}_{\mathrm{KL}}[q(\theta|\boldsymbol{y},\boldsymbol{x}) \| q(\theta|\boldsymbol{x})]} \Bigg)\\
&= G(\boldsymbol{u}) + B(\boldsymbol{u}) + && \text{(19g)}\\
&\quad \underbrace{-\log \tilde{p}(\boldsymbol{u}) + \mathbb{H}\left[q(\boldsymbol{x}|\boldsymbol{u})\right]}_{=\text{const if } \tilde{p}(\boldsymbol{u}) \propto \exp(\mathbb{H}[q(\boldsymbol{x}|\boldsymbol{u})])}\\
&\quad + \sum_{\boldsymbol{x}} q(\boldsymbol{x}|\boldsymbol{u}) \Big(\underbrace{-\log \tilde{p}(\boldsymbol{x}) + \mathbb{H}\left[q(\boldsymbol{y}|\boldsymbol{x})\right]}_{=\text{const if } \tilde{p}(\boldsymbol{x}) \propto \exp(-\mathbb{H}[q(\boldsymbol{y}|\boldsymbol{x})])} \Big)\\
&\quad + \sum_{\boldsymbol{x}} q(\boldsymbol{x}|\boldsymbol{u}) \sum_{\boldsymbol{y}} q(\boldsymbol{y}|\boldsymbol{x}) \Bigg(\underbrace{-\log \tilde{p}(\boldsymbol{y},\boldsymbol{x}) + \mathbb{D}_{\mathrm{KL}}\left[q(\theta|\boldsymbol{y},\boldsymbol{x}) \| q(\theta|\boldsymbol{x})\right]}_{=\text{const if } \tilde{p}(\boldsymbol{y},\boldsymbol{x}) \propto \exp(\mathbb{D}_{\mathrm{KL}}[q(\theta|\boldsymbol{y},\boldsymbol{x}) \| q(\theta|\boldsymbol{x})])} \Bigg)\\
&= G(\boldsymbol{u}) + \mathbb{E}_{q(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})} \left[\log \frac{q(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})}{p(\boldsymbol{y},\boldsymbol{x},\theta|\boldsymbol{u})} \right] + \text{constant} \quad \text{if (8) holds.} && \text{(19h)}
\end{aligned}
$$

□

The term in (19g) works out as follows:

$$
-\log \tilde{p}(\boldsymbol{u}) + \mathbb{H}\left[q(\boldsymbol{x}|\boldsymbol{u})\right] = \begin{cases} 0 & \text{if } \tilde{p}(\boldsymbol{u}) = \exp(\mathbb{H}\left[q(\boldsymbol{x}|\boldsymbol{u})\right]) \\ \text{const} & \text{if } \tilde{p}(\boldsymbol{u}) = \sigma(\mathbb{H}\left[q(\boldsymbol{x}|\boldsymbol{u})\right]) \end{cases} \tag{20a}
$$

where $\text{const} = \log\left(\Sigma_{\boldsymbol{u}'} \exp(\mathbb{H}\left[q(\boldsymbol{x}|\boldsymbol{u}')\right])\right)$ and

$$
\sigma(\mathbb{H}\left[q(\boldsymbol{x}|\boldsymbol{u})\right]) \triangleq \frac{\exp(\mathbb{H}\left[q(\boldsymbol{x}|\boldsymbol{u})\right])}{\Sigma_{\boldsymbol{u}'} \exp(\mathbb{H}\left[q(\boldsymbol{x}|\boldsymbol{u}')\right])} \tag{21}
$$

is the (normalized) softmax function, and similar derivations apply to the other epistemic priors in (8).

In the context of variational inference with Variational Free Energy $F[q]$ as in (6), the normalization of $\tilde{p}(\boldsymbol{u})$ is inconsequential, as the additive constant does not affect the location of the minimum of $F[q]$. As long as $\tilde{p}(\boldsymbol{u}) \propto \exp(\mathbb{H}\left[q(\boldsymbol{x}|\boldsymbol{u})\right])$, the results of VFE minimization will be the same.

B Convergence Diagnostics

Figure 3 shows the Variational Free Energy as a function of optimization iterations for each objective at $T = 4$.

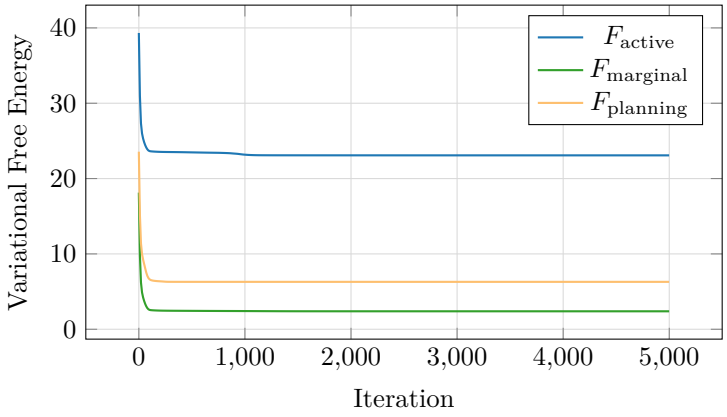

Figure 3: Variational Free Energy during optimization for each objective. The optimization runs for 5000 iterations per planning step; all objectives have converged by this point.

The different asymptotic values reflect the different objectives: F_{marginal} converges to the lowest value as it does not include epistemic prior terms, while F_{active} converges to the highest value due to the additional epistemic contributions. Notably, F_{planning} consistently exceeds F_{marginal}, as shown by Lázaro-Gredilla et al. (2024, Equation 8): the entropy correction terms are always non-negative, so the planning objective provides an upper bound on the marginal objective. These values are not directly comparable across objectives, as each minimizes a different functional.