Safe Human-Robot Interaction using MPC

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***Abstract*—This work presents a model predictive admittance control framework for safe and compliant human–robot interaction. The controller is designed to handle task-space constraints while ensuring robustness against external disturbances. A two-link manipulator case study is considered, where the proposed approach demonstrates effective compliance under interaction forces and reliable performance in the presence of modeling uncertainty.**

***Index Terms*—Compliant control, Admittance Control, Model Predictive Control**

I. Introduction

With the rapid advancement of technology, the pursuit of intelligent and seamless automation through robotics has gained significant attention. Such developments aim to enhance efficiency, precision, and the overall quality of the task. Nevertheless, many tasks still require human intervention, necessitating close cooperation between humans and robots in a shared environment. One of the most fundamental elements of this cooperation is the interaction force, which conveys human intent to the robotic system. In this setup, admittance control [1]allows the robot to interpret the operator's physical interaction and thereby infer the intended motion trajectory.

Admittance control establishes a dynamic relationship between the interaction force and the reference motion by modeling the interaction as a virtual spring–mass–damper system. By appropriately selecting the virtual parameters, the robot can be made to comply with the human's intent, ensuring smooth and intuitive interaction. Building upon this idea, different admittance control strategies exist in the literature, e.g., cite admadap,paul2024codit, to address uncertainties in the robot's physical parameters, improve compliance and safety in the interaction.

There have been attempts to handle interaction by augmenting MPC and impedance/admittance control [2]. In this study, a robust MPC framework is developed to address task-space constraints in robotic manipulators in the presence of external perturbations. Moreover, the idea is to design a task-space robust adaptive MPC for manipulators, where the system knowledge is unknown.

II. Control Background

A. Rigid body dynamics and linearized model

Consider the model of an n-link serial robot manipulator operating in an m-dimensional task space

$$M_q(q)\ddot{q} + C_q(q,\dot{q}) + G(q) = \tau - J(q)^T f_e + d, \qquad (1)$$

where $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ are joint position, velocity and acceleration vector, respectively, $\tau \in \mathbb{R}^n$ is the commanded torque input, $f_e \in \mathbb{R}^m$ is the end-effector wrench that is measured and has a known upper bound, i.e. $\|f_e\| \leq \bar{f}$, d is unwanted external perturbation or lumped modeling uncertainty whose upper bound is finite and known, i.e., $d \leq \bar{d}$ $J(q) \in \mathbb{R}^{m\times n}$ is the robot Jacobian matrix and $M_q \in \mathbb{R}^{n\times n}, C_q \in \mathbb{R}^n$ and $G(q) \in \mathbb{R}^n$ are the inertia matrix, the Coriolis and gravity vector, respectively. Given the accurate model knowledge, the control input can be chosen as

$$\tau = M_q(q)v + C_q(q,\dot{q}) + G(q) + J(q)^T f_e, \qquad (2)$$

with v being an auxiliary control input for the linearized system as given by $\ddot{q} = v$.
To operate in task-space the auxiliary control input can be chosen as

$$v = J^{\dagger}u - J^{\dagger}\dot{J}\dot{q} \qquad (3)$$

where $J^{\dagger}$ is the pseudo-inverse of J and $u \in \mathbb{R}^m$ is the new input resulting in a double integrator model in task space

$$\ddot{p} = u + J(q)d \qquad (4)$$

where $p \in \mathbb{R}^m$ is the end-effector pose of the manipulator.

B. Admittance Control

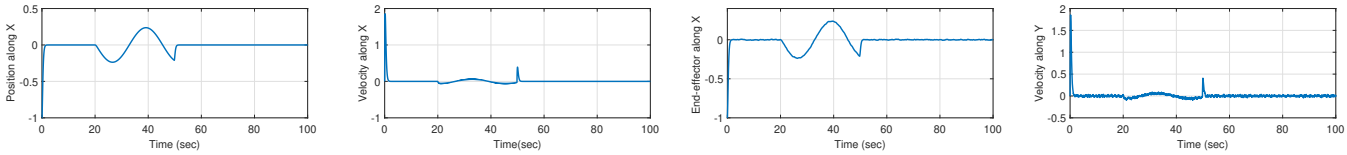
In admittance control, the external force (f_e) is linked with end-effector positions, velocities and accelerations through a linear spring-mass-damper system as given as

$$M_a\ddot{p}_N + D_a(\dot{p}_N - \dot{p}_d) + K_a(p_N - p_d) = f_e \qquad (5)$$

where $p_d \in \mathbb{R}^m$ is the desired trajectory in absence of any external force and $p_N \in \mathbb{R}^m$ is the nominal trajectory of the robot end-effector in presence of the external force,f_e, that is bounded by a known positive constant, i.e. $\|f_e\| \leq \bar{f}_e$. The matrices $M_a, D_a, K_a \in \mathbb{R}^{m\times m}$ are the apparent virtual mass, desired damping and stiffness matrices, respectively.

Assumption 2.1: The matrices M_a, D_a and K_a are chosen to be positive definite diagonal matrices as given by $M_a = diag\{m_1, ..., m_m\}, D_a = diag\{d_1, ..., d_m\}$ and $K_a = diag\{k_1, ..., k_m\}$.
The desired behavior (5) can be achieved by choosing the nominal control input in the absence of any disturbance, i.e. $d = 0$,

$$u_N = M_a^{-1}(-D_a(\dot{p} - \dot{p}_d) - K_a(p - p_d) + f_e). \qquad (6)$$



(a) End-effector position along X axis (b) End-effector velocity along X axis (c) End-effector position along X axis in presence of disturbance (d) End-effector velocity along X axis in presence of disturbance

Fig. 1. Plots of robot's end-effector in different interaction scenario

III. Control Objective

The control objective is to make the end-effector of the manipulator maintain the desired admittance relationship (5) and guarantee to satisfy various constraints despite the action of persistent, unknown, but bounded disturbances. We aim to achieve this goal through the use of the robust MPC framework.

By neglecting the disturbance term d in the linearized model (4) and choosing the control input u_N as given in (6), one can deduce the following structure

$$\dot{\xi}_N = A_a \xi_N + B_a f_e \tag{7}$$

where $\xi_N = \begin{bmatrix} p_N - p_d \\ \dot{p}_N - \dot{p}_d \end{bmatrix} \in \mathbb{R}^{2m}$, $A_a = \begin{bmatrix} 0 & I_m \\ -M_a^{-1}K_a & -M_a^{-1}D_a \end{bmatrix}$ and $B_a = \begin{bmatrix} 0 \\ M_a^{-1} \end{bmatrix}$. To achieve the desired admittance in the actual system, the following MPC controller will be designed to stabilize the tracking error $\xi = \begin{bmatrix} p - p_d \\ \dot{p} - \dot{p}_d \end{bmatrix}$,

$$u = \kappa(\xi, \xi_N, u_N) \tag{8}$$

where $\kappa(\cdot)$ denotes the state-feedback control law which also involves the state and the input of the nominal model. Additionally, the constraint on the tracking error vector is given as follows:

$$\xi \in \mathbb{S} \triangleq \{\xi \in \mathbb{R}^{2m} : \|\xi\| \leq \bar{s}\} \tag{9}$$

The optimization at time t aims to minimize a cost function, which is defined as follows:

$$J(\xi_N(t), u_N(t)) \triangleq \int_t^{t+T} V(\xi_N(\tau|t), u_N(\tau|t))d\tau + T(\xi_N(t+T|t)) \tag{10}$$

with prediction horizon T, where $V(\xi_N(\tau|t), u_N(\tau|t)) \triangleq \|\xi_N(\tau|t)\|_Q^2 + \|u_N(\tau|t)\|_R^2$ is the stage cost, weighting matrix $Q \in \mathbb{R}^{2m\times 2m}$ is a symmetric positive definite matrix, and $R \in \mathbb{R}^{m\times m}$ is a symmetric positive definite. The terminal cost is defined as $T(\xi_N(t+T|t)) \triangleq \|\xi_N(t+T|t)\|_P^2$ with terminal weighting matrix $P \in \mathbb{R}^{2m\times 2m}$ being a positive definite matrix.

A. Problem Statements

The key research objectives motivating this study are as follows:

- To design an appropriate controller to account for the presence of external interaction forces while maintaining state and input constraints.
- To ensure robustness against some perturbations or modeling uncertainties.

IV. Simulation and Discussion

For the preliminary study, a two-link manipulator system is considered, whose dynamics are presented in [3]. The prediction horizon for the model predictive controller is chosen as 50δ, where $\delta = 0.001$ s. The simulation is carried out for a total duration of 100 s, with the desired end-effector trajectory set to the origin. At $t = 20$ s, an external interaction force is applied to the end-effector along the X-direction. The applied force profile during the interval $t \in [20, 50]$ s is defined as

$$f_e = 6\sin(0.05(t-20)). \tag{11}$$

As illustrated in Fig. 1a,1b, the end-effector successfully returns to the origin while maintaining compliance with the external force and maintaining the safety constraint i.e. $\bar{s} = 0.5$. However, when the interaction force magnitude becomes significantly high, the MPC optimizer fails to compute a feasible control input due to the imposed saturation constraint $|u| \leq 10$, which reflects a practical actuation limit.

Furthermore, to evaluate robustness, an external perturbation is introduced with a maximum amplitude of 5. The system response is depicted in Fig. 1c,1d. It can be observed that the proposed MPC maintains satisfactory performance and accurately estimates the states even in the presence of perturbation.

V. Conclusion

A model predictive control-based admittance control framework has been developed to ensure safe and compliant human–robot interaction under task-space constraints. The proposed controller effectively handles external interaction forces and maintains robustness in the presence of disturbances and noise. Future work will focus on extending this framework to adaptive and learning-based schemes for real-time implementation on physical robotic platforms.

References

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