Think before you speak: Training Language Models With Pause Tokens

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Abstract

Language models generate responses by producing a series of tokens in immediate succession: the $(K+1)^{\text{th}}$ token is an outcome of manipulating K hidden vectors per layer, one vector per preceding token. What if instead we were to let the model manipulate say, $K+10$ hidden vectors, before it outputs the $(K+1)^{\text{th}}$ token? We operationalize this idea by performing training and inference on language models with a (learnable) *pause* token, a sequence of which is appended to the input prefix. We then delay extracting the model's outputs until the last pause token is seen, thereby allowing the model to process extra computation before committing to an answer. We empirically evaluate *pause-training* on decoder-only models of 1B and 130M parameters with causal pretraining on C4, and on downstream tasks covering reasoning, question-answering, general understanding and fact recall. Our main finding is that inference-time delays show gains when the model is both pre-trained and finetuned with delays. For the 1B model, we witness gains on eight tasks, most prominently, a gain of 18% EM score on the QA task of SQuAD, 8% on CommonSenseQA and 1% accuracy on the reasoning task of GSM8k. Our work raises a range of conceptual and practical future research questions on making delayed next-token prediction a widely applicable new paradigm.

1 Introduction

Transformer-based causal language models generate tokens one after the other in immediate succession. To generate the $(K+1)^{\text{th}}$ token, the model consumes the K previous tokens, and proceeds layer by layer, computing K intermediate vectors in each hidden layer. Each vector in itself is the output of a module (consisting of self-attention and multi-layer-perceptrons) operating on the previous layer's output vectors. However sophisticated this end-to-end process may be, it abides by a peculiar constraint: the number of operations determining the next token is limited by the number of tokens seen so far. Arguably, this was the most natural design choice when the Transformer was first conceived by [34]. But in hindsight, one may wonder whether for some inputs, the $(K+1)^{\text{th}}$ token demands $K+M$ Transformer operations in each layer (for $M>0$), which cannot be met by the arbitrarily constrained K operations per layer. This paper explores one way to free the Transformer of this arbitrary per-layer computational constraint.

The approach we study is to append dummy tokens into a decoder-only model's input, thereby *delaying* the model's output. Specifically, we select a (learnable) pause token (denoted `<pause>`) and append one or more copies of `<pause>` as a sequence to the input. We simply ignore the model's corresponding outputs until the last `<pause>` token is seen, after which we begin extracting its response.

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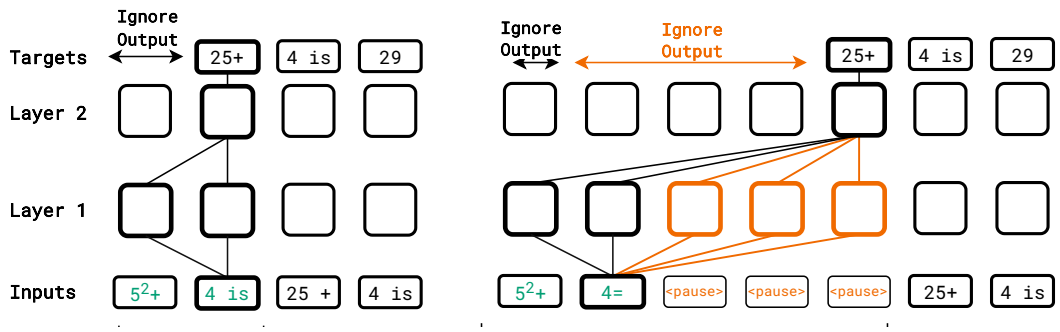


(a) Standard inference and finetuning

(b) Pause-inference and finetuning

Figure 1: **Standard vs. pause-inference (and finetuning)**. We consider a downstream task where, given a prefix, the decoder-only model (bidirectionally) attends to all of the prefix to generate its target answer. The rounded squares denote one Transformer operation (a self-attention and MLP) in a 2-layer Transformer. Any **`Ignore Output`** denotes that during inference, the corresponding output token is not extracted and thus, not fed back autoregressively; during finetuning, this output is not backpropagated through. The connecting lines denote some (not all) of the "computational pathways" within the model. Specifically, we visualize only those pathways that begin at a specific token in the prefix (here arbitrarily chosen to be "`4 is`") and end at an output token (here arbitrarily chosen to be "`25+`"). All differences between the two settings are highlighted in color. (a) In standard inference (finetuning), the model's output is extracted immediately upon seeing the last prefix token. (b) In pause-inference (and pause-finetuning), this is initiated only after appending a manually specified number of `<pause>` tokens. This introduces new computational pathways (the colored lines) between the prefix token and the output token of interest.

Crucially, we consider injecting such delays not just at inference, but also during downstream finetuning (see Fig 1) and pretraining (see Fig 2, which provides additional technical details).

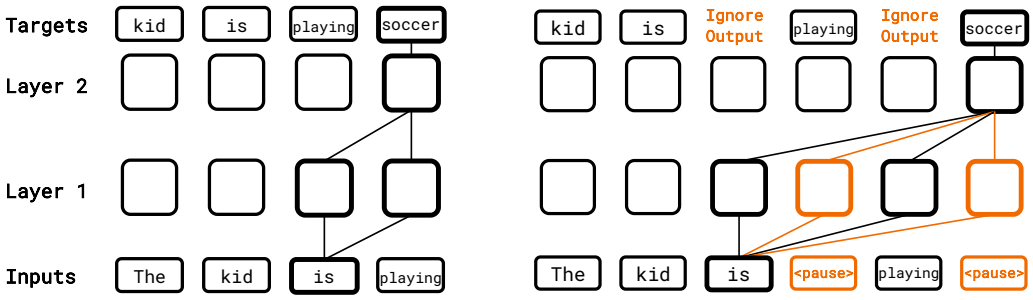
A-priori, it is unclear what this simple change would bring about in practice. Optimistically, the Transformer may take advantage of a "wider" computational pathway induced by the delay. A more mundane outcome though would be that the model simply skips any delays introduced by the `<pause>` tokens. Worse still, these uninformative tokens may drown out informative signals, and hurt the model. Partial answers can be found in the literature, motivated somewhat differently. Lanham et al. [14] append dummy thoughts in the form of periods ('...') at inference but do not observe gains as model is not primed to use additional computational pathways. [5] learn with *prepended* dummy tokens, with the orthogonal motivation of adding memory (rather than extending computation). They train with these tokens only on the target task, and observe minimal performance gains.

What then can we hope for when injecting (appended) delays on all stages of training and inference? We evaluates this, and other key questions that come up when training the Transformer with delays on a 1B and 130M parameter decoder-only model, trained on C4 [26] and finetuned on nine downstream tasks spanning extractive question answering, reasoning, general understanding and fact recall. In summary, we make the following contributions:

(1) We pose the question of *what happens if we delay a model's answer generation, and how can we execute these delays?* We design one way: training with dummy `<pause>` tokens. Accordingly, we design a pause-injected pretraining, downstream finetuning and inference procedure.

(2) We find that on a majority of our downstream tasks, training models with `<pause>` tokens during both pretraining and downstream finetuning, exhibits clear gains compared to standard end-to-end training and inference. Most notably, for the 1B model, in the SQuAD extractive question-answering task, this approach improves the exact match score by 18%. Similarly we observe 8% gains on the general understanding task of CommonSense QA and 1% accuracy gain on the reasoning task of GSM8k over the standard model's accuracy of 7.5%.

(3) On the flip side, when we introduce `<pause>` tokens only in the downstream finetuning stage (on the standard pretrained model), we find that the gains show up in far fewer instances, and are relatively mild. In some instances, we even find a clear drop in performance.



(a) Standard pretraining

(b) Pause-pretraining

Figure 2: **Standard vs. pause-pretraining**. We consider pretraining based on causal language modeling, where each token is predicted given all preceding tokens in the sequence. Here, we visualize the computational pathways beginning from the token "`is`" on the input side of the decoder-only model, to a subsequent token "`soccer`" on the output side. Please see Figure 1 for a guide on how to follow this visualization. (a) In standard pretraining, we compute the model's loss at each output token. (b) In pause-pretraining, we insert multiple copies of `<pause>` tokens at random locations in the input. However, we do not apply a loss on the model to predict these tokens, as indicated by each corresponding **`Ignore Output`** flags. This introduces new computational pathways connecting the input token and the output token of interest.

(4) We conduct a series of key ablations:(a) for any downstream task, there is a optimal number of `<pause>` tokens, (b) prepending the pause tokens is worse than appending, and (c) when decreasing the number of inference `<pause>` tokens, we find a graceful degradation of performance.

2 Pause-training

In the current paradigm of language models, we compute exactly K embeddings $\mathbf{v}_1, \ldots \mathbf{v}_K$ in each layer, before generating the $(K+1)^{\text{th}}$ token, p_{K+1} (detailed working of transform in Appendix A). Our premise is that this limit of K operations is an arbitrary one. Instead, we wish to expend more than K operations towards producing the next token, p_{K+1}. While something to this effect could be achieved by increasing the number of attention heads in each layer, we are interested in an orthogonal approach that introduces hardly any parameters into the network. The idea is to synthetically increase the input sequence length by appending M dummy tokens to the input, thus delaying the model's next response by M tokens of input. In effect, this M-token-delay lets the model manipulate an additional set of M intermediate vectors $\mathbf{v}_{K+1}, \ldots, \mathbf{v}_{K+M}$ before committing to its next (output) token, p_{K+1}. We visualize this in Figure 1.

2.1 Learning and inference with the `<pause>` token

Pretraining with the `<pause>` token The sequences in our pretraining data do not come with an annotation of which suffix constitutes the answer, since every input token also functions as a target output. Thus, it is impossible to execute the simple delaying strategy of appending dummy tokens before extracting the answer. Therefore, for a given pretraining sequence $\mathbf{p}_{1:N}$, we insert multiple `<pause>` tokens (say M_{pt} many) at uniformly *random* locations to obtain a pause-injected sequence, $\tilde{\mathbf{p}}_{1:N+M_{\text{pt}}}$. We visualize this in Figure 2b. We then train the model with the standard next-token prediction loss on this pause-injected sequence, while ignoring any loss term that corresponds to predicting the pause tokens themselves. Formally, let $S_{\text{ignore}} = \{k \colon \tilde{p}_{k+1} = \texttt{<pause>}\}$ denote the positions where the *next* token is a `<pause>` token. Then, for the decoder-only language model f, the pause-training loss is given by:

$$\mathcal{L}_{\text{PausePT}}(f, \tilde{\mathbf{p}}_{1:N+M_{\text{pt}}}) = \sum_{\substack{k=1 \\ k \notin S_{\text{ignore}}}}^{N+M_{\text{pt}}-1} \mathcal{L}_{\text{CE}}(\tilde{p}_{k+1}, f(\tilde{\mathbf{p}}_{1:k})), \tag{1}$$

where $\mathcal{L}_{\text{CE}}$ denotes the cross-entropy loss. Observe that the loss is skipped over indices in S_{ignore}. The rationale is that, we only want to use the <pause> tokens as a way of enforcing a delay in the model's computation; demanding that the model itself produce these tokens, would only be a pointless distraction. Finally, as is standard, we update the parameters of both the model and of all the tokens, including those of the <pause> token. We term this *pause-pretraining* (Algorithm 1).

Finetuning with the <pause> token In downstream finetuning, we are given a prefix $\mathbf{p}_{1:N}$ annotated with a target $\mathbf{t}_{1:T}$. Here, we append M_{ft} copies of the <pause> token to $\mathbf{p}_{1:N}$, to create our new prefix, $\tilde{\mathbf{p}}_{1:N+M_{\text{ft}}}$. We visualize how this introduces new computational pathways in Figure 1. As before, we ignore the model's outputs until the last <pause> token is seen. We apply the standard next-token prediction loss on the target with the new prefix, thus minimizing $\sum_{k=0}^{T-1} \mathcal{L}_{\text{CE}}(t_{k+1}, f([\mathbf{p}_{1:N+M_{\text{ft}}}, \mathbf{t}_{1:k}]))$, where $[\cdot]$ denotes the concatenation operation. Note that for any given downstream task, we fix M_{ft} to be the same across all inputs for that task. We again update both the parameters of the model, and that of the whole vocabulary, including the <pause> token, as is standard. We term this *pause-finetuning* (Algorithm 2).

Pausing during inference During inference on the downstream task, we append M_{inf} <pause> tokens to the prefix and as always, we ignore the output of the model until the last <pause> token (Figure 1). We term this *pause-inference* (Algorithm 3).

2.2 Variants of Pause-Training

We study four training combinations: (a) Standard Pretraining and Standard Finetuning (**StdPT_StdFT**). (b) Standard Pretraining and Pause-Finetuning (**StdPT_PauseFT**): We train with <pause> tokens only during downstream finetuning. *If* this technique helps, it would promise a practically viable approach for pause-training off-the-shelf models. (c) Pause-Pretraining and Standard Finetuning (**PausePT_StdFT**): Here we introduce <pause> tokens during pretraining, but abandon it downstream. This is purely for analytical purposes (See §3.2). (d) Pause-Pretraining and Pause-Finetuning (**PausePT_PauseFT**): We inject delays into both stages. Unless stated otherwise, we use the same number of pause tokens at inference as finetuning i.e., $M_{\text{inf}} = M_{\text{ft}}$.

3 Experiments

We tested decoder-only models of 1B and 130M sizes. For ablations, we used the 1B model. Both standard and pause models were trained on C4 English mixture. With pause-pretraining, a <pause> token was added at 10% of sequence length, maintaining the original sequence size by trimming the now longer sequence to the original size. Both pretraining methods used 200B tokens. We use a *single* <pause> token embedding, effectively increasing the parameter count by 1024 (the token embedding size), a quantity that is dwarfed by the 1 billion total parameter count. Downstream tasks were finetuned with <pause> tokens set at 10 and 50, with the best result reported. For downstream tasks, we averaged results over 5 runs, with randomness from finetuning. We tune the learning rate and batch size for standard end-to-end training baseline and use the best set of all other training variants. More details about experiment setup can be found in Appendix G.

3.1 Downstream datasets

We consider nine varied downstream tasks: (a) reasoning (GSM8k [7]), (b) extractive question answering (SQuAD [27], CoQA [28]), (c) general understanding (CommonSenseQA [31], PhysicalIQA [3]), (d) long term context recall (LAMBADA [24]), (e) natural language inference (HellaSwag [42]), and (f) fact recall (WebQuestions [2], Natural Questions [12]). HellaSwag and PhysicalIQA are scoring tasks. We note that our implementation of CommonSenseQA is as a decoding task, and hence we report Exact Match (EM) scores. Detailed dataset description is in Appendix F.

3.2 Results: Effect of pause-training

We report the performance of the four considered approaches in §2.2 on all our downstream tasks for our 1B model in Figure 3, and for our 130M model in Appendix B.

The benefit of pause-pretraining followed by pause-finetuning (PausePT_PauseFT). Our first core finding is that there are clear gains when <pause> tokens are introduced during both pretraining and finetuning (PausePT_PauseFT), across a majority of the varied tasks we consider. Overall, this outperforms the standard baseline (StdPT_StdFT) on eight tasks for the 1B model, and on six tasks for

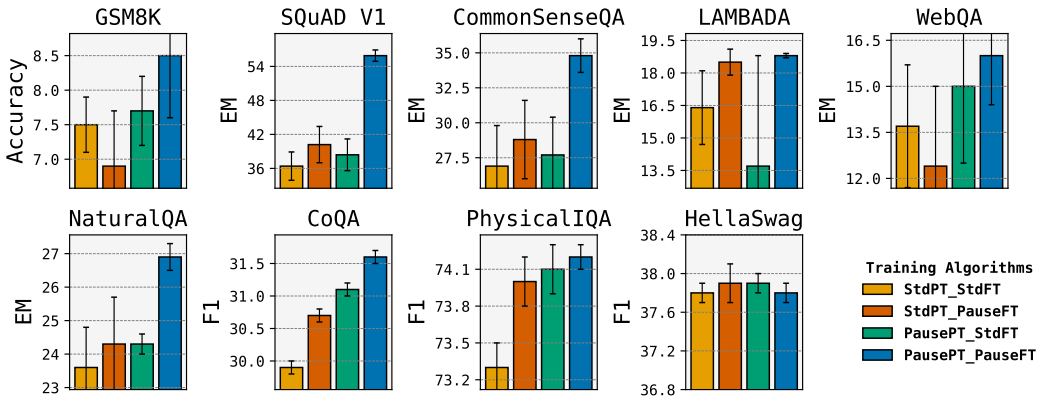


Figure 3: **Downstream performance for a 1B model.** Injecting delays in both stages of training (PausePT_PauseFT) outperforms the standard end-end training StdPT_StdFT on our wide variety of tasks (except HellaSwag). In contrast, introducing delays only in the finetuning stage provides only lukewarm gains, and even hurts in GSM8k.

the 130M model (Appendix Fig 5) albeit to varying extents. Most prominently, for the 1B model on the SQuAD question-answering task, PausePT_PauseFT improves over StdPT_StdFT by an 18% EM score. Similarly, we observe upto 8% gains on the general understanding task of CommonSenseQA. On the reasoning task of GSM8k, PausePT_PauseFT gives an accuracy of 8.5% compared to 7.5% of the standard baseline. Similar gains are observed in other tasks like long-term context understanding (LAMBADA) and also on fact recall tasks like WebQA and NaturalQuestion.

The lukewarm effect of pause-finetuning a standard-pretrained model (StdPT_PauseFT). In contrast to the above observations, introducing delays only during downstream finetuning (StdPT_PauseFT) gives mixed results. While there are gains on about 5 benchmarks, they are comparitively less. On the remaining, the performance mirrors (or is worse than) standard training.

Isolating the benefits of pause-pretraining independent of downstream delay (PausePT_StdFT). Does PausePT_PauseFT learn better pretraining representations itself which cause better downstream performance? To isolate this, we examine the performance of PausePT_StdFT, where we do not inject delays in the downstream task. Here the gains are clear only in two tasks (CoQA and PhysicalIQA). Thus, we conclude that in most tasks, the gains of PausePT_PauseFT come from well-learned delayed computations executed at inference-time.

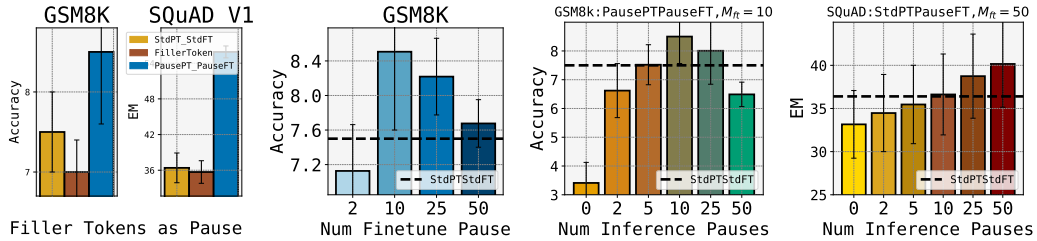
Filler characters as <pause>: For completeness, we also report results for inference on StdPT_StdFT models, delayed with 10 or 50 periods ('.'). Corroborating the observations of Lanham et al. [14], we find no gains in doing this (Figure 4a).

Remark: As a concluding note, we remind the reader that the PausePT_PauseFT model has a (deliberately injected) computational advantage compared to StdPT_StdFT, during finetuning and inference. However, there is no computational advantage during pause-pretraining since we equalize the number of tokens seen. In fact, this only results in a slight statistical disadvantage: the pause-pretrained model sees only 90% of the (meaningful) pretraining tokens that the standard model sees, as the remaining 10% are dummy <pause> tokens.

4 Ablations: Where and how many <pause> tokens to use

4.1 Number of <pause> tokens during finetuning

Recall that we append M_{ft} copies of (the same) <pause> tokens to the prefix during finetuning. We find that for each downstream dataset, there is an optimal value of M_{ft} corresponding to that dataset. For example, on GSM8k, 10 <pause> tokens are optimal with accuracy reducing to that of baseline as <pause> tokens are increased to 50 (Figure 4b). Possibly, for each dataset, there exists a certain threshold of <pause> tokens beyond which the self-attention mechanism becomes overwhelmed.



(a) (b) (c) (d)

Figure 4: **Key Ablations:** (a) We compare a pause-*trained* model vs. a standard model delayed using a filler periods ('...'). As in Lanham et al. [14], the filler periods do not give any gains (§ 3.2). (b) There exists an optimal number of finetuning <pause> tokens (M_{ft}) for a given downstream dataset beyond which gains diminish (§ 4.1). (c) and (d) We test the robustness of pause-trained models to varying number of inference time <pause> tokens (setting M_{inf} not equal to M_{ft}), which exposes the model to a serious test-time distribution shift (§ 4.2). Pause-training degrades gracefully to shifts as wide as $M_{\text{inf}} \in [5, 25]$ for $M_{\text{ft}} = 10$ both for (c) PausePT_PauseFT and (d) StdPT_PauseFT.

4.2 Robustness to a varying number of inference-time pauses

In all our experiments so far, we set the inference-time delay to be the same as what was seen during finetuning ($M_{\text{inf}} = M_{\text{ft}}$), we examine what happens if we vary M_{inf} during inference. PausePT_PauseFT model is robust to a wide range of test-time shift in the number of <pause> tokens (see Figure 4c and Appendix E). The performance remains above the baseline even if pause tokens at inference are half of that seen during training, which is desirable in case of inference-time compute constraints. However, performs drops significantly in the absence of *any* <pause> token at inference.

5 Discussion and key open questions

Enhanced computational width. One hypothesis as to why Transformer delays can help is that it increases the *width* of the computation. We hypothesize that this additional width helps certain downstream tasks. Take for example, comprehension-based question-answering tasks. Here, having a greater number of attention units per layer, would permit a finer distribution of attention across various parts of the supporting context (where the answer resides). We speculate that this would allow the lower layers to extract more precise and diverse representations, which a higher layer can more reliably aggregate to produce a final answer.

Pause-inference vs. Chain-of-Thought. It is worth contrasting the above computational advantage with that enjoyed by chain-of-thought (CoT) prompting. CoT also corresponds to greater computational width, by way of delaying its final answer (albeit with meaningful tokens). However, CoT also increases the computational depth. In particular, each (meaningful) delay token generated by CoT is autoregressively generated by the model. Thus, if there are M such tokens and L layers, the final token arises out of roughly $M \cdot L$ sequentially composed operations. Thus, CoT has a computational depth that is larger by a *multiplicative* factor of M, compared to pause-inference.

Capacity expansion without parameter expansion. There are trivial ways to extend the next-token computation process: add more attention heads, more layers, or more dimensions to the intermediate vectors. However, all these require increasing the parameter count substantially, which pause-training does not. This gives rise to the following learning-theoretic question: how does one formalize the two orthogonal notions of representational capacity, one of raw parameter count, and another of the "computational pathways" through the model?

6 Related Work

Input-only tokens. The idea of using tokens that occur only as an input has found its use in various forms, most commonly as <cls> [6, 9, 19], <sep> or <mask> in BERT [9] and in a line of work on adding memory to transformers [4, 5, 8]. Closest to our work is Burtsev et al. [5] who explore the use of dummy tokens as a way of adding global memory to the Transformer, rather than motivating it as a way of extending its computation. They *prepend* these tokens (rather than

append them) and crucially, introduce them only during training and inference on the target tasks. On smaller scratch-trained models (with parameter counts of 10M, 65M and 277M) and a pretrained BERT model (109M), this reportedly gives minimal gains. This echoes our own mixed results for the `StdPT_PauseFT` variant, and the fact that our smaller model shows gains on fewer datasets. In contrast to their work, we demonstrate that inserting delays both in pretraining and finetuning is crucial to observing clear gains on downstream datasets spanning reasoning, question-answering, fact-recall etc.,

Chain-of-thought prompting and role of intermediate tokens. One (expensive) way to delay the output of a model is through chain-of-thought (CoT) prompting where one prompts the model into generating intermediate reasoning steps (in an autoregressive fashion). This has been shown to significantly improve the reasoning capabilities of large language models [13, 23, 30, 36, 37, 39, 41, 44]. Consequently, there has been a surge of interest in understanding the source of these CoT prompting gains. Recently, Madaan and Yazdanbakhsh [21], Turpin et al. [33], Wang et al. [35] have shown that the generated intermediate reasoning steps can be unfaithful, not representing the model's true reasoning process. Wang et al. [35] empirically show that even incorrect reasoning steps can preserve $80\% - 90\%$ of the performance gains. In turn, [14] analyze whether these gains are simply due to additional attention computations at inference time. For this, they replace the intermediate reasoning steps with filler tokens in the form of repeated periods. They do not however observe *any* performance gains from this. We argue that the model needs to be primed to process such tokens to extend its computation.

Lightweight finetuning techniques. Interestingly, on the face of it, pause-finetuning bears some resemblance to an orthogonal line of work on lightweight finetuning/ensembling techniques [6, 11, 15–18, 20, 25, 29, 38, 43]. Lightweight finetuning is concerned with parameter-efficient techniques that do not update the model's weights, and instead update a series of *multiple distinct* learnable tokens (prepended to the input). While pause-training uses a (single) learnable token too (appended to the input), the goal and effects are significantly different. First, pause-training is not intended for parameter-efficient finetuning. Infact, pause-training tunes slightly more parameters than standard finetuning. Next, in terms of the effect, while pause-training hopes to outperform standard finetuning as it is a less constrained technique, lightweight finetuning typically cannot, as it is a more constrained technique. Finally, note that pause-training crucially benefits from introducing the `<pause>` tokens during pretraining, while lightweight methods do not affect pretraining in any way.

Other feedback loop-based techniques. There have been techniques [1, 10, 22, 40] that can be perceived as delaying the computation of model via more elaborate wrappers. For example, Madaan et al. [22] introduce self-refinement, where a language model provides an initial output which is then refined via feedback. However, note that pause-training and pause-inference preserve the core mechanism of the model itself: the model still produces the $(K+1)^{\text{th}}$ token as a computation over K previous input tokens, and additional *dummy* tokens, without consuming intermediate, *autoregressively* generated outputs.

7 Limitations and Future Work

Pause-training takes a step beyond the paradigm of "immediate" next-token prediction in language models. The key idea is to train models with (dummy) `<pause>` tokens so that the model can learn to harness the additional inference-time computation. We demonstrate that this can improve performance on a variety of tasks, *if* we train with `<pause>` tokens both during pretraining and downstream finetuning. We highlight that pause-pretraining is quite expensive and comes in the way of making this idea more widely accessible. Consequently, we do not study how the gains generalize across more model sizes (beyond 1B and 130M), or to encoder-decoder architectures, or to other pretraining mixtures and objectives. A rigorous study of the underlying mechanism on how `<pause>` tokens help is an interesting direction for future study. We also leave open a variety of follow-up algorithmic questions: pause-training with multiple *different* `<pause>` tokens, better determining the number of `<pause>` tokens (perhaps using model confidence), inducing robustness to shifts in delays, and so on. But the most pressing next step would be to find ways to make delays helpful directly on a standard pretrained model. Overall, we hope that our work opens up many avenues for theoretical and practical work in the paradigm of delayed next-token prediction.

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A Preliminaries: Transformer

Consider a vocabulary $\mathcal{V}$ and an input $\mathbf{p}_{1:K} \in \mathcal{V}^K$ of K tokens, and an L-layer decoder-only language model. The l'th layer of the Transformer produces one intermediate vector for each token here, denoted by $\mathbf{v}_k^{(l)} \in \mathbb{R}^D$ for $k = 1, \ldots, K$. We first describe this operation before outlining the end-to-end next-token generation process.

Consider a Transformer [34] block $T(\cdot) : \mathbb{R}^{K \times D} \to \mathbb{R}^{K \times D}$ that operates over a sequence of K intermediate vectors. The block is defined by H many sets of four matrices each, $W_{\text{query}}^{(h)}, W_{\text{key}}^{(h)}, W_{\text{value}}^{(h)}$ and $W_{\text{out}}^{(h)}, \in \mathbb{R}^{D_{\text{attn}} \times D}$ (for $h = 1, \ldots H$ each denoting an attention head), and a single parameterized feedforward module $f_{\text{FF}} : \mathbb{R}^D \to \mathbb{R}^D$. Let $\Phi_{\text{LN}} : \mathbb{R}^D \to \mathbb{R}^D$ denote the layer-norm operation. Given the input vectors $V_{1:K} \in \mathbb{R}^{D \times K}$, the output $V'_{1:K}$ of the Transformer block $T(\cdot)$ can be expressed in the following steps. For all $k \leq K$,

$$\mathbf{a}_k = \Phi_{\text{LN}}^{(1)} \left(\mathbf{v}_k + \sum_{h=1}^{H} (W_{\text{out}}^{(h)})^T \cdot W_{\text{value}}^{(h)} V_{1:k} \text{softmax} \left(\frac{(W_{\text{key}}^{(h)} V_{1:k})^T W_{\text{query}}^{(h)} \mathbf{v}_k}{\sqrt{D_{\text{attn}}}} \right) \right) \tag{2}$$

$$\mathbf{v}'_k = \Phi_{\text{LN}}^{(2)} \left(\Phi_{\text{FF}}(\mathbf{a}_k) + \mathbf{a}_k \right). \tag{3}$$

Here, the first step computes K different self-attention outputs by attending to all K input vectors, while the second step individually processes each attention output via a feedforward network and other normalization components to produce the final output of the block. Note that here we have assumed a unidirectional attention mechanism; for a bidirectional mechanism over the whole K-length prefix, one simply needs to replace $V_{1:k}$ with $V_{1:K}$ in the above computation.

Given this block, the Transformer generates the next token as follows. Let $\Phi_{\text{token}} : \mathcal{V} \to \mathbb{R}^D$ and $\Phi_{\text{position}} : \mathbb{N} \to \mathbb{R}^D$ denote the token-embedding and position-embedding layers. With an abuse of notation, let the token unembedding layer be denoted as Φ_{token}^{-1}, which maps from $\mathbb{R}^D$ to a probability vector in $\Delta^{|\mathcal{V}|-1}$. Let $T^{(l)}(\cdot)$ denote the lth Transformer layer. Then, the Transformer commits the following operations in sequence to arrive at the $(K+1)^{\text{th}}$ token.

$$\mathbf{v}_k^{(0)} = \Phi_{\text{token}}(p_k) + \Phi_{\text{position}}(k) \tag{4}$$

$$V_{1:K}^{(l)} = T^{(l)}(V_{1:K}^{(l-1)}), \forall l \in [1, L] \tag{5}$$

$$p_{K+1} \sim \Phi_{\text{token}}^{-1}(\mathbf{v}_K^{(L)}). \tag{6}$$

For a more detailed mathematical exposition of the Transformer model, we refer the reader to [32].

Observe here that the kth output $\mathbf{v}'_k$ is obtained by manipulating exactly the k previous hidden embeddings in the same layer, $V_{1:k}$.

B Additional downstream finetuning results

We first report the downstream finetuning performance for the 1B model in Table 1 (numbers corresponding to Figure 3 in §3.2). Further, in Figure 5 we report downstream performance on various tasks for a 130M decoder-only model. Again we observe that `PausePT_PauseFT` clearly outperforms standard training baseline (`StdPT_StdFT`) on GSM8k, CommonSenseQA, LAMBADA and on our fact recall tasks like WebQA and NaturalQA. However, surprisingly, we do not observe gains on SQuAD, in contrast to the gains observed in 1B model. Overall, we see an improvement in six tasks for the smaller model (one of which is PhysicalIQA where the gain is minimal).

C Prepending vs Appending Pause Tokens

In our main experiments, we chose to append `<pause>` tokens since it is the most natural format for a general setting e.g., in long-text-generation as in a conversational agent, one would append `<pause>` tokens to the current text rather than deploying the tokens all at once at the beginning of the

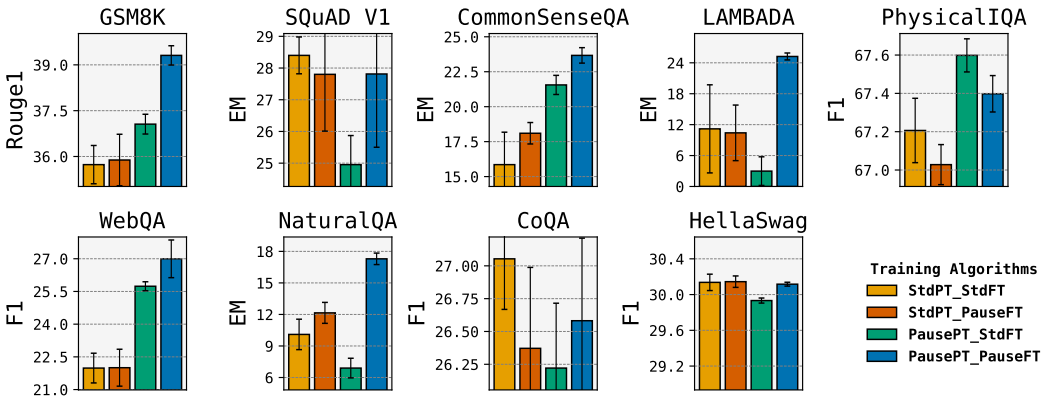


Figure 5: **Downstream performance of pause-training on a 130M decoder-only model.** We find on six out of our nine downstream tasks, the pause-pretrained and pause-finetuned model (PausePT_PauseFT) outperforms standard training (StdPT_StdFT) on a 130M decoder-only model. For example, on the reasoning task of GSM8k, we observe 3% gains in Rouge1 scores (we compare Rouge1 as the final accuracy was too low to be meaningful for our 130M model). Similarly on the general understanding task of CommonSenseQA, we observe upto 8% gains. We note here that we solve CommonsenseQA as a decoding task and not rank classification task, and hence report the Exact Match scores. We also highlight that while pause-trainined on the 1B model showed significant gains on SQuAD, they disappear here.

Dataset	**Metric**	StdPT_StdFT	StdPT_PauseFT		PausePT_StdFT	PausePT_PauseFT	
			10	**50**		**10**	**50**
GSM8k	Acc	7.5 ± 0.5	6.9 ± 1.0	6.5 ± 0.8	7.7 ± 0.5	8.5 ± 0.9	7.7 ± 0.3
	Rouge1	42.3 ± 0.5	41.7 ± 0.7	41.2 ± 1.3	43.5 ± 0.1	44.2 ± 0.2	44.1 ± 0.2
SQuAD	EM	36.4 ± 2.5	36.6 ± 2.2	40.2 ± 3.2	38.4 ± 2.9	51.7 ± 2.3	55.9 ± 1.0
CommonSense QA	EM	26.9 ± 2.9	28.8 ± 2.8	28.7 ± 2.0	27.7 ± 2.7	34.8 ± 1.2	32.3 ± 0.8
LAMBADA	EM	16.4 ± 1.7	18.4 ± 0.3	18.5 ± 0.6	13.7 ± 5.1	18.8 ± 0.1	18.5 ± 0.2
Web Questions	EM	13.7 ± 2.1	9.0 ± 4.4	12.4 ± 2.6	15.0 ± 2.5	13.8 ± 3.7	16.0 ± 1.6
Natural Questions	EM	23.6 ± 1.2	24.3 ± 1.4	23.9 ± 1.3	24.3 ± 7.5	24.9 ± 1.3	26.9 ± 0.4
CoQA	F1	29.9 ± 1.0	30.7 ± 0.5	30.3 ± 0.5	31.1 ± 0.3	31.3 ± 1.1	31.6 ± 0.5
PhysicalIQA	F1	73.3 ± 0.2	73.9 ± 0.2	74.0 ± 0.2	74.1 ± 0.2	74.1 ± 0.1	74.2 ± 0.2
HellaSwag	F1	37.8 ± 0.1	37.9 ± 0.2	37.8 ± 0.2	37.9 ± 0.1	37.7 ± 0.2	37.8 ± 0.2

Table 1: Downstream performance on various tasks for the 1B decoder-only model. We observe that PausePT_PauseFT outperforms the standard training baseline on 8 out of the 9 tasks considered in this work. See §3.2 and Figure 3 for further details.

conversation. Furthermore, when there is unidirectional attention, prepending these tokens should make no difference. Nevertheless, in our downstream tasks which use bidirectional attention on the prefix, it makes sense to consider prepending <pause> tokens. We investigate this in Table 2. Most importantly, we find that, for PausePT_PauseFT, even prepending the <pause> token performs improves over standard end-to-end training. However, appending is still the more optimal choice. This indicates that pause-pretraining induces considerable biases in how readily the delays are used based on their positional embeddings.

D Varying number of pause tokens M_{ft}

In Figure 6, we study the effect of varying the number of pause tokens used during downstream finetuning (M_{ft}) on the downstream performance. We refer the reader to §4.1 for further details.

Dataset	Metric	StdPTStdFT	StdPTPauseFT		PausePTPauseFT	
			Prepending	Appending	Prepending	Appending
GSM8k	Acc.	7.5 ± 0.5	8.0 ± 1.0	6.9 ± 1.0	8.0 ± 0.4	8.5 ± 0.9
SQuAD	EM	36.4 ± 2.5	35.0 ± 1.5	40.2 ± 3.2	44.0 ± 3.2	55.9 ± 1.0
CommonQA	EM	26.9 ± 2.9	31.0 ± 1.3	28.8 ± 1.5	34.5 ± 1.0	34.8 ± 1.2
Lambada	EM	16.4 ± 1.7	17.8 ± 0.4	18.5 ± 0.6	18.0 ± 1.1	18.8 ± 0.1
PhysicalIQA	F1	73.3 ± 0.2	74.0 ± 0.3	74.0 ± 0.3	74.2 ± 0.2	74.2 ± 0.2
NaturalQ	EM	23.6 ± 1.2	24.1 ± 0.6	24.3 ± 1.4	25.7 ± 0.9	26.9 ± 0.4

Table 2: Prepending vs appending the pause tokens (§C). We observe that prepending the pause tokens still outperforms the standard training baseline of `StdPT_StdFT`, but is suboptimal to appending the `<pause>` tokens for `PausePT_PauseFT` training algorithm. However, for `StdPT_PauseFT`, both have equal number wins and losses.

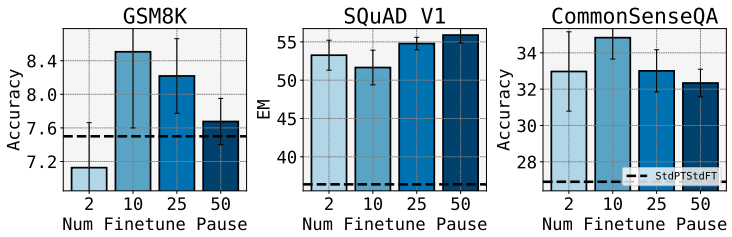


Figure 6: **Varying finetuning delay**: We examine the effect of varying the number `<pause>` tokens used in downstream finetuning (M_{ft}, §4.1) on the performance. Typically, we observe that there exists an optimal number of `<pause>` tokens as expected for each dataset.

Again we observe that there exists an optimal number of pause tokens to be used during downstream finetuning, depending on the task.

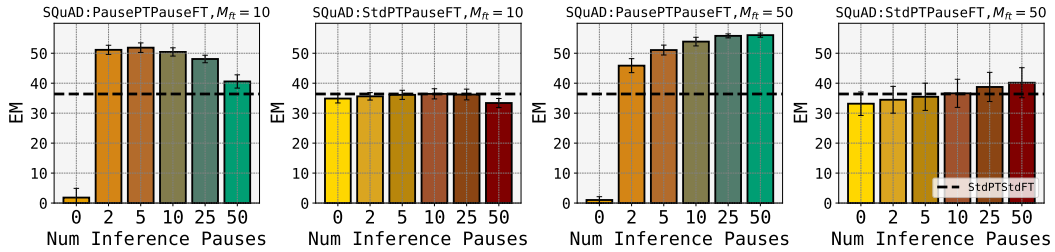
E Robustness to varying number of inference time pauses

Recall in §4.2 and Figure 4c we observed that pause-training is robust to using a different number of inference time pauses compared to that used during finetuning (i.e. $M_{\text{inf}} \neq M_{\text{ft}}$). We present additional results regarding the same in Figure 7a, Figure 7b and Figure 7c. Again, we observe that the performance degrades gracefully for the pause-trained models, even with shifts that halve the number of tokens seen. However, we still find a drastic drop in performance when no delay is given during inference for the `PausePT_PauseFT` model.

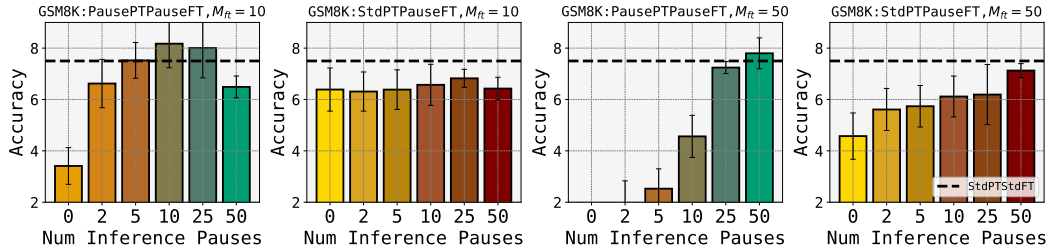
F Downstream Dataset Description

We finetune and evaluate the pretrained models (both standard and pause pretrained) on the following datasets:

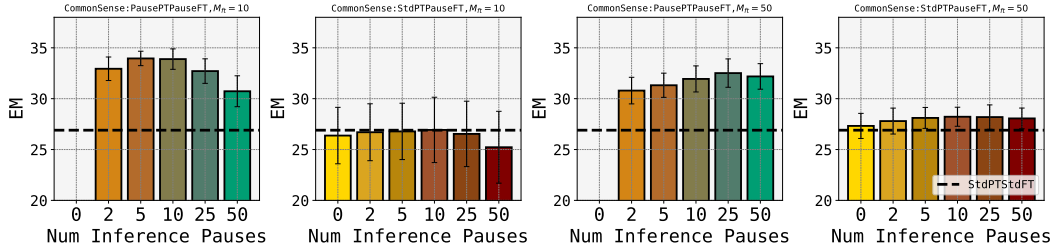
1. GSM8k: A reasoning task with 8.5k grade school math word problems [7].
2. SQuAD V1: Reading-comprehension task based on Wikipedia [27].



(a) SQuAD



(b) GSM8k



(c) CommonSenseQA

Figure 7: **Varying inference-time delays:** We test the robustness of pause-trained models to varying number of inference time <pause> tokens (setting M_{inf} not equal to M_{ft}), which exposes the model to a serious test-time distribution shift (§4.2). Pause-training degrades gracefully to shifts as wide as $M_{\text{inf}} \in [5, 25]$ for $M_{\text{ft}} = 10$ and $M_{\text{ft}} = 50$ both for PausePT_PauseFT and StdPT_PauseFT, apart from GSM8k wherein there is a drop for $M_{\text{ft}} = 50$. In each row, the first and the third column considers the PausePT_PauseFT model for M_{ft} set to 10 and 50, respectively. Likewise, the second and the fourth column show the same for StdPT_PauseFT model.

3. CommonSenseQA: Requires different types of commonsense knowledge to choose the correct answer [31]. Our implementation of CommonSenseQA is as a decoding task, and hence we report Exact Match (EM) scores.

4. LAMBADA: Text-understanding task requiring last-word prediction based on a long context [24].

5. Web Questions: A fact-recall dataset of commonly-asked questions on the web [2].

6. PhysicalIQA: A physical commonsense reasoning dataset, which test the ability to understand interactions with the world [3].

7. Natural Questions: QA task which requires answering fact-based questions from Wikipedia article pages [12]. Since we use the closed-book version of this dataset (no access to helpful context), this is a fact-recall task.

8. HellaSwag: Next-sentence prediction task based on common-sense inference [42].

9. CoQA: Question-answering task based on a context [28].

Dataset	**Learning Rate**	**Warmup Steps**	**Finetuning Steps**	**Batch Size**
SQuAD	1.0E-04	100	10000	256
GSM8k	1.0E-04	200	20000	16
HellaSwag	5.0E-06	100	1000	16
PhysicalIQA	1.0E-06	50	600	32
CoQA	5.0E-05	75	3500	16
CommonSenseQA	5.0E-05	100	4000	16
LAMBADA	5.0E-05	40	2800	16
WebQuestions	5.0E-04	200	2000	16
NaturalQuestions	1.0E-04	100	5000	256

Table 3: Downstream finetuning hyperparameters for the 1B model.

G Hyperparameters: Downstream finetuning

We consider decoder-only models of size 1B and 130M for our main experiments. For our ablations, we stick to the 1B model. Both the standard and pause models are pretrained on the C4 English mixture [26], using the causal next token prediction objective for a total of 200B tokens (slightly more than 1 epoch on C4). For pause-pretraining, we insert the `<pause>` token randomly at 10% of the sequence length (2048) positions, and trim the now-longer sequence to its original sequence length. We then conduct pause-pretraining and standard-pretraining for the same number of total tokens (200B). We use a *single* `<pause>` token embedding, effectively increasing the parameter count by 1024 (the token embedding size), a quantity that is dwarfed by the 1 billion total parameter count (the token constitutes a 10^{-6} fraction of the model size).

Since we expect different downstream tasks to benefit from a different number of finetuning `<pause>` tokens M_{ft}, we run finetuning with M_{ft} (and likewise M_{inf}) set to 10 and 50 and report the best of these two for our consolidated results. However, we provide the values for both $M_{\text{ft}} \in \{10, 50\}$ in Appendix D, in addition to a more finegrained ablation of this hyperparameter in Section 4.1. For all the downstream finetuning experiments, we report mean and standard deviation over 5 runs (with the randomness purely from the finetuning stage). We tune the learning rate and batch size for standard end-to-end training, and use the best hyperparameter for all other training variants as well. We share all the hyperparameters for downstream finetuning in Table 3 (1B model) and Table 4 (130M model). We also provide the decoder-only architecture details for the two models considered in this work in Table 5.

Dataset	Learning Rate	Warmup Steps	Finetuning Steps	Batch Size
SQuAD	1.00E-04	400	40000	16
GSM8k	1.00E-04	75	7500	16
CommonSenseQA	5.00E-05	100	6000	16
LAMBADA	5.00E-05	40	1400	16
WebQuestions	5.00E-04	200	2000	16
NaturalQuestions	5.00E-04	100	5000	256
CoQA	1.00E-04	75	3500	16
PhysicalIQA	1.00E-06	50	600	32
HellaSwag	1.00E-06	100	1000	16

Table 4: Downstream finetuning hyperparameters for the 130M model.

Model	130M	1B
Parameters	136,237,056	1,345,085,440
Transformer Layers	12	24
Attention Heads	12	32
Embedding Dimension	768	2048
Hidden Dimension	3072	8092

Table 5: Architecture details for the models considered in this work

Algorithm 1: Pause-pretraining

Pretraining with Pause

Inputs: Pretraining dataset $\mathcal{D}_{\text{pt}}$, decoder-only model f_θ, number of <pause> tokens M_{pt} to insert, pause token <pause>

$\mathbf{p}_{1:N} \sim \mathcal{D}_{\text{pt}}$ /* Input Sequence from corpus */

$\tilde{\mathbf{p}}_{1:N+M_{pt}} = \texttt{random_insert}(\mathbf{p}_{1:N}, \texttt{<pause>}, M_{\text{pt}})$ /* Insert M_{pt} pause tokens randomly in the original input sequence $\mathbf{p}_{1:N}$, extending its length by M_{pt} */

$S_{\text{ignore}} = \{k \in [0, N + M_{pt} - 1] \colon \tilde{p}_{k+1} = \texttt{<pause>}\}$ /* Identify the set of positions where the next token is <pause> */

$\mathcal{L}_{\text{PausePT}}(f_\theta, \tilde{\mathbf{p}}_{1:N+M_{\text{pt}}}) = \sum_{k=1, k \notin S_{\text{ignore}}}^{N+M_{\text{pt}}-1} \mathcal{L}_{\text{CE}}(\tilde{p}_{k+1}, f_\theta(\tilde{\mathbf{p}}_{1:k}))$ /* Next token prediction error excludes targets which are pause (model isn't made to learn to predict pause itself) */

$\theta = \theta - \nabla_\theta \mathcal{L}_{\text{PausePT}}(f_\theta, \tilde{\mathbf{p}}_{1:N+M_{\text{pt}}})$ /* Update the model */

Algorithm 2: Pause-finetuning

Stage 2: Finetuning with Pause

Inputs: Downstream labeled dataset $\mathcal{D}_{\text{ft}}$, pretrained model f_θ, number of <pause> tokens M_{ft} to insert, pause token <pause>

$\mathbf{p}_{1:N}, \mathbf{t}_{1:T} \sim \mathcal{D}_{\text{ft}}$ /* Sample prefix and target */

$\tilde{\mathbf{p}}_{1:N+M_{\text{ft}}} = \text{Concat}[\mathbf{p}_{1:N}, [\texttt{<pause>}] \times M_{\text{ft}}]$ /* Append prefix and M_{ft} pauses */

$\mathcal{L}_{\text{PauseFT}}(f_\theta, \tilde{\mathbf{p}}_{1:N+M_{\text{ft}}}, \mathbf{t}_{1:T}) = \sum_{k=0}^{T-1} \mathcal{L}_{\text{CE}}(t_{k+1}, f_\theta(\text{Concat}[\tilde{\mathbf{p}}_{1:N+M_{\text{ft}}}, \mathbf{t}_{1:k}])$ /* Next token prediction error on targets */

$\theta = \theta - \nabla_\theta \mathcal{L}_{\text{PauseFT}}(f_\theta, \tilde{\mathbf{p}}_{1:N+M_{\text{ft}}}, \mathbf{t}_{1:T})$

Algorithm 3: Pause-inference

Stage 3: Inference with Pause

Inputs: Prefix $\mathbf{p}_{1:N}$, finetuned model f_θ, number of <pause> tokens M_{inf} to insert, Pause token <pause>

$\tilde{\mathbf{p}}_{[1:N+M_{\text{inf}}]} = [\mathbf{p}_{1:N}, [\texttt{<pause>}] \times M_{\text{inf}}]$ /* Append M_{inf} pauses to prefix */

$\tilde{\mathbf{p}}_{N+M_{\text{inf}}+1} \sim f_\theta(\tilde{\mathbf{p}}_{1:N+M_{\text{inf}}})$ /* Predict the next token in the sequence, and continue in an auto-regressive fashion */

H Inference time cost of Pause tokens

One way to assess the inference-time compute efficiency of a method is by estimating the number of *Floating Point Operations per Second (FLOPS)* it requires. A related, but independent metric, is the *Wall Clock Time* as it affects the latency of deployed systems. Below, we analyze how efficient pause-inference is along these two metrics in this section. Broadly, we make two arguments:

1. Pause-inference offers a more FLOPS-efficient way of increasing performance, as compared to other natural ways of expanding the number of attention operations in a Transformer, such as by adding layers or by adding attention heads.
2. Pause-inference is also wall-clock-efficient compared to the above techniques as it virtually introduces no overhead. When compared to CoT, pause-inference provides a computationally more granular and cheaper way to improve performance (although still upper-bounded by CoT in terms of *performance*).

H.1 Pause tokens allow for a more efficient use of FLOPS

We frame our FLOPS-efficiency analysis as follows. Consider introducing p pause tokens during inference in a given Transformer. How many additional FLOPS does this require? If we spent the same budget of additional FLOPS to introduce more attention operations via other techniques — namely, via an appropriate number of additional layers or additional attention heads — would we find a similar improvement in quality, in terms of metrics like accuracy?

Concretely, we use a running example of the 1B model used in this paper, whose architecture details are provided in Table 5. Specifically, we have number of transformer layers as $l = 24$, input embedding dimension as $h = 2048$, per attention head embedding dimension $d = h/a$, where $a = 32$ is the number of attention heads. We also anchor our analysis for input prompts with $n = 100$ tokens, which represents the average prompt length for many tasks considered in this work. For our analysis, we rely on supporting lemmas deferred to Section H.2.1.

FLOPS-efficiency of p pause tokens vs k additional layers: In the context of the downstream task of SQuAD, appending $p = 10$ pause tokens yields an 18% increase in EM score. Applying these values ($n = 100, p = 10$) and $l = 24$ in the FLOPS-efficiency analysis of Theorem H.3, we can deduce that if we were to allocate the same FLOPS budget to adding more layers to the Transformer stack, we can at most add 2 layers. This enhancement corresponds to a modest 10% rise in parameter count, expanding the model from a 1B parameter model to a 1.1B parameter model. However, in practice, when scaling the parameter count, significant performance improvements (such an 18% increase in EM score) are typically observed only when the parameter count is scaled by much larger factors. Thus we argue that in this case, pause tokens provide a more inference-time-FLOPS-efficient way of increasing performance. Conceptually, this underscores the fact that pause tokens introduce an alternative dimension to representation capacity, distinct from the traditional approach of scaling the parameter count.

Comparing FLOPS with increase in attention heads: In the standard Transformer implementation used in practice, when one increases the attention heads (a), although the number of attention operations increases, the per-attention-head embedding dimension (d) proportionally gets reduced to keep the overall embedding dimension constant (h). Thus, there is effectively no change in the number of FLOPS. In contrast, adding pause tokens increases the number of attention operations, while keeping the per-attention-head embedding dimension d constant. Therefore, for a fair comparison, we consider the case where we increase a, while keeping the per-attention-head dimension d fixed (and we fix it to be equal to h).

Then, from our analysis in Theorem H.4 we have that for an input of length $n = 100$ and $a = 32$ attention heads, appending $p = 10$ pause tokens is equivalent to increasing the number of attention heads by $k = 3$. However, increase the attention heads by 3 in the 1B model configuration adds only 48M parameters (we add $W_q, W_k, W_v, W_{proj} \in \mathbb{R}^{h \times h}$ per additional attention head), bringing the model to a parameter count of 1.048B from 1B. This, we argue cannot account for any significant performance improvement equivalent to the improvements seen under pause-training proposed in this paper.

H.2 Pause tokens do not add sequential compute

Comparing pause tokens with adding layers or attention heads Recall that <pause> tokens are added as a part of the input prompt, where each token is processed in parallel. Thus, <pause> tokens do not add extra serial computations. If there are sufficiently many parallel threads available, the wall clock overhead from pause tokens would be a minimal percentage of the time required for standard inference. However, in contrast, increasing the number of transformer layers increases the length of sequential operations, causing the wall-clock time to increase proportionally to the fraction of layers added. Note that adding attention heads should have a similar effect as adding pause tokens, as they too introduce parallel, not sequential operations.

Comparison with Chain-of-Thought (CoT) prompting Recall that CoT involves auto-regressively decoding a long sequence of tokens involved in the model's reasoning. This requires a significant wall clock time cost, proportional to $O(pl)$, if p is the number of intermediate reasoning tokens and l is the number of layers. In stark contrast, pause tokens do not add extra wall-clock time. Furthermore, CoT prompting offers little flexibility in how large p can be. Pause-inference on the other hand, offers a more direct way for manipulating the number of pause-tokens (even if, in its current version, this adaptivity is not robust beyond a point).

H.2.1 Supporting lemmas for estimating FLOPS efficiency

In Lemma H.1, we present the facts about FLOPS required for basic vector and matrix calculations. Subsequently, in Lemma H.2, we compute the overall FLOPS required for an end-to-end Transformer computation. Finally, in Theorem H.3 and H.4, we derive how different kinds of parameter expansions in the model compare to adding pause tokens, in terms of FLOPS efficiency. Specifically, Theorem H.3 establishes the number of layers one needs to add to a model to realize the same number of FLOPS as adding p pause tokens. Theorem H.4 presents a similar result for adding attention heads.

Lemma H.1. ***(FLOPS for vector and matrix calculations)*** *The number of flops required to compute:*

1. *the dot product* $\mathbf{v}_1 \cdot \mathbf{v}_2$ *where* $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{R}^d$ *is* $O(d)$.
2. *the matrix multiplication* $M_1 \cdot M_2$*, where* $M_1 \in \mathbb{R}^{a\times b}$ *and* $M_2 \in \mathbb{R}^{b\times c}$*, the FLOPS is* $O(a \times b \times c)$.

Proof. For a dot product between $\mathbf{v}_1$ and $\mathbf{v}_2$, both of dimension d, the total number of FLOPS is given by the sum of multiplications and additions required. Specifically, it involves d multiplications and $d-1$ additions, totaling to $2d-1$ FLOPS. For the ease of calculation, we approximate this as $2d$.

For a matrix multiplication of $M_1 \in \mathbb{R}^{a\times b}$ and $M_2 \in \mathbb{R}^{b\times c}$, each element of the resulting matrix is computed by taking the dot product of a row from M_1 and a column from M_2, which requires $2b$ FLOPS . Since there are $a \times c$ such dot products to compute for the entire matrix multiplication, the total FLOPS amount to $(2b) \times a \times c$. However, for simplicity, if we only consider the multiplicative operations, the FLOPS reduce to $a \times b \times c$. □

Lemma H.2. ***(FLOPS for one end-to-end Transformer computation)*** *Consider an* l *layered decoder only language model, where we denote input embedding dimension as* h*, number of attention heads as* a *and per-attention-head embedding dimension as* d*. We assume feed-forward hidden dimension to be* $4h$ *and finally let* n *denote the length of input sequence. Then the total FLOPS are given as:*

$$F(n, h, d, a, l) = (4nadh + 2an^2d + 8nh^2)l \tag{7}$$

We note that standard Transformer implementations assume $d = h/a$, i.e. the per-attention-head embedding dimension decreases as the number of attention heads are increased. However, we treat these are three independent hyperparameters for greater flexibility in our analysis.

Proof. Let us consider the various per-layer operations in a decoder-only model step-by-step and count their FLOPS :

1. **q**, **k**, **v** vector computation: Given input token $x \in \mathbb{R}^h$, for the query vector computation, we have $Q^j = W_q^j x \forall j \in [1, a]$, where $W_q^j \in \mathbb{R}^{d \times h}$. The same extends for key and value vector computations. Thus total FLOPS required is $3nadh$.

2. Self-attention: Given $Q \in \mathbb{R}^{n \times d}$ and $K \in \mathbb{R}^{n \times d}$, QK^T incurs $n^2 d$ flops. The obtained $\alpha = softmax(\frac{QK^T}{\sqrt{d}}) \in \mathbb{R}^{n \times n}$ is multiplied by $V \in \mathbb{R}^{n \times d}$, costing another $n^2 d$ flops. Note that for simplicity, we ignore that FLOPS from softmax or the division by $\sqrt{d}$ operation as they are negligible. Thus, the total FLOPS = $a[n^2 d + n^2 d] = 2an^2 d$.

3. Combining multi-head-attention: The MHA projection matrix concatenates all the outputs from individual attention heads above and projects them to output of dimension h. For simplicity, we ignore the FLOPS from the skip connection as it adds only a relatively minimal number. Thus total FLOPS = $n \times h \times ad = nadh$.

4. Feed-forward network: This adds another $8nh^2$ FLOPS . Again, for simplicity we ignore the FLOPS from the skip connection.

Combining the FLOPS from each of the sub-parts above, we have:

$$FLOPS = (4nadh + 2an^2 d + 8nh^2)l \tag{8}$$

□

Theorem H.3. ***(FLOPS for adding k layers vs. p pause tokens)*** *Consider a l layer decoder only model, with h denoting the input embedding dimension and d denoting the per attention head embedding dimension. Let n be the length of initial prompt. Then, under the assumption that hidden embedding dimensions are much larger then the prompt sequence length and the appended pause tokens i.e. $d, h \gg n, p$; the additional FLOPS from p pause tokens is less than that from k additional layers of transformer if $n > pl/k$.*

Proof. From Lemma H.2 we have that increase in FLOPS due to k additional transformer layer is give by:

$$\begin{aligned} F_{\Delta l=k} &= F(n, h, a, l + k) - F(n, h, a, l) \\ F_{\Delta l=k} &= (4nadh + 2an^2 d + 8nh^2)k \end{aligned} \tag{9}$$

Similarly, increase in FLOPS due to p pause tokens is given by:

$$\begin{aligned} F_{\Delta n=p} &= F(n + p, h, a, l) - F(n, h, a, l) \\ F_{\Delta n=p} &= (4padh + 2ad((n + p)^2 - n^2) + 8ph^2)l \\ F_{\Delta n=p} &= (4adh + 2ad(2n + p) + 8h^2)pl \end{aligned} \tag{10}$$

Now,

$$\begin{aligned} F_{\Delta n=p} &< F_{\Delta l=k} \\ \implies (2adh + ad(2n + p) + 4h^2)pl &< (2adh + and + 4h^2)nk \\ \implies n &> \frac{(2adh + ad(2n + p) + 4h^2)pl}{(2adh + and + 4h^2)k} \\ \implies n &> \frac{pl}{k} \qquad \text{[assuming } h, d \gg n, p\text{]} \end{aligned} \tag{11}$$

□

Next, we derive how many attention heads can be added, to be FLOP-equivalent to adding p pause tokens. Note that in the standard transformer implementation, increasing the attention heads decreases the per attention head embedding dimension (i.e. $d = h/a$). In contrast, adding pause tokens, increases the number of attention computations while keeping the per-attention-head dimension fixed. Thus for a fair comparison, we consider a setting where we increase the number of attention heads, while keeping the per-attention-head dimension fixed. Specifically, we consider the case where per attention head embedding dimension is fixed to be the same of input embedding (i.e. $d = h$).

Theorem H.4. ***(FLOPS for adding k attention heads vs p pause tokens)*** *Consider a decoder only language model, with the per-attention-head embedding dimension d, fixed to be same as the input embedding dimension h. Let n be the length of initial prompt. Then under the assumption that hidden embedding dimension is much larger then the prompt sequence length and the appended pause tokens i.e. $d, h \gg n, p$; the additional FLOPS from p pause tokens is less than that from k additional attention head, if $n > \frac{(a+2)p}{k}$.*

Proof. From Lemma H.2, we have:

$$F(n, h, d, a, l) = (4anh^2 + 2an^2h + 8nh^2)l, \text{ where } d = h. \tag{12}$$

Now, increase in FLOPS due to k additional attention head is given by:

$$\begin{aligned} F_{\Delta a=k} &= F(n, h, a+k, l) - F(n, h, a, l) \\ F_{\Delta a=k} &= (4knh^2 + 2kn^2h)l \end{aligned} \tag{13}$$

Similarly, increase in FLOPS due to p pause tokens is given by:

$$\begin{aligned} F_{\Delta n=p} &= F(n+P, h, a, l) - F(n, h, a, l) \\ F_{\Delta n=p} &= (4aph^2 + 2ah((n+p)^2 - n^2) + 8ph^2)l \\ F_{\Delta n=p} &= (4ah + 2a(2n+p) + 8h)phl \end{aligned} \tag{14}$$

Therefore, we have, for:

$$\begin{aligned} F_{\Delta n=p} &< F_{\Delta a=1} \\ \implies (4ah + 4an + 2ap + 8h)hpl &< (4nh^2 + 2n^2h)lk \\ \implies ((2a+4)h + 2an + ap)p &< (2h+n)kn \\ \implies n &> \frac{(2a+4)h + 2an + ap}{(2h+n)k} \cdot p \\ \implies n &> \frac{(a+2)p}{k}, \qquad \text{[assuming } h, d \gg n, p] \end{aligned} \tag{15}$$

□

I Theoretical Intuition

This section formalizes a broad class of problems where appending pause tokens during inference can be helpful. Our formalization identifies two core insights:

1. *Pause tokens can be critical to solve tasks that require a large number of independent parallel computations that exceed the number of input tokens.* For example, consider a task where the input is a sequence of L numbers $v_1, v_2, \ldots, v_L$, and the target is a polynomial of the form $(v_1 + v_2) \cdot (v_1 + v_3) \ldots (v_5 + v_2)$. If the number of addition operations required (N) scales much larger than the total number of input tokens (L), (and so $N = \omega(L)$) we argue that (under some natural capacity constraints), standard inference fails as it is bottlenecked in terms of its representational capacity: it can conduct only $O(L)$ operations in parallel. Pause-inference however is relieved of this bottleneck.
2. *The attention-feedforward block in any layer has "untapped" representational capacity — that is independent of the input length — which pause-inference taps into.* Specifically, note that the attention-feedforward block can implement many different operations, one for each intermediate vector it generates at each positional index. But crucially, the number of possible such operations (say, K) scales with the parameter count of the block. This quantity is independent of — and in practice, is much larger than — the input sequence length. Unfortunately, standard inference can only help realize at most L such operations (where $L \ll K$), while pause-inference can tap into K different such operations.

Combining the above two insights, our main result stated informally is that, given a fixed (2-layer) Transformer architecture, (a) if the underlying task requires N parallel operations, where N exceeds the number of input tokens L, and (b) as long as N is not much larger than the parameter count K of the attention-feedforward block, pause-inference can solve tasks that standard inference cannot.

We formalize the above insights in the form of assumptions stated in an abstract setting (in order to be as general as possible). We emphasize that the crux of our argument lies within these assumptions themselves, rather than the proof of our theorem. Thus, our main result here should be viewed as identifying precisely what assumptions are required for pause-inference to help.

I.1 Underlying task

We consider an abstract set of tasks that require a first step that involve multiple parallel operations, following by a simple aggregation step to arrive at the solution:

Assumption I.1. ***(structure of the underlying task)*** *Given the vocabulary space $\mathcal{V}$, let $\circ$ be a generic 2-ary operator on $\mathcal{V}$. For an input sequence length L, consider a corresponding function class $\mathcal{F}_L$ that corresponds to all functions $f : \mathcal{V}^L \mapsto \mathcal{V}$ that require applying N $\circ$ operations independently following by a generic aggregation operation $g_{\text{aggr}} : \mathcal{V}^N \to \mathcal{V}$:*

$$\mathcal{F}_L = \Bigg\{ f : \mathcal{V}^L \mapsto \mathcal{V} \Bigg| \exists i_1, \dots i_N, j_1, \dots j_N \in [1, L], \tag{16}$$

$$f(v_1, v_2, \dots, v_L) = g_{aggr}\big(\underbrace{(v_{i_1} \circ v_{j_1}), (v_{i_2} \circ v_{j_2}), \dots, (v_{i_N} \circ v_{j_N})}_{N \textit{ independent } \circ \textit{ operations.}} \big) \Bigg\}. \tag{17}$$

Examples. This structure covers a broad range of examples.

- As a simple mathematical example, this covers learning polynomials of the form $(x_1+x_2)\cdot(x_3+x_4)\cdot(x_1+x_3)$.
- As a natural language example, consider a multi-choice question-answer task with C choices given along with E pieces of evidence in the context. One can then imagine that each v_i corresponds to a piece of evidence, and each v_j a choice. We then require $N = C{\cdot}E$ operations that compare each of the given choices against each of the given pieces of evidence. A final aggregation step would select the choice for which there exists a piece of evidence the most confidently corroborates the choice.

I.2 (Tight) Upper bounds on the Transformer capacity

If the attention-feedforward module had, say, an infinite or exceedingly large capacity, the Transformer would be able to trivially solve any task. It is only when these modules have finite capacity – as they do in practice – that we expect additional operations introduced by the pause tokens to be helpful. Correspondingly, we state these as multiple "tight upper bound" assumptions, each assumption stating that the modules in a Transformer *can* represent objects of a certain complexity, but none any more complex than that.

Our first such assumption is in how much information can be represented by each intermediate vector. Specifically, we assume that each vector can precisely capture one token in $\mathcal{V}$ along with the positional index of the token (akin to positional embeddings injected into each token in practice). The precise form by which this information is represented as a vector is abstracted away for our discussion (e.g., it may be in one-hot form). Also note that our argument can be extended to settings where each intermediate vector could potentially represent more tokens, we discuss this at the end of the section.

Assumption I.2. ***(information bandwidth limit of intermediate Transformer operations)*** *We assume that the i'th intermediate vector in any given layer can be represented as $(u_i, i) \in \mathcal{V} \times \mathbb{N}$.*

Next, we assume a finite limit on the class of functions that each intermediate Transformer operation can represent. To state this, let $\mathbf{u} = ((u_1, 1), (u_2, 2), \dots, (u_L, L))$ be the outputs of an intermediate

layer (corresponding to a L-length input sequence). For convenience, ignoring the residual and layer-norm blocks, let the i'th output of the next layer be expressed as:

$$\phi_{\mathrm{FF}}(\phi_{\mathrm{Attn}}(\mathbf{u}, \mathbf{u}, (u_i, i))) = (u'_i, i) \tag{18}$$

where the first two arguments represent the keys and values, and the third argument the query for the i'th intermediate operation in the considered layer. Note that ϕ_{FF} and ϕ_{Attn} are parameterized modules that can implement a finite set of functions. We assume what this set of functions consists of:

Assumption I.3. ***(representational limits of intermediate Transformer operations)*** *We assume that for each index $i \in \mathbb{N}$, $\phi_{\mathrm{FF}}(\phi_{\mathrm{Attn}}(\cdot, \cdot, (\cdot, i))$ can represent exactly one of two types of functions:*

- *A single $\circ$ operation. Specifically, we assume the self-attention operation can select two indices as $\phi_{\mathrm{Attn}}(\mathbf{u}, \mathbf{u}, (u_i, i)) = (u_{\nu(i)}, u_{\nu'(i)})$, where $\nu, \nu' : \mathbb{N} \to \mathbb{N}$ come from some finite set of "index-selecting" functions $\mathcal{P}$. We then assume ϕ_{FF} can implement $\phi_{\mathrm{FF}}(u_{\nu(i)}, u_{\nu'(i)}) = u_{\nu(i)} \circ u_{\nu'(i)}$.*
- *The aggregating function g_{agg} as $\phi_{\mathrm{FF}}(\phi_{\mathrm{Attn}}(\mathbf{u}, \mathbf{u}, (u_i, i))) = (g_{\mathrm{aggr}}(u_1, \ldots, u_L), i)$.*

We explain why the above sub-assumptions are both reasonable and can hold simultaneously. First, we argue why it is reasonable that each intermediate Transformer operation can implement a limited number of $\circ$ operations, but not any more. Assume that the model needs to represent $u'_{35} = u_1 \circ u_3$. This requires the model to pay attention to the query's positional index 35, and then select the values at two different positional indices 1 and 3. Selecting these two values can be implemented by 2 self-attention heads operating independently. A subsequent feedforward network can then operate on a concatenated input (u_1, u_3). Crucially, implementing any further $\circ$ operations, would require more attention heads. Thus, it is reasonable to assume a (tight) limit on the number of $\circ$ attention operations.

Note that the above assumption can simultaneously hold with the assumption that the Transformer operation can implement the aggregation function g_{aggr}. This is because g_{aggr} does not require preferentially selecting any positional indices: all inputs are aggregated equally. Thus, we only need a single self-attention head that applies equal weight to all the values.

Our last and arguably most insightful assumption stipulates a tight *input-length-independent* upper limit on what the self-attention module can represent. Specifically, above we assume that the self-attention module can implement a finite set of functions $\mathcal{P}$, which help select "indices" $\nu(i)$ and $\nu'(i)$. We assume that there is a limit to this set of index-selecting functions $\mathcal{P}$. *Our key insight is that this limit is purely determined by the parameter count of the module, and is therefore independent of the input-sequence-length, L.* In practice, this quantity is much larger than the number of tokens L, and thus corresponds to untapped capacity in the model which is bottlenecked by the L operations in standard inference.

Assumption I.4. ***(the "untapped" capacity of self-attention operation is independent of input length)*** *We assume that for some $K \gg L$, the self-attention module in each layer has at least $2K \log K$ parameters, and can hence implement the index-selecting functions $\nu, \nu' : [K] \to [K]$ to be any of the K^K many mappings possible. In other words, $\mathcal{P} = \{1, 2, \ldots, K\}^{\{1,2,\ldots,K\}}$.*

From our assumptions, we derive the following result on what pause-inference can implement, which standard inference cannot, given a 2-layer Transformer (we discuss extensions to larger architectures in a subsequent remark):

Theorem I.1. *Under Assumptions I.1, I.2, I.3 and I.4, standard inference on a 2-layer Transformer with no pause tokens can only represent the function class $\mathcal{F}_N$ for $N \leq L$, where N denotes the number of parallel operations required by the function class, and L denotes the length of the input sequence. In contrast, a 2-layer Transformer with $N - L$ appended dummy tokens can represent the function class $\mathcal{F}_N$ for any large $N \geq L$ as long as $N \leq K$, where K scales with the parameter count of the self-attention module as in Assumption I.4.*

The key insight is that the self-attention module has the representational capacity to implement K different $\circ$ operations, a capacity that is independent of, and much larger than the number of input tokens. However the standard Transformer that sees only L tokens, only allows the model to realize L of these.

Proof. Under Assumption I.2 that each intermediate vector can only represent one token from $\mathcal{V}$, the first layer in the Transformer will have to implement all the N $\circ$ operations to represent any given $f \in \mathcal{F}_L$. Therefore, at the k'th index, the model would have to instantiate $\nu(i) = i_k$ and $\nu'(i) = j_k$ (as defined in Assumption I.3), so that the corresponding Transformer operation can compute $v_{i_k} \circ v_{j_k}$. However, by Assumption I.3 (which states that each intermediate Transformer operation can implement only one $\circ$ operation) and by how standard inference in a Transformer is defined (which allows only as many operations as there are input tokens), the first layer can only compute at most L many $\circ$ operations. Hence, a Transformer with standard inference can only represent $\mathcal{F}_N$ for $N \leq L$. On the other hand, from Assumption I.4, we know that the self-attention operator can implement as many as K such operations, where $K \gg L$. Thus as long as $N \leq K$, with $N - L$ dummy tokens, the Transformer can implement $\mathcal{F}_N$ for $N > L$. □

Remark I.2. ***(breaking the information bandwidth assumption)*** *In Assumption I.2, recall that we assume that in each intermediate vector, we are able to communicate precisely one token from $\mathcal{V}$. If this assumption were to break, then standard inference would be able to implement a larger class of functions where $N > L$. However, it would still fall short of what pause inference can do. Specifically, imagine that each layer passes on its computed output, and also all computed outputs from the previous layer. Thus, if the intermediate vector can represent upto N/L tokens, then the Transformer could divide the N required $\circ$ operations over N/L many layers, each layer performing L operations in parallel. This would however be slower as it requires a series of N/L computations. In contrast, pause-inference requires only 1 set of parallel computations, and also a meagre information bandwidth of 1 token per intermediate vector.*

Remark I.3. ***(recurrent depth)*** *An alternative way to exploit the untapped capacity of the attention-feedforward modules, is to repeat these operations sequentially along the depth of the model as done in* **?** *]. This strategy would be helpful in tasks that require recursion. However, our class of problems do not involve recursion of any form. Thus, if these recurrent layers simply repeat the same $\circ$ operations over and over, we may not enjoy any advantages. However, one may argue that, perhaps repeating the same layers over and over somehow implements different operations in each repetition. In this case, we can make an argument similar to the previous remark. Specifically, to fit a task that requires N parallel operations, we would need a model that has an information bandwidth of N/L, and applies a recurrence of N/L layers corresponding to N/L* serial *operations to compute the desired function. This should again be contrasted with pause-inference which requires only 1 set of parallel computations, and also a meagre information bandwidth of 1 token per intermediate vector.*