Advances in Temporal Point Processes: Bayesian, Neural, and LLM Approaches

Anonymous authors
Paper under double-blind review

Abstract

Temporal point processes (TPPs) are stochastic process models used to characterize event sequences occurring in continuous time. Traditional statistical TPPs have a long-standing history, with numerous models proposed and successfully applied across diverse domains. In recent years, advances in deep learning have spurred the development of neural TPPs, enabling greater flexibility and expressiveness in capturing complex temporal dynamics. The emergence of large language models (LLMs) has further sparked excitement, offering new possibilities for modeling and analyzing event sequences by leveraging their rich contextual understanding. This survey presents a comprehensive review of recent research on TPPs from three perspectives: Bayesian, deep learning, and LLM approaches. We begin with a review of the fundamental concepts of TPPs, followed by an in-depth discussion of model design and parameter estimation techniques in these three frameworks. We also revisit classic application areas of TPPs to highlight their practical relevance. Finally, we outline challenges and promising directions for future research.

1 Introduction

Many application scenarios generate time-stamped event sequences, which can be effectively modeled using temporal point processes (TPPs) (Daley & Vere-Jones, 2007). Examples include neural spike train data in neuroscience (Linderman & Adams, 2015), ask and bid orders in high-frequency financial trading (Bacry & Muzy, 2014), as well as tweets and retweets on social media (Kong et al., 2023). These event sequences, composed of asynchronous events, often influence one another and exhibit complex dynamics, making them more challenging to analyze compared to traditional synchronous time series problems. Investigating the underlying dynamic processes of such event sequences not only facilitates the prediction of future events but also helps uncover causal relationships.

In the statistics community, TPPs have a long-standing history of research, with numerous statistical TPP models proposed over the years. Examples include the classic Poisson process (Kingman, 1992), Hawkes process (Hawkes, 1971), and self-correcting process (Isham & Westcott, 1979), among others. Each of these models is particularly well-suited to specific applications. For instance, the Poisson process was used to model telephone call arrivals, while the Hawkes process, due to its ability to capture self-exciting characteristics, has been widely applied to model earthquakes and aftershocks.

Early TPP models are primarily parametric, requiring explicit specification of the parametric form of the model. However, this imposes limitations on their expressive power. To address these limitations, various nonparametric TPPs have been proposed within the statistics community, including approaches from both the frequentist and Bayesian frameworks, enabling more flexible modeling without the constraints of fixed parametric forms. In recent years, driven by rapid advancements in deep learning, the machine learning community has introduced approaches that combine neural network architectures with TPPs, referred to as neural TPPs. These models further enhance expressive power and are often more intuitive, simpler, and easier to train compared to statistical nonparametric TPPs. Over the past several years, the emergence of large language models (LLMs) has brought transformative changes to the field of artificial intelligence. With

their rich contextual understanding and ability to process multimodal data, LLMs offer new possibilities for modeling event sequences.

This paper reviews recent advances in TPPs based on Bayesian methods, deep learning, and LLMs, with a focus on model design and parameter estimation. Due to space limitations, we do not aim to cover every method in detail but instead emphasize fundamental principles and core ideas. We also revisit classic applications of TPPs and discuss key challenges and future research directions in the field. This taxonomy of recent advances in TPPs is illustrated in Figure 1. This survey is expected to give comprehensive background knowledge, research trends and technical insights for TPPs.

Comparison with Existing TPP Surveys Several surveys on TPPs in machine learning already exist, such as Yan (2019) and Shchur et al. (2021). The former summarizes advances in statistical and neural TPPs, while the latter provides a more detailed overview of neural TPPs. Additionally, surveys in other domains, such as finance, include Hawkes (2018). Compared with these works, this paper provides a comprehensive update. In the domain of statistical TPPs, we emphasize recent progress in Bayesian nonparametric TPPs, which has been largely overlooked, as most prior reviews (e.g., Yan (2019)) primarily focus on frequentist approaches. For neural TPPs, Shchur et al. (2021) covers works up to 2020, whereas this survey reviews advances from 2020 to 2025. Furthermore, we include a systematic review of the emerging area of LLM-based TPPs, which has gained significant attention in recent years but has not yet been comprehensively surveyed. A detailed comparison with existing surveys is provided in Table 1.

Table 1: Survey comparison.

Survey	Years Covered	Frequentist TPP	Bayesian TPP	Neural TPP	LLM-based TPP	Training Methods	Applications
Yan (2019)	≤ 2019	✔	✗	✔	✗	Limited	✔
Shchur et al. (2021)	≤ 2021	✔	✗	✔	✗	Limited	✔
Hawkes (2018)	≤ 2018	✔	✗	✗	✗	Limited	Finance
This survey	≤ 2025	✔	✔	✔	✔	✔	✔

2 Background of TPPs

We first briefly review some of the core concepts of TPPs. For readers unfamiliar with TPPs, we recommend Rasmussen (2018) for a comprehensive introduction.

2.1 Unmarked TPP

A TPP is a stochastic process that models the occurrence of events over a time window $[0, T]$. A trajectory from a TPP can be represented as an ordered sequence $\mathcal{T} = (t_1, \ldots, t_N)$, $N(t) = \max\{n : t_n \leq t, t \in [0, T]\}$ represents the associated counting process. Here, N denotes the random number of events within the interval $[0, T]$. TPPs can be defined using different parameterizations. One approach is to specify the distribution of the time intervals between consecutive events. We denote the $f(t_{n+1} \mid \mathcal{H}_{t_n})$ to be the conditional density function of t_{n+1} given the history of previous events $t_1, \ldots, t_n$. In this work, $\mathcal{H}_{t^-}$ denotes the history of events up to but excluding time t, while $\mathcal{H}_t$ includes whether an event occurs at time t. The conditional density function sequentially specifies the distribution of all timestamps. Consequently, the joint distribution of all events can be factorized as:

$$f(t_1, \ldots, t_N) = \prod_{n=1}^{N} f(t_n \mid \mathcal{H}_{t_{n-1}}). \tag{1}$$

A TPP can be defined by specifying the distribution of time intervals. For example, a renewal process assumes that the time intervals are independent and identically distributed (i.i.d.), i.e., $f(t_n|\mathcal{H}_{t_{n-1}}) = g(t_n - t_{n-1})$, where g is a probability density function defined on $(0, \infty)$. If we further specify $g(t_n - t_{n-1})$ to follow an exponential distribution, we obtain a homogeneous Poisson process, where each event occurs independently of the past.

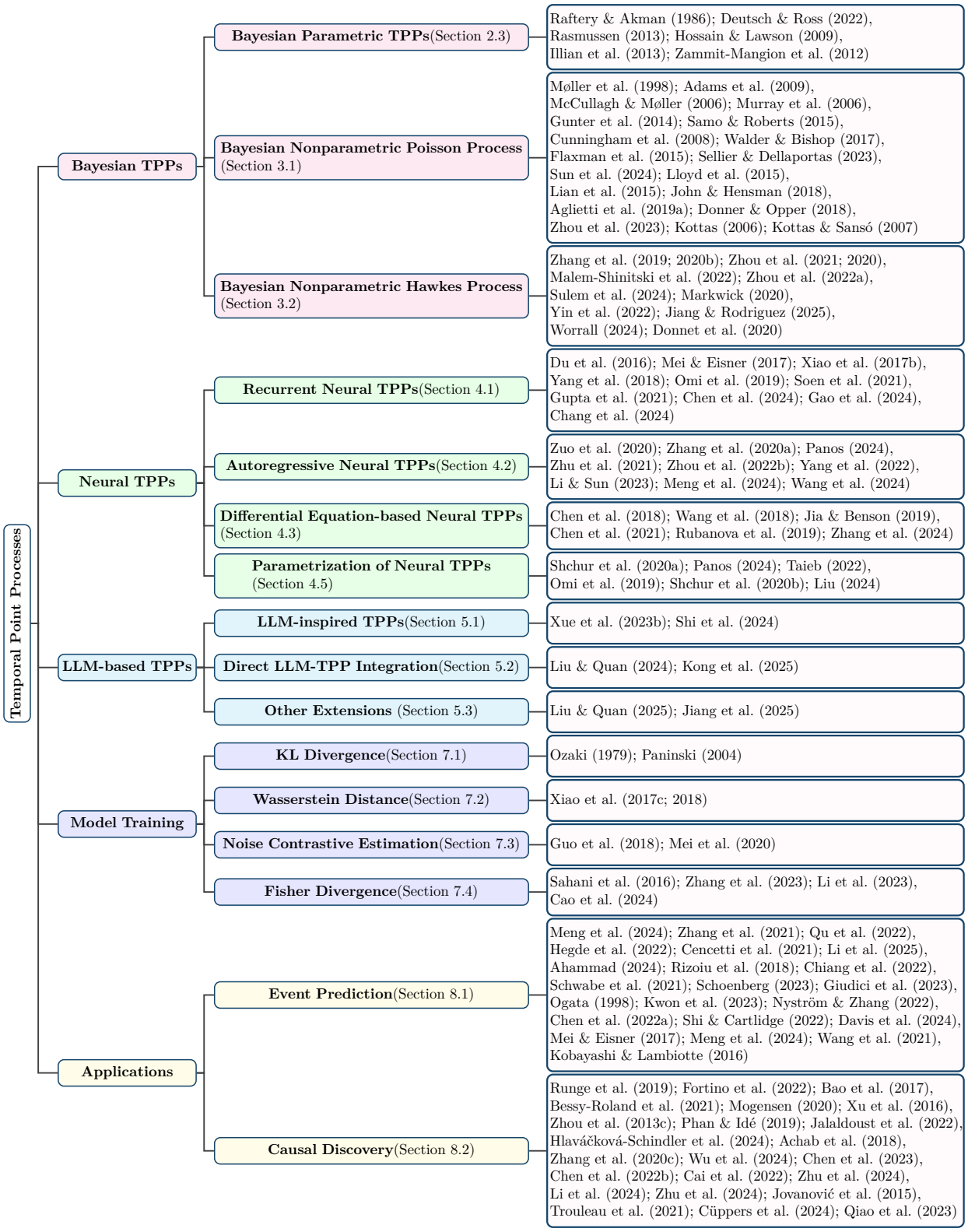

Figure 1: The taxonomy of Bayesian, neural, and LLM-based TPPs.

The above approach can directly define some classic point process models. However, in general cases, event occurrences may depend on the entire history, making it less convenient to specify the model using the probability density function of time intervals. Instead, the conditional intensity function provides a more convenient way to describe how the occurrence of an event depends on its history. The conditional intensity function is defined as:

$$\begin{aligned}\lambda^*(t)dt &= \frac{f(t \mid \mathcal{H}_{t_n})dt}{1 - F(t \mid \mathcal{H}_{t_n})} = \frac{P(t_{n+1} \in [t, t+dt] \mid \mathcal{H}_{t_n})}{P(t_{n+1} \notin (t_n, t) \mid \mathcal{H}_{t_n})} = P(t_{n+1} \in [t, t+dt] \mid t_{n+1} \notin (t_n, t), \mathcal{H}_{t_n}) \\ &= P(t_{n+1} \in [t, t+dt] \mid \mathcal{H}_{t^-}) = \mathbb{E}[N([t, t+dt]) \mid \mathcal{H}_{t^-}],\end{aligned} \tag{2}$$

where $F(t \mid \mathcal{H}_{t_n}) = \int_{t_n}^{t} f(\tau \mid \mathcal{H}_{t_n})d\tau$ denotes the cumulative distribution function. Following tradition, we use * to indicate that the conditional intensity function is based on the history. The conditional intensity function has an intuitive interpretation: it specifies the average number of events in a time interval, conditional on the history up to but not including t. It is worth noting that the history $\mathcal{H}_{t_n}$ in the conditional density function differs from the history $\mathcal{H}_{t^-}$ in the conditional intensity function. This subtle distinction is often overlooked in many TPP works.

The conditional intensity function and the conditional density function are one-to-one[1]. This can be easily proven by inverting Equation (2) to express the conditional density function in terms of the conditional intensity function:

$$\begin{aligned}F(t \mid \mathcal{H}_{t_n}) &= 1 - \exp\left(-\int_{t_n}^{t} \lambda^*(\tau)d\tau\right), \\ f(t \mid \mathcal{H}_{t_n}) &= \lambda^*(t) \exp\left(-\int_{t_n}^{t} \lambda^*(\tau)d\tau\right).\end{aligned} \tag{3}$$

This means we can define new TPP models by directly specifying a particular form of the conditional intensity function. For example, specifying a constant intensity defines a homogeneous Poisson process, while specifying a time-varying intensity function $\lambda^*(t) = \lambda(t)$ defines an inhomogeneous Poisson process. We can also define a Hawkes process by specifying a conditional intensity function:

$$\lambda^*(t) = \mu + \sum_{t_n < t} \phi(t - t_n), \tag{4}$$

where $\mu > 0$ is the baseline intensity, and $\phi(\cdot) : \mathbb{R}^+ \to \mathbb{R}^+$ is the triggering function[2]. The summation of influences from past events increases the likelihood of future events, making it suitable for modeling self-exciting effects. While many other forms of TPPs exist, we primarily focus on the Poisson process (history-independent) and the Hawkes process (history-dependent) in the following due to their widespread use. An illustration of the unmarked temporal point process, along with its conditional density function and conditional intensity function, is shown in Figure 2.

2.2 Marked TPPs

The above discussion focuses on the unmarked TPP, also known as the univariate TPP. However, TPPs can be extended to the marked case, represented as a time-ordered marked sequence $\mathcal{T} = ((t_1, k_1), \ldots, (t_N, k_N))$ over a time window $[0, T]$, where k_n is the mark of the n-th event. The mark space can be continuous or discrete. In practice, discrete marks are more common, often referred to as multivariate TPPs. Similar to the unmarked case, marked TPPs can also be described using the conditional density function:

$$f((t_1, k_1), \ldots, (t_N, k_N)) = \prod_{n=1}^{N} f(t_n, k_n \mid \mathcal{H}_{t_{n-1}}), \tag{5}$$

where $f(t, k \mid \mathcal{H}_{t_n})$ is the joint density of time and mark, conditional on history. The history $\mathcal{H}_{t_n}$ now includes information about both the times and marks of past events.

[1]The conditional intensity function must satisfy certain conditions.

[2]It requires $\int_0^\infty \phi(t)\, dt < 1$ to ensure the Hawkes process does not explode.

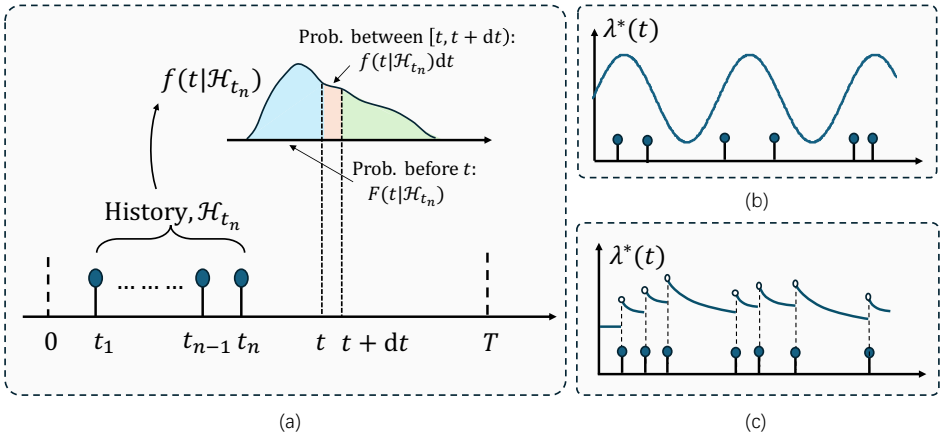

Figure 2: Illustration of the conditional density function and conditional intensity function in TPPs. (a) The conditional density function of the $(n{+}1)$-th event given the history $\{t_1, \ldots, t_n\}$; (b) the intensity function of an inhomogeneous Poisson process; (c) the conditional intensity function of a Hawkes process.

Similarly, when event occurrences depend on the entire history, specifying the model using the probability density function becomes less convenient. In such cases, the conditional intensity function offers a more practical and expressive way to capture the dependence of events on historical information. The conditional intensity function is defined as:

$$\lambda^*(t,k)dtdk = \frac{f(t,k \mid \mathcal{H}_{t_n})dtdk}{1 - F(t \mid \mathcal{H}_{t_n})} = \mathbb{E}[N(dt \times dk) \mid \mathcal{H}_{t^-}],$$

where $F(t \mid \mathcal{H}_{t_n}) = \int_{t_n}^{t} \int_k f(\tau, k \mid \mathcal{H}_{t_n}) dk d\tau$ is the conditional cumulative distribution function and the mark is marginalized out. The conditional intensity function specifies the average number of events with a mark k in a time interval, conditional on the history up to but not including t.

We can also define a marked TPP by specifying a particular conditional intensity function. A classic example is the multivariate Hawkes processes, where the mark $k \in \{1, \ldots, K\}$ represents the event type. The conditional intensity function of multivariate Hawkes processes is given by:

$$\lambda^*(t,k) = \mu_k + \sum_{t_n < t} \phi_{k,k_n}(t - t_n), \tag{6}$$

where μ_k represents the baseline intensity for event type k, and $\phi_{k,k'}$ captures the triggering effects from events of type k' on type k. A comparison of commonly used classical TPP models is provided in Table 2.

Table 2: Comparison of classic TPP models.

Model	History dependence	Marks	Intensity form	Typical use
Poisson	No	No	λ	Independent arrivals
inhomogeneous Poisson	No	No	$\lambda(t)$	Time-varying rate
Hawkes	Yes	No	$\mu + \sum_{t_n<t} \phi(t - t_n)$	Self-excitation
Multivariate Hawkes	Yes	Yes	$\mu_k + \sum_{t_n<t} \phi_{k,k_n}(t - t_n)$	Type interaction

2.3 Inference

There are various methods for estimating the parameters of TPPs. The most common method is maximum likelihood estimation (MLE). In this section, we introduce MLE and Bayesian inference for parametric TPPs. Assume we observe a trajectory of a marked TPP $\mathcal{T} = ((t_1, k_1), \ldots, (t_N, k_N))$ over the time window $[0, T]$.

The unmarked case corresponds to the situation where there is only a single mark. Assume the marked TPP is specified by a parametric conditional intensity function $\lambda^*_\theta(t, k)$. Then, the likelihood function is given by:

$$f(\mathcal{T};\theta) = \prod_{n=1}^{N} \lambda^*_\theta(t_n, k_n) \exp\left(-\int_0^T \lambda^*_\theta(t)dt\right), \tag{7}$$

where $\lambda^*_\theta(t) = \int \lambda^*_\theta(t, k)dk$ is the ground intensity. The proof of Equation (7) is straightforward. Simply substitute Equation (3) into Equation (1) to verify it for the unmarked case. The proof for the marked case follows a similar procedure. We can use numerical methods to maximize the log-likelihood to obtain parameter estimates.

MLE is a widely used parameter estimation method in the frequentist framework. It offers several advantages, such as consistency, asymptotic normality, and asymptotic efficiency. However, as a point estimation method, MLE cannot capture model uncertainty, which limits its applicability in high-stakes domains where understanding uncertainty is crucial. To address this issue, the Bayesian framework has been incorporated into TPPs (Raftery & Akman, 1986). In Bayesian TPPs, we impose suitable priors on the model parameters and then compute their corresponding posterior distribution, equipping the model with the ability to quantify uncertainty. Specifically, a Bayesian TPP is formally expressed as:

$$f(\theta \mid \mathcal{T}) = \frac{f(\mathcal{T} \mid \theta)f(\theta)}{\int f(\mathcal{T} \mid \theta)f(\theta)d\theta}, \tag{8}$$

where $f(\mathcal{T} \mid \theta)$ is the likelihood in Equation (7), $f(\theta)$ is the prior on model parameters, the denominator is the marginal likelihood, and $f(\theta \mid \mathcal{T})$ is the posterior distribution of model parameters. In general, the inference for Bayesian TPPs is more challenging than for frequentist TPPs because the TPP likelihood is not conjugate to any prior. This means that the posterior does not have an analytical expression and can only be obtained through approximation methods, such as Markov chain Monte Carlo (MCMC) (Deutsch & Ross, 2022; Rasmussen, 2013), variational inference (Lloyd et al., 2015; Zammit-Mangion et al., 2012), and Laplace approximation (Hossain & Lawson, 2009; Illian et al., 2013), among others.

3 Bayesian Nonparametric TPPs

Early work on TPPs was limited to parametric models, whether in the frequentist or Bayesian framework. These methods rely heavily on model assumptions, making them inflexible and often performing poorly on complex datasets. To address this limitation, many studies have proposed nonparametric methods. Yan (2019) provides a comprehensive description of frequentist nonparametric methods, while this paper focuses on Bayesian nonparametric TPPs, which not only enhance model flexibility but also incorporate the ability to quantify model uncertainty.

3.1 Bayesian Nonparametric Poisson Process

Bayesian nonparametric TPPs do not parameterize the intensity function into a fixed form. Instead, they treat the intensity function itself as a model parameter with infinite dimensions and impose suitable prior on it. For instance, in the case of an inhomogeneous Poisson process, the intensity function $\lambda(t)$ is treated as the parameter, and a prior $f(\lambda(t))$ is placed on it. The goal is then to compute the posterior distribution. This prior, being a distribution over functions, is commonly modeled using a Gaussian process (GP). Consequently, the Bayesian nonparametric framework is formulated as follows:

$$f(g(t) \mid \mathcal{T}) = \frac{f(\mathcal{T} \mid \lambda(t) = l \circ g(t))\mathcal{GP}(g(t))}{\int f(\mathcal{T} \mid \lambda(t) = l \circ g(t))\mathcal{GP}(g(t))dg}, \tag{9}$$

where $l(\cdot) : \mathbb{R} \to \mathbb{R}^+$ is a link function ensuring the intensity function is non-negative, e.g., exponential (log Gaussian Cox process (Møller et al., 1998)), scaled sigmoid (sigmoidal Gaussian Cox process (Adams et al., 2009)), square (permanental process (McCullagh & Møller, 2006)), etc. It is worth noting that computing Equation (9) is highly challenging, as the posterior is doubly intractable due to an intractable integral over

t in the numerator and another over g in the denominator. This is a well-known problem in the field of Bayesian nonparametric TPPs (Murray et al., 2006).

Many methods have been proposed to solve Equation (9). Some studies focus on utilizing MCMC methods. Adams et al. (2009) proposed an MCMC inference method for a Poisson process with a sigmoidal GP prior. The core idea is to incorporate latent thinned points to make the posterior tractable. However, this method scales cubically with the number of data and thinned points. Gunter et al. (2014) extended this approach to multiple dependent Cox processes using multi-output Gaussian processes, and also derived an MCMC sampler for performing inference. Later, Samo & Roberts (2015) leveraged inducing points, a common technique in GP for reducing time complexity (Titsias, 2009), to derive an MCMC sampler that reduces the computational cost to linear w.r.t. the number of data points.

Some studies focus on methods based on the Laplace approximation. Flaxman et al. (2015) combined Kronecker methods with the Laplace approximation to enable scalable inference. Walder & Bishop (2017) proposed a fast Laplace approximation method relying on the Mercer decomposition of the GP kernel. However, its tractability is limited to standard kernels such as the squared exponential kernel. To address this limitation, Sellier & Dellaportas (2023) introduced an alternative fast Laplace approximation leveraging the spectral representation of kernels. This approach retains tractability while accommodating a broader range of stationary kernels. Furthermore, Sun et al. (2024) extended this method to non-stationary kernels by leveraging sparse spectral representations to overcome the limitations of stationary kernels. This approach provides a low-rank approximation of the kernel, effectively reducing computational complexity from cubic to linear.

Another common approach is variational inference. Lloyd et al. (2015) introduced the first fully variational inference scheme for permanental processes. However, similar to Walder & Bishop (2017), its tractability is limited to certain standard types of kernels. Lian et al. (2015) further extended the method in Lloyd et al. (2015) to a multitask point process model, leveraging information from all tasks via a hierarchical GP. John & Hensman (2018) expanded the approach in Lloyd et al. (2015) to utilize the Fourier representation of the GP, enabling the use of more general stationary kernels. Aglietti et al. (2019a) presented a novel tractable representation of the likelihood through augmentation with a superposition of Poisson processes. This perspective enabled a structured variational approximation that captured dependencies across variables in the model. The method avoided discretization of the domain, did not require numerical integration over the input space, and was not limited to GPs with squared exponential kernels.

It is worth noting that, Donner & Opper (2018) introduced the data augmentation technique based on Pólya-Gamma variables to the field of Bayesian nonparametric TPPs. This technique is an improvement and extension of the method proposed by Adams et al. (2009). The method augments not only thinned points but also Pólya-Gamma latent variables for all data and thinned points in the likelihood. This enables the augmented likelihood to be conditionally conjugate to the GP prior. By leveraging the conditionally conjugacy, we can derive fully analytical Gibbs sampler, EM algorithm, and mean-field variational inference method. This method was later extended to jointly model multiple heterogeneous and correlated tasks—such as classification, regression, and point processes—using multi-output Gaussian processes. This extension facilitated information sharing across heterogeneous tasks while enabling nonparametric parameter estimation (Zhou et al., 2023).

The works discussed above primarily use GP as prior. However, other forms of Bayesian nonparametric priors are also possible. For example, Kottas (2006); Kottas & Sansó (2007) used a Dirichlet process mixture of Beta densities as a prior for the normalized intensity function of a Poisson process. Bayesian nonparametric TPPs based on Dirichlet process mixtures and those based on Gaussian processes represent two orthogonal modeling paradigms, both capable of achieving Bayesian nonparametric inference. Inference for Dirichlet process mixture-based TPPs typically relies on specialized MCMC or variational inference techniques developed within the Dirichlet process framework. A detailed discussion of these methods is beyond the scope of this paper.

3.2 Bayesian Nonparametric Hawkes Process

For the Hawkes process, as shown in Equation (4), the conditional intensity function consists of the baseline intensity $\mu(\cdot)$[3] and the triggering function $\phi(\cdot)$. Therefore, the Bayesian nonparametric Hawkes process typically places GP priors on both $\mu(\cdot)$ and $\phi(\cdot)$ (we take the unmarked case as an example):

$$f(g(t), h(\tau) \mid \mathcal{T}) \propto f(\mathcal{T} \mid \mu(t) = l \circ g(t), \phi(\tau) = l \circ h(\tau))\mathcal{GP}(g(t))\mathcal{GP}(h(\tau)), \tag{10}$$

where $l(\cdot)$ is a link function ensuring $\mu(t)$ and $\phi(\tau)$ are non-negative, similar to Equation (9). Computing Equation (10) is more challenging than Equation (9) because in the likelihood of Hawkes process, $\mu(t)$ and $\phi(\tau)$ are coupled together, which significantly complicates the inference process.

To address this issue, a common approach is to augment a branching latent variable into the Hawkes process likelihood to indicate whether each event is triggered by itself via the baseline intensity or by a previous event via the triggering function (Marsan & Lengline, 2008). The branching variable $\mathbf{X}$ is a lower triangular matrix with Bernoulli entries, where x_{nm} indicates whether the n-th event is triggered by itself or a previous event m:

$$x_{nn} = \begin{cases} 1 & \text{if event } n \text{ is a background event,} \\ 0 & \text{otherwise,} \end{cases}$$

$$x_{nm} = \begin{cases} 1 & \text{if event } n \text{ is caused by event } m, \\ 0 & \text{otherwise.} \end{cases}$$

After augmenting the branching latent variable, the joint likelihood is expressed as:

$$f(\mathcal{T}, \mathbf{X} \mid \mu(t), \phi(\tau)) = \underbrace{\prod_{n=1}^{N} \mu(t_n)^{x_{nn}} \exp\left(-\int_0^T \mu(t)dt\right)}_{\text{baseline intensity part}} \cdot \underbrace{\prod_{n=2}^{N}\prod_{m=1}^{n-1} \phi(t_n - t_m)^{x_{nm}} \prod_{n=1}^{N} \exp\left(-\int_0^{T_\phi} \phi(\tau)d\tau\right)}_{\text{triggering function part}}, \tag{11}$$

where the support of triggering function is assumed to be $[0, T_\phi]$. If the branching variable $\mathbf{X}$ is marginalized out, we obtain the original likelihood.

It is clear that, after introducing the branching variable, the joint likelihood factorizes into two independent components: one corresponding to the baseline intensity and the other to the triggering function. These two components are linked through the branching variable $\mathbf{X}$. To the best of our knowledge, Marsan & Lengline (2008) was the first to identify this structure and subsequently proposed an EM algorithm that leverages it: the E-step computes the posterior distribution of the branching variable, while the M-step estimates the parameters of both the baseline intensity and the triggering function. Later, Lewis & Mohler (2011) extended this approach to the frequentist nonparametric setting by treating the baseline intensity and the triggering function as two flexible, unconstrained functions. They imposed Good's roughness penalty (Goodd & Gaskins, 1971) on both functions and derived the solutions using the Euler–Lagrange equation. Zhou et al. (2013b) further extended the method in Lewis & Mohler (2011) to multivariate Hawkes processes by assuming that the triggering functions in the multivariate setting are linear combinations of a set of basis functions. Each basis function was then estimated using the Euler–Lagrange equation. There also exist frequentist nonparametric approaches that do not rely on the branching variable. For example, Bacry & Muzy (2016) proposed an estimation method based on solving a Wiener–Hopf equation that relates the triggering function to the second-order statistics. Additionally, Eichler et al. (2017); Reynaud-Bouret et al. (2010) attempted to estimate the triggering function by minimizing a quadratic contrast function using a grid-based representation of the triggering function. Bonnet & Sangnier (2024) extended the approach of Flaxman et al. (2017), which formulated Poisson process intensity estimation within a reproducing kernel Hilbert space (RKHS) framework, to the more complex setting of Hawkes processes.

In the Bayesian nonparametric Hawkes process setting, a similar strategy can be adopted. By introducing the branching variable, the Hawkes likelihood factorizes into two independent components, each of which

[3]Some studies treat the baseline intensity as a constant, but here we consider it as a more general function.

can be viewed as an independent Poisson process. Since we place independent GP priors on these two components, we can directly apply the methods discussed in Section 3.1 to compute the posteriors of $\mu(t)$ and $\phi(\tau)$. This naturally leads to an iterative algorithm where, at each iteration, the posterior of $\mathbf{X}$ is used to update the posteriors of $\mu(t)$ and $\phi(\tau)$, which are then used to update the posterior of $\mathbf{X}$ in turn.

In recent years, many GP-based Bayesian nonparametric Hawkes process studies have adopted this iterative framework. Zhang et al. (2019) derived a Gibbs sampler and a maximum a posteriori (MAP) EM algorithm to estimate a nonparametric triggering function. Zhang et al. (2020b) extended the variational inference method from Lloyd et al. (2015) to the Hawkes process for estimating a nonparametric triggering function. Further, Zhou et al. (2021) applied variational inference to simultaneously estimate the nonparametric baseline intensity and triggering function. Zhou et al. (2020); Malem-Shinitski et al. (2022) extended the data augmentation method based on Pólya-Gamma variables from Donner & Opper (2018) to the Hawkes process. This work derived fully analytical Gibbs sampler, EM algorithm, and mean-field variational inference method. This approach was subsequently extended to nonlinear Hawkes processes to account for excitation and inhibition effects between interacting variables (Zhou et al., 2022a). Sulem et al. (2024) analyzed the theoretical properties of this approach; specifically, it provided concentration rates for the posterior distribution of the parameters under mild assumptions on both the prior and the model, and established consistency guarantees for the inferred Granger causal graph.

The works discussed above primarily use GP as prior. However, other forms of Bayesian nonparametric priors are also possible. For instance, Markwick (2020); Yin et al. (2022); Jiang & Rodriguez (2025) extended the Hawkes process to the Bayesian nonparametric setting using Dirichlet process mixture priors. Building on this line of research, Worrall (2024) proposed an online inference method using sequential Monte Carlo techniques. Additionally, Donnet et al. (2020) introduced priors based on piecewise constant functions with either regular or random partitions, as well as priors defined via mixtures of Beta distributions. A comprehensive discussion of alternative approaches lies beyond the scope of this paper. A complete, though not exhaustive, comparison of Bayesian nonparametric Poisson and Hawkes models is provided in Table 3.

Table 3: Comparison of Bayesian nonparametric Poisson and Hawkes models.

Work	Model	Prior	Link Function	Inference	Complexity	Kernel Support
Adams et al. (2009)	Poisson	GP	sigmoid	MCMC	$\mathcal{O}(N^3)$	Any
Gunter et al. (2014)	Poisson	Multi-output GP	sigmoid	MCMC	$\mathcal{O}(N^3)$	Any
Samo & Roberts (2015)	Poisson	Sparse GP	exponential	MCMC	$\mathcal{O}(N)$	Any
Flaxman et al. (2015)	Poisson	GP + Kronecker	exponential	Laplace	near $\mathcal{O}(N)$	Any
Walder & Bishop (2017)	Poisson	Mercer GP	square	Laplace	$\mathcal{O}(N)$	squared exponential
Sellier & Dellaportas (2023)	Poisson	Spectral GP	square	Laplace	$\mathcal{O}(N)$	stationary
Lloyd et al. (2015)	Poisson	GP	square	Variational	$\mathcal{O}(N)$	squared exponential
Aglietti et al. (2019b)	Poisson	Multi-output GP	exponential	Variational	$\mathcal{O}(N)$	Any
Donner & Opper (2018)	Poisson	GP + Pólya-Gamma	sigmoid	Variational/MCMC/EM	$\mathcal{O}(N)$	Any
Kottas & Sansó (2007)	Poisson	DP mixture	–	MCMC	not specified	–
Zhang et al. (2019)	Linear Hawkes	GP for $\phi(\tau)$ only	square	MCMC	$\mathcal{O}(N)$	Any
Zhang et al. (2020b)	Linear Hawkes	GP for $\phi(\tau)$ only	square	Variational	$\mathcal{O}(N)$	squared exponential
Zhou et al. (2021)	Linear Hawkes	GP	square	Variational	$\mathcal{O}(N)$	squared exponential
Zhou et al. (2020)	Linear Hawkes	GP + Pólya-Gamma	sigmoid	Variational/MCMC/EM	$\mathcal{O}(N)$	Any
Malem-Shinitski et al. (2022)	Nonlinear Hawkes	GP + Pólya-Gamma	sigmoid	Variational	$\mathcal{O}(N)$	Any
Zhou et al. (2022a)	Nonlinear Hawkes	GP + Pólya-Gamma	sigmoid	Variational/MCMC/EM	$\mathcal{O}(N)$	Any
Jiang & Rodriguez (2025)	Linear Hawkes	DP mixture	–	MCMC	$\mathcal{O}(N)$	–

4 Neural TPPs

Benefiting from the rapid development of deep learning, another way to enhance the flexibility of TPPs is by using deep models to model TPPs. Compared to frequentist/Bayesian nonparametric TPPs, neural TPPs offer more intuitive and straightforward modeling and parameter estimation. Shchur et al. (2021) provided a comprehensive review of neural TPPs, but it focuses on work prior to 2020. This paper focuses more on the latest advancements from 2020 to 2024. In the following, we categorize neural TPPs into three major types and introduce each in detail. A schematic illustration of these models is shown in Figure 3.

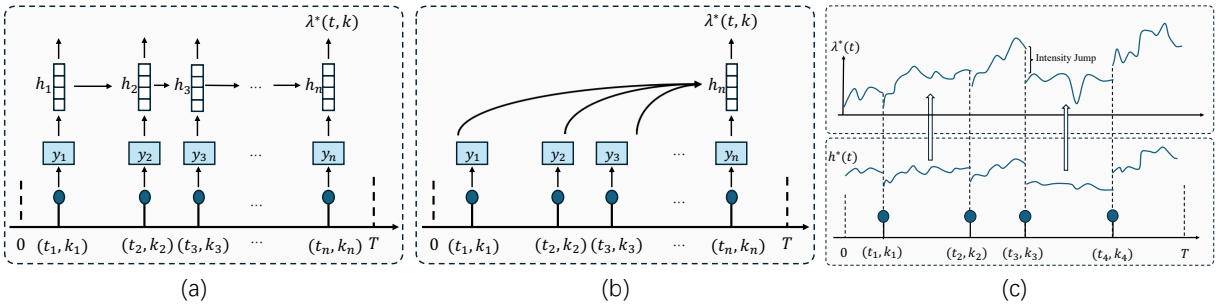

Figure 3: Illustration of neural TPPs. (a) Recurrent neural TPPs, where the hidden state is updated recurrently using the current event and then used to model the conditional intensity function; (b) Autoregressive neural TPPs, where the hidden state is computed by summarizing all previous events and then used to model the conditional intensity function; (c) Differential equation-based neural TPPs, where the hidden state evolves continuously when no events occur and undergoes a jump at event times, and is then used to model the conditional intensity function.

4.1 Recurrent Neural TPPs

The earliest work on neural TPPs can be traced back to Du et al. (2016), which was the first to use a recurrent neural network (RNN) to model TPPs. In that work, each event in $\{(t_n, k_n)\}_{n=1}^N$ is passed through an embedding layer to obtain a compact representation $\mathbf{y}_n$. At each event location, a history embedding $\mathbf{h}_n$ is designed to capture historical information. When a new event occurs, the history embedding is updated based on the feature of the current event:

$$\mathbf{h}_n = \text{Update}(\mathbf{h}_{n-1}, \mathbf{y}_n). \tag{12}$$

Finally, the history embedding $\mathbf{h}_n$ is used to parameterize the conditional distribution of the next event. In Du et al. (2016), $\mathbf{h}_n$ is used to represent the conditional intensity function after t_n:

$$\lambda^*(t,k) = \exp\left(\mathbf{v}_k^\top \mathbf{h}_n + w_k(t - t_n) + b_k\right), \tag{13}$$

where $\mathbf{v}_k$, w_k, and b_k are learnable parameters for k-th mark. The first term captures the influence of past events through the history embedding, the second term is an extrapolation component that models the intensity at time t, and the third term is a bias. The exponential function ensures that the conditional intensity function remains positive. As discussed in Section 2, although the conditional intensity function is a common parameterization, other formulations for characterizing the conditional distribution of the next event are also feasible.

Due to the gradient vanishing or exploding problems of traditional RNNs, Du et al. (2016) was unable to model long-range dependencies. To address this issue, Mei & Eisner (2017) proposed a long short-term memory (Hochreiter, 1997) (LSTM)-based TPP model, which mitigates the shortcomings of traditional RNNs and achieves improved performance. Xiao et al. (2017b) introduced a dual-LSTM framework, where one LSTM models the TPP and the other models a time-series covariate. The history embeddings from both networks are then fused to predict the next event. By leveraging the additional information from covariates, TPPs can achieve improved prediction performance. Yang et al. (2018) further extended the LSTM-based point process model to the spatio-temporal domain. Omi et al. (2019) used RNNs to encode history, but modeled the cumulative conditional intensity instead of the conditional intensity itself to avoid costly numerical integration during MLE. This enables efficient training via differentiation rather than integration, as discussed in Section 4.5. Soen et al. (2021) proposed UniPoint, an RNN-based model that serves as a universal approximator for point process intensity functions. They theoretically proved that RNNs can approximate any valid intensity by leveraging the Stone-Weierstrass theorem. Gupta et al. (2021) addressed missing events by using two RNNs to model the generative processes of observed and missing events, with the latter treated as latent variables. They proposed an unsupervised training method based on variational inference to jointly

learn both models. Chen et al. (2024) established excess risk bounds for RNN-based TPPs under various TPP settings. They showed that a four-layer RNN-TPP can achieve vanishing generalization error.

The main advantage of recurrent models lies in their computational efficiency. During the prediction phase, once the history embedding $\mathbf{h}_n$ is obtained, the model can predict the next event with constant time complexity $O(1)$, regardless of the sequence length. This makes recurrent models particularly suitable for online or streaming settings where fast real-time prediction is essential. Moreover, during training, the time complexity scales linearly with the number of events, i.e., $O(N)$, since the history embedding is updated in an iterative, step-by-step manner along the event sequence. However, recurrent models also suffer from several notable limitations. First, due to their inherently sequential architecture, they cannot be efficiently parallelized during training, which significantly slows down the training process, especially for long event sequences. Second, traditional RNN-based models struggle to capture long-range dependencies due to issues such as vanishing or exploding gradients, which can degrade the model's ability to learn complex temporal dynamics over extended horizons. Although architectures such as LSTMs and GRUs (Chung et al., 2014) partially mitigate these issues, they do not fundamentally resolve them. These limitations have motivated the development of alternative architectures, such as attention-based and Transformer-style models, which offer better support for parallelism and more effective modeling of long-range dependencies.

It is worth noting that in recent years, several powerful recurrent architectures have been proposed in the field of sequential models, such as the Receptance Weighted Key Value (RWKV) (Peng et al., 2023), the Structured State Space Sequence (S4) (Gu et al., 2022), and Mamba (Gu & Dao, 2023). These works aim to design efficient recurrent architectures to replace Transformers, as Transformers have a high time complexity of $O(N^2)$ in training and $O(N)$ in prediction. All these models are recurrent architectures, so they share the same advantages as RNNs, such as $O(1)$ time complexity for prediction and $O(N)$ time complexity for training. However, their key distinction from RNN lies in their ability to support parallelized training and capture long-range dependencies. Integrating these novel and efficient recurrent architectures with TPPs is an important future research direction. This integration could significantly enhance the scalability of neural TPPs for training and prediction on large-scale datasets. Currently, works in this area are limited. Gao et al. (2024) and Chang et al. (2024) explored the idea of combining Mamba or deep state space models with TPPs, offering a promising path forward for scalable TPP modeling.

4.2 Autoregressive Neural TPPs

As stated in Section 4.1, due to the limitations of RNNs, such as the inability to support parallel training and capture long-range dependencies, a large number of studies since 2020 have explored using Transformer architectures to model TPPs. The earliest works include Zuo et al. (2020) and Zhang et al. (2020a), which share similar ideas but differ slightly in certain details.

Each event in the sequence $\{(t_n, k_n)\}_{n=1}^N$ is encoded as a feature vector $\mathbf{y}_n \in \mathbb{R}^M$, combining both temporal and mark information. We adopt sinusoidal positional encodings for the temporal component, defined as:

$$z_j(t_n) = \begin{cases} \cos\left(t_n/10000^{\frac{j-1}{M}}\right), & \text{if } j \text{ is odd,} \\ \sin\left(t_n/10000^{\frac{j}{M}}\right), & \text{if } j \text{ is even,} \end{cases}$$

where $z_j(t_n)$ denotes the j-th entry of the time embedding vector $\mathbf{z}(t_n) \in \mathbb{R}^M$, and $j = 0, \ldots, M-1$. The complete time embedding matrix is denoted by:

$$\mathbf{Z} = [\mathbf{z}(t_1), \ldots, \mathbf{z}(t_N)]^\top \in \mathbb{R}^{N \times M}.$$

Let $\mathbf{U} \in \mathbb{R}^{M \times K}$ be a learnable mark embedding matrix, where K is the total number of event types. The mark k_n is first converted into a one-hot vector $\mathbf{k}_n \in \mathbb{R}^K$, and its embedding is given by:

$$\mathbf{e}(k_n) = \mathbf{U}\mathbf{k}_n \in \mathbb{R}^M.$$

Collecting all event mark embeddings yields:

$$\mathbf{E} = [\mathbf{e}(k_1), \ldots, \mathbf{e}(k_N)]^\top \in \mathbb{R}^{N \times M}.$$

The final embedding for each event is obtained by summing its temporal and mark embeddings:

$$\mathbf{Y} = \mathbf{Z} + \mathbf{E} \in \mathbb{R}^{N \times M},$$

where each row $\mathbf{y}_n$ in $\mathbf{Y}$ represents the complete embedding of the n-th event in the sequence. The matrix $\mathbf{Y}$ is then multiplied by corresponding weight matrices to compute the query, key, and value matrices: $\mathbf{Q} = \mathbf{Y}\mathbf{W}_Q \in \mathbb{R}^{N \times M_K}, \quad \mathbf{K} = \mathbf{Y}\mathbf{W}_K \in \mathbb{R}^{N \times M_K}, \quad \mathbf{V} = \mathbf{Y}\mathbf{W}_V \in \mathbb{R}^{N \times M_V}$. Finally, the attention score is computed as:

$$\mathbf{S} = \text{softmax}\left(\frac{\mathbf{Q}\mathbf{K}^\top}{\sqrt{M_K}}\right)\mathbf{V}. \tag{14}$$

To ensure causality and prevent future events from affecting past events, it applies a mask to the upper triangular entries of $\mathbf{Q}\mathbf{K}^\top$.

The attention score is then used to generate the history embedding $\mathbf{h}_n$, which serves as the history representation for parameterizing the conditional distribution of the next event. In Zuo et al. (2020); Zhang et al. (2020a), $\mathbf{h}_n$ is used to represent the conditional intensity function after t_n:

$$\lambda^*(t, k) = \text{softplus}\left(\mathbf{v}_k^\top \mathbf{h}_n + \frac{w_k(t - t_n)}{t_n} + b_k\right), \tag{15}$$

where $\mathbf{v}_k$, w_k, and b_k are learnable parameters for k-th mark. However, other parameterizations for characterizing the conditional distribution of the next event are also feasible. For example, Panos (2024) used the history embedding $\mathbf{h}_n$ obtained from the Transformer to model the conditional density function of the next event $f(t \mid \mathbf{h}_n)$. This approach makes sampling the next event more efficient.

The advantages and disadvantages of autoregressive models stand in contrast to those of recurrent models. A key strength of autoregressive models lies in their ability to support parallel training and effectively capture long-range dependencies via self-attention mechanisms. However, these benefits come at the cost of computational efficiency. During training, the time and memory complexity scales quadratically with the sequence length, i.e., $\mathcal{O}(N^2)$, due to the self-attention computation across all event pairs. During prediction, autoregressive models typically require all previous hidden states to compute the next event, resulting in a time complexity of $\mathcal{O}(N)$. While key-value (KV) caching techniques can reduce the prediction time to constant time complexity $\mathcal{O}(1)$ by storing intermediate representations, they incur significant memory overhead, which can become prohibitive for long sequences or memory-constrained environments. In contrast, recurrent models maintain a hidden state that is updated incrementally, leading to linear time complexity $\mathcal{O}(N)$ for training and constant time $\mathcal{O}(1)$ for prediction, but at the cost of sequential processing and limited ability to model long-range dependencies.

Between 2020 and 2024, numerous studies have proposed improvements to Transformer TPP models. For instance, Zhu et al. (2021) modified the computation of the attention score in Equation (14), replacing the commonly used dot-product attention with a flexible non-linear attention score based on Fourier kernels, enabling the capture of more complex event similarities. Zhou et al. (2022b) extended Transformer-based point process models to the spatio-temporal domain by first using a Transformer to encode the event history. The resulting history embedding is then mapped to a latent stochastic process, which is subsequently used to generate a set of temporal and spatial kernels. These kernels are linearly combined to construct the spatio-temporal conditional intensity function. Yang et al. (2022) further improved the attention mechanism proposed in Zuo et al. (2020); Zhang et al. (2020a) by introducing future-event-specific query vectors that incorporate continuous positional encodings of time t. As t increases, the attention weights over past events vary smoothly, enabling a more adaptive and temporally aware modeling of historical influence. Mei et al. (2021) proposed a Transformer-based model without setting the form of intensity for flexible modeling. Li & Sun (2023) proposed a sparse Transformer TPP model based on a sliding window mechanism to reduce the quadratic time and space complexity of Transformers. Meng et al. (2024) introduced improvements in the combination of time and mark embeddings, the computation of $\mathbf{Q}$ and $\mathbf{K}$ matrices, and the modeling of the conditional intensity function. These enhancements allowed the Transformer Hawkes process to perfectly align with statistical nonlinear Hawkes processes, thereby improving its interpretability. Wang et al. (2024) combined Transformer TPPs with federated learning, enabling collaborative learning from large amounts of distributed event sequence data.

4.3 Differential Equation-based Neural TPPs

Recurrent and autoregressive neural TPPs share a fundamental limitation: as discrete-time models, they can only compute the history embedding $\mathbf{h}_n$ and the conditional intensity function $\lambda^*(t_n)$ at discrete event times. However, they cannot directly characterize the conditional intensity function over the continuous intervals between events. To address this issue, many studies introduce extrapolation terms to approximate the intensity function over such intervals. For example, in Equation (13), the term $w_k(t - t_n)$, and in Equation (15), the term $w_k(t - t_n)/t_n$, are both designed to serve as extrapolation mechanisms. However, these extrapolation components adopt fixed parametric forms, which limit the expressiveness of the conditional intensity function over event intervals. Specifically, the extrapolation behavior in Equation (15) is approximately linear with respect to t, as the softplus function closely resembles a ReLU. This limitation is visually demonstrated in Meng et al. (2024), where the authors plot the conditional intensity function over intervals to highlight this issue.

Differential equation-based TPPs represent another line of research. These models, being continuous-time, can model the conditional intensity function over continuous time, thus avoiding the above issue. Specifically, these models utilize differential equations (often stochastic differential equations (SDEs)) to describe a history-dependent left-continuous hidden state $\mathbf{h}^*(t)$ over $[0, T]$ with an initial state $\mathbf{h}^*(0)$. For instance, in Jia & Benson (2019), the SDE is defined as:

$$d\mathbf{h}^*(t) = \mathrm{NN}_{\theta_1}(\mathbf{h}^*(t), t)dt + \mathrm{NN}_{\theta_2}(\mathbf{h}^*(t), t)dN(t), \tag{16}$$

where we take the unmarked case as an example. The functions NN_{θ_1} and NN_{θ_2} are two neural networks that govern the flow and jump of $\mathbf{h}^*(t)$, respectively. $N(t)$ is the counting process that records the number of events up to time t. When no event occurs, $\mathbf{h}^*(t)$ evolves smoothly according to NN_{θ_1}. When an event occurs, $\mathbf{h}^*(t)$ undergoes a jump at the event's timestamp, governed by NN_{θ_2}. Then, the history-dependent left-continuous hidden state $\mathbf{h}^*(t)$ is used to define the conditional intensity function:

$$\lambda^*(t) = \mathrm{NN}_{\theta_3}(\mathbf{h}^*(t)), \tag{17}$$

where NN_{θ_3} is another neural network ensuring a non-negative output. Finally, we use MLE to estimate the model parameters. Clearly, since $\mathbf{h}^*(t)$ can vary flexibly over the event intervals—being parameterized by NN_{θ_1}—the resulting conditional intensity function $\lambda^*(t)$ also exhibits flexible dynamics between events. This effectively overcomes the limited expressiveness of recurrent and autoregressive neural TPPs in modeling the conditional intensity function over event intervals.

The earliest work, to the best of our knowledge, that integrates differential equations with point processes is Chen et al. (2018), which considered a simple inhomogeneous Poisson process whose intensity function evolves according to an ordinary differential equation (ODE). However, this model does not account for the discontinuities in the conditional intensity function caused by historical events. To address this limitation, Jia & Benson (2019) proposed a method based on SDE, as shown in Equations (16) and (17), to model the conditional intensity of history-dependent TPPs. This approach allows the conditional intensity function to exhibit jumps at the occurrence of events. A contemporaneous work, Rubanova et al. (2019), combined ODEs with RNNs. In this framework, the hidden state $\mathbf{h}^*(t)$ evolves smoothly according to an ODE when no event occurs, and undergoes a jump at the event time, governed by the RNN update. This enables the model to capture jumps in the conditional intensity function at event times. In fact, Rubanova et al. (2019) shares strong similarities with Mei & Eisner (2017), but is more general. Specifically, Mei & Eisner (2017) assumes that $\mathbf{h}^*(t)$ evolves across event intervals following an exponential law, whereas Rubanova et al. (2019) allows $\mathbf{h}^*(t)$ to evolve flexibly over event intervals according to arbitrary ODEs. The ODE-based approach was later extended to spatio-temporal TPPs by Chen et al. (2021). Wang et al. (2018) adopted a SDE-based approach to directly model the conditional intensity function of TPPs. Additionally, they used an SDE to model users' opinions, enabling the incorporation of user feedback into the TPP framework. This results in a closed-loop model that jointly captures the dynamics of users' opinions and event generation. More recently, Zhang et al. (2024) proposed a novel SDE-based method for modeling TPPs. Instead of modeling a latent hidden state via an SDE and mapping it to the conditional intensity function, their approach directly models the conditional intensity function with an SDE, and further provides a theoretical analysis on the existence and uniqueness of its solution.

Although differential equation-based neural TPPs offer greater flexibility than recurrent and autoregressive models in characterizing the conditional intensity function over event intervals, they suffer from significant computational inefficiencies during both training and sampling. During training, when using MLE, the model requires numerical integration to compute the integral of the conditional intensity function. This necessitates evaluating the intensity at multiple time points, which can only be obtained by solving the ODE or SDE defined in Equation (16) using a numerical solver. As a result, training is considerably slower compared to recurrent or autoregressive neural TPPs. Similarly, during the sampling phase—such as when applying the thinning algorithm—intensity values at various time points must also be computed via ODE or SDE solvers, further increasing the computational cost and slowing down the sampling process. A comparison among the three mainstream neural TPP architectures is provided in Table 4.

Table 4: Comparison of neural TPP architectures.

Model Type	Parallel Training	Long-range	Training Complexity	Prediction Complexity	Continuous-time	Strengths	Limitations
Recurrent	✗	Limited	$O(N)$	$O(1)$	No	efficient prediction	slow training
Transformer	✔	Strong	$O(N^2)$	$O(N)/O(1)$[4]	No	parallel training	high complexity
ODE/SDE-based	✗	Strong	depend on solver	depend on solver	Yes	continuous-time, expressive	slow training/prediction

4.4 Diffusion-based TPPs

Recently, diffusion-based generative modeling has emerged as a promising new direction for TPPs. Diffusion models have achieved remarkable success in a wide range of generative tasks, including image generation, video synthesis, and tabular data modeling. In these models, data are generated by reversing a gradual noising process through iterative denoising steps (Song et al., 2020).

Inspired by these advances, diffusion models have recently been introduced into temporal point process modeling. Unlike conventional neural TPPs that typically model the conditional intensity function or inter-event distribution in an autoregressive manner, diffusion-based approaches aim to learn the distribution of an entire event sequence through iterative denoising. This perspective enables non-autoregressive sequence generation and can mitigate the error accumulation problem that often arises in long-horizon prediction.

One of the first works in this direction is Lüdke et al. (2023), which develops a diffusion framework specifically tailored to temporal point processes. Instead of applying standard Gaussian diffusion directly, the method constructs a noising and denoising process through point addition and thinning operations, which are classical mechanisms in point-process simulation. The reverse diffusion process therefore naturally preserves the semantics of event sequences while learning the underlying intensity structure. Subsequent work has further extended diffusion modeling for event sequences. For instance, Kerrigan et al. (2024) proposes a diffusion-based approach that predicts multiple future events within a prediction horizon through a single denoising trajectory, rather than repeatedly predicting the next event in an autoregressive fashion. This design allows the model to capture global dependencies among future events. Other studies have explored diffusion modeling for more complex point-process structures. For example, Yuan et al. (2023) introduce a diffusion-based framework for jointly modeling spatial and temporal point processes, enabling unified generation of spatiotemporal event patterns while addressing the conditional dependence between spatial and temporal dynamics. Meanwhile, Lüdke et al. (2024) formulate point processes as unordered sets and apply point-cloud denoising techniques to enable permutation-invariant event generation and direct likelihood evaluation.

Compared with traditional intensity-based or autoregressive neural TPPs, diffusion-based approaches offer a complementary modeling paradigm. By generating entire event sequences through iterative denoising, they provide a more global view of future event trajectories and are potentially better suited for long-horizon forecasting and sequence simulation. However, diffusion models typically involve higher training and sampling costs due to their multi-step generation process, and their theoretical foundations and benchmarking in the context of point processes remain relatively underdeveloped. Overall, diffusion-based TPPs represent an important emerging direction that bridges modern generative modeling techniques with event-sequence analysis.

[4] with KV cache.

4.5 Parametrization of Neural TPPs

Both recurrent and autoregressive neural TPPs extract a history embedding from past event information and use it to model the conditional distribution of the next event. As discussed in Section 2, there are multiple ways to represent the conditional distribution of the next event, such as the conditional density function $f(t \mid \mathcal{H}_{t_n})$ in Equation (1), the cumulative distribution function $F(t \mid \mathcal{H}_{t_n})$ in Equation (2), the conditional intensity function $\lambda^*(t)$ in Equation (2), and the cumulative intensity function $\Lambda^*(t) = \int_0^t \lambda^*(\tau)d\tau$. All of these parameterizations are equivalent and can be used to characterize the conditional distribution of the next event, since, as proved in Equation (3), the conditional intensity function and the conditional density function are in a one-to-one correspondence. A schematic illustration of the four common parameterizations of TPPs is shown in Figure 4.

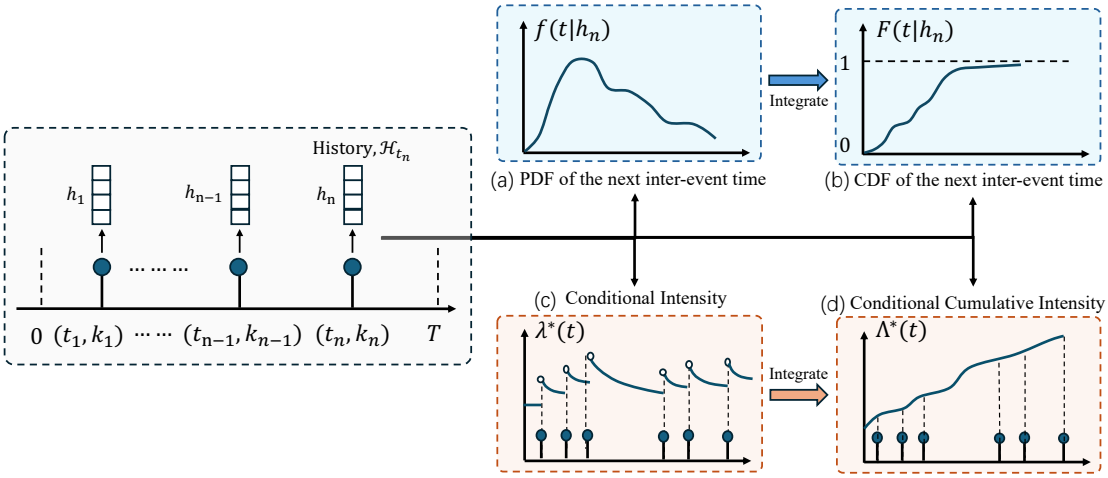

Figure 4: Four common parameterizations of TPPs: (a) the probability density function (PDF) of the next event, (b) the cumulative distribution function (CDF) of the next event, (c) the conditional intensity function, and (d) the conditional cumulative intensity function.

Most of the works discussed above adopt the conditional intensity function as the primary parameterization. The main advantage of this approach lies in its conceptual and implementation simplicity—the model only needs to ensure that the output is non-negative, which can be easily achieved using functions like ReLU, softplus, or exponential mappings. However, a notable drawback arises during MLE: the log-likelihood requires evaluating the integral of the conditional intensity function over the observation window. In most practical cases, this integral does not admit a closed-form solution and must be approximated using numerical integration techniques, such as Monte Carlo sampling or quadrature. This introduces additional computational overhead and can compromise both the estimation accuracy and the overall training efficiency, especially for complex or high-dimensional models.

In recent years, several studies have explored alternative parameterizations for TPPs. Shchur et al. (2020a) and Panos (2024) modeled the conditional density function $f(t \mid \mathcal{H}_{t_n})$ directly as a mixture of log-normal distributions, using history embeddings as inputs. The key difference between the two lies in how the history embeddings are extracted: Shchur et al. (2020a) used a RNN, while Panos (2024) employed a Transformer-based encoder. One major advantage of modeling the conditional density directly is that it eliminates the need to compute the integral of the conditional intensity function during MLE, earning it the name "intensity-free" modeling. Another benefit is efficient sampling: given the history, the next event can be sampled directly from a mixture of log-normal distributions, which admits a simple and tractable form. Taieb (2022) modeled the inverse of the cumulative distribution function $F^{-1}(t \mid \mathcal{H}_{t_n})$ using the history embedding with monotonic rational-quadratic splines. This method also avoids numerical integration and supports efficient sampling. Omi et al. (2019); Shchur et al. (2020b); Liu (2024) proposed modeling the cumulative intensity function $\Lambda^*(t)$ conditioned on history embeddings using monotonic neural networks or splines. This enables a reformulation of the log-likelihood in Equation (7), which, for the unmarked case,

becomes:

$$\log f(\mathcal{T};\theta) = \sum_{n=1}^{N} \log \frac{d}{dt}\Lambda_\theta^*(t_n^-) - \Lambda_\theta^*(T), \tag{18}$$

where t_n^- denotes the left-hand limit of the derivative at t_n. By modeling the cumulative intensity function directly, this parameterization transforms the integral in the log-likelihood into a derivative, allowing for efficient computation using automatic differentiation. Consequently, it eliminates the need for numerical integration during MLE training, improving both accuracy and efficiency. The comparison of four parameterizations of TPPs is provided in Table 5.

Table 5: Comparison of neural TPP parameterizations.

Parameterization	Numerical Integration	Sampling Efficiency	Training Efficiency	Requirement	Representative Works
Conditional Intensity Function $\lambda^*(t \mid \mathcal{H})$	Yes	Low	Low	Nonnegative	(Du et al., 2016; Mei & Eisner, 2017; Zuo et al., 2020)
Conditional Density Function $f(t \mid \mathcal{H})$	No	High	High	Nonnegative, normalized	(Shchur et al., 2020a; Panos, 2024)
Cumulative Distribution Function $F(t \mid \mathcal{H})$	No	High	High	Monotonically increasing between $(0, 1)$	(Taieb, 2022)
Cumulative Intensity Function $\Lambda^*(t \mid \mathcal{H})$	No	Low	High	Nonnegative, monotonically increasing	(Omi et al., 2019; Shchur et al., 2020b; Liu, 2024)

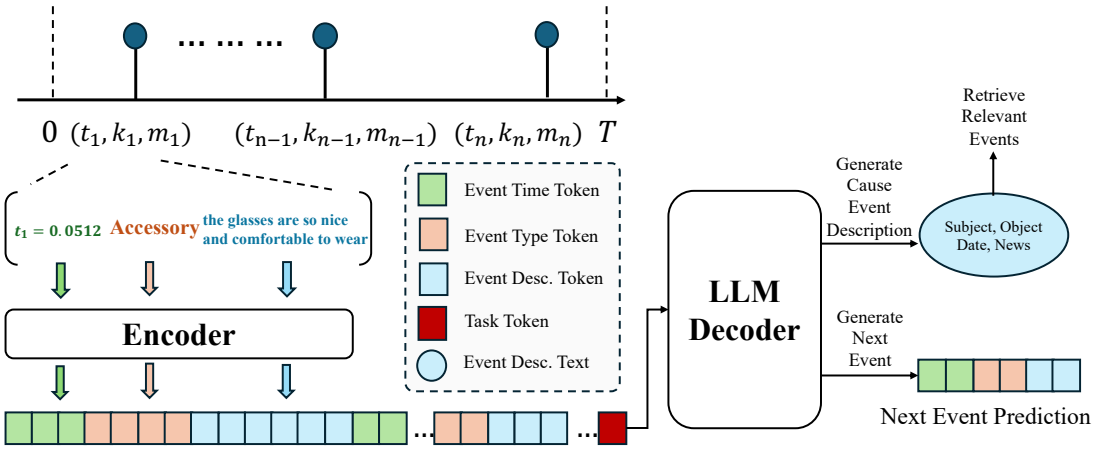

Figure 5: An overview of LLM-based TPPs. The event time t, type k, and associated multimodal data m are first encoded into tokens via an encoder. These tokens are then fed into a LLM, which can be used to generate the next event or produce a textual description of the causal events. The encoder design varies across different works. For example, Liu & Quan (2024) uses temporal positional encodings as in standard Transformers; Shi et al. (2024) treats event times as text and leverages the LLM's built-in tokenizer; and Kong et al. (2025) converts event times into floating numbers and encodes them as byte-level tokens.

5 LLM-based TPPs

LLM-based temporal point processes can be broadly categorized into two paradigms depending on the role played by large language models. One line of work uses LLM-inspired mechanisms to enhance conventional neural TPPs without replacing their temporal modeling backbone, while another line directly integrates LLMs as the core sequence model for representing and predicting event streams. Although both aim to leverage the representational power of LLMs, they differ fundamentally in architecture, training strategy, and how semantic and temporal information are encoded. More broadly, this line of research lies at the intersection of temporal point processes, sequential foundation models, multimodal event understanding, and retrieval-augmented reasoning. Unlike classical TPPs, which mainly focus on continuous-time likelihood modeling and prediction, LLM-based approaches naturally connect event streams with free text, external knowledge, and heterogeneous modalities. This makes them relevant not only to TPP modeling, but also to adjacent areas such as temporal representation learning, event-sequence retrieval, multimodal reasoning, and question answering over time-stamped observations. From this perspective, LLM-based TPPs should be understood not merely as a new model family, but as an expansion of the TPP research agenda toward semantically rich event understanding. A schematic illustration of LLM-based TPPs is shown in Figure 5.

5.1 LLM-inspired TPPs

LLM-inspired TPPs retain a neural temporal point process as the primary model of event dynamics but borrow ideas from LLMs—such as prompt learning or reasoning—to improve adaptation and interpretability. PromptTPP (Xue et al., 2023b) is a representative example that introduces prompt learning into neural TPPs to enable continual learning under distribution shifts. Instead of retraining the base TPP when new data arrive, PromptTPP prepends a small set of learnable temporal prompts to the event sequence. These prompts are retrieved from a continuously updated memory pool and optimized jointly with the TPP, allowing the model to adapt to new patterns without storing past data or introducing task-specific modules. This design is computationally efficient and particularly attractive in streaming or privacy-constrained scenarios. However, because the prompts only modulate an existing neural TPP, the approach does not introduce higher-level semantic or causal reasoning.

A different LLM-inspired strategy is adopted in LAMP (Shi et al., 2024), which augments neural TPPs with abductive reasoning capabilities provided by an LLM. LAMP uses a multi-stage pipeline in which a base event sequence model first proposes candidate future events. A fine-tuned LLM then generates plausible causes for each proposed event, after which a retrieval module searches the historical sequence for events matching these causes. A scoring function evaluates whether the retrieved events can reasonably explain the proposed future events. In contrast to PromptTPP, which focuses on efficient adaptation via prompts, LAMP aims to improve prediction quality and interpretability through causal-style reasoning. However, this comes at the cost of increased complexity and reduced robustness, since prediction depends on multiple interacting components, while the temporal dynamics themselves are still modeled by a conventional TPP.

5.2 Direct LLM-TPP Integration

In contrast, direct LLM–TPP integration methods use LLMs as the primary model for representing and predicting event sequences, embedding both semantic and temporal information into the LLM input space. TPP-LLM (Liu & Quan, 2024) follows this paradigm by representing events through their textual descriptions rather than categorical marks, enabling pretrained LLMs to capture rich semantic relations between events. Temporal information is injected via positional-style temporal embeddings, and parameter-efficient fine-tuning methods such as LoRA are employed to adapt the LLM to temporal prediction tasks. The LLM produces contextualized representations of the event history, which are then passed to an intensity head for predicting the next event time and type. Compared to LLM-inspired approaches, TPP-LLM tightly couples semantic understanding with temporal modeling, but it still relies on external temporal embeddings to represent continuous time.

Language-TPP (Kong et al., 2025) proposes a more unified representation by encoding continuous time intervals directly as byte-level tokens that are processed by the LLM in the same way as natural language. This eliminates the need for positional or temporal embeddings and allows the LLM to model time and semantics in a single token sequence. As a result, Language-TPP supports standard TPP tasks such as event time and type prediction, while also enabling new capabilities such as generating natural-language event descriptions. Compared with TPP-LLM, this token-based temporal encoding provides a tighter integration of time and language but leads to longer sequences and higher computational cost.

Overall, LLM-inspired TPPs such as PromptTPP and LAMP enhance existing neural TPPs with adaptation and reasoning mechanisms, whereas direct LLM–TPP approaches such as TPP-LLM and Language-TPP redefine TPP modeling by using LLMs as the core event sequence model. This distinction highlights different trade-offs in efficiency, expressiveness, interpretability, and scalability across the emerging landscape of LLM-based temporal point processes.

5.3 Other Extensions

Liu & Quan (2025) introduced TPP-Embedding for temporal event sequence retrieval from textual descriptions, targeting applications in e-commerce behavior analysis, social media monitoring, and criminal incident tracking. The authors developed TESRBench, a comprehensive benchmark with diverse real-world datasets and synthesized textual descriptions. Their model leverages the TPP-LLM (Liu & Quan, 2024) framework

to integrate LLMs with TPPs, encoding both event texts and temporal information through pooling representations and contrastive loss to align sequence-level embeddings with textual descriptions. This approach outperforms baseline models across TESRBench datasets and establishes foundations for retrieval-augmented generation in TPP domains.

Extending beyond textual data, Jiang et al. (2025) introduced DanmakuTPPBench, a comprehensive multi-modal TPP benchmark addressing the gap in datasets with temporal, textual, and visual information. The benchmark comprises two components: DanmakuTPP-Events, derived from Bilibili's user-generated bullet comments (Danmaku) that form multi-modal events with timestamps, textual content, and video frames; and DanmakuTPP-QA, a question-answering dataset constructed via a multi-agent pipeline using state-of-the-art LLMs and multi-modal LLMs (MLLMs). Targeting temporal-textual-visual reasoning tasks requiring multi-modal event dynamics understanding, extensive evaluations of classical TPP models and recent MLLMs reveal significant performance gaps in modeling multi-modal event sequences. This work establishes baselines and opens research directions for integrating TPP modeling into the multi-modal language modeling landscape. An illustration of DanmakuTPPBench is shown in Figure 6.

On the boundary of the TPP framework. Recent LLM-based extensions significantly broaden the scope of TPP research by introducing tasks such as event-sequence retrieval, question answering, and multimodal reasoning. While these tasks are highly valuable, they do not always fit neatly into the traditional definition of a temporal point process as a stochastic process over event times and marks. In classical TPPs, the central object is a probability law over event occurrences in continuous time, and core tasks focus on likelihood modeling, prediction, simulation, or causal structure discovery. By contrast, retrieval or QA tasks often require high-level semantic understanding, external knowledge integration, and cross-modal reasoning that go beyond the standard probabilistic objectives of TPPs.

This observation does not diminish the importance of these new directions; rather, it highlights a conceptual shift. LLM-based TPP research is gradually moving from *modeling event occurrence processes* to *understanding temporally indexed event data*. As a result, future work may need to distinguish more clearly between tasks that are fundamentally point-process problems and tasks that use TPPs as one component within a broader temporal reasoning system. Clarifying this boundary would help the community better define evaluation protocols, modeling assumptions, and the scope of claimed contributions.

Overall, this research direction is rapidly evolving, with approaches ranging from LLM-inspired techniques to direct integration and multi-modal extensions. The field shows promise for further exploration in areas such as retrieval-augmented TPPs, multi-modal event understanding, and more sophisticated temporal reasoning capabilities.

6 Datasets, Benchmarks, and Evaluation Protocols

Besides model design, progress in TPPs also depends critically on datasets, benchmark protocols, and evaluation metrics. In practice, however, the empirical study of TPPs has long suffered from fragmented datasets, inconsistent preprocessing, different train/validation/test splits, and heterogeneous metric definitions. As a result, performance comparisons across papers are often not directly comparable. Recent efforts such as EasyTPP (Xue et al., 2023a) have started to address this issue by providing a unified benchmarking framework, standardized implementations, and common evaluation pipelines. In this section, we summarize representative datasets, typical evaluation tasks, and commonly used metrics.

Representative datasets. Existing TPP datasets cover a wide range of domains, including social interactions, e-commerce, finance, healthcare, and multimodal user behavior. Classical datasets frequently used in neural TPP studies include Retweet (Zhou et al., 2013a), StackOverflow (Du et al., 2016), and Taxi (Whong, 2014), which are typically medium-scale and mostly focus on timestamp–type prediction. More recently, larger-scale datasets have become available. For example, Amazon review data (Ni et al., 2019) provide large collections of user-item interactions with timestamps and rich semantic information, making them suitable for studying large-scale event modeling and language-enhanced TPPs. Taobao-based datasets (Alibaba, 2018) further provide realistic industrial event streams with dense user behaviors and are partic-

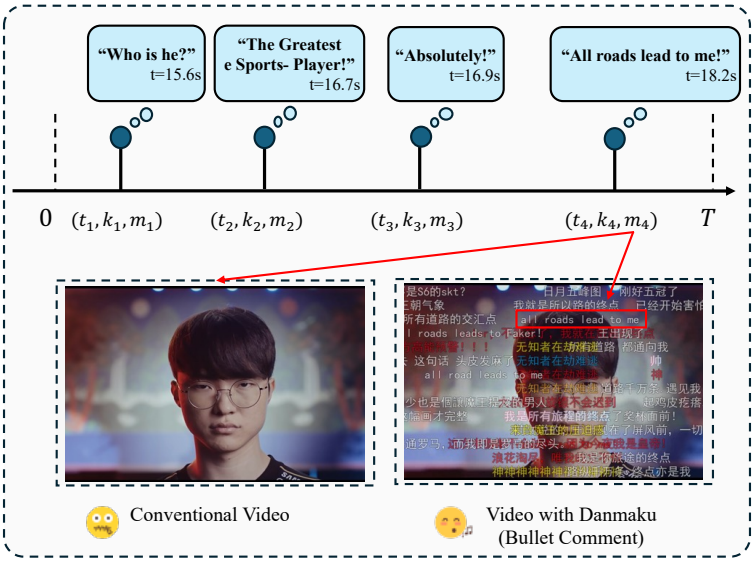

Figure 6: An illustration of DanmakuTPPBench (Jiang et al., 2025), a multi-modal benchmark for TPPs. The dataset consists of time-stamped events with associated event types k and multimodal information m, where m includes both the textual content of user comments (danmaku) and the corresponding video frames. This benchmark establishes standard baselines and opens new research directions for integrating TPP modeling into the broader landscape of multimodal language modeling.

ularly useful for evaluating long-horizon prediction and large-scale sequential decision settings. In addition, benchmark-oriented resources such as DanmakuTPPBench (Jiang et al., 2025) extend the scope of TPP evaluation to text retrieval, question answering, and multimodal temporal reasoning, broadening the traditional view of event sequence modeling.

Benchmark standardization. A major recent development is the emergence of unified benchmark toolkits. EasyTPP is one of the first systematic efforts toward open benchmarking for TPPs, providing standardized data preprocessing, model implementations, training pipelines, and evaluation scripts (Xue et al., 2023a). Such a benchmark is valuable not only for fair comparison, but also for improving reproducibility and lowering the entry barrier for researchers new to the field. From the perspective of a survey paper, benchmark standardization is as important as new model classes, because it determines whether empirical conclusions can be trusted and accumulated across studies.

Evaluation tasks. Existing TPP evaluation protocols can be roughly divided into four categories. The first is *next-event prediction*, where the goal is to predict the time and/or mark of the next event. This is the most common setting in the literature. The second is *long-horizon prediction*, where the model predicts multiple future events over an extended time window rather than only the next one. This task is substantially more difficult because errors accumulate autoregressively and the model must capture longer-term uncertainty. HyPro (Xue et al., 2022) explicitly highlights this setting and proposes a benchmark protocol for evaluating long-horizon event-sequence prediction. The third category is *semantic or multimodal tasks*, such as sequence retrieval, question answering, and multimodal reasoning, which arise in recent LLM-based TPP benchmarks such as DanmakuTPPBench (Jiang et al., 2025).

Evaluation metrics. The choice of metrics depends on the task. For next-event prediction, common metrics include time prediction error (e.g., MAE or RMSE of the next-event time), and classification metrics such as accuracy or macro-F1 for mark prediction. For long-horizon prediction, one often needs sequence-level metrics that assess the quality of multi-step forecasts over a future window, rather than only one-step accuracy. In this case, both temporal and mark-distribution alignment become important. One may consider distributional metrics such as negative log-likelihood, Wasserstein-style discrepancies or statistics derived

from inter-event times and event-type frequencies. For retrieval and QA benchmarks in LLM-based TPPs, information-retrieval metrics and task-specific accuracy measures are also needed. Therefore, a complete evaluation of TPP models should align the metric with the intended downstream use, rather than relying only on likelihood.

7 Model Training

In this section, we focus on frequentist methods for estimating model parameters in neural TPPs and LLM-based TPPs. Let $\mathcal{T} = \{(t_n, k_n)\}_{n=1}^N$ denote an observed event sequence and let $f_\theta(\mathcal{T})$ be the model distribution. Parameter learning is commonly formulated as minimizing a discrepancy between the empirical data distribution and the model distribution:

$$\hat{\theta} = \arg\min_\theta \mathrm{D}(f(\mathcal{T}) \,\|\, f_\theta(\mathcal{T})). \tag{19}$$

Depending on the choice of D, one obtains different estimators with distinct statistical and computational trade-offs. To make these differences explicit, we next summarize the objective functions and practical properties of four representative training principles.

7.1 KL Divergence

KL divergence is a commonly used training criterion. Minimizing the KL divergence $\mathrm{KL}(f(\mathcal{T}) \,\|\, f_\theta(\mathcal{T}))$ is equivalent to maximizing the log-likelihood:

$$\hat{\theta} = \arg\max_\theta \log f_\theta(\mathcal{T}).$$

For a marked TPP with conditional intensity $\lambda_\theta^*(t, k)$, the log-likelihood of an observed sequence over $[0, T]$ can be written as

$$\log f_\theta(\mathcal{T}) = \sum_{n=1}^{N} \log \lambda_\theta^*(t_n, k_n) - \int_0^T \sum_{k=1}^{K} \lambda_\theta^*(\tau, k)\, d\tau. \tag{20}$$

Therefore, minimizing the KL divergence is equivalent to maximizing equation 20. The first term rewards high intensity at observed events, while the second term normalizes the process by penalizing excessive intensity mass over the observation window.

This approach necessitates explicit evaluation of the conditional intensity function and its integral over time, which is typically intractable and must be approximated via numerical integration. Despite its computational burden, MLE remains asymptotically efficient and statistically optimal. Due to its solid theoretical foundation and general applicability, MLE has been adopted in the vast majority of TPP studies (Ozaki, 1979; Paninski, 2004).

7.2 Wasserstein Distance

In Wasserstein-based training, the model is learned by minimizing a distributional distance between model and data distributions:

$$\hat{\theta} = \arg\min_\theta W\big(f(\mathcal{T}), f_\theta(\mathcal{T})\big), \tag{21}$$

where $W(\cdot, \cdot)$ denotes the Wasserstein distance. Since the Wasserstein distance is generally difficult to compute directly, it is typically approximated using an adversarial framework with a critic network based on the Kantorovich–Rubinstein dual formulation. For example, Xiao et al. (2017a) proposed leveraging the Wasserstein distance within a Wasserstein GAN (WGAN) framework for TPPs. In this setup, the TPP model acts as a generator, and a critic network learns to distinguish between real and generated event sequences by minimizing the Wasserstein distance.

Compared with KL divergence, the Wasserstein distance avoids the numerical integration required in MLE, yields smoother gradients for optimization, and is generally more robust to mode collapse. However, the

adversarial training procedure used in WGANs may introduce additional instability and increase training complexity. To further improve upon this approach, Xiao et al. (2018) introduced a likelihood-free training method based directly on the Wasserstein distance between point processes. Unlike the earlier WGAN-based framework, which primarily learns the aggregate intensity over a dataset, their method enables individual-level, in-sample forward prediction of event sequences conditioned on historical context. Moreover, Wasserstein distance captures the underlying geometric structure of event sequences more effectively than KL divergence, leading to more robust alignment between the generated and real sequences.

7.3 Noise Contrastive Estimation

Noise contrastive estimation (NCE) reframes parameter learning as a binary classification problem that distinguishes observed data from artificially generated noise samples, thereby bypassing direct likelihood computation and making it suitable for models with intractable likelihoods such as TPPs. A generic objective takes the form

$$\hat{\theta} = \arg\max_{\theta} \mathbb{E}_{\mathcal{T}\sim f}\big[\log \sigma(s_\theta(\mathcal{T}))\big] + \mathbb{E}_{\widetilde{\mathcal{T}}\sim q}\big[\log(1-\sigma(s_\theta(\widetilde{\mathcal{T}})))\big], \tag{22}$$

where f denotes the observed TPP, q a noise TPP, $s_\theta(\cdot)$ a score function, and $\sigma(\cdot)$ the sigmoid function.

Both Guo et al. (2018) and Mei et al. (2020) applied NCE techniques to estimate the parameters of TPPs. The key difference between them lies in the specific NCE variant they adopt. Guo et al. (2018) employed the original binary classification formulation proposed by Gutmann & Hyvärinen (2012), which treats the learning task as discriminating between real and noise sequences. In contrast, Mei et al. (2020) adopted a ranking-based NCE variant introduced by Jozefowicz et al. (2016), which is better suited for modeling conditional distributions—a natural fit for TPPs where future events are conditioned on historical ones.

7.4 Fisher Divergence

Fisher divergence, also known as score matching (Hyvärinen, 2005), measures discrepancies between distributions through their score functions (i.e., gradients of log-densities). Since it does not require evaluating normalizing constants, it is particularly attractive for models with intractable likelihoods. In general form, the objective can be written as

$$\hat{\theta} = \arg\min_{\theta} \mathbb{E}_f\left[\|\nabla \log f(\mathcal{T}) - \nabla \log f_\theta(\mathcal{T})\|^2\right], \tag{23}$$

although defining the score operator for TPPs is more subtle than in the standard Euclidean-density setting.

Several prior works have introduced score matching into the context of point processes. For example, Sahani et al. (2016) derived score matching estimators for classical Poisson processes. Zhang et al. (2023) extended this framework to deep covariate-based spatio-TPPs, while Li et al. (2023) further generalized it to multivariate Hawkes processes. These efforts have significantly advanced the applicability of score matching in point process modeling. However, recent work by Cao et al. (2024) highlights a critical limitation: the estimators proposed in these earlier studies are incomplete and only valid for specific classes of point processes. In more general cases—including some simple parametric models—these methods fail to produce accurate parameter estimates. To address this issue, Cao et al. (2024) proposed a weighted (autoregressive) score matching estimator that generalizes to a broader class of point process models, offering improved theoretical soundness and practical applicability.

7.5 Comparison between Different Estimators

Among these methods, only MLE requires computing the integral of the conditional intensity function. The others avoid this step, potentially improving training efficiency. All methods are consistent under mild conditions, but MLE remains asymptotically optimal in terms of variance. Alternative methods may exhibit slower convergence and higher asymptotic variance, but offer better scalability and computational simplicity. A systematic comparison of representative training objectives for TPPs is summarized in Table 6, highlighting their computational requirements, statistical properties, and practical trade-offs.

Table 6: Comparison of TPP training objectives.

Estimator	Divergence Type	Likelihood Required	Numerical Integration	Statistical Efficiency	Representative Works
MLE	KL divergence	Yes	Yes	Asymptotically optimal	(Ozaki, 1979; Paninski, 2004)
Wasserstein	Wasserstein distance	No	No	Consistent, higher variance	(Xiao et al., 2017a; 2018)
NCE	Classification-based	No	No	Consistent, higher variance	(Guo et al., 2018; Mei et al., 2020)
Score Matching	Fisher divergence	No	No	Consistent, higher variance	(Sahani et al., 2016; Li et al., 2023; Cao et al., 2024)

Table 7: Comparison of TPP applications in event prediction and causal discovery.

Application Type	Domain	Primary Objective	Event Representation	Typical Models	Representative Works
Event Prediction	Social Networks	Predict future event time/type	User actions (posts, retweets)	Hawkes, Neural TPPs	(Kobayashi & Lambiotte, 2016; Zhang et al., 2021)
Event Prediction	Epidemiology	Forecast disease spread	Infection times, locations	Hawkes, Spatio-temporal TPPs	(Rizoiu et al., 2018; Chiang et al., 2022)
Event Prediction	Earthquakes	Aftershock forecasting	Time–location of quakes	Spatio-temporal Hawkes	(Ogata, 1998; Kwon et al., 2023)
Event Prediction	Finance	Market event prediction	Trades, orders	Hawkes, Neural Hawkes	(Bacry & Muzy, 2014; Shi & Cartlidge, 2022)
Event Prediction	Recommendation	User behavior prediction	Purchases, clicks	Neural TPPs	(Mei & Eisner, 2017; Wang et al., 2021)
Causal Discovery	Neuroscience	Infer functional connectivity	Neural spike trains	Multivariate Hawkes	(Linderman & Adams, 2015; Zhou et al., 2022a)
Causal Discovery	Finance	Discover buy–sell influence	Order book events	Hawkes with sparsity	(Bacry & Muzy, 2014; Xu et al., 2016)
Causal Discovery	AI Operations	Root cause analysis	System failure events	Topological Hawkes	(Cai et al., 2022)
Causal Discovery	Healthcare	Analyze treatment interactions	Symptoms, drug events	Hawkes-based causal models	(Bao et al., 2017)
Causal Discovery	Cybersecurity	Attack pattern analysis	Security alerts	Neural Hawkes	(Fortino et al., 2022)

8 Applications

TPPs have a wide range of applications, including in seismology, finance, neuroscience, social networks, and epidemiology. Broadly speaking, these applications can be categorized into two main types: event prediction and causal discovery. To provide a structured overview, Table 7 summarizes representative TPP applications across different domains, highlighting their objectives, modeling choices, and typical use cases.

8.1 Application in Event Prediction

Event prediction leverages historical data to forecast the timing, frequency, and types of future events, with applications spanning social networks, epidemiology, earthquake forecasting, finance, and recommendation systems. In social networks, Hawkes processes and neural TPPs are widely employed to model temporal interactions and information diffusion. For instance, Zhang et al. (2021) introduced a neural TPP for detecting coordinated behavior, while Kobayashi & Lambiotte (2016); Hegde et al. (2022) applied Hawkes processes to retweet dynamics prediction, and Cencetti et al. (2021) further analyze the higher-order social interactions. Recent work enhances interpretability (Meng et al., 2024) and predicting information popularity (Li et al., 2025). Additionally, these methods aid in anomaly detection, such as identifying fake accounts (Qu et al., 2022), and Ahammad (2024) integrated sentiment analysis to detect fake news trends during the COVID-19 pandemic.

In epidemiology, Hawkes processes effectively model disease propagation, as demonstrated by Rizoiu et al. (2018). Studies such as Chiang et al. (2022) incorporated mobility data to predict COVID-19 transmission patterns, while Schwabe et al. (2021) leveraged similar data for early outbreak forecasting. Further contributions include estimating transmission times (Schoenberg, 2023) and assessing pandemic impacts using self-exciting processes (Giudici et al., 2023). In earthquake forecasting, spatio-temporal Hawkes processes are instrumental in capturing aftershock sequences. For instance, Ogata (1998) introduced spatio-temporal TPP models for earthquake occurrences, with recent advancements enhancing flexibility (Kwon et al., 2023) and refining decay rate modeling (Davis et al., 2024). In financial markets, Hawkes processes analyze market microstructure and limit order book dynamics (Chen et al., 2022a). Neural Hawkes processes (Shi & Cartlidge, 2022) and (Nyström & Zhang, 2022) further improve predictive accuracy in high frequency financial data. Recommendation systems leverage Hawkes processes to model sequential user behavior. Wang et al. (2021) combined them with attention mechanisms for sequential recommendations. These techniques can also be used to predict a user's future shopping times and item types based on their past purchase history, enabling targeted promotional strategies (Mei & Eisner, 2017; Meng et al., 2024).

Beyond event prediction, TPPs can also be used to analyze heterogeneous event streams and uncover latent temporal patterns. One representative task is event sequence clustering, which aims to group event sequences according to their underlying temporal dynamics. This is useful in applications such as user behavior analysis, patient stratification, and social activity mining, where different groups may exhibit distinct triggering

patterns or interaction structures. A representative approach is the Dirichlet mixture model of Hawkes processes proposed by Xu & Zha (2017), which assumes that each cluster corresponds to a different Hawkes process and learns cluster-specific excitation patterns from asynchronous event sequences. Compared with standard feature-based clustering methods, TPP-based clustering explicitly models temporal dependencies and excitation structures, leading to more interpretable clusters from a dynamical perspective.

8.2 Application in Causal Discovery

In Hawkes process, causal discovery aim to recover the causal structure among difference event types from observed event sequence data. Applications in this category are prevalent in areas like neuroscience, finance, AI for operations, social network, healthcare, and cybersecurity. Here, the focus is not on predicting future events but on uncovering dependencies between event types, often referred to as Granger causality (Granger, 1969). These causal relationships enable better decision-making and provide mechanistic understanding of complex event dynamics. For example, in neuroscience, each neuron can be considered as an event type, and its spike train forms a univariate point process. The spike trains of multiple neurons naturally constitute a multivariate point process. The goal is to determine whether there exists functional connectivity between neurons (Linderman & Adams, 2015; Zhou et al., 2022a). Similarly, in high-frequency financial trading, a large number of asks (sell orders) and bids (buy orders) occur within short periods. Here, all sell orders are treated as one event type, and all buy orders as another. The primary interest lies in understanding the mutual influence between buy and sell orders in the order book (Bacry & Muzy, 2014). In AI operations, TPPs identify system failure root causes by distinguishing primary triggers from secondary effects through their causal structure, guiding prioritized fixes (Cai et al., 2022). Social network analysis similarly leverages Hawkes processes to quantify mutual influence patterns, where user actions (posts, likes, shares) as distinct event types reveal how influential users trigger reaction cascades (Zhou et al., 2013c). Healthcare applications employ TPPs to analyze drug reactions and symptom interactions for improved treatment strategies (Bao et al., 2017). Cybersecurity implementations further demonstrate their value in attack pattern analysis for enhanced defense mechanisms (Bessy-Roland et al., 2021; Fortino et al., 2022).

The modeling framework for these applications in causal discovery builds upon multivariate Hawkes processes (MHP) where a K-varaite MHP can be formulated as a collection of K univariate TPPs with the conditional intensity function taking the form of Equation (6). Crucially, we say process k' does not Granger-cause process k if and only if $\phi_{k,k'}(\cdot) = 0$. Therefore, estimating these triggering functions thus directly translates to learning the causal structure. While MLE offers a straightforward solution for learning causal structure, it often produces spurious connections due to finite samples and the lack of sparsity constraints. To tackle this issue, various types of sparsity approaches have been developed, typically categorized into constraint-based and score-based methodologies. Constraint-based approaches address the problem through statistical testing to prune spurious edges. For instance, Runge et al. (2019) proposed a general constraint-based framework for learning causal structure in time series data using conditional independence tests, but it is only applicable to discrete-time processes. Later, Mogensen (2020) proposed a screening algorithm for the Hawkes process, extending it to the continuous-time domain.

In contrast, score-based approaches learn causal structure by optimizing a well-defined criterion with various sparsity regularizations to enforce structural sparsity. For instance, Xu et al. (2016) proposed a nonparametric Hawkes process model with group sparsity regularization, while Zhou et al. (2013c) employed both nuclear norm and ℓ_1 norm as sparsity regularizations. Idé et al. (2021) used ℓ_0-regularization via an ϵ-sparsity approach (Phan & Idé, 2019). Alternatively, based on data compression techniques, Jalaldoust et al. (2022) proposed a minimum description length (MDL) criterion for Hawkes processes. Using a similar approach, Hlaváčková-Schindler et al. (2024) introduced a minimum message length criterion, which extends the MDL framework. This extension incorporates prior distributions—such as expert knowledge—over model parameters, enhancing flexibility in structure-related penalization.

By further assuming process stability, Achab et al. (2018) demonstrated that the inference procedure can be accelerated using a cumulant matching strategy based on the analytical form of cumulants (Jovanović et al., 2015), thereby eliminating the need to estimate the triggering function. Recently, several deep point process-based methods have been proposed to move beyond static parametric triggering functions. For example, Zhang & Yan (2021) introduced a variational neural relation inference framework that combines

multivariate TPPs with message-passing graphs for probabilistic relation discovery. Yang & Zha (2024) further proposed a variational autoencoder with dynamic latent graphs to capture time-varying dependencies among event types. Wang et al. (2025) modeled multivariate TPPs through neural jump stochastic differential equations with latent graphs, offering a flexible continuous-time framework for relation-aware intensity dynamics. Zhang et al. (2020c) introduced an attribution method to uncover Granger causality, and Wu et al. (2024) explored instance-wise causal structures using Transformer self-attention, aligning the mechanism with Granger causality principles. These works illustrate that neural causal discovery in TPPs is increasingly shifting toward latent-graph and graph-neural formulations.

Another line of research addresses more realistic scenarios where processes may exhibit nonstationarity, topological dependence, and insufficient temporal resolution. First, nonstationary dynamics frequently emerge when event interactions and background intensities vary over time (Chen et al., 2023). Chen et al. (2022b) further demonstrated that the causal mechanisms can change over time. Second, network effects often exist, where events are influenced not only by their own history but also by their topological neighbors. Failure to account for these dependencies can lead to biased causal estimates. To handle topological dependencies, Cai et al. (2022) proposed the Topological Hawkes Process (THP), which extends temporal convolution to graph-time convolution while employing an EM-based inference approach. Subsequent improvements by Li et al. (2024) enhanced THP's scalability through gradient-based optimization, and Zhu et al. (2024) further generalized the framework using causal-attention Transformers to capture complex network relationships.

For low-resolution scenarios, Trouleau et al. (2021) demonstrated the robustness of cumulant-based methods to observational noise. Later, Cüppers et al. (2024) incorporated delayed effects using a delay-aware MDL criterion to handle observation delays. Furthermore, Qiao et al. (2023) showed that low resolution may lead to instantaneous effects and thus developed a discrete-time structural Hawkes process to handle such instantaneous causal relationships.

9 Challenges

Despite the rapid progress of temporal point processes (TPPs), several fundamental challenges remain unresolved. Beyond empirical performance, these challenges stem from deeper issues in data curation, model structure, optimization, and deployment, which jointly limit the reliability, interpretability, and scalability of current TPP systems.

Data and Model A major bottleneck for neural TPP research is the absence of standardized and well-curated benchmarks. Existing datasets differ widely in preprocessing, time resolution, event definitions, and evaluation protocols, which makes reported improvements difficult to compare and often irreproducible. This inconsistency is not merely a logistical issue: it induces implicit distribution shifts that confound model evaluation and can lead to misleading conclusions about architectural superiority. While large-scale studies such as Bosser & Taieb (2023) have taken important steps toward unified evaluation, the field still lacks community-agreed benchmarks with fixed data splits, standardized metrics, and clear task definitions. A promising direction is to establish shared repositories similar to those in vision and NLP, where models are evaluated under identical data and metric pipelines, enabling principled and cumulative progress. Some initial efforts have already been made. For example, EasyTPP (Xue et al., 2023a) provides a unified repository that implements a wide range of classical TPP models, ensuring consistency across implementations. Meanwhile, several standardized benchmark datasets have also been introduced, such as Jiang et al. (2025).

Model Interpretability The interpretability gap between classical and neural TPPs arises from a fundamental mismatch between model structure and domain semantics. In Poisson or Hawkes processes, parameters directly correspond to meaningful quantities such as background rates and triggering kernels, whereas neural TPPs represent temporal dynamics implicitly in high-dimensional latent states. As a result, even when neural models achieve superior predictive accuracy, their decisions are difficult to explain or validate in high-stakes applications such as healthcare or finance. While recent works propose attention-based or post-hoc attribution methods, these explanations are often heuristic and lack formal guarantees. A more principled direction is to design neural architectures with built-in inductive biases that preserve interpretable components, for example by constraining intensities, kernels, or latent dynamics to have physically mean-

ingful structure. There have also been some initial efforts in this direction. For instance, Meng et al. (2024); Zhou & Yu (2023) attempt to align neural TPPs with traditional statistical TPPs to improve interpretability.

Model Scalability Scalability limitations originate from the quadratic complexity of attention-based architectures, which makes modeling long event histories prohibitively expensive. This is particularly problematic in domains such as online systems or medical records, where event sequences may span tens of thousands of timestamps. Although linear-complexity alternatives such as state space models and Mamba (Chang et al., 2024; Gao et al., 2024) have shown promise in sequence modeling, their theoretical properties and inductive biases for point process data remain poorly understood. Future work must go beyond simply replacing attention with efficient modules and instead study how these architectures represent hazard functions, history dependence, and long-term temporal causality.

Sampling Efficiency Most existing TPPs are optimized for likelihood evaluation rather than for fast generation. However, many downstream applications—such as simulation, forecasting, and decision support—require sampling large numbers of future event sequences in real time. The root cause is that dominant sampling methods, including thinning and inverse transform sampling, are inherently sequential, making parallelization difficult. Bridging this gap will require rethinking the generative paradigm of TPPs. In recent years, a number of highly interesting works have emerged in this direction. For example, flow-based methods (Lüdke et al., 2025; Kerrigan et al., 2024; Shou, 2025), diffusion-based approaches (Yuan et al., 2023; Lüdke et al., 2024; Zhang et al., 2024), and speculative decoding techniques (Gong et al., 2025; Biloš et al., 2025) have been developed to enable the parallel generation of multiple events or time steps while preserving temporal consistency.

Multimodal Modeling Real-world event streams rarely consist of timestamps alone; instead, they are accompanied by rich contextual signals such as text, images, or sensor measurements. Current TPPs struggle to integrate such heterogeneous data due to misaligned representations, missing modalities, and high-dimensional inputs. While LLM-based TPPs provide a promising direction by leveraging pretrained multimodal representations, they also introduce new challenges in representation alignment, uncertainty calibration, and controllability. A key open problem is how to jointly model temporal dynamics and multimodal semantics in a way that is both statistically grounded and computationally efficient. This direction is still emerging, but initial efforts have already been made to integrate textual and visual information into TPP modeling (Jiang et al., 2025; Liu & Quan, 2024; Zhang et al., 2023; Kong et al., 2025).

10 Conclusions

TPPs provide a powerful mathematical framework for modeling asynchronous event sequences across diverse domains such as neuroscience, finance, and social media. Over the years, the field has evolved from traditional parametric models to increasingly flexible nonparametric and neural approaches. In this survey, we reviewed recent progress in three major directions of TPP research: Bayesian TPPs, neural TPPs, and LLM-based TPPs. Each paradigm offers distinct modeling philosophies and strengths—Bayesian methods emphasize uncertainty quantification and principled inference, neural methods prioritize expressive power and scalability, while LLM-based approaches open new avenues for handling complex multimodal sequences. We also provided a taxonomy of representative works, highlighted core modeling principles and estimation strategies, and discussed ongoing challenges including model interpretability, scalability, and sampling efficiency. In particular, we emphasized recent developments since 2020 that have not been fully covered in earlier surveys, including Bayesian nonparametric models and the emerging trend of applying large language models to TPPs. Looking ahead, we believe that continued progress will hinge on deeper integration across statistical rigor, neural flexibility, and language-model capabilities. Addressing key challenges and enabling broader application of TPPs in real-world, multimodal, and high-resolution settings remain central to advancing the field.

References

Massil Achab, Emmanuel Bacry, Stéphane Gaïffas, Iacopo Mastromatteo, and Jean-François Muzy. Uncovering causality from multivariate hawkes integrated cumulants. *Journal of Machine Learning Research*, 18(192):1–28, 2018.

Ryan Adams, Iain Murray, and David MacKay. Tractable nonparametric Bayesian inference in Poisson processes with Gaussian process intensities. In *International Conference on Machine Learning*, 2009.

Virginia Aglietti, Edwin V Bonilla, Theodoros Damoulas, and Sally Cripps. Structured variational inference in continuous cox process models. *Advances in Neural Information Processing Systems*, 32, 2019a.

Virginia Aglietti, Theodoros Damoulas, and Edwin V Bonilla. Efficient inference in multi-task Cox process models. In *The 22nd International Conference on Artificial Intelligence and Statistics*, pp. 537–546. PMLR, 2019b.

Tanvir Ahammad. Identifying hidden patterns of fake covid-19 news: An in-depth sentiment analysis and topic modeling approach. *Natural Language Processing Journal*, 6:100053, 2024.

Alibaba. User behavior data from taobao for recommendation, 2018. URL `https://tianchi.aliyun.com/dataset/dataDetail?dataId=649`.

Emmanuel Bacry and Jean-François Muzy. Hawkes model for price and trades high-frequency dynamics. *Quantitative Finance*, 14(7):1147–1166, 2014.

Emmanuel Bacry and Jean-François Muzy. First-and second-order statistics characterization of Hawkes processes and non-parametric estimation. *IEEE Transactions on Information Theory*, 62(4):2184–2202, 2016.

Yujia Bao, Zhaobin Kuang, Peggy Peissig, David Page, and Rebecca Willett. Hawkes process modeling of adverse drug reactions with longitudinal observational data. In *Machine learning for healthcare conference*, pp. 177–190. PMLR, 2017.

Yannick Bessy-Roland, Alexandre Boumezoued, and Caroline Hillairet. Multivariate hawkes process for cyber insurance. *Annals of Actuarial Science*, 15(1):14–39, 2021. doi: 10.1017/S1748499520000093.

Marin Biloš, Anderson Schneider, and Yuriy Nevmyvaka. Speculative sampling for parametric temporal point processes. *arXiv preprint arXiv:2510.20031*, 2025.

Anna Bonnet and Maxime Sangnier. Nonparametric estimation of hawkes processes with rkhss. *arXiv preprint arXiv:2411.00621*, 2024.

Tanguy Bosser and Souhaib Ben Taieb. On the predictive accuracy of neural temporal point process models for continuous-time event data. *arXiv preprint arXiv:2306.17066*, 2023.

Ruichu Cai, Siyu Wu, Jie Qiao, Zhifeng Hao, Keli Zhang, and Xi Zhang. Thps: Topological hawkes processes for learning causal structure on event sequences. *IEEE Transactions on Neural Networks and Learning Systems*, 35(1):479–493, 2022.

Haoqun Cao, Zizhuo Meng, Tianjun Ke, and Feng Zhou. Is score matching suitable for estimating point processes? In *Advances in Neural Information Processing Systems*, 2024.

Giulia Cencetti, Federico Battiston, Bruno Lepri, and Márton Karsai. Temporal properties of higher-order interactions in social networks. *Scientific reports*, 11(1):7028, 2021.

Yuxin Chang, Alex Boyd, Cao Xiao, Taha Kass-Hout, et al. Deep linear Hawkes processes. *arXiv preprint*, 2024.

Jing Chen, Nick Taylor, Steve Yang, and Qian Han. Hawkes processes in finance: market structure and impact. *The European Journal of Finance*, 28(7):621–626, 2022a.

Ricky TQ Chen, Yulia Rubanova, Jesse Bettencourt, and David K Duvenaud. Neural ordinary differential equations. *Advances in neural information processing systems*, 31, 2018.

Ricky TQ Chen, Brandon Amos, and Maximilian Nickel. Neural spatio-temporal point processes. In *International Conference on Learning Representations*, 2021.

Wei Chen, Jibin Chen, Ruichu Cai, Yuequn Liu, and Zhifeng Hao. Learning granger causality for non-stationary hawkes processes. *Neurocomputing*, 468:22–32, 2022b.

Yu Chen, Fengpei Li, Anderson Schneider, Yuriy Nevmyvaka, Asohan Amarasingham, and Henry Lam. Detection of short-term temporal dependencies in hawkes processes with heterogeneous background dynamics. In *Uncertainty in Artificial Intelligence*, pp. 369–380. PMLR, 2023.

Zhiheng Chen, Guanhua Fang, and Wen Yu. On non-asymptotic theory of recurrent neural networks in temporal point processes. *arXiv preprint arXiv:2406.00630*, 2024.

Wen-Hao Chiang, Xueying Liu, and George Mohler. Hawkes process modeling of covid-19 with mobility leading indicators and spatial covariates. *International journal of forecasting*, 38(2):505–520, 2022.

Junyoung Chung, Caglar Gulcehre, KyungHyun Cho, and Yoshua Bengio. Empirical evaluation of gated recurrent neural networks on sequence modeling. *arXiv preprint arXiv:1412.3555*, 2014.

John P Cunningham, Krishna V Shenoy, and Maneesh Sahani. Fast Gaussian process methods for point process intensity estimation. In *International Conference on Machine Learning*, 2008.

Joscha Cüppers, Sascha Xu, Ahmed Musa, and Jilles Vreeken. Causal discovery from event sequences by local cause-effect attribution. *Advances in Neural Information Processing Systems*, 37:24216–24241, 2024.

Daryl J Daley and David Vere-Jones. *An introduction to the theory of point processes: volume II: general theory and structure*. Springer Science & Business Media, 2007.

Louis Davis, Boris Baeumer, and Ting Wang. A fractional hawkes process model for earthquake aftershock sequences. *Journal of the Royal Statistical Society Series C: Applied Statistics*, 73(5):1185–1202, 2024.

Isabella Deutsch and Gordon J Ross. Bayesian estimation of multivariate hawkes processes with inhibition and sparsity. *arXiv preprint arXiv:2201.05009*, 2022.

Christian Donner and Manfred Opper. Efficient Bayesian inference of sigmoidal Gaussian Cox processes. *Journal of Machine Learning Research*, 19(1):2710–2743, 2018.

Sophie Donnet, Vincent Rivoirard, and Judith Rousseau. Nonparametric bayesian estimation for multivariate hawkes processes. *The Annals of statistics*, 48(5):2698–2727, 2020.

Nan Du, Hanjun Dai, Rakshit Trivedi, Utkarsh Upadhyay, Manuel Gomez-Rodriguez, and Le Song. Recurrent marked temporal point processes: embedding event history to vector. In *International Conference on Knowledge Discovery and Data Mining*, 2016.

Michael Eichler, Rainer Dahlhaus, and Johannes Dueck. Graphical modeling for multivariate Hawkes processes with nonparametric link functions. *Journal of Time Series Analysis*, 38(2):225–242, 2017.

Seth Flaxman, Andrew Wilson, Daniel Neill, Hannes Nickisch, and Alex Smola. Fast Kronecker inference in Gaussian processes with non-Gaussian likelihoods. In *International Conference on Machine Learning*, pp. 607–616. PMLR, 2015.

Seth Flaxman, Yee Whye Teh, Dino Sejdinovic, et al. Poisson intensity estimation with reproducing kernels. *Electronic Journal of Statistics*, 11(2):5081–5104, 2017.

Giancarlo Fortino, Claudia Greco, Antonella Guzzo, and Michele Ianni. Neural network based temporal point processes for attack detection in industrial control systems. In *2022 IEEE international conference on cyber security and resilience (CSR)*, pp. 221–226. IEEE, 2022.

Anningzhe Gao, Shan Dai, and Yan Hu. Mamba hawkes process. *arXiv preprint*, 2024.

Paolo Giudici, Paolo Pagnottoni, and Alessandro Spelta. Network self-exciting point processes to measure health impacts of covid-19. *Journal of the Royal Statistical Society Series A: Statistics in Society*, 186(3): 401–421, 2023.

Shukai Gong, YIYANG FU, Fengyuan Ran, Quyu Kong, and Feng Zhou. Tpp-sd: Accelerating transformer point process sampling with speculative decoding. In *The Thirty-ninth Annual Conference on Neural Information Processing Systems*, 2025.

IJ Goodd and Ray A Gaskins. Nonparametric roughness penalties for probability densities. *Biometrika*, 58 (2):255–277, 1971.

Clive WJ Granger. Investigating causal relations by econometric models and cross-spectral methods. *Econometrica: journal of the Econometric Society*, pp. 424–438, 1969.

Albert Gu and Tri Dao. Mamba: Linear-time sequence modeling with selective state spaces. *arXiv preprint*, 2023.

Albert Gu, Karan Goel, and Christopher Re. Efficiently modeling long sequences with structured state spaces. In *International Conference on Learning Representations*, 2022.

Tom Gunter, Chris Lloyd, Michael A Osborne, and Stephen J Roberts. Efficient Bayesian nonparametric modelling of structured point processes. In *Conference on Uncertainty in Artificial Intelligence*, 2014.

Ruocheng Guo, Jundong Li, and Huan Liu. Initiator: noise-contrastive estimation for marked temporal point process. In *Proceedings of the 27th International Joint Conference on Artificial Intelligence*, pp. 2191–2197, 2018.

Vinayak Gupta, Srikanta Bedathur, Sourangshu Bhattacharya, and Abir De. Learning temporal point processes with intermittent observations. In *International Conference on Artificial Intelligence and Statistics*, pp. 3790–3798. PMLR, 2021.

Michael U Gutmann and Aapo Hyvärinen. Noise-contrastive estimation of unnormalized statistical models, with applications to natural image statistics. *The journal of machine learning research*, 13(1):307–361, 2012.

Alan G Hawkes. Spectra of some self-exciting and mutually exciting point processes. *Biometrika*, 58(1): 83–90, 1971.

Alan G Hawkes. Hawkes processes and their applications to finance: a review. *Quantitative Finance*, 18(2): 193–198, 2018.

Sampad Dinesh Hegde, Akhilesh Shetty, NM Manoj, Abhigna Kalasad, and R Bharathi. Framework for detecting fake retweets using deep neural network. In *2022 IEEE 7th International conference for Convergence in Technology (I2CT)*, pp. 1–6. IEEE, 2022.

Katerina Hlaváčková-Schindler, Anna Melnykova, and Irene Tubikanec. Granger causal inference in multivariate hawkes processes by minimum message length. *Journal of Machine Learning Research*, 25(133): 1–26, 2024.

S Hochreiter. Long short-term memory. *Neural Computation MIT-Press*, 1997.

Md Monir Hossain and Andrew B Lawson. Approximate methods in bayesian point process spatial models. *Computational statistics & data analysis*, 53(8):2831–2842, 2009.

Aapo Hyvärinen. Estimation of non-normalized statistical models by score matching. *J. Mach. Learn. Res.*, 6:695–709, 2005.

Tsuyoshi Idé, Georgios Kollias, Dzung Phan, and Naoki Abe. Cardinality-regularized hawkes-granger model. *Advances in Neural Information Processing Systems*, 34:2682–2694, 2021.

Janine B Illian, Sara Martino, Sigrunn H Sørbye, Juan B Gallego-Fernández, María Zunzunegui, M Paz Esquivias, and Justin MJ Travis. Fitting complex ecological point process models with integrated nested laplace approximation. *Methods in Ecology and Evolution*, 4(4):305–315, 2013.

Valerie Isham and Mark Westcott. A self-correcting point process. *Stochastic Processes and Their Applications*, 8(3):335–347, 1979.

Amirkasra Jalaldoust, Kateřina Hlaváčková-Schindler, and Claudia Plant. Causal discovery in hawkes processes by minimum description length. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 36, pp. 6978–6987, 2022.

Junteng Jia and Austin R Benson. Neural jump stochastic differential equations. In *Advances in Neural Information Processing Systems*, 2019.

Alex Ziyu Jiang and Abel Rodriguez. Semiparametric estimation for multivariate hawkes processes using dependent dirichlet processes: An application to order flow data in financial markets. *arXiv preprint arXiv:2502.17723*, 2025.

Yue Jiang, Jichu Li, Yang Liu, Dingkang Yang, Feng Zhou, and Quyu Kong. Danmakutppbench: A multi-modal benchmark for temporal point process modeling and understanding. *arXiv preprint arXiv:2505.18411*, 2025.

ST John and James Hensman. Large-scale Cox process inference using variational Fourier features. In *International Conference on Machine Learning*, 2018.

Stojan Jovanović, John Hertz, and Stefan Rotter. Cumulants of hawkes point processes. *Physical Review E*, 91(4):042802, 2015.

Rafal Jozefowicz, Oriol Vinyals, Mike Schuster, Noam Shazeer, and Yonghui Wu. Exploring the limits of language modeling. *arXiv preprint arXiv:1602.02410*, 2016.

Gavin Kerrigan, Kai Nelson, and Padhraic Smyth. Eventflow: Forecasting temporal point processes with flow matching. *arXiv preprint arXiv:2410.07430*, 2024.

John Frank Charles Kingman. *Poisson processes*, volume 3. Clarendon Press, 1992.

Ryota Kobayashi and Renaud Lambiotte. Tideh: Time-dependent hawkes process for predicting retweet dynamics. In *Proceedings of the international AAAI conference on web and social media*, volume 10, pp. 191–200, 2016.

Quyu Kong, Pio Calderon, Rohit Ram, Olga Boichak, and Marian-Andrei Rizoiu. Interval-censored Transformer Hawkes: Detecting information operations using the reaction of social systems. In *The Web Conference*, 2023.

Quyu Kong, Yixuan Zhang, Yang Liu, Panrong Tong, Enqi Liu, and Feng Zhou. Language-tpp: Integrating temporal point processes with language models for event analysis. *arXiv preprint arXiv:2502.07139*, 2025.

Athanasios Kottas. Dirichlet process mixtures of Beta distributions, with applications to density and intensity estimation. In *Workshop on Learning with Nonparametric Bayesian Methods, ICML*, 2006.

Athanasios Kottas and Bruno Sansó. Bayesian mixture modeling for spatial Poisson process intensities, with applications to extreme value analysis. *Journal of Statistical Planning and Inference*, 137(10):3151–3163, 2007.

Junhyeon Kwon, Yingcai Zheng, and Mikyoung Jun. Flexible spatio-temporal hawkes process models for earthquake occurrences. *Spatial Statistics*, 54:100728, 2023.

Erik Lewis and George Mohler. A nonparametric EM algorithm for multiscale Hawkes processes. *Journal of Nonparametric Statistics*, 1(1):1–20, 2011.

Junliang Li, Yajun Yang, Yujia Zhang, Qinghua Hu, Alan Zhao, and Hong Gao. Public opinion field effect and hawkes process join hands for information popularity prediction. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 39, pp. 12076–12083, 2025.

Mingjia Li, Shuo Liu, Hong Qian, and Aimin Zhou. A simple yet scalable granger causal structural learning approach for topological event sequences. *Advances in Neural Information Processing Systems*, 37:97124–97140, 2024.

Zhuoqun Li and Mingxuan Sun. Sparse Transformer Hawkes process for long event sequences. In *Joint European Conference on Machine Learning and Knowledge Discovery in Databases*, 2023.

Zichong Li, Yanbo Xu, Simiao Zuo, Haoming Jiang, Chao Zhang, Tuo Zhao, and Hongyuan Zha. Smurf-thp: score matching-based uncertainty quantification for transformer hawkes process. In *International Conference on Machine Learning*, pp. 20210–20220. PMLR, 2023.

Wenzhao Lian, Ricardo Henao, Vinayak Rao, Joseph Lucas, and Lawrence Carin. A multitask point process predictive model. In *International Conference on Machine Learning*, 2015.

Scott W Linderman and Ryan P Adams. Scalable Bayesian inference for excitatory point process networks. *arXiv preprint*, 2015.

Bingqing Liu. Cumulative hazard function based efficient multivariate temporal point process learning. *arXiv preprint*, 2024.

Zefang Liu and Yinzhu Quan. TPP-LLM: Modeling temporal point processes by efficiently fine-tuning large language models. *arXiv preprint*, 2024.

Zefang Liu and Yinzhu Quan. Retrieval of temporal event sequences from textual descriptions. In *Proceedings of the 4th International Workshop on Knowledge-Augmented Methods for Natural Language Processing*, pp. 37–49, 2025.

Chris Lloyd, Tom Gunter, Michael Osborne, and Stephen Roberts. Variational inference for Gaussian process modulated Poisson processes. In *International Conference on Machine Learning*, 2015.

David Lüdke, Marin Biloš, Oleksandr Shchur, Marten Lienen, and Stephan Günnemann. Add and thin: Diffusion for temporal point processes. *Advances in Neural Information Processing Systems*, 36:56784–56801, 2023.

David Lüdke, Enric Rabasseda Raventós, Marcel Kollovieh, and Stephan Günnemann. Unlocking point processes through point set diffusion. *arXiv preprint arXiv:2410.22493*, 2024.

David Lüdke, Marten Lienen, Marcel Kollovieh, and Stephan Günnemann. Edit-based flow matching for temporal point processes. *arXiv preprint arXiv:2510.06050*, 2025.

Noa Malem-Shinitski, César Ojeda, and Manfred Opper. Variational bayesian inference for nonlinear hawkes process with gaussian process self-effects. *Entropy*, 24(3):356, 2022.

Dean Markwick. *Bayesian nonparametric Hawkes processes with applications*. PhD thesis, UCL (University College London), 2020.

David Marsan and Olivier Lengline. Extending earthquakes' reach through cascading. *Science*, 319(5866): 1076–1079, 2008.

Peter McCullagh and Jesper Møller. The permanental process. *Advances in applied probability*, 38(4): 873–888, 2006.

Hongyuan Mei and Jason Eisner. The neural Hawkes process: A neurally self-modulating multivariate point process. In *Advances in Neural Information Processing Systems*, 2017.

Hongyuan Mei, Tom Wan, and Jason Eisner. Noise-contrastive estimation for multivariate point processes. In *Advances in Neural Information Processing Systems*, 2020.

Hongyuan Mei, Chenghao Yang, and Jason Eisner. Transformer embeddings of irregularly spaced events and their participants. In *International conference on learning representations*, 2021.

Zizhuo Meng, Ke Wan, Yadong Huang, Zhidong Li, Yang Wang, and Feng Zhou. Interpretable Transformer Hawkes processes: Unveiling complex interactions in social networks. In *International Conference on Knowledge Discovery and Data Mining*, 2024.

Søren Wengel Mogensen. Causal screening in dynamical systems. In *Conference on Uncertainty in Artificial Intelligence*, pp. 310–319. PMLR, 2020.

Jesper Møller, Anne Randi Syversveen, and Rasmus Plenge Waagepetersen. Log Gaussian Cox processes. *Scandinavian journal of statistics*, 25(3):451–482, 1998.

Iain Murray, Zoubin Ghahramani, and David JC MacKay. MCMC for doubly-intractable distributions. In *Conference on Uncertainty in Artificial Intelligence*, 2006.

Jianmo Ni, Jiacheng Li, and Julian McAuley. Justifying recommendations using distantly-labeled reviews and fine-grained aspects. In *Proceedings of the 2019 conference on empirical methods in natural language processing and the 9th international joint conference on natural language processing (EMNLP-IJCNLP)*, pp. 188–197, 2019.

Kaj Nyström and Changyong Zhang. Hawkes-based models for high frequency financial data. *Journal of the Operational Research Society*, 73(10):2168–2185, 2022.

Yosihiko Ogata. Space-time point-process models for earthquake occurrences. *Annals of the Institute of Statistical Mathematics*, 50(2):379–402, 1998.

Takahiro Omi, Naonori Ueda, and Kazuyuki Aihara. Fully neural network based model for general temporal point processes. In *Advances in Neural Information Processing Systems*, 2019.

Tohru Ozaki. Maximum likelihood estimation of Hawkes' self-exciting point processes. *Annals of the Institute of Statistical Mathematics*, 31(1):145–155, 1979.

Liam Paninski. Maximum likelihood estimation of cascade point-process neural encoding models. *Network: Computation in Neural Systems*, 15(4):243–262, 2004.

Aristeidis Panos. Decomposable Transformer point processes. In *Advances in Neural Information Processing Systems*, 2024.

Bo Peng, Eric Alcaide, Quentin Gregory Anthony, Alon Albalak, et al. RWKV: Reinventing RNNs for the Transformer era. In *Conference on Empirical Methods in Natural Language Processing*, 2023.

Dzung T Phan and Tsuyoshi Idé. ℓ_0-regularized sparsity for probabilistic mixture models. In *Proceedings of the 2019 SIAM International Conference on Data Mining*, pp. 172–180. SIAM, 2019.

Jie Qiao, Ruichu Cai, Siyu Wu, Yu Xiang, Keli Zhang, and Zhifeng Hao. Structural hawkes processes for learning causal structure from discrete-time event sequences. In *Proceedings of the Thirty-Second International Joint Conference on Artificial Intelligence*, pp. 5702–5710, 2023.

Zheng Qu, Chen Lyu, and Chi-Hung Chi. Mush: Multi-stimuli hawkes process based sybil attacker detector for user-review social networks. *IEEE Transactions on Network and Service Management*, 19(4):4600–4614, 2022.

Adrian E Raftery and Volkan E Akman. Bayesian analysis of a Poisson process with a change-point. *Biometrika*, pp. 85–89, 1986.

Jakob Gulddahl Rasmussen. Bayesian inference for hawkes processes. *Methodology and Computing in Applied Probability*, 15:623–642, 2013.

Jakob Gulddahl Rasmussen. Lecture notes: Temporal point processes and the conditional intensity function. *arXiv preprint*, 2018.

Patricia Reynaud-Bouret, Sophie Schbath, et al. Adaptive estimation for Hawkes processes; application to genome analysis. *The Annals of Statistics*, 38(5):2781–2822, 2010.

Marian Andrei Rizoiu, Swapnil Mishra, Quyu Kong, Mark Carman, and Lexing Xie. Sir-hawkes: on the relationship between epidemic models and Hawkes point processes. In *The Web Confernce*, 2018.

Yulia Rubanova, Ricky TQ Chen, and David K Duvenaud. Latent ordinary differential equations for irregularly-sampled time series. *Advances in Neural Information Processing Systems*, 2019.

Jakob Runge, Peer Nowack, Marlene Kretschmer, Seth Flaxman, and Dino Sejdinovic. Detecting and quantifying causal associations in large nonlinear time series datasets. *Science advances*, 5(11):eaau4996, 2019.

Maneesh Sahani, Gergo Bohner, and Arne Meyer. Score-matching estimators for continuous-time point-process regression models. In Francesco A. N. Palmieri, Aurelio Uncini, Kostas I. Diamantaras, and Jan Larsen (eds.), *26th IEEE International Workshop on Machine Learning for Signal Processing, MLSP 2016, Vietri sul Mare, Salerno, Italy, September 13-16, 2016*, pp. 1–5. IEEE, 2016.

Yves-Laurent Kom Samo and Stephen Roberts. Scalable nonparametric Bayesian inference on point processes with Gaussian processes. In *International Conference on Machine Learning*, 2015.

Frederic Schoenberg. Estimating covid-19 transmission time using hawkes point processes. *The Annals of Applied Statistics*, 17(4):3349–3362, 2023.

Amray Schwabe, Joel Persson, and Stefan Feuerriegel. Predicting covid-19 spread from large-scale mobility data. In *Proceedings of the 27th ACM SIGKDD Conference on Knowledge Discovery & Data Mining*, pp. 3531–3539, 2021.

Jeremy Sellier and Petros Dellaportas. Sparse spectral Bayesian permanental process with generalized kernel. In *International Conference on Artificial Intelligence and Statistics*, 2023.

Oleksandr Shchur, Marin Bilos, and Stephan Günnemann. Intensity-free learning of temporal point processes. In *International Conference on Learning Representations*, 2020a.

Oleksandr Shchur, Nicholas Gao, Marin Bilos, and Stephan Günnemann. Fast and flexible temporal point processes with triangular maps. In *Advances in neural information processing systems*, 2020b.

Oleksandr Shchur, Ali Caner Türkmen, Tim Januschowski, and Stephan Günnemann. Neural temporal point processes: A review. *arXiv preprint*, 2021.

Xiaoming Shi, Siqiao Xue, Kangrui Wang, Fan Zhou, James Zhang, Jun Zhou, Chenhao Tan, and Hongyuan Mei. Language models can improve event prediction by few-shot abductive reasoning. In *Advances in Neural Information Processing Systems*, 2024.

Zijian Shi and John Cartlidge. State dependent parallel neural hawkes process for limit order book event stream prediction and simulation. In *Proceedings of the 28th ACM SIGKDD Conference on Knowledge Discovery and Data Mining*, pp. 1607–1615, 2022.

Xiao Shou. Unified flow matching for long horizon event forecasting. *arXiv preprint arXiv:2508.04843*, 2025.

Alexander Soen, Alexander Mathews, Daniel Grixti-Cheng, and Lexing Xie. Unipoint: Universally approximating point processes intensities. In *AAAI conference on artificial intelligence*, 2021.

Yang Song, Jascha Sohl-Dickstein, Diederik P Kingma, Abhishek Kumar, Stefano Ermon, and Ben Poole. Score-based generative modeling through stochastic differential equations. *arXiv preprint arXiv:2011.13456*, 2020.

Deborah Sulem, Vincent Rivoirard, and Judith Rousseau. Bayesian estimation of nonlinear hawkes processes. *Bernoulli*, 30(2):1257–1286, 2024.

Zicheng Sun, Yixuan Zhang, Zenan Ling, Xuhui Fan, and Feng Zhou. Nonstationary sparse spectral permanental process. In *Advances in Neural Information Processing Systems*, 2024.

Souhaib Ben Taieb. Learning quantile functions for temporal point processes with recurrent neural splines. In *International Conference on Artificial Intelligence and Statistics*, 2022.

Michalis Titsias. Variational learning of inducing variables in sparse Gaussian processes. In *Artificial Intelligence and Statistics*, pp. 567–574, 2009.

William Trouleau, Jalal Etesami, Matthias Grossglauser, Negar Kiyavash, and Patrick Thiran. Cumulants of hawkes processes are robust to observation noise. In *International Conference on Machine Learning*, pp. 10444–10454. PMLR, 2021.

Christian J Walder and Adrian N Bishop. Fast Bayesian intensity estimation for the permanental process. In *International Conference on Machine Learning*, 2017.

Dongjing Wang, Xin Zhang, Zhengzhe Xiang, Dongjin Yu, Guandong Xu, and Shuiguang Deng. Sequential recommendation based on multivariate hawkes process embedding with attention. *IEEE transactions on cybernetics*, 52(11):11893–11905, 2021.

Xinyu Wang, Feng Qiang, Li Ma, Peng Zhang, Hong Yang, Zhao Li, and Ji Zhang. Federated Transformer Hawkes processes for distributed event sequence prediction. In *International Joint Conference on Neural Networks*, 2024.

Yichen Wang, Evangelos Theodorou, Apurv Verma, and Le Song. A stochastic differential equation framework for guiding online user activities in closed loop. In *International Conference on Artificial Intelligence and Statistics*, pp. 1077–1086. PMLR, 2018.

Yuchen Wang, Dongpeng Hou, Chao Gao, and Xianghua Li. Learning neural jump stochastic differential equations with latent graph for multivariate temporal point processes. In *Proceedings of the Thirty-Fourth International Joint Conference on Artificial Intelligence*, pp. 3444–3452, 2025.

Chris Whong. Foiling nyc's taxi trip data. `https://chriswhong.com/open-data/foil_nyc_taxi/`, 2014.

John Worrall. *Online Nonparametric Bayesian Hawkes Processes*. PhD thesis, Queensland University of Technology, 2024.

Dongxia Wu, Tsuyoshi Idé, Georgios Kollias, Jiri Navratil, Aurelie Lozano, Naoki Abe, Yian Ma, and Rose Yu. Learning granger causality from instance-wise self-attentive hawkes processes. In *International Conference on Artificial Intelligence and Statistics*, pp. 415–423. PMLR, 2024.

Shuai Xiao, Mehrdad Farajtabar, Xiaojing Ye, Junchi Yan, et al. Wasserstein learning of deep generative point process models. In *Advances in Neural Information Processing Systems*, 2017a.

Shuai Xiao, Junch Yan, Xiaokang Yang, Hongyuan Zha, and Stephen Chu. Modeling the intensity function of point process via recurrent neural networks. In *AAAI Conference on Artificial Intelligence*, 2017b.

Shuai Xiao, Junchi Yan, Xiaokang Yang, Hongyuan Zha, and Stephen M. Chu. Modeling the intensity function of point process via recurrent neural networks. In Satinder Singh and Shaul Markovitch (eds.), *Proceedings of the Thirty-First AAAI Conference on Artificial Intelligence, February 4-9, 2017, San Francisco, California, USA*, pp. 1597–1603. AAAI Press, 2017c.

Shuai Xiao, Hongteng Xu, Junchi Yan, Mehrdad Farajtabar, Xiaokang Yang, Le Song, and Hongyuan Zha. Learning conditional generative models for temporal point processes. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 32, 2018.

Hongteng Xu and Hongyuan Zha. A dirichlet mixture model of hawkes processes for event sequence clustering. *Advances in neural information processing systems*, 30, 2017.

Hongteng Xu, Mehrdad Farajtabar, and Hongyuan Zha. Learning granger causality for hawkes processes. In *International conference on machine learning*, pp. 1717–1726. PMLR, 2016.

Siqiao Xue, Xiaoming Shi, James Zhang, and Hongyuan Mei. Hypro: A hybridly normalized probabilistic model for long-horizon prediction of event sequences. *Advances in Neural Information Processing Systems*, 35:34641–34650, 2022.

Siqiao Xue, Xiaoming Shi, Zhixuan Chu, Yan Wang, Hongyan Hao, Fan Zhou, Caigao JIANG, Chen Pan, James Y Zhang, Qingsong Wen, et al. Easytpp: Towards open benchmarking temporal point processes. In *The Twelfth International Conference on Learning Representations*, 2023a.

Siqiao Xue, Yan Wang, Zhixuan Chu, Xiaoming Shi, et al. Prompt-augmented temporal point process for streaming event sequence. In *Advances in Neural Information Processing Systems*, 2023b.

Junchi Yan. Recent advance in temporal point process: from machine learning perspective. *SJTU Technical Report*, 2019.

Chenghao Yang, Hongyuan Mei, and Jason Eisner. Transformer embeddings of irregularly spaced events and their participants. In *Proceedings of the Tenth International Conference on Learning Representations (ICLR)*, 2022.

Guolei Yang, Ying Cai, and Chandan K Reddy. Recurrent spatio-temporal point process for check-in time prediction. In *International Conference on Information and Knowledge Management*, 2018.

Sikun Yang and Hongyuan Zha. A variational autoencoder for neural temporal point processes with dynamic latent graphs. In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 38, pp. 16343–16351, 2024.

Fan Yin, Jieying Jiao, Jun Yan, and Guanyu Hu. Bayesian nonparametric learning for point processes with spatial homogeneity: A spatial analysis of nba shot locations. In *International Conference on Machine Learning*, pp. 25523–25551. PMLR, 2022.

Yuan Yuan, Jingtao Ding, Chenyang Shao, Depeng Jin, and Yong Li. Spatio-temporal diffusion point processes. In *Proceedings of the 29th ACM SIGKDD Conference on Knowledge Discovery and Data Mining*, pp. 3173–3184, 2023.

Andrew Zammit-Mangion, Guido Sanguinetti, and Visakan Kadirkamanathan. Variational estimation in spatiotemporal systems from continuous and point-process observations. *IEEE Transactions on Signal Processing*, 60(7):3449–3459, 2012.

Qiang Zhang, Aldo Lipani, Ömer Kirnap, and Emine Yilmaz. Self-attentive Hawkes process. In *International Conference on Machine Learning*, 2020a.

Rui Zhang, Christian J. Walder, Marian-Andrei Rizoiu, and Lexing Xie. Efficient non-parametric Bayesian Hawkes processes. In *International Joint Conference on Artificial Intelligence*, 2019.

Rui Zhang, Christian Walder, and Marian-Andrei Rizoiu. Variational inference for sparse Gaussian process modulated Hawkes process. In *AAAI Conference on Artificial Intelligence*, 2020b.

Shuai Zhang, Chuan Zhou, Yang Aron Liu, Peng Zhang, Xixun Lin, and Zhi-Ming Ma. Neural jump-diffusion temporal point processes. In *International Conference on Machine Learning*, 2024.

Wei Zhang, Thomas K. Panum, Somesh Jha, Prasad Chalasani, and David Page. CAUSE: Learning granger causality from event sequences using attribution methods. In *Proceedings of the 37th International Conference on Machine Learning*, 2020c.

Yixuan Zhang, Quyu Kong, and Feng Zhou. Integration-free training for spatio-temporal multimodal covariate deep kernel point processes. In *Advances in Neural Information Processing Systems*, 2023.

Yizhou Zhang, Karishma Sharma, and Yan Liu. Vigdet: Knowledge informed neural temporal point process for coordination detection on social media. *Advances in Neural Information Processing Systems*, 34:3218–3231, 2021.

Yunhao Zhang and Junchi Yan. Neural relation inference for multi-dimensional temporal point processes via message passing graph. In *IJCAI*, pp. 3406–3412, 2021.

Feng Zhou, Zhidong Li, Xuhui Fan, Yang Wang, Arcot Sowmya, and Fang Chen. Efficient inference for nonparametric Hawkes processes using auxiliary latent variables. *Journal of Machine Learning Research*, 21(241):1–31, 2020.

Feng Zhou, Simn Luo, Zhidong Li, Xuhu Fan, Yang Wang, Arcot Sowmya, and Fang Chen. Efficient EM-variational inference for nonparametric Hawkes process. *Statistics and Computing*, 31(4):46, 2021.

Feng Zhou, Quyu Kong, Zhijie Deng, Jichao Kan, Yixuan Zhang, Cheng Feng, and Jun Zhu. Efficient inference for dynamic flexible interactions of neural populations. *Journal of Machine Learning Research*, 23(211):1–49, 2022a.

Feng Zhou, Quyu Kong, Zhijie Deng, Fengxiang He, Peng Cui, and Jun Zhu. Heterogeneous multi-task Gaussian Cox processes. *Machine Learning*, 112(12):5105–5134, 2023.

Ke Zhou, Hongyuan Zha, and Le Song. Learning triggering kernels for multi-dimensional hawkes processes. In *Proceedings of the 30th International Conference on International Conference on Machine Learning - Volume 28*, ICML'13, pp. III–1301–III–1309. JMLR.org, 2013a.

Ke Zhou, Hongyuan Zha, and Le Song. Learning triggering kernels for multi-dimensional Hawkes processes. In *International Conference on Machine Learning*, 2013b.

Ke Zhou, Hongyuan Zha, and Le Song. Learning social infectivity in sparse low-rank networks using multi-dimensional hawkes processes. In *Artificial Intelligence and Statistics*, pp. 641–649, 2013c.

Zihao Zhou and Rose Yu. Automatic integration for fast and interpretable neural point processes. In *Learning for Dynamics and Control Conference*, pp. 573–585. PMLR, 2023.

Zihao Zhou, Xingyi Yang, Ryan Rossi, Handong Zhao, and Rose Yu. Neural point process for learning spatiotemporal event dynamics. In *Learning for Dynamics and Control Conference*, 2022b.

Hua Zhu, Hong Huang, Kehan Yin, Zejun Fan, Hai Jin, and Bang Liu. Causalnet: Unveiling causal structures on event sequences by topology-informed causal attention. In *Proceedings of the IJCAI*, pp. 7144–7152, 2024.

Shixiang Zhu, Minghe Zhang, Ruyi Ding, and Yao Xie. Deep fourier kernel for self-attentive point processes. In *International Conference on Artificial Intelligence and Statistics*, 2021.

Simiao Zuo, Haoming Jiang, Zichong Li, Tuo Zhao, and Hongyuan Zha. Transformer Hawkes process. In *International Conference on Machine Learning*, 2020.