Entropy Meets Importance: A Unified Head Importance–Entropy Score for Stable and Efficient Transformer Pruning

Table R.1: **Additional Baseline Evaluation** (in response to *Reviewer egVn*, *Reviewer EJiM*). LLaMA-2-7B reasoning benchmark comparison at iso-FLOP budget (matched to HIES 30% head pruning, ≈12% total FLOPs reduction). Reasoning Avg. is the macro-average of HellaSwag, Winogrande, ARC-e, and ARC-c (0-shot).

Method	Granularity	Pruning	Weight Restoration	Reasoning Avg.
ShortGPT	layer	4 layers (12.5%)	No Required	0.493
SparseGPT	weight	12.95% sparse	Required	0.544
HIES (ours)	head	10/32 heads (31.25%)	No Required	0.534

Table R.2: **Systematic Analysis of Preserved Heads** (in response to *Reviewer DBbk*). ERASER faithfulness metrics (comprehensiveness ↑, sufficiency ↓; DeYoung et al., 2020) of heads retained by each pruning method on SST-2 and MNLI. HIES uses a single fixed $\alpha = 0.5$ (no per-task tuning). At 50% pruning, HIES retains heads with strictly better faithfulness than HIS.

	HIS		HIES (ours, α=0.5)	
Pruning	Comp↑	Suff↓	Comp↑	Suff↓
SST-2				
0%	0.2405	0.2783	0.2405	0.2783
10%	0.2331	0.2804	0.2351	0.2790
30%	0.2328	0.2806	0.2371	0.2797
50%	0.1805	0.3129	**0.1907**	**0.3027**
MNLI				
0%	0.1018	0.2124	0.1018	0.2124
10%	0.1021	0.2137	0.0994	0.2211
30%	0.1010	0.2267	0.0996	0.2206
50%	0.0955	0.2510	**0.0983**	**0.2352**

Table R.3: **Ablation on Fusion Strategies** (in response to *Reviewer DBbk, Reviewer MGYd*). Accuracy comparison across fusion methods on BERT-base GLUE.

Ratio	Method	SST-2	CoLA	MRPC	QQP	STS-B	QNLI	MNLI	RTE	Avg.
10%	Multiplicative fusion	88.65	**76.69**	90.65	90.57	93.99	90.52	82.75	**72.20**	85.75
	Rank-based fusion	88.88	**76.69**	89.88	90.82	93.69	90.76	83.50	70.76	85.62
	HIES (ours)	**92.66**	75.48	**91.04**	**91.93**	**94.00**	**91.03**	**84.04**	71.84	**86.50**
30%	Multiplicative fusion	84.17	**75.23**	87.86	87.62	92.68	85.52	78.75	64.26	82.01
	Rank-based fusion	85.78	69.98	87.52	89.08	92.88	84.99	77.00	62.45	81.21
	HIES (ours)	**91.86**	74.97	**88.81**	**90.37**	**93.89**	**89.13**	**82.50**	**70.04**	**85.20**
50%	Multiplicative fusion	81.54	**74.85**	84.07	**81.78**	91.65	69.85	66.25	54.15	75.52
	Rank-based fusion	82.00	50.00	84.48	80.53	82.13	62.64	49.25	59.21	68.78
	HIES (ours)	**90.71**	68.52	**86.80**	85.73	**92.65**	**82.68**	**79.00**	**65.34**	**81.43**

Table R.4: **Ablation on Fusion Strategies** (in response to *Reviewer DBbk, Reviewer MGYd*). Stability comparison across fusion methods on BERT-base GLUE.

Ratio	Method	SST-2	CoLA	MRPC	QQP	STS-B	QNLI	MNLI	RTE	Avg.
10%	Multiplicative fusion	94.84	97.12	**94.36**	96.78	82.20	95.97	95.00	**94.22**	93.81
	Rank-based fusion	96.67	**97.22**	93.87	96.98	79.07	95.99	94.75	93.14	93.46
	HIES (ours)	**98.17**	96.36	94.12	**97.80**	**96.67**	**97.25**	**95.74**	93.50	**96.20**
30%	Multiplicative fusion	85.55	90.80	86.03	88.04	66.07	83.27	81.25	85.56	83.32
	Rank-based fusion	89.22	82.84	85.05	89.78	61.60	87.55	88.25	83.39	83.46
	HIES (ours)	**97.13**	**93.38**	**89.46**	**94.97**	**86.67**	**93.23**	**91.56**	**88.81**	**91.90**
50%	Multiplicative fusion	82.22	**88.78**	76.72	84.66	23.60	69.06	76.00	67.51	71.07
	Rank-based fusion	82.91	81.21	79.17	82.96	27.87	63.52	**79.00**	66.79	70.43
	HIES (ours)	**95.07**	83.13	**84.56**	**88.27**	**75.20**	**85.54**	78.29	**81.23**	**83.91**

Table R.5: **Overhead Analysis of Entropy Computation** (in response to *Reviewer DBbk*). Computational overhead of HIES compared to HIS on an NVIDIA RTX 3090, showing that the additional cost is negligible.

Stage	BERT-base	LLaMA-2-7B
HIS computation (shared)	$1.49 \pm 0.22\,\text{s}$	$6.42 \pm 0.07\,\text{s}$
AE computation (HIES additional)	$26.9\,\text{ms}$	$3.19\,\text{s}$
α grid search (HIES-only)	$9.37\,\text{ms}$	$65.6\,\text{ms}$
Total HIES additional cost	**36.3 ms**	**3.26 s**

Table R.6: **Extended Orthogonality Analysis** (in response to *Reviewer MGYd, Reviewer egVN*). Orthogonality of projected gradient directions $(\tilde{u}_h, \tilde{v}_h)$ across model scales. All values confirm near-orthogonality ($\cos(\theta) = 0.06$–0.11, corresponding to $84°$–$87°$), supporting Lemma 4.3.

Model	# Heads	Mean $\cos(\theta)$	Std $\cos(\theta)$
BERT-base	144	0.1102	0.0447
LLaMA-2-7B	1024	0.1090	0.1365
LLaMA-2-13B	1600	0.0994	0.1020
LLaMA-2-70B	5120	0.0574	0.0712

Table R.7: **Bridging Theoretical Analysis and the HIES Formulation** (in response to *Reviewer MGYd*). Theoretical grounding of HIES, showing how each component follows from the corresponding analytical results.

Theoretical result	Section	Consequence for HIES
Loss-increase bound via HIS	§4.2.1, Lemma 1	Motivates $\widehat{\text{HIS}}_h$
Generalization gap bound via AE	§4.2.2	Motivates $(1 - \widehat{\text{AE}}_h)$
Composite risk minimization	§4.2.3	Proves linear HIES is globally optimal
Orthogonality of signals	§4.2.4, Lemma 4.3	Guarantees non-redundancy

Table R.8: **Real Hardware Speedup** (in response to *Reviewer egVn*). Inference latency and speedup under head pruning on LLaMA-2-7B, demonstrating larger gains at longer sequence lengths.

Model	SeqLen	Prune%	Heads	Latency (ms)	Speedup
LLaMA-2-7B (fp16)		0%	1024	293.57 ± 1.25	1.00×
	1024	30%	704	256.55 ± 1.26	1.14×
		50%	512	232.33 ± 0.92	1.26×
		0%	1024	683.36 ± 1.67	1.00×
	2048	30%	704	570.09 ± 1.01	1.20×
		50%	512	505.20 ± 0.85	1.35×
		0%	1024	1586.07 ± 1.70	1.00×
	4096	30%	704	1287.03 ± 1.35	1.23×
		50%	512	1118.82 ± 0.62	1.42×

Table R.9: **GQA Model Evaluation** (in response to *Reviewer egVn*). Reasoning benchmarks on recent GQA-based architectures at 10/30/50% head pruning. A single fixed α is used per model without per-task tuning (α=0.9 for Qwen2.5-7B, α=0.7 for Qwen3-8B). HIES outperforms HIS across all pruning ratios on both models.

Pruning	Method	HellaSwag	ARC-C	ARC-E	Winogrande	Avg.
Qwen2.5-7B (GQA, 28Q/4KV, α=0.9)						
10%	HIS	57.25	38.98	59.56	66.46	55.56
	HIES (ours)	57.25	39.66	61.20	66.77	**56.22**
30%	HIS	53.10	35.59	55.74	61.80	51.56
	HIES (ours)	52.85	39.66	57.56	60.38	**52.61**
50%	HIS	45.00	33.56	54.64	53.20	46.60
	HIES (ours)	45.25	38.98	56.65	54.46	**48.84**
Qwen3-8B (GQA, 32Q/8KV, α=0.7)						
10%	HIS	53.65	41.36	57.92	62.12	53.76
	HIES (ours)	54.45	40.68	56.83	63.30	**53.81**
30%	HIS	48.75	35.93	50.64	60.85	49.04
	HIES (ours)	51.70	37.63	53.73	62.83	**51.47**
50%	HIS	41.35	32.88	50.46	57.30	45.50
	HIES (ours)	42.00	35.25	53.55	56.67	**46.87**