TemplateRL: Structured Template-Guided Reinforcement Learning for LLM Reasoning

Abstract

Reinforcement learning (RL) has emerged as an effective paradigm for enhancing model reasoning. However, existing RL methods like GRPO often rely on unstructured self-sampling to fit scalar rewards, often producing inefficient rollouts that fail to capture transferable problem-solving strategies. To address these limitations, we propose **TemplateRL**, a structured **template**-guided **RL** framework that augments policy optimization with explicit template guidance. Our approach first constructs a problem-solving template library via MCTS on a small seed set, then seamlessly integrates this high-level structured guidance into RL training. By guiding rollout generation to align with proven template structures, TemplateRL significantly improves high-quality trajectory hit rates while reducing ineffective exploration. This structure-guided design steers the policy toward validated strategic patterns, stabilizing training dynamics, and enhancing RL sampling efficiency. Notably, the explicit template library is interpretable, editable, and supports online updates-enabling continuous updates during both training and inference. Extensive experiments demonstrate that TemplateRL outperforms GRPO by 99% on AIME and 41% on AMC, with superior stability on weak models and remarkable cross-domain generalization, highlighting its potential for broader tasks.

1 Introduction

Reinforcement learning (RL) has demonstrated remarkable success in enhancing the reasoning capabilities of large language models (LLMs), as exemplified by OpenAI-o1 (Jaech et al., 2024), DeepSeek-R1 (Guo et al., 2025), and Kimi-1.5 (Team et al., 2025). In contrast to traditional approaches that rely on human-curated annotations (Achiam et al., 2023; Grattafiori et al., 2024), contemporary RL training paradigms (Shao et al., 2024) directly optimize base models using auto-

Figure 1: **Paradigm comparison: Teacher-Student Analogy for RL.** Standard RL like GRPO (**left**) provides only sparse answer rewards, while TemplateRL (**right**) offers structured templates encoding problem-solving thought patterns, enabling effective learning of both concrete steps and underlying strategic logic.

matically computable reward signals. This enables models to develop sophisticated capabilities, including problem decomposition and self-reflection (Gandhi et al., 2025; Yue et al., 2025).

Recent RL research primarily focuses on three directions: (1) Algorithmic Refinements that address inherent limitations such as length bias, KL divergence constraints, and advantage estimation (Yu et al., 2025; Liu et al., 2025); (2) Data-Level Optimizations that reduce annotation dependencies and improve data organization to enable minimally supervised training (Zuo et al., 2025; Wang et al., 2025b); (3) Exploration Enhancement through entropy regularization or token-level learning to encourage distributional diversity (Wang et al., 2025a; Cui et al., 2025b).

While promising, they remain fundamentally constrained by unstructured self-sampling within the model's internal distribution, lacking mechanisms to leverage structured guidance for more effective policy learning. This leads to three critical limitations: **(a) Inefficient trajectory sampling:** without structured guidance, models struggle to consistently generate high-quality trajectories, re-

sulting in low hit rates and training instability on weak models; **(b) Difficulty in learning transferable strategies:** existing RL methods tends to implicitly learn from surface-level steps rather than extract generalizable high-level thought patterns (e.g., divide-and-conquer), hindering knowledge transfer and accumulation across domains; **(c) Lack of interpretability:** existing RL optimization often produce reasoning trajectories without explicit strategic structure, hindering error diagnosis and expert intervention in critical applications.

To this end, we propose **TemplateRL**, a novel template-guided reinforcement learning framework that elegantly integrates structured thought templates into policy optimization (Figure 1). Our approach consists of three parts: **(1) template construction:** We employ Monte Carlo Tree Search on a small seed set to explore diverse solutions, then abstract successful paths into structured templates encoding high-level problem-solving patterns; **(2) template-guided training:** During RL training, we adaptively retrieve relevant templates as structured guidance for rollout generation, steering the policy toward proven strategic patterns; **(3) optional template updates:** The template library supports continuous updates by automatically abstracting new patterns from successful RL rollouts. This design delivers three key advantages: For **(a)**, structured guidance improves solution hit rates and stabilizes training on weaker models; For **(b)**, explicit templates enable learning both concrete steps and underlying strategic logic, facilitating cross-domain generalization and flexible knowledge updates; For **(c)**, template-guided trajectories form interpretable decision chains that support error diagnosis and expert refinement.

Extensive experiments validate TemplateRL's effectiveness across four dimensions: **(1) Improved Performance:** TemplateRL significantly outperforms GRPO by 99% on AIME, and 41% on AMC on a 7B backbone, surpassing representative baselines across benchmarks, model scales and architectures; **(2) Enhanced Training Stability:** Unlike GRPO which easily collapses on weaker models (e.g., Llama-3.2-3B), TemplateRL achieves stable training dynamics; **(3) Broad Generalization:** Our method demonstrates strong cross-domain performance on out-of-distribution benchmarks, including BALROG (agent), GPQA-D (science), and MMLU-Pro (knowledge). It consistently improves across different modalities (text, vision); **(4) Dynamic Extensibility:** Template library supports continuous updates during both training and inference, enabling incremental strategy incorporation. Further analysis reveal that TemplateRL produces clear and interpretable reasoning trajectories.

2 Preliminaries

LLMs as Markov Decision Processes. LLM generation can be formulated as a token-level Markov Decision Process $\mathcal{M}(\mathcal{S}, \mathcal{A}, r, p_{\mathcal{Q}})$ (Sutton et al., 1998; Liu et al., 2025), where $\mathcal{S}$ represents states (observation sequences) and $\mathcal{A}$ denotes the action space (vocabulary). At each step t, state $s_t \in \mathcal{S}$ consists of the input question $\mathbf{q}$ concatenated with all tokens generated so far $\mathbf{o}_{<t}$. This state serves as input to policy model $\pi_\theta(\cdot | s_t)$. Specifically, the policy processes $s_t = (\mathbf{q}, \mathbf{o}_{<t}) = (q_1, q_2, \ldots, q_l, o_1, o_2, \ldots, o_{t-1})$, where q_i denotes the i-th token of question $\mathbf{q}$ and $o_{j,<t}$ represents the token generated by π_θ at step j. In the RL framework, the policy then samples the next token from $\mathcal{A}$. The entropy-regularized optimization objective (Schulman et al., 2017) is:

$$
\begin{aligned}
\mathcal{J}(\pi_\theta) = \mathbb{E}_{\mathbf{q} \sim p_{\mathcal{Q}}} \big[\mathbb{E}_{\mathbf{o} \sim \pi_\theta(\cdot | \mathbf{q})}[R(\mathbf{q}, \mathbf{o})] - \\
\beta \cdot \mathbb{D}_{KL}[\pi_\theta(\cdot | \mathbf{q}) || \pi_{\text{ref}}(\cdot | \mathbf{q})] \big],
\end{aligned} \tag{1}
$$

where $R(\mathbf{q}, \mathbf{o}) = \sum_{t=1}^{|\mathbf{o}|} r(s_t, o_t)$ is the return of trajectory $(\mathbf{q}; \mathbf{o})$, $r(\cdot)$ is the reward model, and π_{ref} is a reference policy. The KL regularization term prevents π_θ from deviating excessively from π_{ref}.

Template Action. A template action is a prompt a designed to elicit a specific type of reasoning from a model, given a partial solution as input. It encapsulates an abstract human-reasoning procedure, such as proposing the next sub-question or generating the subsequent inference step (Guan et al., 2025). More precisely, given a partial solution $\mathbf{p}$, a induces the policy model π_θ to generate the next partial solution $\mathbf{p}'$:

$$
\mathbf{p}' = \pi_\theta(a(\mathbf{p})). \tag{2}
$$

3 Template-Guided RL

As shown in Figure 2, TemplateRL consists of three parts: (1) template construction (Section 3.1); (2) template-guided training (Section 3.2); and (3) optional template updates (Section 3.3).

3.1 Template Construction

We first describe how to construct a template library, which guides RL training in Section 3.2.

Figure 2: **Flowchart of TemplateRL.** This framework consists of three components: (1) template construction (Section 3.1); (2) template-guided training (Section 3.2); and (3) optional template updates (Section 3.3).

Previous work (Kahneman, 2011) reveals that humans solve complex reasoning tasks by applying universal guidelines ("templates") induced from similar problems rather than starting from scratch. These high-level templates help address unfamiliar tasks by leveraging previously successful strategies. Inspired by this, we introduce "template library", a lightweight hub of structured templates that provide high-level strategic guidance for RL.

MCTS Exploration. Starting with a small seed set $\mathcal{S} = \{\mathbf{s}_1, \ldots, \mathbf{s}_s\}$, we employ Monte Carlo Tree Search (MCTS) (Kocsis and Szepesvári, 2006a) to generate solution trees. For each question $\mathbf{s}_i \in \mathcal{S}$, given a predefined action set $\mathcal{A} = \{a_1, \ldots, a_{|\mathcal{A}|}\}$ and model π_θ, MCTS build a search tree $\mathcal{T}_i$ where: the root node represents question $\mathbf{s}_i$, each edge denotes an action $a \in \mathcal{A}$, and each child node n contains partial solutions generated by π_θ under the corresponding action. A path from root $\mathbf{s}_i$ to leaf node $\mathbf{n}_{i,j,d_{i,j}}$ forms a solution trajectory:

$$\mathbf{t}_{i,j} = \left(\mathbf{s}_i, a_{i,j,1}, \mathbf{n}_{i,j,1}, \ldots, a_{i,j,d_{i,j}}, \mathbf{n}_{i,j,d_{i,j}}\right), \quad (3)$$

Each intermediate node $\mathbf{n}_{i,j,\ell}$ is generated based on the cumulative context of its parent nodes and current action:

$$\mathbf{n}_{i,j,\ell} = \pi_\theta([\mathbf{s}_i, a_{i,j,1}, \mathbf{n}_{i,j,1}, \ldots, a_{i,j,\ell}]). \quad (4)$$

Through this process, for each seed question $\mathbf{s}_i$, we obtain multiple solution traces $\mathbb{T}_i = \{\mathbf{t}_{i,1}, \ldots, \mathbf{t}_{i,|\mathbb{T}_i|}\}$ (e.g., Figure 3). The MCTS algorithm assigns a final reward $R(\mathbf{t}_{i,j} \mid \mathbf{s}_i)$ to each trace $\mathbf{t}_{i,j} \in \mathbb{T}_i$. More details are in Appendix C.

Rejection Sampling. After MCTS exploration, we obtain multiple solution traces for each seed question $\mathbf{s}_i$. To identify the optimal path, we use a simple path selection metric from Wu et al. (2024):

$$\text{Score}(\mathbf{s}_i, \mathbf{t}_{i,j}) = b \cdot R(\mathbf{t}_{i,j} \mid \mathbf{s}_i) - (1-b) \cdot C(\mathbf{t}_{i,j}), \quad (5)$$

where $C(\mathbf{t}_{i,j})$ denotes path complexity (e.g, action count, $C(\mathbf{t}_{i,j}) = d_{i,j}$), and $b \in [0, 1]$ balances solution quality and complexity. This scoring function selects paths that maximize accuracy while maintaining procedural conciseness. For each question $\mathbf{s}_i \in \mathcal{S}$, we select the optimal trace via:

$$\mathbf{t}_{i,\text{best}} = \arg \max_{\mathbf{t}_{i,j} \in \mathbb{T}_i} \text{Score}(\mathbf{s}_i, \mathbf{t}_{i,j}). \quad (6)$$

Aggregation. Since each node in $\mathbf{t}_{i,\text{best}}$ corresponds to an instantiated action $a_{i,\ell} \in \mathcal{A}$, we retain the action sequence as a high-level thought template $T_i = (a_{i,1}, \ldots, a_{i,d_{i,\text{best}}})$. We then aggregate identical patterns to construct a template library $\mathcal{L} = \{\hat{T}_1, \ldots, \hat{T}_{|\mathcal{L}|}\}$. Aggregation is guided by Problem Condition Complexity (PCC) (Lee and Heyworth, 2000), defined as the number of prior conditions in $\mathbf{s}_i$; we obtain $\text{PCC}(\mathbf{s}_i) \in \mathbb{R}_{\geq 0}$ by prompting π_θ with $\mathbf{s}_i$. Each template stores both a structured thought pattern (e.g., $a_1 \rightarrow a_2$) and the average PCC of questions sharing this pattern:

$$\hat{T}_j = \left(\text{PCC}_{T_j}, T_j\right), \text{PCC}_{T_j} = \frac{1}{|\mathcal{I}_j|} \sum_{i \in \mathcal{I}_j} \text{PCC}(\mathbf{s}_i). \quad (7)$$

Figure 3: Structure of action-chain solution trajectories.

where $\mathcal{I}_j = \{i : T_i = T_j\}$. These templates represent generalized problem-solving strategies and serve as structured guidance in Section 3.2. More details are in Appendix C.

Notably, while we use MCTS to build reasoning trees and abstract them into structured templates, such templates can also be obtained through other ways, such as expert-crafted general solutions for problem categories. We leave this for future work.

3.2 Template-Guided Training

Taking GRPO as an example, we now integrate high-level structured template guidance into RL to enhance policy learning. We exploit the observation that for multi-step reasoning, models are more likely to generate one correct step than to complete the entire reasoning steps in a single inference.

Rollout with Structured Guidance. Following meta-reasoning principles (Russell and Wefald, 1991), we adaptively retrieve relevant templates for each training question $\mathbf{q}_t$. We compute its $\mathrm{PCC}(\mathbf{q}_t)$, define the distance $d(\mathbf{q}_t, \hat{T}_j) = |\mathrm{PCC}(\mathbf{q}_t) - \mathrm{PCC}_{T_j}|$ to each template $\hat{T}_j \in \mathcal{L}$, and select the top-$k$ most similar $\{\hat{T}_{i_1}, \ldots, \hat{T}_{i_k}\}$.

Given policy π_θ, query $\mathbf{q}_t$, and a retrieved template $T_i = (a_1, \ldots, a_{d_i})$, we roll out iteratively. Let $\oplus$ denote sequence concatenation. Define the history $\mathbf{p}_0 = \mathbf{q}_t$ and for $\ell = 1, \ldots, d_i$,

$$\mathbf{s}_\ell \sim \pi_\theta\big(\cdot \mid \mathbf{p}_{\ell-1}, a_\ell \big), \mathbf{p}_\ell = \mathbf{p}_{\ell-1} \oplus a_\ell \oplus \mathbf{s}_\ell. \quad (8)$$

The final template-guided rollout is $\mathbf{o} = \mathbf{p}_{d_i}$.

Multi-Guidance RL. Taking GRPO as an example, we now investigate how multiple template guidance decomposes RL's learning objective into templated sub-objectives, and elucidate how templates enhance sampling quality and RL training.

As illustrated above, given diverse templates $\{T_1, \ldots, T_{|g|}\}$, we independently sample G_i rollouts $\{\mathbf{o}_{i,1}, \ldots, \mathbf{o}_{i,G_i}\}$ for each template T_i and use

all rollouts to update the policy. Omitting KL term for simplicity, the optimization objective becomes

$$\tilde{\mathcal{J}}_{\mathrm{GRPO}}(\pi_\theta) = \frac{1}{\sum_{i=1}^{|g|} G_i} \sum_{i=1}^{|g|} \sum_{j=1}^{G_i} \frac{1}{|\mathbf{o}_{i,j}|} \sum_{t=1}^{|\mathbf{o}_{i,j}|} \quad (9)$$
$$\min\left[\rho_{i,j,t} A_{i,t}, \hat{\rho}_{i,j,t} A_{i,t}\right],$$

where the probability ratio is $\rho_{i,j,t} = \frac{\pi_\theta(o_{i,j,t}|\mathbf{q}, \mathbf{o}_{i,j,<t})}{\pi_{\theta_{\mathrm{old}}}(o_{i,j,t}|\mathbf{q}, \mathbf{o}_{i,j,<t})}$, and the clipped ratio is $\hat{\rho}_{i,j,t} = \mathrm{clip}(\rho_{i,j,t}; 1 - \epsilon, 1 + \epsilon)$.

For convenience, define the template-specific objective as:

$$\mathcal{J}_i(\pi_\theta) = \frac{1}{G_i} \sum_{j=1}^{G_i} \frac{1}{|\mathbf{o}_i|} \sum_{t=1}^{|\mathbf{o}_i|} \min\left[\rho_{i,j,t} A_{i,t}, \hat{\rho}_{i,j,t} A_{i,t}\right], \quad (10)$$

Then, Equation (9) can be rewritten as

$$\tilde{\mathcal{J}}_{\mathrm{GRPO}}(\pi_\theta) = \frac{1}{\sum_{i=1}^{|g|} G_i} \sum_{i=1}^{|g|} G_i \mathcal{J}_i(\pi_\theta). \quad (11)$$

which shows that template-guided training optimizes the model across multiple structured strategic patterns, with G_i weighting each template's contribution. We provide theoretical analysis below, and detailed proofs in Appendix B.

Proposition 3.1 (Stability and positive-sample guarantee. Proof in Appendix B.2). *Consider guided GRPO with $|g|$ independent guidance groups, each containing $G_i = G/|g|$ trajectories. If the per-trajectory positive-advantage probability in template group i is p_i, then the probability of at least one positive trajectory is*

$$P_{\mathrm{pos}} = 1 - P_0 = 1 - \prod_{i=1}^{|g|} (1 - p_i)^{G_i}. \quad (12)$$

Since $\log(1 - p)$ is concave, we have $\prod_i (1 - p_i)^{G_i} \leq (1 - \bar{p})^G$ for $\bar{p} = \frac{1}{|g|} \sum_i p_i$. Hence, grouping increases P_{pos} and reduces gradient variance, leading to improved training stability compared with the unguided setting.

Proposition 3.2 (Template transfer improves success probability. Proof in Appendix B.3). *Let $T^\star(\mathbf{q}')$ denote a template obtained from MCTS for a seed question $\mathbf{q}'$ with positive advantage. If the advantage function $A(\tau; \mathbf{q})$ is L-Lipschitz continuous in $\mathbf{q}$*

$$A(T^\star(\mathbf{q}'); \mathbf{q}) \geq A(T^\star(\mathbf{q}'); \mathbf{q}') - Ld(q, q'), \quad (13)$$

Table 1: **Performance comparison on Qwen2.5-Math-7B-Base**. The best results on each benchmark are highlighted in **bold**. ∗ represents significantly better than baselines (p<0.05).

Model	MATH500 ↑	AIME24 ↑	AMC ↑	Minerva ↑	Olympiad ↑	Avg. ↑
Qwen2.5-Math-7B-Base (Yang et al., 2024a)	50.8	13.3	42.5	12.1	17.2	27.2
Qwen2.5-Math-7B-Ins. (Yang et al., 2024a)	81.0	13.3	55.0	32.7	38.8	44.1
SimpleRL-Zero (Zeng et al., 2025)	74.6	26.7	60.0	27.6	35.8	44.9
OpenReasoner-Zero (Hu et al., 2025a)	81.0	16.7	57.5	32.7	43.2	46.2
PRIME-Zero (Cui et al., 2025a)	79.0	20.0	60.0	36.4	40.6	47.2
Oat-Zero (Liu et al., 2025)	79.6	30.0	60.0	34.2	39.9	48.7
RLOO (Ahmadian et al., 2024)	73.8	30.0	50.0	35.5	36.0	45.1
TemplateRL (Ours)	**78.8**	**33.3**	**67.5**	**36.7**	**41.2**	**51.5**∗
△ (↑)	+6.8%	+11.0%	+35.0%	+3.4%	+14.4%	+14.2%
GRPO (Shao et al., 2024)	76.2	16.7	55.0	32.7	38.1	43.8
TemplateRL (Ours)	**83.4**	**33.3**	**77.5**	**38.2**	**46.2**	**55.8**∗
△ (↑)	+9.4%	+99.4%	+40.9%	+16.8%	+21.2%	+27.4%

then for any new query **q** satisfying

$$A(T^\star(\mathbf{q}'); \mathbf{q}') > L\, d(\mathbf{q}, \mathbf{q}'), \qquad (14)$$

we have $A(T^\star(\mathbf{q}'); \mathbf{q}) > 0$. Therefore, retrieving a high-quality template from similar questions yields a mini-group on **q**, with positive advantage probability $r_h > p_h^{\text{policy}}$ (random rollouts).

3.3 Optional Template Update

During RL training, we optionally leverage correct rollouts to continually enrich the template library. Specifically, given a correct rollout **o**, we apply keyword-based or lightweight model-based pattern extraction to automatically identify the underlying action sequence (template) $T' = (a'_1, \ldots, a'_d)$. Such templates are then incorporated into the library for continual expansion. We empirically validate the effectiveness of dynamic template updates during both training and inference in Section 4.5.

4 Experiments

This section presents experimental settings, results, and analysis. We analyze TemplateRL from five aspects: (1) improved performance, (2) enhanced training stability, (3) broad generalization, (4) dynamic extensibility, and (5) ablation study.

4.1 Experimental Setup

Training Datasets. We use training data exclusively from MATH (Hendrycks et al., 2021) level 3-5 problems, yielding 5.5K examples. We randomly sample 500 instances to construct the template library, with the remaining 5K for training.

Evaluation. We evaluate on widely used reasoning benchmarks, including MATH500 (Hendrycks et al., 2021), AIME 2024 (Li et al., 2024), AMC (Li et al., 2024), Minerva (Lewkowycz et al., 2022), OlympiadBench (He et al., 2024), GSM8K (Cobbe et al., 2021), College Math (Tang et al., 2024), and Gaokao23 (Zhang et al., 2023). Since our training data focuses on math, we further assess out-of-domain generalization on BALROG (Paglieri et al., 2025) for agent tasks, GPQA-D (Rein et al., 2024) for graduate-level science, and MMLU-Pro (Wang et al., 2024b) for general knowledge answering.

Baseline Methods. For fair comparison, we benchmark TemplateRL with multiple representative baselines, including *Standard GRPO* (Shao et al., 2024), *SimpleRL-Zero* (Zeng et al., 2025), *OpenReasoner-Zero* (Hu et al., 2025b), *PRIME-Zero* (Cui et al., 2025a), and *Oat-Zero* (Liu et al., 2025). More details are provided in Appendix D.

Implementation Details. Following previous work (Cui et al., 2025a; Liu et al., 2025), we use Qwen2.5-Math-7B-Base (Yang et al., 2024a) as the default model. For RL training, we set $\beta = 0$ (no KL loss) and employ Dr.GRPO loss (Liu et al., 2025). Our configuration includes batch size 128, 16 samples per question, and 2 guidance templates ($|g| = 2$). The reward is binary accuracy verified by Math-Verify. We train for 500 steps on 8 A100 GPUs. More details are provided in Appendix D.

4.2 Improved Performance

Main Results. Table 1 presents performance comparisons across five competition-level reasoning benchmarks. The results reveal three findings:

Figure 4: **Performance comparison across different model scales and architectures.** For better visualization, MATH500 results are adjusted by subtracting 20 points for all models.

(a) Qwen2.5-Math-7B-Base   (b) Llama-3.2-3B-Base

Figure 5: **Training stability verification.** We evaluate on Qwen2.5-Math-7B-Base and Llama-3.2-3B-Base.

◇ *TemplateRL Significantly Outperforms Baselines.* TemplateRL achieves an average score of 55.8, outperforming the best baseline (Oat-Zero) by 7.1 points and GRPO by 12.0 points. On AIME24, TemplateRL achieves 33.3% versus GRPO's 16.7% (+99.4% relative gain), demonstrating the effectiveness of structure-guided policy optimization.

◇ *Greater Improvements on More Challenging Benchmarks.* Performance gains scale with task difficulty: MATH500 shows +9.4% improvement, while more challenging AMC and AIME24 exhibit +40.9% and +99.4%, respectively. This suggests explicit template provide greater benefits for problems requiring complex strategic decomposition.

◇ *Structure-Guided Training Generalizes Across RL Algorithms.* When applied to other RL algorithms (e.g., RLOO), TemplateRL maintains substantial improvements. This consistency further confirms that structure-guided training is a general apporach applicable to diverse RL algorithms.

Scalability Across Different Models. As shown in Figure 4, TemplateRL maintains consistent improvements across: (1) different scales, from 1.5B (Qwen2.5-Math-1.5B) to 8B parameters (Llama-3.1-8B-Instruct); (2) diverse architectures, both Qwen and Llama families show substantial improvements; and (3) various model types, both base models and instruction-tuned models benefit from structure-guided training (details in Table 6).

4.3 Enhanced Training Stability

We analyze training stability by comparing reward curves on Qwen2.5-Math-7B-Base and Llama-3.2-3B-Base. As shown in Figure 5, TemplateRL achieves higher training rewards than GRPO on both models overall. Critically, GRPO exhibits training collapse after 100 steps on Llama-3.2-3B-Base, with rewards dropping from 0.3 to near 0.0. This aligns with prior findings (Gandhi et al., 2025; Liu et al., 2025) that GRPO frequently fails on weaker models. In contrast, TemplateRL maintains stable dynamics, with rewards above 0.25.

TemplateRL's enhanced stability mainly stems from its structured template guidance during RL sampling. By providing explicit strategic patterns, templates equips weaker models with advanced decomposition capabilities typically exclusive to stronger models. This enables effective learning from challenging examples, which would otherwise yield sparse positive signal under standard GRPO training, as detailed in Section 3.

4.4 Broad Generalization

As shown in Table 2 and Figure 6, we examine TemplateRL's broad generalization through out-of-domain evaluation and multimodal extensions.

Cross-Domain Generalization. To assess out-of-domain (OOD) performance, we evaluate TemplateRL on diverse reasoning tasks beyond mathematical domains: BALROG (agentic reasoning in games), GPQA-D (graduate-level science), and MMLU-Pro (general knowledge). As shown in Figure 6, TemplateRL consistently outperforms GRPO across OOD tasks, with +6.1% performance gains on complex agentic scenarios (BALROG). This reveals that high-level template guidance effectively enhances model generalization to practical applications. Detailed results are in Appendix E.1.

Table 2: **Multimodal Extension.** We extend TemplateRL to multimodal domains with Qwen2.5-VL-3B-Instruct, and evaluate on multimodal reasoning benchmarks. TemplateRL consistently improves over GRPO across mathematical reasoning (MathVision, MathVerse, MathVista), general understanding (MMMU), and visual perception (BLINK).

Method	MathVision ↑	MathVerse ↑	MathVista ↑	MMMU ↑	BLINK ↑	Avg. ↑
CoT	27.7	27.3	61.1	51.2	45.1	42.5
GRPO	29.0	29.2	64.3	52.0	45.9	44.1
TemplateRL (Ours)	**29.8**	**30.9**	**65.4**	**52.8**	**47.2**	**45.2**
△ (↑)	+2.8%	+5.8%	+1.7%	+1.5%	+2.8%	+2.5%

Figure 6: **Cross-domain generalization verification.** We provide OOD results on Qwen2.5-Math-7B-Base.

Table 3: **Training-time dynamic extensibility.** We compare the fixed library baseline with dynamic expansion that continuously updates from successful rollouts.

Method	AIME24 ↑	AMC ↑	GPQA-D ↑	Avg. ↑
Fixed-5k	33.3	77.5	37.9	49.6
Dynamic-1k	16.7	52.5	20.2	29.8
Dynamic-2k	23.3	65.0	25.8	38.0
Dynamic-5k	36.7	80.0	38.9	51.9

Table 4: **Test-time dynamic extensibility.** We process test samples sequentially and expand the template library with majority-voting pseudo-supervision signals.

Method	AIME24 ↑	AMC ↑	GPQA-D ↑	Avg. ↑
w/o update	33.3	77.5	37.9	49.6
w/ update	36.7	77.5	40.4	51.6
△ (↑)	+10.2%	+0%	+6.6%	+4.1%

Multimodal Extension. Beyond textual reasoning, we extend TemplateRL to multimodal domains. We use the Geometry3K dataset (Lu et al., 2021) as training data and evaluate on diverse benchmarks, including mathematical reasoning (MathVision, MathVerse, MathVista), general understanding (MMMU), and visual perception (BLINK). Our settings are: batch size 128, 16 samples per question, and 2 template guidance. As shown in Table 2, TemplateRL consistently outperforms GRPO across all benchmarks, achieving +2.5% average improvement. This indicates that structured template guidance effectively generalizes beyond text modality to multimodal scenarios. More implementation details are provided in Appendix E.3.

4.5 Dynamic Extensibility

We further explore the dynamic extensibility of the template library, enabling continuous knowledge updates during both training and inference.

Training-time Template Expansion. We explore dynamic library expansion during training by continuously incorporating new templates from successful rollouts. After training on 1k, 2k, 3k, and 5k samples, we extract high-level action sequences from correct samples and add them to the library. As shown in Table 3, dynamic expansion achieves substantial improvements across all benchmarks, with notable gains on AIME24 (+10.2%).

This indicates that our explicit structured library enables continuous updates and reuse of successful exploration experiences, benefiting experience-intensive domains such as medical reasoning where iterative knowledge updates are necessary.

Test-time Template Expansion. Beyond training, we explore library expansion during inference. For each test sample, we retrieve five templates to generate multiple paths, apply majority voting for the answer, and immediately abstract patterns from voted outputs into the library before processing the next sample. Table 4 shows substantial improvements, demonstrating progressive library enrichment during inference using majority-voted pseudo-supervision. This continuous expansion capability helps for test-time scaling scenarios where models leverage accumulated knowledge from earlier predictions to improve subsequent ones.

4.6 Ablation Study and Analysis

Ablation Study. We investigate the effect of template guidance quantity ($|g| = 1, 2, 4$) on training dynamics with fixed total rollouts. Figure 8 shows training rewards and evaluation results across vary-

Figure 7: **Case Study.** TemplateRL produces a more structured and interpretable reasoning chain with clear steps.

(a) Training Reward Curve (b) Evaluation Performance

Figure 8: **Ablation study.** We provide results with different numbers of thought patterns (template guidance).

ing templates. For clearer visualization, we uniformly adjust performance on AMC and MATH by subtracting a fixed value without affecting conclusions. We observe that more diverse template guidance often yields higher rewards. We choose $|g| = 2$ by default for flexible consideration.

Case Study. As shown in Figure 7 on a geometry problem, while GRPO produces less readable outputs, TemplateRL first identifies its strategy (e.g., divide-and-conquer), then addresses each subproblem with clear solutions. This demonstrates how template-guided RL training enhances system readability and interpretability.

Other Analysis. We provide additional results and analysis in Appendix, including comprehensive OOD results (E.1), sensitivity to training data and pattern-matching metrics (E.6,E.7), ablation study (E.9), statistical analysis (E.10), and case study (F).

5 Related Work

RL for LLMs. Reinforcement learning (RL) has become a popular paradigm for enhancing LLM reasoning (Guo et al., 2025; Team et al., 2025; Yang et al., 2025). Recent research mainly focuses on: (1) algorithmic refinements that address RL's inherent limitations (Yu et al., 2025; Liu et al., 2025); (2) data-level optimizations (Zuo et al., 2025; Wang et al., 2025b); and (3) exploration enhancement (Wang et al., 2025a; Cui et al., 2025b). However, these methods remain constrained by unstructured self-sampling, lacking mechanisms to leverage structured guidance (Yue et al., 2025). TemplateRL addresses this by integrating explicit reasoning templates into policy optimization, steering training toward validated strategic patterns.

Reasoning with Template Guidance. Template-based guidance has been explored primarily at inference time, including retrieval-augmented generation (Asai et al., 2023; Zhao et al., 2024), task decomposition (Khot et al., 2022), and abstract thought patterns (Yang et al., 2024b; Wu et al., 2024). Few works systematically integrate structured templates into RL training. Our approach bridges this gap by constructing a template library and incorporating it during policy optimization. This enables effective learning of both concrete steps and underlying strategic logic, achieving superior performance, flexibility, and transferability.

6 Conclusion

We introduce TemplateRL, a template-guided RL framework for LLM reasoning. By constructing structured templates and integrating them during policy optimization, TemplateRL steers training toward validated strategic patterns. Experiments demonstrate significant improvements across models, datasets, and modalities. The framework exhibits cross-domain generalization, dynamic extensibility, and produces interpretable decision chains.

Limitations

In this paper, we primarily explore TemplateRL on text and multimodal reasoning tasks. In future work, we plan to extend the framework to more practical domains such as robotic control, scientific discovery, and embodied AI agents. Applying structured template guidance to real-world scenarios is expected to yield broader impact and demonstrate the generalizability of structured guidance across physical and interactive environments.

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Appendix

In this section, we provide a comprehensive elaboration of the TemplateRL algorithm's technical details. We describe the specific implementation of the Monte Carlo Tree Search algorithm, the construction process of our thought template library, and the adaptive retrieval and instantiation mechanisms for thought patterns during reasoning. Additionally, we provide supplementary experiments and case studies to illustrate our points further. The contents are organized as follows:

Contents

A TL;DR: Main Contributions and Takeaways

> **Takeaways**
>
> 1. **Structure-Guided RL Paradigm**: Augmenting policy optimization with explicit structured templates significantly improves training efficiency, stability, and generalization over unstructured self-sampling.
>
> 2. **Transferable High-Level Strategic Patterns**: Abstract reasoning templates exhibit remarkable cross-domain and cross-task transferability, enabling knowledge accumulation and flexible expert intervention through interpretable, editable structures.
>
> 3. **Effective Learning on Weaker Models**: Template-guided training compensates for limited model capacity, achieving stable training where standard RL frequently collapses.

Existing RL methods for LLM reasoning rely primarily on unstructured self-sampling to fit scalar rewards, producing inefficient rollouts that fail to capture transferable problem-solving strategies. While recent work has explored algorithmic refinements, data-level optimizations, and exploration enhancement, no prior approach has systematically integrated structured guidance into policy optimization to address the fundamental limitations of trajectory quality, strategy transferability, and interpretability. Our core contributions are:

- **Novel Structure-Guided RL Framework**: We introduce TemplateRL, which constructs an explicit reasoning template library via offline MCTS on minimal seed data, then seamlessly integrates these structured templates as guidance during online RL training. This design steers policy optimization toward validated strategic patterns.

- **Superior Performance and Stability**: TemplateRL outperforms GRPO by +99% on AIME, +41% on AMC, and +17% on Minerva Math on a 7B backbone, and consistently improves across scales (1.5B-8B) and architectures (Qwen, Llama). It maintains stable

training on weaker models (e.g., Llama-3.2-3B) where GRPO typically collapses.

- **Broad Generalization**: Structured templates transfer effectively across diverse domains (BALROG for agents, GPQA-D for science, MMLU-Pro for knowledge) and modalities (text, vision), highlighting the universality of high-level thought patterns.

- **Interpretability and Dynamic Extensibility**: TemplateRL produces structured reasoning trajectories with identifiable strategic patterns, enabling error diagnosis and expert intervention. The template library is interpretable, directly editable, and supports continuous updates during both training and inference.

B   Theoretical Analysis: Multi-Guidance RL and Template Transfer

B.1   Notation and assumptions

We summarize the notation used in the propositions below.

- π_θ: parametric policy, π_{ref}: fixed reference policy.

- For a single training query $\mathbf{q}$, GRPO collects a total of G trajectories (trials). These G trajectories are partitioned into m groups (small groups), where $m = |g|$ and we assume for simplicity that G is divisible by m; denote the per-group size by $g_s := G/m$.

- Within group $h \in \{1, \ldots, m\}$, trajectories may be dependent (e.g., because they share the same guidance template or retrieved seed); different groups are assumed approximately independent (e.g., they come from different guidance templates or independent retrievals).

- For group h, let $X_{h,j}$ be the indicator that the j-th trajectory in group h is positive (i.e., has $A > 0$). Define the group-success event $Y_h := \mathbf{1}\{\exists j \in [1, g_s] : X_{h,j} = 1\}$ (i.e., at least one positive in group h).

- Denote the per-group success probability by $p_h := \Pr(Y_h = 1)$. In a symmetric setting we write p_{grp} for the common per-group success probability.

- Other notation (seed database $\mathcal{S}$, retrieval Retr_k, seed-transfer success probability r, etc.) follows the earlier appendix.

B.2   Proof of Proposition 3.1

Proposition 3.1. (Stability and positive-sample guarantee) *Under the grouped sampling setup above, suppose different groups are independent (or weakly dependent so that group-level covariances are negligible). Then:*

1. *The probability that the entire batch of G trajectories contains at least one positive sample equals*

$$P_{\geq 1}^{\text{batch}} = 1 - \prod_{h=1}^{m}(1 - p_h).$$

In particular, if groups are symmetric ($p_h \equiv p_{\text{grp}}$), then

$$P_{\geq 1}^{\text{batch}} = 1 - (1 - p_{\text{grp}})^m,$$

which is strictly increasing in m and approaches 1 as $m \to \infty$ for any fixed $p_{\text{grp}} > 0$.

2. *Let g_h denote the (vector-valued) gradient contribution from group h (e.g., the average of per-trajectory contributions within the group). Assume group contributions are independent with $\mathbb{E}[g_h] = \bar{g}_{\text{grp}}$ and $\text{Var}[g_h] = \Sigma_{\text{grp}}$ (bounded). Then the variance of the batch-averaged estimator $\hat{g} = \frac{1}{m}\sum_{h=1}^{m} g_h$ satisfies*

$$\text{Var}[\hat{g}] \preceq \frac{1}{m}\Sigma_{\text{grp}} + o\left(\frac{1}{m}\right),$$

i.e. the leading-order variance scales as $\mathcal{O}(1/m)$ (controlled by the number of independent groups), even if within-group samples are dependent.

Proof sketch. (1) For each group h define the Bernoulli variable Y_h indicating whether the group contains at least one positive trajectory. By definition $\Pr(Y_h = 1) = p_h$. If groups are independent, the probability that no group contains a positive sample equals $\prod_{h=1}^{m}\Pr(Y_h = 0) = \prod_{h=1}^{m}(1 - p_h)$, and the complement yields the stated formula. In the symmetric case set $p_h \equiv p_{\text{grp}}$.

Monotonicity in m holds because multiplying an additional factor $(1 - p_{\text{grp}})$ decreases the product, increasing its complement; the limit follows as $(1 - p_{\text{grp}})^m \to 0$ when $p_{\text{grp}} > 0$.

(2) Let g_h denote the aggregated gradient contribution from group h, e.g., $g_h = \frac{1}{g_s}\sum_{j=1}^{g_s} g_{h,j}$ where $g_{h,j}$ is the per-trajectory contribution. Even if the $g_{h,j}$ within a group are dependent, g_h is a

single random vector (with finite second moments). Under independence across groups,

$$\mathrm{Var}\left(\frac{1}{m}\sum_{h=1}^{m} g_h\right) = \frac{1}{m^2}\sum_{h=1}^{m}\mathrm{Var}[g_h]$$

$$= \frac{1}{m}\left(\frac{1}{m}\sum_{h=1}^{m}\mathrm{Var}[g_h]\right)$$

$$\preceq \frac{1}{m}\Sigma_{\mathrm{grp}},$$

where Σ_{grp} is a uniform bound on $\mathrm{Var}[g_h]$. If group covariances are weak (mixing), their contribution is lower-order, giving the $o(1/m)$ term. Thus the dominant variance reduction is governed by the number of independent groups m, not the raw total sample size G when within-group dependence is present. Intuitively: dependent samples within a group provide limited additional independent information, so averaging across independent groups is the effective variance reducer. □

Remark. The theorem clarifies that when data are collected in dependent clusters (groups) the effective sample size is the number of independent clusters m. Therefore, to achieve variance reduction it is valuable to increase the number of independent groups (e.g., by using multiple distinct guidance templates or distinct retrievals) rather than only increasing correlated samples within a single group.

B.3 Proof of Proposition 3.2

Proposition 3.2. (Template transfer improves success probability under grouped sampling) *Consider the grouped sampling with m groups, and suppose group h uses PCC retrieval $\mathrm{Retr}_k^{(h)}(\mathbf{q})$ returning k templates whose best trajectories are used as candidate rollouts within that group. Let $T(\mathbf{q})$ be a template produced by MCTS for a seed query $\mathbf{q}$, the per-mini-group template-transfer success probability is:*

$$r_h = \mathrm{Pr}\left(\exists\, T \in \mathrm{Retr}_k^{(h)}(\mathbf{q})\ s.t.\ A(T(\mathbf{q}),\mathbf{q}) > 0\right).$$

Assume different groups are independent. Then:

1. *Per-mini-group: the event that mini-group h contains at least one positive trajectory due to template transfer occurs with probability at least r_h. If $r_h > p_h^{\mathrm{policy}}$ (the per-group success probability under policy rollouts), template transfer strictly improves per-group success.*

2. *Batch-level: the probability that the full batch has at least one positive sample is*

$$P_{\geq 1}^{\mathrm{template}} = 1 - \prod_{h=1}^{m}(1 - r_h),$$

which exceeds the policy-rollout probability $1 - \prod_{h=1}^{m}(1 - p_h^{\mathrm{policy}})$ whenever $r_h \geq p_h^{\mathrm{policy}}$ for all h and strict for some h.

Proof sketch. (1) and (2) follow from the same complement argument as in the proof of Proposition 3.1 (B.2). □

B.4 Why template transfer increases success probability

The key reason why template transfer improves the chance of positive samples is that MCTS produces high-quality trajectories that approximately maximize expected reward for their original problem. If a new query $\mathbf{q}$ is similar to a seed query $\mathbf{q}'$, then the action sequence that solves $\mathbf{q}'$ remains near-optimal for $\mathbf{q}$ under mild regularity assumptions.

Lemma B.1 (Transfer success under smoothness). *Let $T^\star(\mathbf{q}')$ be a template produced by MCTS for a seed query $\mathbf{q}'$ such that $A(T^\star(\mathbf{q}'); \mathbf{q}') > 0$. Suppose the advantage function $A(T; \mathbf{q})$ is L-Lipschitz continuous in $\mathbf{q}$ for all trajectories τ:*

$$\left|A(T; \mathbf{q}) - A(T; \mathbf{q}')\right| \leq L\, d(\mathbf{q}, \mathbf{q}')$$

for some distance metric d. Then, if

$$A(T^\star(\mathbf{q}'); \mathbf{q}') > L\, d(\mathbf{q}, \mathbf{q}'),$$

it follows that $A(T^\star(\mathbf{q}'); \mathbf{q}) > 0$.

Proof sketch. By Lipschitz continuity:

$$A(T^\star(\mathbf{q}'); \mathbf{q}) \geq A(T^\star(\mathbf{q}'); \mathbf{q}') - L\, d(\mathbf{q}, \mathbf{q}').$$

If the right-hand side is positive, the inequality implies the transferred trajectory still has positive advantage on $\mathbf{q}$. □

Remark (Template transfer as a high-probability event). MCTS templates $T^\star(\mathbf{q}')$ are approximately optimal for $\mathbf{q}'$ and often satisfy $A(T^\star(\mathbf{q}'); \mathbf{q}') \gg 0$. Thus, for new queries $\mathbf{q}$ that are close to $\mathbf{q}'$ (small $d_{\mathrm{PCC}}(\mathbf{q}, \mathbf{q}')$), the inequality in Lemma B.1 is likely to hold. This implies that retrieving high-quality templates from similar seed problems leads to transferred trajectories with positive advantage on the new problem. Consequently, the per-group template success probability r_h is strictly higher than the baseline policy success probability p_h^{policy}.

Figure 9: An illustration of four phases in an iteration of MCTS for complex reasoning tasks.

C More Details about TemplateRL

C.1 Monte Carlo Tree Search (MCTS)

As a heuristic search algorithm, MCTS has demonstrated remarkable success in complex reasoning and decision-making environments (Chaslot et al., 2008; Ye et al., 2021; Zhou et al., 2024). The algorithm conceptualizes search spaces as tree structures and has achieved significant breakthroughs across various domains, most notably in game-playing AI such as AlphaGo and AlphaZero (Dong et al., 2020). As described in Section 2.3 in the main text, we employ MCTS to generate solution trees based on a small set of 500 seed samples.

To implement MCTS effectively, we first define a predefined action set. Understanding human complex reasoning is crucial for modeling cognitive processes (Jaffe et al., 2023). Existing studies distinguish between two cognitive systems: System 1 and System 2 (Kahneman, 2011; Da Silva, 2023). While "System 1" represents fast, intuitive, yet error-prone thinking, "System 2" involves slow, deliberative thinking with superior performance. With the emergence of advanced models like OpenAI's o1, developing efficient "System 2" approaches to emulate human cognitive processes has gained significant research attention (Qin et al., 2024; Guo et al., 2025).

Inspired by this and following previous work (Wu et al., 2024; Qi et al., 2024), we introduce five human-like reasoning actions to bridge the gap between model reasoning and human cognition:

- **Divide and Conquer (DC, a_1)**: Approaching complex problems by breaking them into manageable sub-problems for easier resolution.

- **Self-Reflection (SR, a_2)**: Assessing and refining prior solutions during the reasoning process to ensure correctness.

- **System Analysis (SA, a_3)**: Analyzing the overall structure of the problem and identifying the constraints and conditions before addressing it, thereby clarifying task requirements effectively.

- **One-Step Thought (OST, a_4)**: Aiming to address a single aspect of the problem through a focused and concise reasoning step.

- **Chain-of-Thought (CoT, a_5)**: Adopting a sequential reasoning process that builds a series of connected logical steps.

Based on the above predefined action set $\mathcal{A} = \{a_1, ..., a_{|\mathcal{A}|}\}$ and model π_θ, for each question $\mathbf{s}_i \in \mathcal{S}$, MCTS builds a search tree $\mathcal{T}_i$ where: the root node represents question $\mathbf{s}_i$, each edge denotes an action $a \in \mathcal{A}$, and each child node n contains partial solutions generated by π_θ under the corresponding action. A path from root $\mathbf{s}_i$ to leaf node $\mathbf{n}_{i,j,d_{i,j}}$ forms a solution trajectory:

$$\mathbf{t}_{i,j} = \big(\mathbf{s}_i, a_{i,j,1}, \mathbf{n}_{i,j,1}, \ldots, a_{i,j,d_{i,j}}, \mathbf{n}_{i,j,d_{i,j}}\big),$$

Each intermediate node $\mathbf{n}_{i,j,\ell}$ is generated based on the cumulative context of its parent nodes and current action:

$$\mathbf{n}_{i,j,\ell} = \pi_\theta([\mathbf{s}_i, a_{i,j,1}, \mathbf{n}_{i,j,1}, \ldots, a_{i,j,\ell}]).$$

Specifically, the MCTS algorithm involves an iterative search process with four key steps: selection, expansion, simulation, and backpropagation.

Figure 10: Thought Template Visualization. On the left, the seed dataset contains 3 questions with the same reasoning pattern, "DC→DC→CoT", corresponding to template k on the right. The template includes both the reasoning pattern and characteristics used for matching during testing, based on cognitive complexity metrics. When a test question q matches template k, reasoning follows the corresponding pattern "DC→DC→CoT".

(1) Selection. Starting from the root node, we identify optimal nodes for expansion by traversing the tree level-by-level until reaching a leaf node (maximum depth or final answer). To balance exploration and exploitation, we employ Upper Confidence Bounds applied to Trees (UCT) (Kocsis and Szepesvári, 2006b):

$$\text{UCT}(\mathbf{n}) = Q(\mathbf{n}) + w\sqrt{\frac{\ln N(\mathbf{n}_p)}{N(\mathbf{n})}},$$

where $Q(\mathbf{n})$ is the reward value, $N(\mathbf{n})$ is the visit count, $\mathbf{n}_p$ is the parent node, and w is the exploration weight. The node with the highest UCT value is selected.

(2) Expansion. The selected node $\mathbf{n}$ is expanded by sampling n actions from π_θ and generating corresponding reasoning outcomes. These n child nodes are then added to the tree.

(3) Simulation. Starting from the selected node, we iteratively sample and expand nodes until reaching a terminal state. To enhance efficiency, we implement an early termination strategy based on self-consistency (Wang et al., 2023): if the model's consistency score exceeds threshold c (i.e., $\text{SC}(\mathbf{n}) > c$), the simulation terminates early.

(4) Backpropagation. Upon simulation completion, node information is updated along the simulation path. Visit counts are incremented ($N(\mathbf{n}) \leftarrow N(\mathbf{n}) + 1$), and node value $Q(\mathbf{n})$ is propagated backward to its parent node $\mathbf{n}_p$:

$$Q(\mathbf{n}_p) \leftarrow (1-\alpha)Q(\mathbf{n}_p) + \alpha Q(\mathbf{n}),$$

where α is a discount factor. For terminal nodes, following Qi et al. (2024), we use the confidence of self-consistency majority voting as the reward value.

Through this process, we obtain diverse solution traces $\mathbb{T} = \{\mathbf{t}_1, \mathbf{t}_2, \ldots, \mathbf{t}_t\}$ for each question $\mathbf{s}_i \in \mathcal{S}$. MCTS assigns a final reward $R(\mathbf{t}_j|\mathbf{s}_i)$ to each trace $\mathbf{t}_j \in \mathbb{T}$.

Figure 9 illustrates the four phases in an iteration, expanding the tree and then updating reward values.

C.2 Thought Template Library Construction

After MCTS exploration, we obtain multiple solution paths for each question $\mathbf{s}_i \in \mathcal{S}$. Following Section 3.1, we identify the best reasoning path and abstract it into generalizable templates. To select the optimal trajectory, we employ the balanced scoring metric:

$$\text{Score}(\mathbf{s}_i, \mathbf{t}_j) = b \cdot R(\mathbf{t}_j|\mathbf{s}_i) - (1-b) \cdot C(\mathbf{t}_j),$$

where $C(\mathbf{t}_j)$ represents trajectory complexity (action count), and $b = 0.95$ balances solution quality against complexity. This function selects trajectories that maximize accuracy while maintaining conciseness.

For each question $\mathbf{s}_i \in \mathcal{S}$, we select the optimal trace $\mathbf{t}_{i,\text{best}} = \arg\max_{\mathbf{t}_j} \text{Score}(\mathbf{s}_i, \mathbf{t}_j)$. Since each node in $\mathbf{t}_{i,\text{best}}$ corresponds to an action $a_{j,\ell} \in \mathcal{A}$, we extract the action sequence as a high-level template $T_j = (a_1, \ldots, a_d)$. For instance, a successful solution trajectory might follow the pattern $a_1 \rightarrow a_2 \rightarrow a_4$ (System Analysis → One-Step Thought → Divide and Conquer).

> **System Prompt:** A conversation between User and Assistant. The user asks a question, and the Assistant solves it step by step. The assistant first thinks about the reasoning process in the mind and then provides the user with the answer, i.e., reasoning process here ... the answer is: \boxed{answer}.

Figure 11: The system prompt used for all experiments.

To organize extracted patterns, we use Problem Condition Complexity (PCC) (Lee and Heyworth, 2000; Embretson and Daniel, 2008) as a categorization metric. PCC quantifies the number of prior conditions in question $\mathbf{s}_i$ and can be computed by model π_θ. Similar problems tend to share similar PCC values, making this metric effective for pattern aggregation.

Through this process, each question $\mathbf{s}_i \in \mathcal{S}$ is associated with its optimal template, with some questions naturally sharing identical patterns. Our final template library $\mathcal{L} = \{\hat{T}_1, \ldots, \hat{T}_s\}$ consists of entries where each template $\hat{T}_j$ contains both a high-level pattern and the average PCC of questions sharing this pattern: $\hat{T}_j = (\text{PCC}_{T_j}, T_j)$. These templates represent generalized problem-solving strategies and serve as structured guidance during RL training.

For better visualization, we provide a detailed example thought template in Figure 10.

C.3 Adaptive Template Retrieval

During RL training, we employ an adaptive retrieval mechanism to identify relevant reasoning strategies from our template library. This approach follows meta-reasoning principles (Russell and Wefald, 1991; De Sabbata et al., 2024), which emphasize selecting appropriate strategies based on problem characteristics.

Template Retrieval. For each training question $\mathbf{q}_t$, we first compute its PCC metric, which characterizes the question's structure and complexity. We then calculate the distance to each template $\hat{T}_j \in \mathcal{L}$:

$$d_j = |\text{PCC}_{\mathbf{q}_t} - \text{PCC}_{T_j}|.$$

This distance quantifies similarity between the current question and problems from which each template was derived.

Template Selection and Application. We rank templates by distance and select the top-k most similar: $\{\hat{T}_{i_1}, \ldots, \hat{T}_{i_k}\}$. These selected templates contain high-level patterns $(a_1, \ldots, a_d)$ proven effective for problems with similar complexity profiles.

During rollout generation, retrieved templates serve as structured guidance that balances exploiting known successful strategies with model-internal exploration. This adaptive mechanism ensures the model leverages appropriate reasoning strategies based on problem characteristics, enabling more targeted and effective sampling across diverse problem types.

D Experimental Details

In addition to the implementation details in the main paper, we provide supplementary details here:

Baseline Methods. For fair comparison, we benchmark TemplateRL with the following baselines on Qwen2.5-Math-7B: (1) *GRPO* (Shao et al., 2024), using the same 5.5k training samples; (2) *SimpleRL-Zero* (Zeng et al., 2025), applying GRPO to 24k samples from GSM8K and MATH; (3) *OpenReasoner-Zero* (Hu et al., 2025b), employing PPO with 129k samples; (4) *PRIME-Zero* (Cui et al., 2025a), utilizing implicit process rewards on 150k NuminaMath (Li et al., 2024) queries; (5) *Oat-Zero* (Liu et al., 2025), introducing Dr.GRPO to mitigate length bias on 8k MATH questions.

Implementation Details. During training, we generate with rollout parameters of temperature=0.8 and top-p=0.95, and a maximum generation of 1500 tokens. The reward function is a binary accuracy metric verified by Math-Verify, defined as $r(\mathbf{o}) = \mathbb{1}\{\mathbf{o} \text{ contains the correct final answer}\}$. Moreover, we employ a cosine learning rate decay with warm-up. The maximum learning rate is set at 3×10^{-6}, and the warm-up ratio is set at 0.1. We use the same system prompt in Figure 11 for all experiments. All experiments were conducted on a machine running Ubuntu 22.04, equipped with NVIDIA A100-80GB GPUs.

Table 5: Detailed results on three representative out-of-distribution benchmarks (Qwen2.5-Math-7B-Base).

Model	GPQA-D↑	MMLU-Pro↑	BALROG↑
Qwen2.5-Math-7B-Base (Yang et al., 2024a)	15.2	19.2	12.5
Qwen2.5-Math-7B-Instruct (Yang et al., 2024a)	28.8	36.1	15.4
SimpleRL-Zero (Zeng et al., 2025)	21.2	36.9	17.4
OpenReasoner-Zero (Hu et al., 2025a)	30.3	54.6	28.3
PRIME-Zero (Cui et al., 2025a)	20.2	34.3	24.1
Oat-Zero (Liu et al., 2025)	25.8	41.8	26.2
GRPO (Shao et al., 2024)	31.3	42.1	25.4
TemplateRL (Ours)	**37.9**	**49.5**	**31.5**
△ (↑)	+21.1%	+17.6%	+24.0%

E Supplementary Results

E.1 Out-of-Domain Results

We present supplementary out-of-distribution generalization results in Table 5.

E.2 Extension to More Models

In addition to the results presented in the main text based on Qwen2.5-Math-1.5B-Base, Qwen2.5-7B-Instruct, and Llama3.1-8B-Instruct, we further evaluate TemplateRL on two additional models: Qwen2.5-Math-1.5B-Instruct and Llama3.2-3B-Instruct. As illustrated in Table 6, TemplateRL achieves an average performance improvement of 3.2% on Qwen2.5-Math-1.5B-Instruct and 5.6% on Llama3.2-3B-Instruct.

E.3 Extension to Multimodal Domains

As shown in Table 2 in the main text, we extend TemplateRL to multimodal domains. We provide more details here. We employ the Geometry3K dataset (Lu et al., 2021), which contains geometry problems with both textual descriptions and visual diagrams, providing a challenging testbed for multimodal reasoning capabilities. We utilize Qwen2.5-VL-3B-Instruct as the base model and follow the EasyR1 pipeline (Zheng et al., 2025) for RL training. Our training configuration maintains consistency with the text-only setting: batch size of 128, generating 16 samples per prompt, with the number of guidance patterns set to 2 (i.e., $|g| = 2$). The generation length is configured to 4096 tokens to accommodate detailed multimodal reasoning processes. We evaluate the multimodal extension across five diverse benchmarks spanning different reasoning domains: (1) **Multimodal Mathematical reasoning**: MathVision (Wang et al., 2024a),

MathVerse (Zhang et al., 2025), and MathVista (Lu et al., 2023) for multimodal mathematical problem solving; (2) **General reasoning**: MMMU (Yue et al., 2024) for broad multimodal understanding; and (3) **Visual perception**: BLINK (Fu et al., 2025) for visual comprehension tasks.

As shown in Table 2, TemplateRL demonstrates consistent improvements over both CoT baseline and vanilla GRPO across most benchmarks. Notably, TemplateRL achieves the best performance on MathVision (29.8 vs. 27.7 CoT baseline, +7.6% improvement) and MathVerse (30.9 vs. 27.3 CoT baseline, +13.2% improvement), indicating that thought-augmented training effectively transfers to multimodal mathematical reasoning scenarios. While performance on MathVista shows some variation, the overall trend suggests that template guidance integration remains beneficial in multimodal settings. These results demonstrate that TemplateRL's core principle lies in integrating structured reasoning guidance with policy optimization, which generalizes beyond pure text domains to multimodal reasoning tasks. This extension highlights the framework's potential for broader applications across diverse modalities and reasoning scenarios.

E.4 Extension to Other RL Algorithms

To demonstrate the generalizability of our thought-augmented framework beyond GRPO, we investigate its compatibility with alternative policy optimization algorithms. Specifically, we extend TemplateRL to REINFORCE Leave-One-Out (RLOO) (Ahmadian et al., 2024), a variance-reduced policy gradient method that has shown effectiveness in model fine-tuning. Following our default configuration, we set the number of guidance

Table 6: Accuracy (%) results of different LLMs across eight benchmarks. The best results in each box are highlighted in **bold**. We provide the relative improvement of our method compared to GRPO.

Method	AIME24↑	AMC↑	MATH500↑	GSM8K↑	Minerva↑	Olympiad↑	CollegeMath↑	Gaokao23↑	Avg.↑
				Qwen2.5-Math-1.5B-Base (Yang et al., 2024a)					
CoT	10.0	42.5	59.0	74.6	24.3	27.6	39.5	49.6	40.9
GRPO	13.3	40.0	66.4	74.7	25.0	30.1	40.5	52.7	42.8
Ours	**16.7**	**55.0**	**69.0**	**84.2**	**31.6**	**33.6**	**47.3**	**54.8**	**49.0**
△ (↑)	+25.6%	+37.5%	+3.9%	+12.7%	+26.4%	+11.7%	+16.8%	+4.0%	+14.5%
				Qwen2.5-Math-1.5B-Instruct (Yang et al., 2024a)					
CoT	6.7	47.5	68.2	76.8	28.3	36.9	47.1	63.1	46.8
GRPO	13.3	52.5	**76.8**	85.9	28.3	36.7	45.9	65.2	50.6
Ours	**16.7**	**55.0**	76.0	**86.5**	**29.4**	**39.7**	**48.3**	**65.7**	**52.2**
△ (↑)	+25.6%	+4.8%	-1.0%	+1.0%	+3.9%	+8.2%	+5.3%	+1.0%	+3.2%
				Qwen2.5-7B-Instruct (Qwen Team, 2024)					
CoT	13.3	47.5	73.2	90.0	30.5	38.8	46.9	64.2	50.5
GRPO	13.3	57.5	76.6	90.1	32.4	36.1	44.5	62.9	51.6
Ours	**16.7**	**67.5**	**78.0**	**91.5**	**36.8**	**40.6**	**50.6**	**65.2**	**55.8**
△ (↑)	+25.6%	+17.4%	+1.9%	+1.6%	+13.6%	+12.5%	+13.7%	+3.7%	+8.1%
				Llama-3.1-8B-Instruct (Grattafiori et al., 2024)					
CoT	3.3	20.0	36.6	77.2	16.2	15.9	13.3	29.9	26.5
GRPO	3.3	22.5	45.0	82.9	21.0	16.1	31.7	40.8	32.9
Ours	**6.7**	**30.0**	**52.2**	**85.2**	**26.8**	**17.3**	**34.1**	**42.6**	**36.7**
△ (↑)	+103.0%	+33.3%	+16.0%	+2.8%	+27.7%	+7.5%	+7.6%	+4.5%	+11.6%
				Llama-3.2-3B-Instruct (Grattafiori et al., 2024)					
CoT	6.7	20.0	38.3	69.3	11.8	12.6	23.8	33.5	27.6
GRPO	3.3	25.0	47.8	75.2	17.6	14.5	34.1	40.8	32.2
Ours	**6.7**	**27.5**	**48.8**	**78.8**	**18.4**	**16.0**	32.5	**43.1**	**34.0**
△ (↑)	+103.0%	+10.0%	+2.1%	+4.8%	+4.6%	+10.4%	-4.7%	+5.7%	+5.6%

Table 7: Extension to other RL algorithms. Results demonstrate consistent improvements when integrating thought-augmented training with alternative RL algorithms, highlighting the generalizability of our approach.

Method	AIME24↑	AMC↑	MATH500↑	GSM8K↑	Minerva↑	Olympiad↑	CollegeMath↑	Gaokao23↑	Avg.↑
				Qwen2.5-Math-7B-Base (Yang et al., 2024a)					
RLOO	30.0	50.0	73.8	82.7	35.5	36.0	39.8	64.2	51.5
TemplateRL	**33.3**	**67.5**	**78.8**	**85.8**	**36.7**	**41.2**	**41.8**	**68.1**	**56.7**
△ (↑)	+11.0%	+35.0%	+6.8%	+3.7%	+3.4%	+14.4%	+5.0%	+6.1%	+10.1%
				Qwen2.5-Math-1.5B-Base (Yang et al., 2024a)					
RLOO	20.0	50.0	68.0	82.6	28.7	32.0	41.4	51.2	46.7
TemplateRL	**26.7**	**57.5**	**75.6**	**84.7**	**33.8**	**33.8**	**41.8**	**52.9**	**50.9**
△ (↑)	+33.5%	+15.0%	+11.2%	+2.5%	+17.8%	+5.6%	+1.0%	+3.3%	+9.0%

patterns to 2 (i.e., $|g| = 2$) and maintain consistent experimental settings across both model scales.

As shown in Table 7, TemplateRL consistently enhances RLOO performance across all benchmarks and model configurations. For Qwen2.5-Math-7B-Base, RLOO+TemplateRL achieves substantial improvements over vanilla RLOO, with notable gains on AMC (+17.5 points, +35.0% relative improvement) and AIME24 (+3.3 points, +11.0% relative improvement). Similar trends emerge for the smaller Qwen2.5-Math-1.5B-Base model, where RLOO+TemplateRL delivers consistent improvements across all metrics, achieving an average

performance boost of +4.2 points (+9.0% relative improvement). These results demonstrate that the benefits of thought-augmented training extend beyond GRPO to other RL algorithms, highlighting the framework's versatility and broad applicability.

E.5 Ablation Study on the Action Space

To evaluate the sensitivity of our method to the action space used for constructing the thought template library, we conduct a comprehensive ablation study using Qwen2.5-Math-7B-Base as the backbone model. We systematically test various action subsets, ranging from a single action to the

Table 8: Performance comparison across different action spaces on Qwen2.5-Math-7B-Base. Results demonstrate consistent improvements with expanded spaces.

Action Space	MATH500 ↑	AIME24 ↑	AMC ↑	Avg. ↑
$\{a_5\}$ (i.e. CoT)	76.8	23.3	67.5	55.9
$\{a_1, a_5\}$	80.6	26.7	70.0	59.1
$\{a_1, a_4, a_5\}$	82.0	30.0	72.5	61.5
$\{a_1, a_3, a_4, a_5\}$	82.8	30.0	77.5	63.4
$\{a_1, a_2, a_3, a_4, a_5\}$	83.4	33.3	77.5	64.7

Table 9: Performance scaling with training data size on Qwen2.5-Math-7B-Base. 'Syn.' represents synthetic data. Results demonstrate positive scaling behavior, with notable improvements on challenging benchmarks.

Data Size	MATH500 ↑	AIME24 ↑	AMC ↑	Avg. ↑
5K (Original)	83.4	33.3	77.5	64.7
10K (+ 5K Syn.)	84.0	36.7	77.5	66.1
15K (+ 10K Syn.)	84.6	36.7	80.0	67.1

complete five-action set, across three representative benchmarks.

As shown in Table 8, expanding the action space consistently yields performance improvements, with the full five-action set $\{a_1, a_2, a_3, a_4, a_5\}$ achieving optimal results (64.7 average). Notably, even minimal subsets demonstrate reasonable performance, with the single-action baseline $\{a_5\}$ (equivalent to standard Chain-of-Thought) achieving 55.9 average accuracy. The incremental gains from $\{a_5\}$ to $\{a_1, a_5\}$ (+3.2 points) and subsequent expansions demonstrate that each additional action contributes meaningfully to the reasoning process. These results highlight two key insights: (1) TemplateRL's robustness to action space configuration, enabling deployment in resource-constrained scenarios with reduced action sets, and (2) the complementary nature of different reasoning actions, where diverse problem-solving strategies collectively enhance performance. This flexibility allows practitioners to adapt the framework based on task-specific requirements.

E.6 Sensitivity to Training Data Size

As mentioned in the main paper, we select training data exclusively from the MATH dataset (Hendrycks et al., 2021), using only 5k MATH level 3-5 problems for RL training. Given the prevalence of large-scale datasets, we investigate whether there exists a scaling law for performance versus training data size. Specifically, we augment our original 5K sample set by leveraging the OpenAI-o1 model to generate an additional

10K synthetic problems of similar difficulty, along with their solutions. To ensure solution quality, we employ deepseek-R1 for further verification and correction, filtering out erroneous samples and refining incorrect solutions. This process yields 10k high-quality synthetic data that maintains consistency with the original MATH distribution. We then conduct experiments using mixed datasets of varying sizes: 5k (original), 10k (original + 5k synthetic), and 15k (original + 10k synthetic).

As shown in Table 9, TemplateRL exhibits positive scaling behavior with increased training data. Performance improves from 64.7% (5K) to 66.1% (10K) and further to 67.1% (15K), representing consistent gains of +1.4 and +2.4 percentage points respectively. Notably, the most substantial improvements occur on the challenging AIME24 benchmark (+3.4 percentage points from 5K to 10K), suggesting that additional training data particularly benefits complex reasoning tasks. While the scaling trend is encouraging, we observe that TemplateRL can achieve competitive performance even with limited training data, making it practical for resource-constrained scenarios. More comprehensive investigations into sample diversity and scaling to larger datasets are left for future work.

E.7 Sensitivity to Pattern-Matching Metrics

Beyond the Problem Condition Complexity (PCC) metric (Lee and Heyworth, 2000; Salado and Nilchiani, 2014) employed in the main paper for retrieving optimal thought patterns, we explore alternative pattern-matching strategies to assess the robustness of our approach. Specifically, we investigate three additional metrics: (1) **Subquestion-count (SC)**: the number of subquestions an unstructured problem can be decomposed into; (2) **Problem semantics (PS)**: semantic similarity-based matching using sentence embeddings; and (3) **Unified ranking**: a composite metric that computes PCC, SC, and PS for each incoming question, performs triple relevance ranking across thought templates, and selects the top candidates through weighted averaging.

As shown in Table 11, while the unified ranking metric achieves the highest overall performance (66.0 average), the PCC-only approach demonstrates competitive results (64.7 average) with lower computational overhead. The semantic-based matching (PS-only) shows the weakest performance, likely due to the abstract nature of mathematical reasoning patterns that may not be

Table 10: Statistics of reasoning actions: proportion of each action relative to all actions in the template library.

Action	DC (a_1)	SR (a_2)	SA (a_3)	OST (a_4)	CoT (a_5)
Ratio	31.3%	7.2%	5.7%	30.3%	25.5%

Table 11: Performance comparison across different pattern-matching metrics on Qwen2.5-Math-7B-Base. "Equal" denotes equal weights for three indicators.

Metric	MATH500 ↑	AIME24 ↑	AMC ↑	Avg. ↑
SC-only	81.2	30.0	72.5	61.2
PS-only	80.8	26.7	72.5	60.0
PCC-only	83.4	33.3	77.5	64.7
Unified (Equal)	83.9	36.7	77.5	66.0

Table 12: Impact of seed data size.

Seed Data	MATH500	AIME24 ↑	AMC ↑	Avg. ↑
100	77.4	20.0	65.0	54.1
200	81.0	26.7	72.5	60.0
500	83.4	33.3	77.5	64.7
1000	85.2	36.7	77.5	66.5

well-captured by surface-level semantic similarity. Given that PCC provides a simple yet effective solution with minimal computational cost, we adopt it as our default pattern-matching metric. Future work could explore more sophisticated weighting schemes for the unified approach to further enhance performance while maintaining efficiency.

E.8 Sensitivity to Seed Data Size.

As shown in Table 12, performance scales with seed data size: average accuracy improves from 38.7% (100 samples) to 51.4% (1,000 samples). Notably, TemplateRL achieves competitive performance even with 100 samples, highlighting practicality in resource-constrained settings. We use 500 samples by default, balancing performance enhancement and data efficiency.

E.9 Statistics on Reasoning Actions

We present the occurrence statistics of different reasoning actions in the constructed thought template library in Table 10. For brevity, DC, SR, SA, OST, and CoT denote DIVIDE_AND_CONQUER, SELF_REFLECTION, SYSTEM_ANALYSIS, ONE_STEP_THOUGHT, and CHAIN_OF_THOUGHT, respectively. The results reveal a relatively balanced distribution across actions, with DC and OST demonstrating the highest frequencies (31.4% and 30.3%), followed by CoT (25.5%), while SR and SA show lower occurrences (7.2% and 5.7%). This distribution pattern indicates that during MCTS exploration, OST, DC, and CoT are more effective at generating correct solutions and resulting in their elevated ratio within the thought template library.

E.10 Statistical Analysis

To statistically evaluate the significance of performance differences between TemplateRL and baseline methods, we apply the nonparametric Wilcoxon signed-rank test (Kotz and Johnson, 1992). This statistical test is specifically designed to compare paired observations when data may not follow a normal distribution, making it particularly suitable for performance analysis across multiple benchmarks. The Wilcoxon signed-rank test evaluates whether there is a significant difference between paired observations through the following procedure:

1. **Calculate differences:** For each benchmark pair, compute the difference $D_i = X_i - Y_i$ where X_i represents TemplateRL performance and Y_i represents baseline performance.

2. **Rank differences:** Take absolute values $|D_i|$ and rank them from smallest to largest as R_i, with average ranks assigned for ties.

3. **Assign signs to ranks:** For each difference D_i, assign its sign to the corresponding rank: $R'_i = \text{sign}(D_i) \cdot R_i$.

4. **Calculate rank sums:** Compute positive and negative rank sums: $W^+ = \sum_{D_i > 0} R'_i$ and $W^- = \sum_{D_i < 0} R'_i$.

5. **Determine test statistic:** The test statistic is $W = \min(W^+, W^-)$.

6. **Calculate p-value:** Derive the p-value from the distribution of test statistic W.

We test the null hypothesis H_0: no significant difference between methods against the alternative hypothesis H_1: significant difference exists.

Comparing TemplateRL with GRPO on Qwen2.5-Math-7B-Base across all benchmarks, we obtain a p-value of 0.0156. Using a significance level of $\alpha = 0.05$, we reject the null hypothesis ($p < 0.05$), providing strong statistical evidence that TemplateRL significantly enhances reasoning performance compared to standard GRPO training.

F   Case Study

To further analyze the improvements of TemplateRL over conventional GRPO, we compare their reasoning processes on representative mathematical problems from the MATH dataset in Figure 12 and Figure 13.

In Figure 12, we observe how both methods approach an arithmetic mean problem. GRPO produces a solution with scattered notation and repetitive statements, particularly evident in its final steps where it repeatedly states "The problem involves ... final answer is 46." In contrast, TemplateRL demonstrates a more structured approach by explicitly introducing a "step by step" thinking process. The thought-augmented process methodically builds upon each reasoning step, clearly identifying the relationship between the original sum (984), the highest score (98), and the remaining scores to derive the lowest score (46). This structured approach results in a more readable and logically coherent solution.

Figure 13 presents a more challenging problem involving complex numbers and Vieta's formulas. Here, the limitations of GRPO become more pronounced. While GRPO initially applies the correct formula, its reasoning process deteriorates into incoherent text fragments and coding artifacts (e.g., "Tre localVEC?" and various non-mathematical expressions). This demonstrates how GRPO struggles with maintaining coherent reasoning for complex problems. In contrast, TemplateRL maintains its structured approach throughout, clearly stating the problem context, applying Vieta's formulas with proper explanation, and presenting a clean, coherent solution without extraneous text or errors.

G   Additional Discussion

Justification for Choosing MCTS.   The selection of MCTS is driven by both task-specific requirements and its comparison with other search algorithms.

Task Requirements: Our task involves constructing complex reasoning chains from a vast combinatorial action space. This setting demands an algorithm that can efficiently search and optimize within such a large space. MCTS is particularly well-suited for this, as it incrementally explores the most promising reasoning trajectories while maintaining flexibility through stochastic sampling.

Limitations of Other Search Methods: We investigated alternatives such as Breadth-First Search (BFS) and Depth-First Search (DFS), and identified several issues:

- DFS is vulnerable to early-stage errors. A suboptimal initial action can lead the search deep into unproductive paths, making recovery and backtracking difficult in multi-step reasoning where early decisions are often uncertain.

- BFS is computationally expensive, exploring the reasoning space level by level with exponential growth in candidate states, making it intractable for practical problems.

MCTS Advantages: MCTS offers a superior balance between exploration and exploitation:

- Compared to DFS, MCTS does not commit to a single path. It uses rollout and backpropagation to update the estimated value of nodes, allowing the search to adaptively shift away from poor paths.

- Compared to BFS, MCTS grows the search tree asymmetrically, guided by UCT (Upper Confidence Bound for Trees). This selective expansion enables it to focus computation on the most promising subtrees, significantly improving efficiency.

Given its ability to handle large search spaces, its robustness to early decision errors, and its structural alignment with sequential reasoning, MCTS emerged as an effective and efficient approach for our reasoning framework.

Question The arithmetic mean of 12 scores is 82. When the highest and lowest scores are removed, the new mean becomes 84. If the highest of the 12 scores is 98, what is the lowest score?

GRPO

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$\sum_{i=1}^{12} x_i = 12 \times 82 = 984$

2. Given that the highest score is 98:

$$\sum_{i=1}^{12} x_i - x_1 - x_{12} = 984 - x_1 - x_{12}$$

3. After removing the highest and lowest scores, there are 10 scores left:

$$\sum_{i=2}^{11} x_i = 10 \times 84 = 840$$

So, we have: $984 - x_1 - x_{12} = 840$

4. Solve for the sum of the highest and lowest scores:

$x_1 + x_{12} = 984 - 840 = 144$

5. We know the highest score is 98: $x_1 = 98$

6. Substitute x_1 into the equation for the sum of the highest and lowest scores:

$98 + x_{12} = 144$

Thus, the lowest score is: 46

The lowest score is 46. The problem involves … final answer is 46. The problem involves … final answer is 46.

…(repeat)

TemplateRL

Let's think step by step. We **consider the existing reasoning steps, then give the next one-step thought**.

1. The arithmetic mean of 12 scores is 82. This means that the sum of the 12 scores is $12 \times 82 = 984$.

2. When the highest and lowest scores are removed, the new mean becomes 84. Since there are now 10 scores, the sum of these 10 scores is $10 \times 84 = 840$.

3. The highest score is 98. Therefore, the sum of the remaining 11 scores (excluding the highest and lowest scores) is $984 - 98 - \text{lowest score} = 840$. Solving for the lowest score, we get:

$984 - 98 - \text{lowest score} = 840$

$\implies$ lowest score $= 984 - 98 - 840 = 46$

4. Check what the question asks. The question asks for the lowest score, and we found it to be 46.

5. The answer is: 46

Figure 12: Comparison of GRPO and TemplateRL for a simple algorithm problem from the MATH dataset.

Question

$$2a^3 + 7a^2 - 8a + 5 = 0,$$

Let a, b, c be distinct complex numbers such that $2b^3 + 7b^2 - 8b + 5 = 0,$ Compute the value of the product

$$2c^3 + 7c^2 - 8c + 5 = 0.$$

GRPO

To solve for the product abc, we start by noting that a, b, and c are the roots of the polynomial equation
$2x^3 + 7x^2 - 8x + 5 = 0.$
According to Vieta's formulas, for a cubic polynomial of the form $ax^3 + bx^2 + cx + d = 0$, the product of the roots taken one at a time (i.e., the product of the roots) is given by $-\frac{d}{a}$. Here, the polynomial is $2x^3 + 7x^2 - 8x + 5 = 0$, so we have $a = 2, b = 7, c = -8$, and $d = 5$.

Therefore, the product of the roots is $abc = -\frac{5}{2}$.

Thus, the value of the product abc is $abc = -\frac{5}{2}$

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TemplateRL

Let's think step by step. The numbers a, b, and c are the roots of the polynomial equation
$2x^3 + 7x^2 - 8x + 5 = 0.$
By Vieta's formulas, for the polynomial
$ax^3 + bx^2 + cx + d = 0$, the product of the roots taken one at a time is given by $-\frac{d}{a}$, provided the roots are distinct. In this case, the polynomial is
$2x^3 + 7x^2 - 8x + 5 = 0$, so the product of the roots
abc is given by $abc = -\frac{5}{2}$.

Therefore, the answer is: $abc = -\frac{5}{2}$

Figure 13: Comparison of GRPO and TemplateRL for a difficult algorithm problem from the MATH dataset.