Correlation-Aware Ranking and Filtering for Heterogeneous Small-Model Judge Ensembles

Abstract

Small language model judges are often selected by accuracy alone, but majority voting can help only when selected judges are individually competent and do not fail on the same examples. We introduce Judge CIA (Correlation-Informed Agents), a selection framework that trains heterogeneous LoRA judge adapters and constructs juries by using held-out data to approximate competence and bounded-redundancy conditions. Our analysis gives an independent Condorcet-style bound, a bounded-redundancy diagnostic with variance-equivalent effective jury size, and a slice-wise decomposition for high-stakes risk. Across 12 RAGBench model–dataset settings, Judge CIA voting is non-worse than the strongest held-out-selected adapter in 10 settings by point estimate, with an average signed accuracy difference of $+1.85$. On Gemma-3-1B/TAT-QA, adding the correlation filter improves full-jury accuracy from 59.91% to 64.22% while reducing mean selected-pair phi correlation from 0.311 to 0.025. On a compliance dataset, slice-weighted ranking reduces average target-slice and critical-slice error proxies in the selected setting, while revealing heterogeneous effects across slices.

1 Introduction

Small language models (SLLMs) are increasingly used as task-specific judges for retrieval support, policy compliance, and domain-specific audits. A common approach is to fine-tune several adapters and deploy the one with the best accuracy. This can be brittle: adapters often share prompts, training data, architectures, and domain artifacts, so they may fail on the same examples. For judge deployment, especially in high-stakes settings, reliability therefore depends not only on individual accuracy but also on whether judge errors are sufficiently non-redundant.

We study Judge CIA (Correlation-Informed Agents), a selection framework for constructing small-model judge juries from heterogeneous LoRA adapters [10]. Judge CIA trains N adapters, filters adapters by competence and pairwise empirical error correlation, and aggregates the selected odd-sized jury by majority vote. The method is a feasibility filter rather than a scalar subset optimizer: it admits adapters only when selection statistics suggest the operating regime in which majority voting is expected to help.

Our analysis formalizes this regime. An independent Condorcet-style bound clarifies the competence condition for majority voting, while a bounded-redundancy diagnostic shows how correlated errors reduce the effective size of the jury. We also decompose high-stakes risk into slice prevalence, slice-wise error, and slice-specific amplification, motivating slice-weighted ranking when unweighted accuracy is not the right deployment proxy.

Empirically, we evaluate Judge CIA on the RAGBench release [6] of TAT-QA [24], FinQA [2], and PubMedQA [11], with an internally constructed compliance dataset for the high-stakes slice study.

Across 12 public model–dataset settings, Judge CIA voting is non-worse than the strongest selected adapter in 10 settings by point estimate and improves by $+1.85$ percentage points on average. In the Gemma-3-1B [8]/TAT-QA diagnostic, adding the correlation filter improves full-jury accuracy from 59.91% to 64.22% while reducing mean selected-pair error correlation from 0.311 to 0.025. These results support the central message: useful small-model judge ensembles should be selected for both competence and non-redundant errors, not accumulated by size alone.

2 Related work

Judge CIA builds on classical ensemble learning, where gains depend on both individual competence and non-redundant errors [4, 7]. Prior work studies diversity measures such as pairwise correlation, disagreement, and double-fault statistics [12], as well as pruning large ensembles into compact high-performing subsets [15]. Our contribution is not a new diversity statistic, but a specialization of these ideas to small-model judge ensembles with feasibility filtering, correlation diagnostics, and high-stakes slices.

LLM-based evaluators are widely used for scalable evaluation of generated text and assistant behavior [13, 23], and recent work replaces a single large judge with a heterogeneous panel of LLM evaluators to improve agreement with human judgments, reduce intra-model bias, and lower evaluation cost [19]. RAGBench provides explainable retrieval-augmented generation evaluation data [6]. We focus on small, fine-tuned judge adapters and on selecting juries with held-out competence and error-correlation filters rather than using a fixed panel composition. For high-stakes deployment, we also draw on cost-sensitive and slice-aware evaluation, where average accuracy may not match operational loss [5].

3 Judge CIA jury and correlation-aware filtering

3.1 Jury definition and independent baseline

We first present an idealized independence baseline. The theorem below is not assumed to hold for LoRA adapters, but clarifies why the selection rule first filters for individually competent judges. We begin with a tail bound for abstract binary random variables.

Theorem 3.1 (Hoeffding–Chernoff tail bound). *Let $Z_1, \ldots, Z_n \in \{0, 1\}$ be independent random variables, and define*

$$S := \sum_{i=1}^{n} Z_i, \qquad \bar{\varepsilon} := \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}[Z_i], \qquad \Delta := \tfrac{1}{2} - \bar{\varepsilon}.$$

If $\bar{\varepsilon} < 1/2$, then

$$\mathbb{P}(S \geq n/2) \leq \exp\big(-2n\Delta^2\big).$$

The proof is given in Appendix B.

Definition 3.2 (Judge CIA jury). *Let $\mathcal{X}$ be an input space, $\mathcal{D}$ a data distribution on $\mathcal{X}$, and $f : \mathcal{X} \to \{0, 1\}$ the ground-truth oracle. An agent $A_i : \mathcal{X} \to \{0, 1\}$ is a randomized decision rule.*

Given an odd number n of agents $\{A_1, \ldots, A_n\}$ producing decisions $\hat{y}_i = A_i(x) \in \{0, 1\}$, the Judge CIA collective decision is the majority vote

$$\hat{y}(x) = \mathbf{1}\Big\{\sum_{i=1}^{n}(2\hat{y}_i - 1) > 0\Big\}.$$

The odd-cardinality convention avoids ties.

For the Judge CIA jury, define the agent error indicators and total error count by

$$Z_i := \mathbf{1}\{A_i(x) \neq f(x)\}, \qquad S := \sum_{i=1}^{n} Z_i,$$

Unless stated otherwise, probabilities are over $x \sim \mathcal{D}$ and any internal randomness of the agents, and we use Z_i for agent i's error indicator and $S = \sum_i Z_i$ for the total error count.

Corollary 3.3 (Average-error Condorcet-style bound for Judge CIA). *If the agent error indicators $Z_1, \ldots, Z_n$ are independent and their average marginal error satisfies*

$$\bar{\varepsilon} := \frac{1}{n} \sum_{i=1}^{n} \mathbb{P}(A_i(x) \neq f(x)) < \tfrac{1}{2},$$

then the Judge CIA majority vote satisfies

$$\mathbb{P}(\hat{y}(x) \neq f(x)) \leq \exp(-2n\Delta^2) \quad \text{for the majority margin } \Delta := \tfrac{1}{2} - \bar{\varepsilon}.$$

In particular, $\mathbb{P}(\hat{y}(x) \neq f(x)) \to 0$ as $n \to \infty$ whenever $\Delta > 0$ is fixed.

The proof is given in Appendix B. This is a heterogeneous, average-error version of the classical *Condorcet Jury Theorem*: independence and an aggregate margin $\Delta > 0$ make majority error decay, even when the individual marginal errors are not identical [22].

3.2 Error dependence and bounded redundancy

Independence is an idealized regime. In practice, adapters share architectures, training data, prompts, and domain artifacts, so their errors can be dependent. Judge CIA therefore does not try to enforce independence, common marginal error rates, or exact equicorrelation. Instead, it targets a weaker selection-relevant regime: each selected adapter should be individually competent, and selected pairs should not have overly redundant sample-wise error patterns. This aligns with classic ensemble theory that aggregation can outperform a single expert when errors are not perfectly correlated [4, 7].

Assumption 3.4 (Bounded competence and bounded redundancy). *For a selected jury $\mathcal{K}$ with $|\mathcal{K}| = k$, suppose each selected judge satisfies*

$$\mathbb{P}(Z_i = 1) \leq \varepsilon_0 < 1/2 \quad (i \in \mathcal{K}),$$

and each selected pair satisfies

$$\mathrm{Corr}(Z_i, Z_j) \leq \phi \quad (i \neq j, \ i, j \in \mathcal{K}),$$

for some $0 \leq \phi \leq 1$. In practice, the selection rule in Section 5.3 approximates these population conditions using validation estimates: empirical error below $1 - \tau$ and empirical pairwise error correlation below a user-specified filtering threshold ρ.

Proposition 3.5 (Diagnostic majority bound under bounded redundancy). *Let $S_{\mathcal{K}} := \sum_{i \in \mathcal{K}} Z_i$ and $\Delta_0 := 1/2 - \varepsilon_0$. Under Assumption 3.4,*

$$\mathrm{Var}(S_{\mathcal{K}}) \leq k\varepsilon_0(1 - \varepsilon_0)\big(1 + (k-1)\phi\big),$$

and Cantelli's inequality gives the diagnostic bound

$$\mathbb{P}(S_{\mathcal{K}} \geq k/2) \leq \frac{\varepsilon_0(1 - \varepsilon_0)\big(1 + (k-1)\phi\big)}{\varepsilon_0(1 - \varepsilon_0)\big(1 + (k-1)\phi\big) + k\Delta_0^2} = \frac{\varepsilon_0(1 - \varepsilon_0)}{\varepsilon_0(1 - \varepsilon_0) + k_{\mathrm{eff}}(\phi)\Delta_0^2}$$

with

$$k_{\mathrm{eff}}(\phi) := \frac{k}{1 + (k-1)\phi}.$$

The proof is given in Appendix B. This is a conservative diagnostic upper bound, not a tight prediction of majority-vote error. Appendix A gives the corresponding redundancy interpretation.

To explain one source of error correlation, let θ denote a common condition affecting all agents, such as a distribution shift, shared prompt artifact, or shared training-data bias. Agent i responds through its error propensity $p_i(\theta)$. If

$$Z_i \mid \theta \ \sim \ \mathrm{Bernoulli}(p_i(\theta)), \qquad Z_i \perp Z_j \mid \theta \ (i \neq j).$$

then conditional independence holds only after θ is fixed; marginally, agents may still be correlated because their propensities depend on the same θ. By the law of total covariance,

$$\mathrm{Cov}(Z_i, Z_j) = \mathrm{Cov}\big(p_i(\theta), p_j(\theta)\big) \quad (i \neq j),$$

so heterogeneity helps when it makes the propensity functions $p_i(\theta)$ and $p_j(\theta)$ less aligned. In our experiments, heterogeneity is induced by varying prompt template, LoRA rank/scaling settings, target-module set, and sampling seed. These axes are intended to produce adapters whose error propensities respond differently to shared latent difficulty factors, thereby reducing alignment among the $p_i(\theta)$ functions. We use Pearson correlation on binary error-indicator vectors, equivalently the phi correlation coefficient, as the empirical redundancy statistic [12]. Empirically, we estimate pairwise error correlations $\widehat{\rho}_{ij}$ from adapter error vectors and report mean and maximum selected-jury pairwise error correlation as scalar redundancy diagnostics. Pairwise correlations involving zero-variance error vectors are handled using the convention described in Appendix D.1.

4 Catastrophic risk

Majority aggregation can reduce error, but high-stakes deployment also depends on which errors are operationally severe. The selection criterion should therefore reflect deployment-relevant slice risk rather than average error alone [3]. Throughout, a slice denotes a measurable subset of examples defined by task metadata, operational context, or a high-stakes attribute. In the catastrophic-risk decomposition below, $\mathcal{H} \subseteq \mathcal{X}$ is the high-stakes subset and $(\mathcal{H}_s)_{s \in \mathcal{S}}$ denotes a disjoint measurable partition of $\mathcal{H}$. If monitored metadata slices overlap in an application, the decomposition can be applied to a disjoint refinement of those slices. In experiments, compliance slices are defined by criterion criticality, policy flags, and violation or failure-type metadata; details are given in Appendix C.

Definition 4.1 (Catastrophic event and risk). *A catastrophic event is specified by a predicate*

$$\mathcal{C}(x, \hat{y}(x), f(x)) \in \{0, 1\},$$

where $\mathcal{C} = 1$ denotes that the realized decision induces a domain-specified severe adverse consequence. The predicate may depend on the input, the decision, and the ground truth. For $x \sim \mathcal{D}$, define the catastrophic risk of a decision rule $\hat{y}$ as

$$\mathrm{CatRisk}(\hat{y}) := \mathbb{P}\big(\mathcal{C}(x, \hat{y}(x), f(x)) = 1 \;\wedge\; \hat{y}(x) \neq f(x) \;\wedge\; x \in \mathcal{H}\big).$$

Here, $\mathcal{C}$ identifies the adverse consequence itself, while the additional conditions $\hat{y}(x) \neq f(x)$ and $x \in \mathcal{H}$ restrict evaluation to high-stakes errors. Examples include falsely approving a non-compliant consultation or falsely flagging a compliant consultation as severe.

Definition 4.2 (Slice-wise catastrophic amplification). *Let $(\mathcal{H}_s)_{s \in \mathcal{S}}$ be the high-stakes slice partition above, with $\mathbb{P}(x \in \mathcal{H}_s) > 0$. Define the slice-wise misclassification rate*

$$\mathrm{Err}_s(\hat{y}) := \mathbb{P}\big(\hat{y}(x) \neq f(x) \mid x \in \mathcal{H}_s\big),$$

and, for a fixed decision rule $\hat{y}$, the slice-wise catastrophic amplification factor

$$\kappa_s(\hat{y}) := \mathbb{P}\big(\mathcal{C}(x, \hat{y}(x), f(x)) = 1 \mid x \in \mathcal{H}_s \;\wedge\; \hat{y}(x) \neq f(x)\big) \in [0, 1].$$

When the conditioning event has probability zero, we set $\kappa_s(\hat{y}) := 0$. Thus $\kappa_s(\hat{y})$ is the decision-rule-dependent probability that an error *becomes* catastrophic *on slice $\mathcal{H}_s$.*

Lemma 4.3 (Slice-specific catastrophic risk decomposition).

$$\mathrm{CatRisk}(\hat{y}) = \sum_{s \in \mathcal{S}} \mathbb{P}(x \in \mathcal{H}_s)\, \kappa_s(\hat{y})\, \mathrm{Err}_s(\hat{y}).$$

Thus reducing catastrophic risk requires controlling slice-wise *error rates and, when possible, reducing the decision-rule-dependent amplification factor $\kappa_s(\hat{y})$ (changing the decision policy on that slice), not merely reducing average error.*

The proof is given in Appendix B.

Remark 4.4 (Why this is different from misclassification). *Even when average error is small,* $\mathrm{CatRisk}$ *can be dominated by rare, high-impact slices. The decomposition separates slice-wise error control from the domain policy question of whether an error becomes catastrophic.*

5 Filtering and Slice-Weighted Ranking

Lemma 4.3 identifies slice-wise error as the part of the high-stakes risk proxy that can be addressed by agent selection. Building on the slice terminology in Section 4, we call $\mathcal{H}_s$ the target slice when it is assigned larger weight during slice-weighted ranking. The target-slice weighting protocol is detailed in Appendix D.4. We first formalize the statistics used to rank adapters: ordinary error and slice-weighted scores that emphasize high-stakes regions.

5.1 Ranking stability

Definition 5.1 (Selection sample and empirical error)**.** *Let* $V := \{x^{(1)}, \ldots, x^{(m)}\}$ *be an i.i.d. selection sample set with* $x^{(k)} \sim \mathcal{D}$. *For agent* A_i, *define its true statistical error and empirical error by*

$$\varepsilon_i := \mathbb{P}(A_i(x) \neq f(x)), \qquad \hat{\varepsilon}_i := \frac{1}{m} \sum_{k=1}^{m} \mathbf{1}\{A_i(x^{(k)}) \neq f(x^{(k)})\}.$$

Equivalently, $\varepsilon_i = \mathbb{E}[Z_i]$ *without assuming a common error rate across agents.*

Definition 5.2 (Top-k ranking rule)**.** *Fix an integer* $k \in \{1, \ldots, n\}$. *Compute empirical errors* $\hat{\varepsilon}_1, \ldots, \hat{\varepsilon}_n$ *on the selection sample set and keep the* k *agents with the smallest empirical error. Formally, let* $\hat{\sigma}$ *be a permutation such that*

$$\hat{\varepsilon}_{\hat{\sigma}(1)} \leq \hat{\varepsilon}_{\hat{\sigma}(2)} \leq \cdots \leq \hat{\varepsilon}_{\hat{\sigma}(n)}.$$

Define the kept set

$$\widehat{\mathcal{K}}_k := \{\hat{\sigma}(1), \ldots, \hat{\sigma}(k)\}.$$

Proposition 5.3 (Ranking stability of top-k under a separation margin)**.** *Let* σ *be a permutation that sorts the* true *errors:*

$$\varepsilon_{\sigma(1)} \leq \varepsilon_{\sigma(2)} \leq \cdots \leq \varepsilon_{\sigma(n)}, \qquad \mathcal{K}_k := \{\sigma(1), \ldots, \sigma(k)\}.$$

Assume a separation margin at the boundary:

$$\varepsilon_{\sigma(k)} \leq \varepsilon_{\sigma(k+1)} - 2\gamma \quad \textit{for some } \gamma > 0.$$

Then the empirical top-k ranking recovers the true top-k set with probability at least

$$\mathbb{P}\big(\widehat{\mathcal{K}}_k = \mathcal{K}_k\big) \geq 1 - 2n \exp(-2m\gamma^2).$$

Equivalently, with probability at least $1 - 2n \exp(-2m\gamma^2)$,

$$\max_{i \in \mathcal{K}_k} \hat{\varepsilon}_i \leq \min_{j \notin \mathcal{K}_k} \hat{\varepsilon}_j.$$

The proof is given in Appendix B.

Risk note.   This stability result controls only ranking error on the selection sample distribution. If V under-samples operationally important sub-slices (especially high-stakes or rare slices), then the recovered ranking may be optimal for $\mathcal{D}$ yet suboptimal for deployment. In high-stakes settings, V should be stratified to include the catastrophic slices defined in Section 4.

5.2 Slice-weighted ranking

Definition 5.4 (Slice-weighted empirical error)**.** *Let* $(\mathcal{H}_s)_{s \in \mathcal{S}}$ *be a measurable partition of a high-stakes subset* $\mathcal{H} \subseteq \mathcal{X}$ *(as in Section 4). Let* $w : \mathcal{X} \to [0, W]$ *be a bounded weight function that upweights high-stakes slices; for example, one may use*

$$w(x) = 1 + \sum_{s \in \mathcal{S}} (w_s - 1)\mathbf{1}\{x \in \mathcal{H}_s\}, \qquad 1 \leq w_s \leq W.$$

Given an i.i.d. selection sample set $V = \{x^{(1)}, \ldots, x^{(m)}\}$ *with* $x^{(k)} \sim \mathcal{D}$, *define the slice-weighted empirical error of agent* A_i *as*

$$\hat{\varepsilon}_i^{(w)} := \frac{\sum_{k=1}^{m} w(x^{(k)}) \mathbf{1}\{A_i(x^{(k)}) \neq f(x^{(k)})\}}{\sum_{k=1}^{m} w(x^{(k)})}.$$

174 *For ranking agents, we use the equivalent unnormalized weighted score*

$$\widehat{L}_i^{(w)} := \frac{1}{m} \sum_{k=1}^{m} w(x^{(k)}) \, \mathbf{1}\{A_i(x^{(k)}) \neq f(x^{(k)})\}.$$

175 *This score induces the same ordering as $\hat{\varepsilon}_i^{(w)}$ whenever the common denominator is positive, and*
176 *remains well-defined when the empirical denominator is zero. Define the corresponding weighted*
177 true *error*

$$\varepsilon_i^{(w)} := \frac{\mathbb{E}_{x\sim\mathcal{D}}[w(x) \, \mathbf{1}\{A_i(x) \neq f(x)\}]}{\mathbb{E}_{x\sim\mathcal{D}}[w(x)]}.$$

178 **Definition 5.5** (Top-k ranking with slice-weighted empirical error)**.** *Replace $\hat{\varepsilon}_i$ in the top-k ranking*
179 *rule by $\widehat{L}_i^{(w)}$ and keep the k agents with the smallest weighted empirical scores. Denote the kept set*
180 *by $\widehat{\mathcal{K}}_k^{(w)}$.*

181 **Proposition 5.6** (Ranking stability of weighted top-k under a separation margin)**.** *Let σ be a*
182 *permutation that sorts the* weighted true *errors:*

$$\varepsilon_{\sigma(1)}^{(w)} \leq \varepsilon_{\sigma(2)}^{(w)} \leq \cdots \leq \varepsilon_{\sigma(n)}^{(w)}, \qquad \mathcal{K}_k^{(w)} := \{\sigma(1), \ldots, \sigma(k)\}.$$

183 *Assume a separation margin at the boundary:*

$$\varepsilon_{\sigma(k)}^{(w)} \leq \varepsilon_{\sigma(k+1)}^{(w)} - 2\gamma \quad \text{for some } \gamma > 0.$$

184 *Assume further that the weight function is bounded as $w(x) \in [0, W]$ and that the* mean weight *is*
185 *uniformly bounded below:*

$$\mu_w := \mathbb{E}_{x\sim\mathcal{D}}[w(x)] \geq \mu_{\min} > 0.$$

186 *Then the weighted empirical top-k ranking recovers the true weighted top-k set with probability at*
187 *least*

$$\mathbb{P}\big(\widehat{\mathcal{K}}_k^{(w)} = \mathcal{K}_k^{(w)}\big) \geq 1 - 2n \exp\Big(-\frac{2m\,\mu_{\min}^2\,\gamma^2}{W^2}\Big).$$

188 The proof is given in Appendix B.

189 **Remark 5.7** (Direct control of slice-wise error)**.** *Choosing $w(x)$ concentrated on high-stakes slices*
190 *makes top-k selection prioritize lower slice-weighted errors. In particular, if $w(x) = \sum_s w_s \mathbf{1}\{x \in$*
191 *$\mathcal{H}_s\}$, then ranking by $\varepsilon_i^{(w)}$ emphasizes the slices with larger w_s. This is the relevant proxy statistic*
192 *when high-stakes risk is dominated by specific slices, since $\mathrm{CatRisk}$ decomposes through slice-wise*
193 *error terms rather than through average error alone.*

194 Domain routing can be incorporated by conditioning the catastrophic-risk decomposition on the true
195 and predicted domains. We give the corresponding router–jury–slice decomposition in Appendix F.

5.3 Correlation-aware selection rule

197 We now combine the ranking scores above with the redundancy statistic from Section 3. Given a
198 ranking score r_i for each adapter, where r_i is either the ordinary error $\hat{\varepsilon}_i$ or the slice-weighted score
199 $\widehat{L}_i^{(w)}$, Judge CIA first forms the competent candidate pool

$$P_\tau = \{i : \widehat{\mathrm{Acc}}_i \geq \tau\}.$$

200 Here $\widehat{\mathrm{Acc}}_i = 1 - \hat{\epsilon}_i$, so the threshold τ is equivalently an empirical-error threshold $1 - \tau$. It then
201 traverses P_τ in increasing order of r_i and accepts an adapter only if its empirical error correlation
202 with every previously accepted adapter is at most ρ:

$$i \text{ is accepted only if } \max_{j \in \widehat{K}} \widehat{\rho}_{ij} \leq \rho.$$

203 The resulting set $\widehat{K}$ is therefore an approximation to the competence and bounded-redundancy
204 conditions in Assumption 3.4. The rule is intentionally a feasibility filter rather than a fixed-size
205 subset optimizer: it preserves high-ranked competent adapters whose error patterns are not too
206 redundant with the adapters already selected.

207 Fixed-size juries use the first k accepted adapters from this traversal, while the main voting results
208 use the full feasible set, with the lowest-ranked selected adapter removed only to ensure odd cardi-
209 nality. Slice weighting changes only the ranking score, not the unweighted competence threshold τ.
210 Appendix D.1 gives pseudocode for this rule as Algorithm 1.

Table 1: Main performance across SLLMs and RAGBench datasets. Δ denotes the signed accuracy difference Vote − Top-1; positive values favor Vote. Values use the same percentage scale as the accuracy columns. Model-level average rows summarize Δ across the three datasets; confidence intervals are reported in Appendix D.2.

	FinQA				TAT-QA				PubMedQA				
Model	Vanilla	Top-1	Vote	Δ	Vanilla	Top-1	Vote	Δ	Vanilla	Top-1	Vote	Δ	Avg. Δ
Qwen3-0.6B	49.49	56.38	61.22	4.85	43.97	61.21	58.62	-2.59	16.00	61.20	61.20	0.00	0.75
Qwen3-1.7B	48.72	65.31	67.09	1.79	45.69	66.81	68.10	1.29	35.60	59.60	62.20	2.60	1.89
Llama-3.2-1B	54.08	57.40	57.40	0.00	53.02	63.36	62.50	-0.86	50.80	57.20	60.40	3.20	0.78
Gemma-3-1B	50.00	52.81	53.32	0.51	45.26	59.05	64.22	5.17	50.40	52.60	58.80	6.20	3.96
Average	50.57	57.97	59.76	1.79	46.99	62.61	63.36	0.75	38.20	57.65	60.65	3.00	1.85

6 Experiments

The SLLMs are Qwen3-0.6B and Qwen3-1.7B [21], Llama-3.2-1B [16], and Gemma-3-1B [8]. We evaluate majority-vote improvements, pairwise error correlation, jury-size curves, ranking stability, and high-stakes slice weighting. We use RAGBench TAT-QA, FinQA, and PubMedQA for the main experiments [6], and an internally constructed telephone-consultation compliance dataset for the high-stakes study.

We organize the experiments around four hypotheses: **H1** heterogeneous adapter construction lowers pairwise phi correlation; **H2** among competent adapters, phi-correlation filtering improves majority-vote behavior relative to accuracy-only selection; **H3** filtering and ranking decisions become more stable as selection sample size increases; and **H4** slice-weighted ranking can reduce target-slice error, possibly trading off non-target critical-slice performance.

6.1 Experimental setup

For each base SLLM, we train a heterogeneous pool of LoRA adapters [10] on the training split, select adapters on the validation split, and evaluate single-adapter and majority-vote decisions on the test split. In this experimental protocol, the selection sample set V from Section 5 is the validation split. Detailed hyperparameters, adapter-pool dimensions, and additional comparison tables are in Appendix D–E. Because our main question is how to select among heterogeneous small-adapter judges from the same candidate pool, we keep single-backbone comparison results in Appendix E as external context rather than treating them as mechanism-level baselines.

6.2 Main performance

We first ask whether a selected heterogeneous jury improves over the validation-selected Top-1 adapter in the candidate pool. Table 1 compares the unfine-tuned base model (Vanilla), the strongest validation-selected adapter (Top-1), and the validation-selected correlation-aware jury (Vote) across four SLLMs and three RAGBench datasets. Section 6.4 isolates the contribution of the correlation filter by comparing this rule against random and accuracy-only jury construction.

Across the 12 model–dataset cells, voting is usually competitive with or better than the validation-selected Top-1 adapter, but the gains are not uniform. Gemma-3-1B shows the clearest benefit, while two TAT-QA settings lose slightly relative to Top-1. This pattern is consistent with the theory: majority voting is useful only when the selected agents are both competent and non-redundant, so Table 1 should be read together with the diagnostics below.

6.3 Heterogeneity and error correlation

H1 tests the prediction from Section 3 that majority voting is most useful when agent errors are not strongly correlated. We test whether heterogeneous adapter design reduces shared failure modes and shifts the pairwise error-correlation distribution downward. Figure 1 compares a heterogeneous Qwen3-1.7B/TAT-QA pool against a fixed-axis pool that retains adapter-level training variation but holds the main design choices fixed. The heterogeneous pool shifts the pairwise error-correlation

Figure 1: Pairwise error-correlation distributions. Fixed-axis varies only the seed, while heterogeneous varies prompt, LoRA rank/scale settings, target-module, and seed choices.

Table 2: Pool-selection comparison across random, accuracy-only, and correlation-aware jury construction. The Random row averages over 50 random draws from the competent-adapter pool. Accuracy differences are measured relative to Top-1 on the same percentage scale as full-jury accuracy. Lower pairwise error correlation is associated with the strongest majority-vote result. Here, $\hat{\rho}$ denotes empirical phi correlation between selected adapters' error-indicator vectors.

Selection rule	Full-pool acc. (%)	Jury size	Accuracy difference	Mean $\hat{\rho}$	Max $\hat{\rho}$
Random	57.84	23	-1.22	0.087	0.846
Accuracy-only	59.91	23	0.86	0.311	0.761
Accuracy+corr.	64.22	23	5.17	0.025	0.398

distribution downward, with lower mean and median phi correlations and more low-correlation pairs. Thus H1 is supported: varying prompt and LoRA design axes produces visibly less redundant adapter errors than varying only training randomness.

6.4 Correlation versus jury error

H2 asks whether lower empirical correlation translates into lower majority-vote error. This subsection explains the mechanism behind Table 1 rather than replacing the headline performance comparison. On TAT-QA with Gemma-3-1B, we compare three ways to form juries: random selection from competent adapters averaged over 50 draws (Random), validation-accuracy ranking without the correlation filter (Accuracy-only), and the proposed rule with the additional pairwise error-correlation filter (Accuracy+corr.). Table 2 summarizes the full-pool comparison, and Appendix D.3 shows the full jury-size curve. Thus Table 1 reports the main accuracy outcome, while this diagnostic comparison tests whether the strongest gains are associated with fewer shared error patterns. The rule with the additional correlation filter has fewer shared error patterns in the selected jury and the best full-pool test accuracy among the three rules. These results support H2 for this diagnostic setting and explain why the Gemma-3-1B/TAT-QA cell in Table 1 is one of the strongest majority-vote gains.

6.5 Filtering and ranking stability

H3 asks whether filtering and ranking decisions become more stable as validation size increases. Motivated by Proposition 5.3, Figure 2 recomputes the top-k adapter set on growing validation prefixes and compares it with a post hoc full-test top-k proxy. This diagnostic is run on the Qwen3-0.6B/TAT-QA adapter pool, with the test-set proxy used only for analysis and never for adapter selection. Small prefixes are unstable, but the prefix-selected set increasingly overlaps with the full-test proxy as more validation data are used. This supports H3 qualitatively and shows why the procedure should be treated as validation-size dependent rather than as a fixed property of the adapter pool.

(a) Exact top-k recovery rate

(b) Partial overlap and gap

(c) Original-order alignment

(d) Boundary gap

Figure 2: Prefix-alignment diagnostics for validation-based filtering and ranking. The reference set is the post hoc full-test top-k proxy. Panels (a) and (b) aggregate over shuffled validation-prefix orders: (a) reports the strict exact-recovery rate, while (b) reports partial top-k overlap and the test-accuracy gap to that proxy. Panel (c) shows the same recovery and overlap diagnostics in the original validation order; panel (d) reports the empirical boundary gap.

Table 3: High-stakes slice-weighted selection results. Columns report test error rates: All is the full test set, Target is the upweighted high-stakes slice, Other crit. pools the remaining critical slices, Critical is the full critical subset, and Target mass is the target-slice contribution to total test error. Rows aggregate over the five target-slice runs reported individually in Table 7.

	Setting		Test error rate (%)				
Rule	k	w_{target}	All	Target	Other crit.	Critical	Target mass
Unweighted top-k	9	1	2.60	2.59	2.26	2.26	0.52
Slice-weighted top-k	9	1.5	2.48	2.38	1.70	1.65	0.48

6.6 High-stakes slice weighting

Lemma 4.3 shows that average error can be misleading when rare high-stakes slices dominate risk. **H4** evaluates whether slice-weighted validation ranking reduces target-slice error, possibly trading off non-target critical-slice performance. Since this dataset does not provide a fully observed catastrophic predicate, this experiment evaluates the controllable component of the decomposition: slice-wise error on operationally critical compliance slices. Using the internally constructed compliance dataset with Qwen3-1.7B, we compare average validation-error ranking with slice-weighted ranking on the same candidate pool. Table 3 reports overall error, target-slice error, error on other critical slices, full critical-subset error, and target-slice mass error. The slice-weighted rule reduces the target-slice proxy without increasing average error in the selected setting. Appendix D.4 shows that this effect is heterogeneous across slices, so H4 is partially supported: slice weighting can move the jury toward high-stakes slices, but it should be monitored slice by slice rather than summarized only by the average.

7 Conclusion

Judge CIA builds small-model judge juries by selecting for the two properties that majority voting requires but accuracy alone does not capture: competence and non-redundant errors. The theory explains this dependence through an independent majority bound and a bounded-redundancy diagnostic, while the experiments show that correlation-aware voting is usually competitive with or better than the strongest held-out-selected adapter. The high-stakes study further shows that slice-weighted ranking can target operationally important error proxies when average accuracy is insufficient.

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A Equicorrelated Diagnostic Special Case

The main text uses Assumption 3.4 because it matches the filtering rule: selected adapters need only have bounded empirical error and bounded pairwise empirical error correlation. For intuition, the bounded-redundancy diagnostic reduces to the following common-error, equicorrelated special case. This appendix result is not an assumption enforced by Judge CIA.

Assumption A.1 (Equicorrelated diagnostic model). *Assume a common marginal error $\mathbb{E}[Z_i] = \varepsilon$ and*

$$\text{Corr}(Z_i, Z_j) = \rho \quad \text{for all } i \neq j,$$

where ρ lies in the admissible range for the joint Bernoulli distribution and $1 + (n-1)\rho > 0$. The main regime of interest is $\rho \geq 0$, where redundant errors reduce the usable jury size.

Proposition A.2 (Equicorrelated effective jury size diagnostic). *Under Assumption A.1, assume $\varepsilon < 1/2$ and let $\Delta_{\text{diag}} := 1/2 - \varepsilon$. Define*

$$n_{\text{eff}} := \frac{n}{1 + (n-1)\rho}.$$

Then $\text{Var}(S) = n\varepsilon(1 - \varepsilon)(1 + (n-1)\rho)$, and

$$\mathbb{P}(S \geq n/2) \leq \frac{\text{Var}(S)}{\text{Var}(S) + n^2\Delta_{\text{diag}}^2} = \frac{\varepsilon(1-\varepsilon)}{\varepsilon(1-\varepsilon) + n_{\text{eff}}\Delta_{\text{diag}}^2}.$$

The same variance-equivalent reading applies to Proposition 3.5 in the main text. There, $k_{\text{eff}}(\phi) = k/(1 + (k-1)\phi)$ is a diagnostic for how much pairwise redundancy inflates the variance of the jury error count. It should not be read as a calibrated prediction of test error or as an assumption that selected adapters have common error rates or exact equicorrelation.

B Proofs

Proof of Theorem 3.1. We have $\mathbb{E}[S] = n\bar{\varepsilon}$, so $\{S \geq n/2\} = \{S - \mathbb{E}[S] \geq n\Delta\}$. Since the $Z_i \in [0, 1]$ are independent, Hoeffding's inequality gives

$$\mathbb{P}(S - \mathbb{E}[S] \geq n\Delta) \leq \exp(-2n\Delta^2).$$

$\square$

Proof of Corollary 3.3. A majority-vote error can occur only when at least half of the agents err:

$$\{\hat{y}(x) \neq f(x)\} \subseteq \{S \geq n/2\}.$$

The result follows from Theorem 3.1. $\square$

Proof of Proposition A.2. First,

$$\text{Var}(S) = \sum_i \text{Var}(Z_i) + 2\sum_{i<j} \text{Cov}(Z_i, Z_j) = n\varepsilon(1 - \varepsilon)(1 + (n - 1)\rho).$$

Cantelli's inequality gives

$$\mathbb{P}(X - \mu \geq t) \leq \frac{\sigma^2}{\sigma^2 + t^2}.$$

Apply it with $X = S$, $\mu = \mathbb{E}[S] = n\varepsilon$, $t = n\Delta_{\text{diag}}$, and $\sigma^2 = \text{Var}(S)$. $\square$

Proof of Proposition 3.5. For each $i \in \mathcal{K}$, $\mathbb{P}(Z_i = 1) \leq \varepsilon_0 < 1/2$, so

$$\text{Var}(Z_i) \leq \varepsilon_0(1 - \varepsilon_0).$$

For $i \neq j$ in $\mathcal{K}$,

$$\text{Cov}(Z_i, Z_j) \leq \phi\sqrt{\text{Var}(Z_i)\text{Var}(Z_j)} \leq \phi\,\varepsilon_0(1 - \varepsilon_0).$$

Therefore

$$\text{Var}(S_{\mathcal{K}}) \leq k\varepsilon_0(1 - \varepsilon_0) + k(k - 1)\phi\varepsilon_0(1 - \varepsilon_0).$$

Also $\mathbb{E}[S_{\mathcal{K}}] \leq k\varepsilon_0$, hence

$$\{S_{\mathcal{K}} \geq k/2\} \subseteq \{S_{\mathcal{K}} - \mathbb{E}[S_{\mathcal{K}}] \geq k\Delta_0\}.$$

Applying Cantelli's inequality with the variance bound above gives the result. $\square$

Proof of Lemma 4.3. For readability, write $\mathcal{C} = 1$ for $\mathcal{C}(x, \hat{y}(x), f(x)) = 1$. Since $(\mathcal{H}_s)_{s \in \mathcal{S}}$ partitions $\mathcal{H}$,

$$\text{CatRisk}(\hat{y}) = \sum_{s \in \mathcal{S}} \mathbb{P}(\mathcal{C} = 1 \wedge \hat{y} \neq f \wedge x \in \mathcal{H}_s).$$

For a fixed slice s,

$$\mathbb{P}(\mathcal{C} = 1 \wedge \hat{y} \neq f \wedge x \in \mathcal{H}_s) = \mathbb{P}(x \in \mathcal{H}_s)\mathbb{P}(\mathcal{C} = 1 \wedge \hat{y} \neq f \mid x \in \mathcal{H}_s).$$

If $\text{Err}_s(\hat{y}) = 0$, this term is zero. Otherwise, the chain rule gives

$$\mathbb{P}(\mathcal{C} = 1 \wedge \hat{y} \neq f \mid x \in \mathcal{H}_s) = \mathbb{P}(\mathcal{C} = 1 \mid x \in \mathcal{H}_s \wedge \hat{y} \neq f)\,\mathbb{P}(\hat{y} \neq f \mid x \in \mathcal{H}_s)$$
$$= \kappa_s(\hat{y})\,\text{Err}_s(\hat{y}).$$

Summing over $s \in \mathcal{S}$ gives the equality. $\square$

Proof of Proposition 5.3. If $|\hat{\varepsilon}_i - \varepsilon_i| < \gamma$ holds for all $i \in \{1, \ldots, n\}$, then

$$\max_{i \in \mathcal{K}_k} \hat{\varepsilon}_i \leq \varepsilon_{\sigma(k)} + \gamma \leq \varepsilon_{\sigma(k+1)} - \gamma \leq \min_{j \notin \mathcal{K}_k} \hat{\varepsilon}_j.$$

Thus the empirical ordering cannot swap any element across the $k/(k + 1)$ boundary, implying $\widehat{\mathcal{K}}_k = \mathcal{K}_k$. By the single-agent Hoeffding lemma and a union bound,

$$\mathbb{P}(\exists i : |\hat{\varepsilon}_i - \varepsilon_i| \geq \gamma) \leq 2n \exp(-2m\gamma^2),$$

which proves the claim. $\square$

Proof of Proposition 5.6. Recall the numerator score

$$L_i^{(w)} := \mathbb{E}_{x \sim \mathcal{D}}[w(x)\,\mathbf{1}\{A_i(x) \neq f(x)\}].$$

The denominator in $\hat{\varepsilon}_i^{(w)}$ is common to all agents, so the weighted empirical-error ordering is identical to the ordering by $\widehat{L}_i^{(w)}$ whenever the denominator is positive. Similarly,

$$L_i^{(w)} = \mu_w\,\varepsilon_i^{(w)},$$

where $\mu_w = \mathbb{E}_{x \sim \mathcal{D}}[w(x)]$ is common to all agents, so the true ordering by $\varepsilon_i^{(w)}$ is identical to the true ordering by $L_i^{(w)}$. Each summand in $\widehat{L}_i^{(w)}$ lies in $[0, W]$, and Hoeffding's inequality gives

$$\mathbb{P}\big(|\widehat{L}_i^{(w)} - L_i^{(w)}| \geq \mu_w\gamma\big) \leq 2\exp\Big(-\frac{2m\,\mu_w^2\gamma^2}{W^2}\Big) \leq 2\exp\Big(-\frac{2m\,\mu_{\min}^2\gamma^2}{W^2}\Big).$$

Union bound over $i \in \{1, \ldots, n\}$ gives that, with probability at least

$$1 - 2n\exp\Big(-\frac{2m\,\mu_{\min}^2\gamma^2}{W^2}\Big),$$

we have $|\widehat{L}_i^{(w)} - L_i^{(w)}| < \mu_w\gamma$ for all i. The separation condition implies

$$L_{\sigma(k)}^{(w)} = \mu_w\varepsilon_{\sigma(k)}^{(w)} \leq \mu_w\varepsilon_{\sigma(k+1)}^{(w)} - 2\mu_w\gamma = L_{\sigma(k+1)}^{(w)} - 2\mu_w\gamma.$$

Thus the same boundary-swap argument as in the unweighted case implies that no agent can cross the $k/(k+1)$ boundary, hence $\widehat{\mathcal{K}}_k^{(w)} = \mathcal{K}_k^{(w)}$. $\square$

C Compliance Dataset Details

The high-stakes dataset is an internally constructed telephone-consultation audit dataset for securities-domain compliance. It is generated by a script-based data-construction pipeline from normalized criterion items, violation templates, policy flags, and deterministic audit metadata. The canonical sample unit is a single-unit script audit centered on one normalized criterion item, rather than a full dialogue as the primary training unit. Each example contains dialogue turns, evidence turns, item labels, and metadata describing the intended compliance criterion and any injected fault. The full constructed split contains 4720 training rows, 590 validation rows, and 590 test rows, with balanced binary labels in each split. Following the common evaluation protocol in Appendix D, the experiments cap validation and test evaluation at 500 examples each.

The data specification defines high-stakes examples through criterion-level criticality and policy flags. Examples of policy flags include explicit-confirmation requirements, numerical-precision requirements, prohibitions on leading the customer's answer, waiting-for-answer requirements, and mandatory explanation-on-request rules. Violations are organized by operational fault types corresponding to coercive inducement, deceptive bypass, topic evasion, vulnerable-group handling, and fact omission. These fields induce slices that correspond to concrete compliance failures, which is why the dataset is used for the slice-wise high-stakes error proxy experiment in Section 6.6. For the slice-weighted experiment, validation and test results are stored as a single bundle containing sample metadata and adapter-level correctness vectors. Each metadata row records the high-stakes flag and slice label, and each adapter row stores a binary correctness vector aligned to the metadata order. Thus a value of 1 means that the adapter is correct on the corresponding sample, while 0 means that it is incorrect.

D Experimental Details

Table 4 reports hyperparameters shared by the RAGBench and internally constructed compliance experiments. The model and dataset choices are described in Section 6; this appendix records implementation-level settings needed to reproduce the adapter pools and selection pipeline.

Table 4: Training, evaluation, and selection hyperparameters used for the main experiments.

Stage	Setting	Value
Data	Training split	At most 1000 training examples per task
	Evaluation splits	At most 500 validation examples and 500 test examples per task
	Reported metric	Accuracy on balanced validation/test binary-label evaluation sets for RAGBench
Training	Adapter method	LoRA; dropout 0.05; bias disabled
	Optimization	1 epoch; learning rate 2×10^{-4}; per-device train batch size 4; gradient accumulation 4; bf16 enabled
	Runtime setup	Single NVIDIA RTX 5090 GPU; maximum sequence length 1024; greedy decoding with temperature 0.0 and maximum 4 new tokens
Adapter pool	Size	252 adapters per task and base model
	Heterogeneity axes	Prompt template, LoRA rank/scale settings, target-module set, and adapter-specific sampling seed
	Grid values	Two prompt variants; $r \in \{4, 8, 12, 16, 24, 32\}$; $\alpha/r \in \{1, 1.5, 2\}$; seven target-module sets; base data seed 500
Selection	Candidate filter	Validation accuracy threshold $\tau = 0.5$ and pairwise correlation threshold $\rho = 0.4$
	Ranking and voting	Sort candidates by the chosen validation ranking score, break ties by validation accuracy, and enforce odd cardinality for majority voting

D.1 Correlation-Aware Jury Construction

Pairwise correlations are computed on error vectors. If both error vectors have zero variance and are identical, the implementation treats their correlation as 1.0 for redundancy filtering; if exactly one vector has zero variance, the pair is treated as non-blocking with correlation 0.0. This convention is conservative for duplicate no-error vectors and avoids excluding otherwise competent adapters solely because one error vector is constant.

Algorithm 1 Correlation-aware jury construction

Require: adapters $\{A_i\}_{i=1}^N$, selection sample set V, accuracy threshold τ, correlation threshold ρ, ranking scores r_i, optional jury size k

Ensure: selected jury $\widehat{K}$

1: Compute accuracies $\widehat{\mathrm{Acc}}_i$, error vectors, and pairwise empirical phi correlations $\widehat{\rho}_{ij}$ on V.
2: Form $P_\tau \leftarrow \{i : \widehat{\mathrm{Acc}}_i \geq \tau\}$.
3: Sort P_τ by increasing r_i, using accuracy as a tie-breaker.
4: Initialize $\widehat{K} \leftarrow \emptyset$.
5: **for** i in the sorted order **do**
6:    **if** $\widehat{\rho}_{ij} \leq \rho$ for all $j \in \widehat{K}$ **then**
7:       Add i to $\widehat{K}$.
8:    **end if**
9:    **if** optional fixed size k is specified and $|\widehat{K}| = k$ **then**
10:       **break**
11:    **end if**
12: **end for**
13: If odd cardinality is required and $|\widehat{K}|$ is even, remove the lowest-ranked adapter in $\widehat{K}$.
14:
15: **return** $\widehat{K}$

D.2 Main Performance Confidence Intervals

Table 5 reports the same point estimates as Table 1, with 95% paired bootstrap confidence intervals over test examples.

Table 5: Main performance with 95% bootstrap confidence intervals over test examples. Δ denotes the signed accuracy difference $\text{Vote} - \text{Top-1}$; positive values favor Vote. Difference intervals use paired bootstrap resampling.

Model	Dataset	Accuracy (%)			Accuracy difference	
		Vanilla	Top-1	Vote	Δ	Avg. Δ
Qwen3-0.6B	FinQA	49.49	56.38	61.22	4.85	0.75
		[44.39, 54.34]	[51.53, 61.22]	[56.38, 66.07]	[0.77, 8.93]	[-1.71, 3.22]
	TAT-QA	43.97	61.21	58.62	-2.59	
		[37.50, 50.43]	[55.17, 67.24]	[52.16, 64.67]	[-7.33, 2.16]	
	PubMedQA	16.00	61.20	61.20	0.00	
		[13.00, 19.40]	[57.00, 65.40]	[57.00, 65.40]	[-3.80, 3.80]	
Qwen3-1.7B	FinQA	48.72	65.31	67.09	1.79	1.89
		[43.62, 53.83]	[60.46, 69.90]	[62.50, 71.68]	[-2.04, 5.61]	[-0.65, 4.49]
	TAT-QA	45.69	66.81	68.10	1.29	
		[39.22, 52.16]	[60.78, 72.84]	[62.07, 74.14]	[-3.88, 6.47]	
	PubMedQA	35.60	59.60	62.20	2.60	
		[31.40, 39.80]	[55.20, 63.80]	[58.00, 66.40]	[-1.80, 7.00]	
Llama-3.2-1B	FinQA	54.08	57.40	57.40	0.00	0.78
		[49.23, 58.93]	[52.55, 62.24]	[52.30, 62.24]	[-0.77, 0.77]	[-1.68, 3.25]
	TAT-QA	53.02	63.36	62.50	-0.86	
		[46.55, 59.48]	[56.90, 69.40]	[56.47, 68.53]	[-6.90, 5.17]	
	PubMedQA	50.80	57.20	60.40	3.20	
		[46.40, 55.20]	[52.80, 61.40]	[56.20, 64.60]	[-1.00, 7.40]	
Gemma-3-1B	FinQA	50.00	52.81	53.32	0.51	3.96
		[45.15, 54.85]	[47.96, 57.65]	[48.47, 58.16]	[-4.85, 5.87]	[0.82, 7.17]
	TAT-QA	45.26	59.05	64.22	5.17	
		[38.79, 51.72]	[52.59, 65.52]	[58.19, 70.26]	[-1.29, 11.64]	
	PubMedQA	50.40	52.60	58.80	6.20	
		[46.00, 54.80]	[48.20, 57.00]	[54.40, 63.00]	[1.80, 10.60]	

D.3 Jury-Size Curve Diagnostic

Figure 3 shows the full jury-size curve for the Gemma-3-1B/TAT-QA diagnostic summarized in Table 2.

D.4 Compliance Dataset Slice-Weighted Experiments

For the compliance-dataset slice-weighted filtering experiment, we evaluated a small grid of jury sizes and target-slice weights before fixing the main-text configuration. All results in Tables 6 and 7 use the Qwen3-1.7B heterogeneous adapter pool. Specifically, we varied the jury size over $k \in \{3, 9\}$ and the target-slice weight over $w_{\text{target}} \in \{1.5, 2, 3, 4\}$, while keeping the candidate pool fixed. For each target slice $\mathcal{H}_s$, slice-targeted selection used

$$w(x) = \begin{cases} w_{\text{target}}, & x \in \mathcal{H}_s, \\ 1, & x \notin \mathcal{H}_s. \end{cases}$$

This weighting increases the importance of validation errors on the targeted slice while leaving all other examples at unit weight. Each weighted ranking was compared against the unweighted top-k baseline on the same candidate pool, so the only difference was whether target-slice validation errors were upweighted during ranking.

For each (k, w_{target}) pair, we report the same five test metrics used in the main text. They measure full-set error, target-slice error, pooled error on the other critical slices, error over the full critical subset, and the mass of target-slice mistakes in the overall test distribution. The last quantity is used as a simple proxy for slice-specific deployment risk.

We report $k = 9$ and $w_{\text{target}} = 1.5$ as a representative moderate-weight setting, and provide the full sweep as a sensitivity analysis. The sweep is intended to show qualitative robustness rather than to tune the main reported test result. Smaller weights tended to behave more similarly to the unweighted baseline, while larger weights more aggressively specialized the jury to the target slice and could

Figure 3: Jury-size curve for the Gemma-3-1B/TAT-QA diagnostic. Test accuracy is shown across odd jury sizes for random competent subsets, accuracy-only top-k selection, and the correlation-aware selection rule. The correlation-aware rule achieves the strongest full-jury result, although gains vary with jury size.

increase non-target critical error. Likewise, increasing k could improve aggregation in some settings, but could also dilute the benefit of slice targeting by admitting weaker or more correlated agents. The full sweep is therefore reported in this appendix to show that the qualitative comparison between unweighted and slice-targeted filtering is not tied to a single isolated hyperparameter choice. Table 6 provides a compact summary across (k, w_{target}) settings, while Table 7 reports the corresponding slice-level results.

D.5 Existing Assets and Licenses

Our experiments use existing public datasets, pretrained model checkpoints, and standard open-source training libraries. For the public benchmark experiments, we use the RAGBench release of TAT-QA, FinQA, and PubMedQA [6]. RAGBench is released under the CC BY 4.0 license. The original TAT-QA [24] and FinQA [2] datasets are released under CC BY 4.0, and PubMedQA [11] is released under the MIT license. We cite RAGBench and the corresponding original datasets where applicable, and we do not redistribute the original datasets.

For pretrained backbones, we use public checkpoints from the Qwen3 [21], Llama 3.1/3.2 [9, 16], Gemma 3 [8], Phi-4 [1], and Ministral 3 [17] families. Qwen3 and Ministral 3 checkpoints are released under Apache 2.0, Phi-4 checkpoints under the MIT license, Llama 3.1/3.2 checkpoints under the corresponding Meta Llama Community Licenses and acceptable use policies, and Gemma 3 checkpoints under the Gemma Terms of Use. We use these checkpoints only for research fine-tuning and evaluation, and we do not redistribute the original model weights. Any derived adapter artifacts would be subject to the corresponding base-model terms.

The implementation uses standard open-source libraries, including PyTorch [18], Hugging Face Transformers [20], and Hugging Face PEFT [14]. PyTorch is distributed under a BSD-style license, while Transformers and PEFT are distributed under Apache 2.0. We respect the licenses and terms of use of all assets used in the paper.

E   Additional Comparison Results

The main performance table compares Judge CIA majority voting with the strongest validation-selected adapter from the same heterogeneous small-model pool. For completeness, Table 8 reports single-backbone comparison models, each evaluated before and after LoRA fine-tuning under the same binary-answer extraction and validation label-orientation protocol. These rows provide external context for the small-model jury results, but they are **not the central mechanism studied in this**

Table 6: Summary sensitivity sweep for compliance-dataset slice-targeted filtering. Each row corresponds to one (k, w_{target}) configuration evaluated on the same fixed candidate pool, averaged over the five target-slice runs. Target error reduction is measured relative to the corresponding unweighted top-k baseline and uses the same percentage scale as the error-rate columns. The remaining columns report average test error rates; All is the full test set, Critical is the full critical subset, and Target mass is the target-slice contribution to total test error. The selected setting is highlighted in bold.

Setting		Target error reduction	Average test error rate (%)		
k	w_{target}		All	Critical	Target mass
3	1.5	-0.20	2.84	2.11	0.60
3	2	0.64	2.84	2.26	0.44
3	3	0.45	2.84	2.11	0.48
3	4	0.24	2.88	2.41	0.52
9	**1.5**	**0.21**	**2.48**	**1.65**	**0.48**
9	2	0.00	2.52	1.80	0.52
9	3	0.02	2.44	2.11	0.52
9	4	-0.19	2.56	2.11	0.56

Table 7: Detailed slice-level results for the selected compliance-dataset setting $k = 9$ and $w_{\text{target}} = 1.5$. Each target slice corresponds to one failure-type slice within the critical subset. Target count is the number of examples for that slice in the 500-example test evaluation subset. Columns use the same shorthand as Table 3: All, Target, Other crit., Critical, and Target mass are test error rates in percent. The target-error reductions summarized in Table 6 are heterogeneous across slices; bold entries mark reductions relative to the corresponding unweighted top-k row.

Target slice	Target count	Rule	Setting		Test error rate (%)				
			k	w_{target}	All	Target	Other crit.	Critical	Target mass
Coercive inducement	100	Unweighted top-k	9	1	2.60	1.00	2.88	2.26	0.20
		Slice-weighted top-k	9	1.5	2.60	1.00	**1.92**	**1.50**	0.20
Deceptive bypass	105	Unweighted top-k	9	1	2.60	4.76	1.90	2.26	1.00
		Slice-weighted top-k	9	1.5	**2.40**	**3.81**	**0.95**	**1.50**	**0.80**
Topic evasion	95	Unweighted top-k	9	1	2.60	3.16	0.92	2.26	0.60
		Slice-weighted top-k	9	1.5	**2.20**	**2.11**	0.92	**1.50**	**0.40**
Vulnerable group	104	Unweighted top-k	9	1	2.60	1.92	2.86	2.26	0.40
		Slice-weighted top-k	9	1.5	2.60	2.88	2.86	2.26	0.60
Fact omission	96	Unweighted top-k	9	1	2.60	2.08	2.75	2.26	0.40
		Slice-weighted top-k	9	1.5	2.60	2.08	**1.83**	**1.50**	0.40

paper: they are not jury-selection or majority-vote results. For each comparison backbone, we use a LoRA configuration adapted to that model family, including the corresponding projection-module names and architecture-compatible target modules. Thus the comparison is intended to measure the effect of fine-tuning each backbone under a reasonable model-specific adapter setup, rather than forcing all models to share an identical target-module recipe.

E.1 High-Stakes Comparison Backbone Check

For completeness, Table 9 reports high-stakes compliance results for Qwen3-8B under the same validation-calibrated label-orientation protocol used for the comparison models. This check compares the vanilla and LoRA fine-tuned backbone, not a slice-weighted adapter-pool sweep.

F Routing–jury composition for catastrophic risk

This appendix is the only place where we need domain-indexed notation. This extension is not used in the main experiments, but records how the same slice-risk decomposition composes with an

Table 8: Additional comparison-model results on RAGBench. Very low vanilla scores for some instruction-tuned backbones reflect failures under the fixed binary-answer extraction protocol, not a claim about the backbone's general capability.

| | Accuracy (%) | | | | | |
| | FinQA | | TAT-QA | | PubMedQA | |
Comparison model	Vanilla	Fine-tuned	Vanilla	Fine-tuned	Vanilla	Fine-tuned
Gemma-3-4B	50.00	57.91	50.43	65.95	51.80	49.20
Gemma-3-12B	50.00	76.28	50.00	75.43	51.80	58.80
Llama-3.1-8B	30.87	50.00	41.38	81.47	37.80	54.00
Llama-3.2-3B	50.00	68.62	50.00	75.43	51.80	49.20
Phi-4	1.02	49.49	1.29	50.43	0.00	52.00
Phi-4-mini	50.00	50.26	50.00	50.00	51.80	52.00
Ministral-3-3B	52.04	58.16	51.29	55.60	51.20	63.80
Ministral-3-8B	50.00	61.22	50.00	55.60	51.80	54.20
Ministral-3-14B	50.77	83.16	50.43	84.05	52.00	55.60
Qwen3-4B	7.91	53.06	4.74	49.14	11.80	58.60
Qwen3-8B	50.00	52.30	50.00	50.00	51.80	51.60
Qwen3-14B	11.48	85.97	12.50	83.19	9.60	67.00

Table 9: Detailed high-stakes slice results for the Qwen3-8B comparison backbone. Each target slice corresponds to one failure-type slice in the 500-example test evaluation subset, matching the target-count convention in Table 7. All, Target, Other crit., Critical, and Target mass are test error rates in percent; Other crit. and Critical use critical-subset masks. Average rows aggregate over the five target-slice rows.

Target slice	Target count	Model	All	Target	Other crit.	Critical	Target mass
Coercive inducement	100	Qwen3-8B vanilla	41.60	39.00	45.19	44.36	7.80
		Qwen3-8B fine-tuned	**1.80**	**1.00**	**2.88**	**3.01**	**0.20**
Deceptive bypass	105	Qwen3-8B vanilla	41.60	43.81	43.81	44.36	9.20
		Qwen3-8B fine-tuned	**1.80**	**3.81**	**2.86**	**3.01**	**0.80**
Topic evasion	95	Qwen3-8B vanilla	41.60	35.79	44.95	44.36	6.80
		Qwen3-8B fine-tuned	**1.80**	**2.11**	**2.75**	**3.01**	**0.40**
Vulnerable group	104	Qwen3-8B vanilla	41.60	40.38	43.81	44.36	8.40
		Qwen3-8B fine-tuned	**1.80**	**0.00**	**3.81**	**3.01**	**0.00**
Fact omission	96	Qwen3-8B vanilla	41.60	48.96	44.04	44.36	9.40
		Qwen3-8B fine-tuned	**1.80**	**2.08**	**2.75**	**3.01**	**0.40**
Average	–	Qwen3-8B vanilla	41.60	41.59	44.36	44.36	8.32
		Qwen3-8B fine-tuned	**1.80**	**1.80**	**3.01**	**3.01**	**0.36**

upstream domain router. We combine (i) domain routing error, (ii) within-domain jury error, and (iii) catastrophic slice structure into a single decomposition.

Definition F.1 (Domain family, router, and mixed decision rule). *Let $\mathfrak{D}$ be an index set of domains. For each $d \in \mathfrak{D}$, let $(\mathcal{X}_d, \mathcal{D}_d, f_d)$ denote the input space, data distribution, and ground-truth oracle for that domain. Let π be a probability distribution on $\mathfrak{D}$, so a random input is generated by first sampling $d \sim \pi$ and then sampling $x \sim \mathcal{D}_d$. A router is a possibly randomized map*

$$R : \bigsqcup_{d \in \mathfrak{D}} \mathcal{X}_d \to \mathfrak{D},$$

with predicted domain label $\hat{d} := R(x)$. For each domain d, let $\hat{y}^{(d)}$ be the decision rule of the corresponding domain jury. The mixed decision rule is

$$\hat{y}(x) := \hat{y}^{(\hat{d})}(x), \qquad \hat{d} := R(x).$$

518 *In this appendix, $\mathrm{CatRisk}(\hat{y})$ is evaluated under the domain mixture $d \sim \pi$, $x \sim \mathcal{D}_d$.*

519 **Definition F.2** (True domain label and routing error on slices)**.** *Under the domain mixture, let $d \sim \pi$*
520 *be the true domain label and $x \sim \mathcal{D}_d$ the input. Given a router R, define $\hat{d} := R(x)$. For any*
521 *measurable slice $\mathcal{H}_{d,s} \subseteq \mathcal{X}_d$ with positive probability (typically $\mathcal{H}_{d,s} = \mathcal{H}_s \cap \mathcal{X}_d$), define the slice*
522 *routing error*

$$\alpha_{d,s} := \mathbb{P}(\hat{d} \neq d \mid x \in \mathcal{H}_{d,s}).$$

523 *Define also the slice* misroute catastrophic-error rate

$$\eta_{d,s} := \mathbb{P}\big(\mathcal{C}(x, \hat{y}(x), f_d(x)) = 1 \ \wedge \ \hat{y}(x) \neq f_d(x) \mid x \in \mathcal{H}_{d,s} \ \wedge \ \hat{d} \neq d\big) \in [0,1].$$

524 *When the conditioning event has probability zero, we set $\eta_{d,s} := 0$.*

525 **Definition F.3** (Correct-route jury error and catastrophic amplification)**.** *For each domain d, let $\hat{y}^{(d)}$*
526 *be the domain jury decision rule (possibly after filtering). On correctly routed inputs from slice $\mathcal{H}_{d,s}$,*
527 *define*

$$\mathrm{Err}_{d,s}(\hat{y}^{(d)}) := \mathbb{P}\big(\hat{y}^{(d)}(x) \neq f_d(x) \mid x \in \mathcal{H}_{d,s} \ \wedge \ \hat{d} = d\big),$$

528 *and the corresponding catastrophic amplification factor*

$$\kappa_{d,s} := \mathbb{P}\big(\mathcal{C}(x, \hat{y}^{(d)}(x), f_d(x)) = 1 \mid x \in \mathcal{H}_{d,s} \ \wedge \ \hat{d} = d \ \wedge \ \hat{y}^{(d)}(x) \neq f_d(x)\big) \in [0,1].$$

529 *When the conditioning event has probability zero, we set $\kappa_{d,s} := 0$.*

530 **Proposition F.4** (Router + jury + slice composition)**.** *Let $\hat{y}(x) := \hat{y}^{(\hat{d})}(x)$ be the mixed decision rule.*
531 *Let $\mathcal{H} \subseteq \bigsqcup_{d \in \mathfrak{D}} \mathcal{X}_d$ be the high-stakes set, and let $(\mathcal{H}_{d,s})_{d \in \mathfrak{D}, \, s \in \mathcal{S}}$ be a measurable partition of $\mathcal{H}$*
532 *across domains and slices. Then*

$$\mathrm{CatRisk}(\hat{y}) = \sum_{d \in \mathfrak{D}} \sum_{s \in \mathcal{S}} \mathbb{P}(x \in \mathcal{H}_{d,s}) \Big[\alpha_{d,s} \, \eta_{d,s} + (1 - \alpha_{d,s}) \, \kappa_{d,s} \, \mathrm{Err}_{d,s}(\hat{y}^{(d)}) \Big].$$

533 *Equivalently, using $\mathbb{P}(x \in \mathcal{H}_{d,s}) = \pi(d)\mathbb{P}(x \in \mathcal{H}_{d,s} \mid d)$,*

$$\mathrm{CatRisk}(\hat{y}) = \sum_{d \in \mathfrak{D}} \pi(d) \sum_{s \in \mathcal{S}} \mathbb{P}(x \in \mathcal{H}_{d,s} \mid d) \Big[\alpha_{d,s} \, \eta_{d,s} + (1 - \alpha_{d,s}) \, \kappa_{d,s} \, \mathrm{Err}_{d,s}(\hat{y}^{(d)}) \Big].$$

534 *Proof.* For readability, write $\mathcal{C} = 1$ for $\mathcal{C}(x, \hat{y}(x), f_d(x)) = 1$ and $\hat{y} \neq f_d$ for $\hat{y}(x) \neq f_d(x)$.
535 Decompose the catastrophic-error event on each slice $\mathcal{H}_{d,s}$ and split by whether routing is correct:

$$\mathbb{P}(\mathcal{C} = 1 \wedge \hat{y} \neq f_d \wedge x \in \mathcal{H}_{d,s}) = \mathbb{P}(x \in \mathcal{H}_{d,s})\mathbb{P}(\mathcal{C} = 1 \wedge \hat{y} \neq f_d \mid x \in \mathcal{H}_{d,s}).$$

536 By conditioning on the routing outcome, we have

$$\mathbb{P}(\mathcal{C} = 1 \wedge \hat{y} \neq f_d \mid x \in \mathcal{H}_{d,s}) = \alpha_{d,s} \, \mathbb{P}(\mathcal{C} = 1 \wedge \hat{y} \neq f_d \mid x \in \mathcal{H}_{d,s}, \hat{d} \neq d)$$
$$+ (1 - \alpha_{d,s}) \, \mathbb{P}(\mathcal{C} = 1 \wedge \hat{y} \neq f_d \mid x \in \mathcal{H}_{d,s}, \hat{d} = d).$$

537 The misrouting term is exactly controlled by $\eta_{d,s}$: $\mathbb{P}(\mathcal{C} = 1 \wedge \hat{y} \neq f_d \mid x \in \mathcal{H}_{d,s} \wedge \hat{d} \neq d) = \eta_{d,s}$, so
538 the first branch contributes $\alpha_{d,s}\eta_{d,s}$. On $\{\hat{d} = d\}$ we have $\hat{y}(x) = \hat{y}^{(d)}(x)$, so the chain rule gives

$$\mathbb{P}(\mathcal{C} = 1 \wedge \hat{y} \neq f_d \mid x \in \mathcal{H}_{d,s} \wedge \hat{d} = d) = \mathbb{P}(\mathcal{C} = 1 \mid x \in \mathcal{H}_{d,s} \wedge \hat{d} = d \wedge \hat{y}^{(d)}(x) \neq f_d(x))$$
$$\cdot \mathbb{P}(\hat{y}^{(d)}(x) \neq f_d(x) \mid x \in \mathcal{H}_{d,s} \wedge \hat{d} = d)$$
$$= \kappa_{d,s} \, \mathrm{Err}_{d,s}(\hat{y}^{(d)}).$$

539 Combine and sum over (d, s). $\square$