A Logical Characterization of Filter Bubbles under Personalized Feeds

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Abstract

The filter bubble phenomenon, generated by algorithmic personalization that secludes users within ideologically similar groups, has emerged as a significant issue in social media. Existing logical frameworks typically focus on opinion dynamics through social network relations (the internal perspective) rather than on how platform algorithms contribute to polarization via personalized content feeds. In this paper, we propose a logical framework that formalizes the process of filter bubble formation from an external perspective, emphasizing the role of algorithmic recommendations over personal social interactions. We formally characterize filter bubbles by three criteria: intra-cluster homogeneity below a threshold, inter-cluster isolation above a threshold, and resistance to conflicting information. We characterize how repeated personalized feeds shrink within-cluster distances while increasing between-cluster isolation. Consequently, we show that filter bubble formation can be determined by formulas that capture both internal homogeneity and external isolation.

Introduction

The notion of a **Filter Bubble** (FB) was popularized to describe how algorithmic personalization can isolate users in ideologically homogeneous enclaves, restricting exposure to different viewpoints (see (Pariser 2011; Flaxman, Goel, and Rao 2016; Nguyen et al. 2014) among others). Empirical analyses of social media platforms show that algorithmic ranking and user selection jointly shape the information a user encounters (e.g. (Bakshy, Messing, and Adamic 2015; Nguyen et al. 2014)). Typically, users' prior selections guide algorithmic content selection, which in turn reinforces those preferences by presenting similar viewpoints, gradually narrowing exposure to alternative perspectives, and leading to increasingly homogeneous attitude clusters.

From a logical perspective, FB formation can be seen as a dynamic process of opinion diffusion or aggregation. Existing logical studies have explored similar dynamics in a social setting. For example, (Christoff and Hansen 2015) introduces a modal language with diffusion operators for belief propagation along network edges. In (Shi 2021), collective opinion is modeled as a Markov chain over influence structures to characterize group convergence or oscillation. In these studies, a core assumption is that social relations between agents are straightforward and serve as the primary influence on belief updates.

A more fine-grained framework accounts for the fact that both beliefs and relations can change through homophily. For example, (Baccini, Christoff, and Verbrugge 2024) introduces a logic with separate operators for threshold-based belief diffusion and threshold-based link rewiring. In this model, social relations and opinions evolve together, showing how similarity and influence shape both the network and beliefs. (Pedersen, Smets, and Ågotnes 2019) uses a two-sorted hybrid logic that marks strong and weak ties. They show that when similar agents form ties, then tight, uniform clusters—echo chambers—naturally emerge.

All these works focus on how ties inside a social network drive opinion changes. This perspective provides a natural model for echo chambers, where polarization arises from within-group relations (e.g. (Pedersen, Smets, and Ågotnes 2019; Pedersen 2019)). We call this the internal perspective.

However, the FB phenomenon highlights how platform algorithms push content that fuels polarization. This needs an external perspective: agents are "netizens" who may have no direct ties but share views, so the algorithm treats them as homogeneous and feeds them duplicate content. Each personalized recommendation reinforces their bias, and over time, they form a bubble[1].

In this paper, we formalize filter-bubble formation from an external perspective, with dynamics driven by personalized information feeds rather than social relations. We focus on the questions: How do individual agents update their opinions in response to personalized algorithmic feeds? How can we logically characterize the formation of an FB under these dynamics?

To address these questions, drawing on the above works

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[1]This dynamic is like the CTL-based evolution in (Wang and Dong 2024), but there the driving force still comes from ties inside the network.

on social network logic, we propose a logical framework in which each agent assigns a numeric endorsement degree to each proposition. These degrees represent initial opinions, and then we can measure distances between agents in an opinion space. In our model, each personalized feed is an update to an agent's endorsement, and agents with similar endorsements will receive duplicate content. We capture the dynamic with an update operator that models three update mechanisms: the algorithmic selection, the relation among content, and each agent's learning rate.

Each update directly adjusts agents' endorsement degrees, thereby changing the distances among them. With repeated updates, distances shrink, and agents form like-minded clusters, until further feeds have little effect as the learning rate approaches zero. We then define an FB by three criteria: intra-cluster distances fall below a given threshold, inter-cluster distances exceed a given threshold, and clusters ignore conflicting information.

Static group models

Clusters in group models

We begin with a static model, called a **group model**, before any personalized-feed updates. We consider a finite set of agents, each of which assigns endorsement degrees to every proposition in a fixed proposition set. For each agent a, the map $p \mapsto V(a,p)$ can be seen as an endorsement vector in the cube $[0,1]^{AT}$. Agents' endorsement profiles then induce a distance metric that captures pairwise homogeneity. This distance serves as a parameter for personalized feeds. So the closer two agents are, the more similar their messages are. A threshold determines when two agents are sufficiently close to be regarded as a cluster sharing an ideological frame.

Definition 1 (Group Model) *Let AT be a finite set of propositional atoms, and $\theta \in \mathbb{R}_{\geq 0}$ be a threshold. The static model is a tuple*

$$\mathsf{M} = (AG, V, \mathcal{C}^\theta),$$

where:

- *AG is a finite non-empty set of agents.*
- *$V : AG \times AT \to [0,1]$ is the endorsement function. For each agent a and each proposition $p \in AT$, $V(a,p)$ indicates how strongly a endorses p. Values near 1 mean strong endorsement (tending to accept p as true); values near 0 mean weak endorsement (tending to reject p or withhold acceptance).*
- *$\mathcal{C}^\theta$ is a partition on AG satisfying:*

  1. $\bigcup_i Cl_i = AG$;
  2. $\forall\, a, b \in Cl \in \mathcal{C}^\theta,\ d(a,b) \leq \theta$; *(Homogeneity)*
  3. $\forall c \in AG$, *if* $\forall a \in Cl, d(a,c) \leq \theta$, *then* $c \in Cl$. *(Maximality)*

  where for any $a, b \in AG$, the distance between a and b is defined as

$$d(a,b) \;=\; \sqrt{\sum_{p \in AT} \big(V(a,p) - V(b,p)\big)^2}.$$

A smaller $d(a,b)$ means higher similarity in endorsement profiles. Condition Homogeneity states that any two agents in the same cluster are "close enough" in this metric, while Maximality says that clusters are inclusion-maximal with respect to this closeness: if an outside agent c is within distance θ of every member of Cl, then c is already in Cl. Equivalently, each $Cl \in \mathcal{C}^\theta$ is a maximal set of agents whose pairwise distances do not exceed θ. Thus $\mathcal{C}^\theta$ can be viewed as the family of maximal "ideological components" at resolution θ.

In applications, $V(a,p)$ may initially arise from surveys, historical behaviour, or an initialization procedure that estimates user preferences. Given a fixed θ, every agent belongs to exactly one cluster. As θ increases, clusters may merge, reflecting a coarser notion of similarity; as θ decreases, clusters may split, reflecting a finer distinction between endorsement profiles. The parameter θ therefore controls how strictly we individuate like-minded groups in the opinion space.

We now introduce the basic language and will later enrich it with dynamic operators.

Definition 2 (Language $\mathcal{L}_0$) *Let AT be a finite set of propositional atoms, written $p, q, r, \ldots$. The language $\mathcal{L}_0$ is given by:*

$$\varphi ::= D^\theta_{ab} \mid E^\tau_a p \mid \neg\varphi \mid \varphi \wedge \varphi,$$

where $a \in AG$ and $p \in AT$.

In the language $\mathcal{L}_0$, there are two kinds of atomic formulas. The formula D^θ_{ab} says that the distance between agents a and b does not exceed θ; the formula $E^\tau_a p$ says that agent a's endorsement degree for proposition p is at least τ. Finite conjunctions of such atoms can describe, for example, that all members of a given finite set of agents are mutually close, or that a group of agents all strongly endorse a certain proposition.

The semantic clauses for formulas in $\mathcal{L}_0$ are as follows.

Definition 3 (Satisfaction Relation) *Let $\mathsf{M} = (AG, V, \mathcal{C}^\theta)$ be a group model. The relation $\mathsf{M} \models \varphi$ is defined by:*

$$\begin{aligned}
\mathsf{M} \models E^\tau_a p \quad &\textit{iff} \quad V(a,p) \geq \tau \in [0,1],\\
\mathsf{M} \models D^\theta_{ab} \quad &\textit{iff} \quad d(a,b) \leq \theta,\\
\mathsf{M} \models \neg\psi \quad &\textit{iff} \quad \mathsf{M} \not\models \psi,\\
\mathsf{M} \models \psi \wedge \chi \quad &\textit{iff} \quad \mathsf{M} \models \psi \textit{ and } \mathsf{M} \models \chi.
\end{aligned}$$

Thus $E^\tau_a p$ is read as: agent a's endorsement degree for proposition p is at least $\tau \in [0,1]$, and D^θ_{ab} is read as: the distance between agents a and b is not greater than θ; in other words, a and b are sufficiently similar. At this stage, formulas are evaluated at the level of the whole group model rather than at individual "worlds".

Distance between clusters

In group models, we have defined clusters of agents. The next question is how to measure the distance between these clusters. This is important in identifying bubble formation. Intuitively, a filter bubble should not only be internally cohesive but also well separated from the rest of the group. To

make this precise, we distinguish internal and external distance measures.

We first define a distance between clusters by lifting the agent distance d.

Definition 4 (Cluster distance) *Let Cl_i and Cl_j be two distinct clusters in a group model M. The distance between Cl_i and Cl_j is*

$$d(Cl_i, Cl_j) \;:=\; \min\big\{\, d(a,b) \mid a \in Cl_i,\; b \in Cl_j \big\}.$$

This is a "nearest–neighbour" measure: $d(Cl_i, Cl_j)$ looks at the pair of agents, one from each cluster, that are closest in opinion space. The smaller this value, the easier it is for information or attitudes to "bridge" the two clusters.

On this basis, we can now measure how far a given cluster stands apart from all the others.

Definition 5 (Isolation degree) *Let Cl be a cluster in the model M. The isolation degree of Cl is defined as*

$$d(Cl) \;:=\; \min\big\{\, d(a,b) \mid b \in Cl,\; a \notin Cl \big\}.$$

Equivalently, we may write

$$d(Cl) \;=\; \min\{\, d(Cl, Cl') \mid Cl' \in \mathcal{C}^{\theta},\; Cl' \neq Cl \,\}.$$

Intuitively, the isolation degree of a cluster measures how well it stands apart from the rest of the population. The minimum distance $d(Cl)$ tells us how close any outsider can get to the cluster in opinion space. A high isolation degree means no outsiders come close, indicating a tight, well-separated group.

In addition, we can also define a notion of internal diameter in this way, corresponding to θ.

Definition 6 (Internal diameter) *Let Cl be a cluster in M. The internal diameter of Cl is defined as*

$$\mathrm{diam}(Cl) \;:=\; \max\big\{\, d(a,b) \mid a, b \in Cl \big\}.$$

While $\mathrm{diam}(Cl)$ captures how diverse opinions can be *inside* the cluster, the isolation degree $d(Cl)$ captures how far the cluster is from all outsiders. For filter bubbles, only clusters with both a small internal diameter and a large isolation degree qualify as genuine bubbles. They are internally cohesive but externally insulated.

Updates by personalized feeds

Under personalized feeds, agents receive large streams of information that update their endorsement degrees, change the distances between them, and cause clusters to reconfigure over time.

Prolonged exposure to personalized feeds can shrink distances within specific subgroups while pushing them farther from others. This growing isolation creates filter bubbles. In this section, we extend the baseline model by adding personalized recommendation mechanisms to capture these dynamics. The guiding idea is that the platform selects which piece of information q is shown to which agents at each step, and agents then partially adjust their endorsement degrees in response to q.

When an agent receives personalized information feeds from the system, its endorsement degrees for propositions update depending on (1) whether the agent takes the information to be relevant and (2) the impact of the information on its current state. We formalize this process using a relating function f and a learning rate function β, where β dynamically adapts based on the agent's current state.

For example, if an agent repeatedly accepts information supporting p, its endorsement degree for p will increase over time, e.g., toward 0.9. Once the endorsement is that high, opposing information is likely rejected. Even if it is accepted, it only reduces the endorsement slightly—reflecting a much lower learning rate β for negative updates at high endorsement levels. The exact mechanism also models selective openness, i.e., agents with middling endorsements have higher learning rates and are hence more easily swayed.

Dynamic language

We first extend the static language by adding atoms that describe how information bears on propositions for a given agent, together with a dynamic modality for personalized feeds.

Definition 7 (Language $\mathcal{L}_1$) *Let AT be a finite set of propositional atoms, and let **Info** be a set of informational atoms disjoint from AT. The language $\mathcal{L}_1$ is given by:*

$$\varphi ::= D^{\theta}_{ab} \mid E^{\tau}_{a} p \mid q \bigoplus_a p \mid q \bigominus_a p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle PF(q) \rangle \varphi,$$

*where $a, b \in AG$, $p \in AT$, and $q \in$ **Info**.*

The static atoms $E^{\tau}_{a} p$ and D^{θ}_{ab} are interpreted as before. The new atoms $q \bigoplus_a p$ and $q \bigominus_a p$ talk about how agent a evaluates the informational item q with respect to proposition p (roughly: q supports or weakens p for a). The dynamic modality $\langle PF(q) \rangle \varphi$ says that φ holds after a personalized feed step in which q is recommended.

Dynamic group models

Definition 8 (Dynamic Group Model) *Let $\theta \in \mathbb{R}_{\geq 0}$ be a threshold. A dynamic group model is a tuple*

$$\mathsf{M} = (AG, V, f, \mathcal{C}^{\theta}),$$

where:

- *AG is a finite non-empty set of agents.*
- *$V : AG \times AT \to [0,1]$ is the endorsement function. For each agent a and each proposition $p \in AT$, $V(a,p)$ indicates how strongly a endorses p. Values near 1 mean strong endorsement, tending to accept p as true; values near 0 mean weak endorsement, tending to reject p or withhold acceptance.*
- *$f : AG \times$ **Info** $\times AT \to [0,1]$ is the relating function, where $f(a,q,p)$ indicates to what extent agent a takes the informational item q to support (or undermine) p.*
- *$\mathcal{C}^{\theta}$ is a partition on AG satisfying:*

  1. $\bigcup_i Cl_i = AG$;
  2. $\forall\, a, b \in Cl \in \mathcal{C}^{\theta},\; d(a,b) \leq \theta$; *(Homogeneity)*
  3. $\forall c \in AG$, *if* $\forall a \in Cl, d(a,c) \leq \theta$, *then* $c \in Cl$. *(Maximality)*

where for any $a, b \in AG$, the social distance between a and b is defined as

$$d(a,b) \;=\; \sqrt{\sum_{p \in AT} \big(V(a,p) - V(b,p)\big)^2}.$$

For dynamic evolution, we consider a sequence of models $M_0, M_1, \ldots$ that describe the group's state after successive rounds of personalized recommendations. For convenience, we write $B^n(a,p)$ for the endorsement degree of a for p in M_n, i.e., $B^n(a,p) = V_n(a,p)$.

Personalized feeds and learning

At each stage n, a personalized feed function selects which agents receive a given informational item q.

$$\alpha^n : \textbf{Info} \to \mathcal{P}(AG).$$

Intuitively, $\alpha^n(q)$ is the set of agents to whom the system recommends q at step n. Only these agents will update their endorsements in response to q; other agents keep their current values for that step.

Given $p \in AT$ and $q \in \textbf{Info}$, the relating function

$$f : AG \times \textbf{Info} \times AT \to [0,1]$$

encodes how a interprets q with respect to p. If $f(a,q,p)$ is close to 1, then a regards q as strongly supporting p; if it is close to 0, a takes q to strongly support $\neg p$. When $f(a,q,p)$ is close to $B^n(a,p)$, a treats q as largely irrelevant for p.

We use a learning function β^n to characterize the agent's willingness to learn in state M_n:

$$\beta^n : AG \times [0,1] \to [0,1].$$

Here $\beta^n(a,x)$ is the learning rate of agent a when its current endorsement for p is x. In many applications, one requires that $\beta^n(a,x)$ is higher for moderate values of x and lower for values of x very close to 0 or 1, reflecting the idea that agents with extreme opinions are harder to move.

The update rule for information q at stage n is then

$$B^n(a,p) =$$

$$B^{n-1}(a,p) + \beta^{n-1}\big(a, B^{n-1}(a,p)\big)\big(f(a,q,p) - B^{n-1}(a,p)\big)$$

for all $a \in \alpha^{n-1}(q)$, while $B^n(a,p) = B^{n-1}(a,p)$ for $a \notin \alpha^{n-1}(q)$.

Let $M_0 = (AG, V_0, \mathcal{C}_0^\theta)$ be the initial model, $M_{n-1} = (AG, V_{n-1}, \mathcal{C}_{n-1}^\theta)$, and $M_n = (AG, V_n, \mathcal{C}_n^\theta)$. We write

$$M_{n-1} \xrightarrow{PF(q)} M_n \quad \text{iff}$$

$$V_n(a,p) =$$

$$V_{n-1}(a,p) + \beta^{n-1}(a, V^{n-1}(a,p))(f(a,q,p) - V^{n-1}(a,p))$$

M_n is obtained from M_{n-1} by applying the above update rule for q to all agents in $\alpha^{n-1}(q)$, and recomputing the partition $\mathcal{C}^\theta$ from the new distances.

If the learning rate is constant, i.e. $\beta^n(a,x) = \beta \in (0,1]$ for all n and x, and if $f(a,q,p)$ is fixed, then the sequence $B^0(a,p), B^1(a,p), \ldots$ is a standard exponential smoothing process that converges to $f(a,q,p)$. In other words, repeated exposure to the same informational item q will eventually align a's endorsement of p with $f(a,q,p)$, modelling the "locking in" of attitudes under highly repetitive feeds.

Semantics of the dynamic operators

We now give the truth conditions of the new operators.

Definition 9 (Satisfaction Relation) *Let $M = (AG, V, f, \mathcal{C}^\theta)$ be a dynamic model. The relation $M \models \varphi$ is defined as before for the static connectives, and for the new atoms and modality as follows:*

$$M \models q \oplus_a p \quad \textit{iff} \quad f(a,q,p) > B^n(a,p)$$
$$M \models q \ominus_a p \quad \textit{iff} \quad f(a,q,p) < B^n(a,p)$$
$$M \models \langle PF(q)\rangle\varphi \quad \textit{iff} \quad \exists M' \big(M \xrightarrow{PF(q)} M' \textit{ and } M' \models \varphi\big).$$

The formula $q \oplus_a p$ expresses that q is currently a positive signal for p for agent a, while $q \ominus_a p$ expresses that q is a negative signal. The modality $\langle PF(q)\rangle$ allows us to reason about properties that hold after a single recommendation step. By iteratively applying this modality, we can describe the cumulative effect of entire recommendation histories on distances and cluster structure, which will be crucial in the next section for characterizing when filter bubbles arise.

A logical characterization of FB

The filter bubble has the following characteristics:

-C1: The agents within the cluster exhibit strong homogeneity;

-C2: The cluster is highly isolated within the group.

Condition C1 reflects the idea that members of a bubble share a highly similar endorsement profile. In the opinion space they lie very close to each other. Condition C2 captures the complementary idea that, as a group, they are far away from outsiders. In terms of the isolation degree introduced above, genuine bubbles are those clusters with both small internal distances and a large isolation degree.

Formally, for a family of agents $\mathrm{F} \subseteq AG$ and thresholds θ_e, θ_d with $\theta_e < \theta_d$, we define the following formulas:

$$\text{FB-I} \;:=\; \bigwedge_{a \neq b,\ a,b \in \mathrm{F}} D_{ab}^{\theta_e} \qquad \text{and}$$

$$\text{FB-O} \;:=\; \bigwedge_{a \in \mathrm{F},\ b \in AG \setminus \mathrm{F}} \neg D_{ab}^{\theta_d}.$$

Intuitively, FB-I says that any two agents in F are within distance θ_e (internal homogeneity), while FB-O says that every member of F is at least θ_d away from any outsider (external isolation). The gap between θ_e and θ_d represents a separation band that must be crossed in order to leave or enter the bubble.

We can now state the logical characterization of filter bubbles in the static setting.

Theorem 1 (Static characterization) *A family of agents F forms a filter bubble in M iff $M \models$ FB-I $\wedge$ FB-O.*

The dynamic setting connects this static notion to sequences of personalized updates. Let M_0 be the initial group model before any personalized feeds, and let M_n be the model obtained after a sequence of personalized information $q_1, \ldots, q_n$ has been applied.

Theorem 2 (Dynamic characterization) *Let M_0 be the initial model and M_n the model obtained after a sequence of personalized information $q_1, \cdots, q_n$. A family of agents F forms a filter bubble after these updates iff $M_n \models$ FB-I $\wedge$ FB-O.*

A toy example

To see how filter bubbles arise in our framework, consider the simple case with a single propositional atom p and three agents $AG = \{a, b, c\}$. Endorsement degrees are one-dimensional, so distances reduce to absolute differences $d(x, y) = |V(x, p) - V(y, p)|$.

Initially, suppose

$$V_0(a, p) = 0.4, \quad V_0(b, p) = 0.5, \quad V_0(c, p) = 0.6,$$

so all pairwise distances are small and no non-trivial family forms a filter bubble. Now assume that, over several steps, the platform repeatedly recommends a pro-p informational item q to a and b, but not to c. Under the update rule, $V_n(a, p)$ and $V_n(b, p)$ move towards a high support value (e.g. 0.9), while $V_n(c, p)$ drifts towards a low value (e.g. 0.2) due to different or opposing content.

After enough iterations, we obtain a model M_n with

$$V_n(a, p) \approx 0.9, \quad V_n(b, p) \approx 0.9, \quad V_n(c, p) \approx 0.2.$$

For suitable thresholds θ_e and θ_d with $\theta_e < \theta_d$, the family $F = \{a, b\}$ then satisfies FB-I $\wedge$ FB-O in M_n: a and b are very close to each other but far away from c. In other words, a bubble has formed around $\{a, b\}$ through repeated personalized feeds.

Figure 1 shows this evolution in the opinion space.

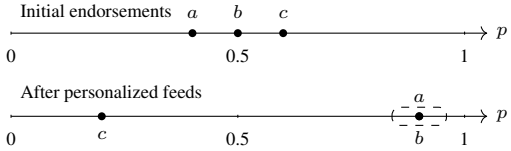

Figure 1: A simple example of bubble formation in the endorsement space: initially all three agents are close; after repeated personalized feeds, a and b become a tightly clustered, isolated pair.

Conclusion

We have proposed a logical framework for modeling filter bubbles from an external perspective, where polarization is driven by personalized information feeds rather than by direct social ties. Starting from group models, we treat agents as points in an opinion space, represented by endorsement degrees for propositions. The resulting metric on endorsement profiles generates cluster structures that reflect ideological similarity at a given threshold.

We introduced dynamic group models in which personalized feeds are formalized as update steps. These steps employ two ingredients: a relating function that captures how incoming information supports or undermines propositions for each agent, and a learning-rate function that depends on the agent's current endorsement state. Each feed adjusts endorsement degrees, and thereby changes social distances and cluster boundaries. Repeated feeds may shrink distances within some parts of the population while increasing distances to others, leading to stable, tightly knit clusters. We finally defined two cluster properties that jointly characterize filter bubbles formation.

We leave the model-checking problem and its computational complexity for future work.

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