Invariance and Allocation: When Subgroup Representation Matters for Subgroup Performance

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Abstract

Unequal representation of demographic groups in training data poses challenges to model generalisation across populations. Standard practice assumes that balancing subgroup representation optimises performance. However, recent empirical results contradict this assumption: in some cases, imbalanced data distributions actually improve subgroup performance, while in others subgroup performance remains unaffected by the absence of an entire subgroup during training. We conduct a systematic study of subgroup allocation across four vision and language models, varying training data composition to characterise the sensitivity of subgroup performance to data balance. We propose the *latent separation hypothesis*, which states that a partially fine-tuned model's dependence on subgroup representation is determined by the degree of separation between subgroup latent space representation of the pre-trained model. We formalise this hypothesis, provide theoretical analysis, and validate it empirically. Finally, we present a practical application to foundation model fine-tuning, demonstrating that quantitative analysis of latent subgroup separation can inform data collection and balancing decisions.

1 Introduction

There is a wide consensus in machine learning that model performance improves monotonically with increasing training data (Rosenfeld et al., 2020; Kaplan et al., 2020). This principle, formalised through dataset scaling laws, has guided much of the recent progress in model training. However, real-world data rarely satisfies the assumption of being independent and identically distributed (i.i.d.) (Arjovsky et al., 2020; Wang et al., 2023). Instead, datasets are composed of clusters of correlated samples, corresponding to subgroups or domains. In the medical domain, clusters may correspond to demographic categories, while in image datasets they may reflect camera types, and in multilingual corpora they may represent language varieties.

In such cases, the question becomes more nuanced: how does model performance on a particular subgroup scale as its representation in the training data increases? While intuition suggests that a higher proportion of subgroup-specific data should directly improve performance on that subgroup, recent studies have revealed surprising counterexamples (Rolf et al., 2021; Weng et al., 2023; Čevora et al., 2025), where increasing subgroup allocation had little or even no effect. This challenges the widely held view that dataset rebalancing is always a reliable solution (Idrissi et al., 2022).

Therefore, understanding the relationship between subgroup allocation and subgroup performance remains an important open question. When concerned about model fairness, practitioners must decide whether to conduct certain interventions, like collecting balanced data across demographic groups, or resampling or augmenting their dataset, potentially at the cost of reduced overall performance (Raji & Buolamwini, 2019; Idrissi et al., 2022). In domain generalisation, practitioners must weigh whether fine-tuning on a smaller set of domain-specific data will yield better deployment performance than

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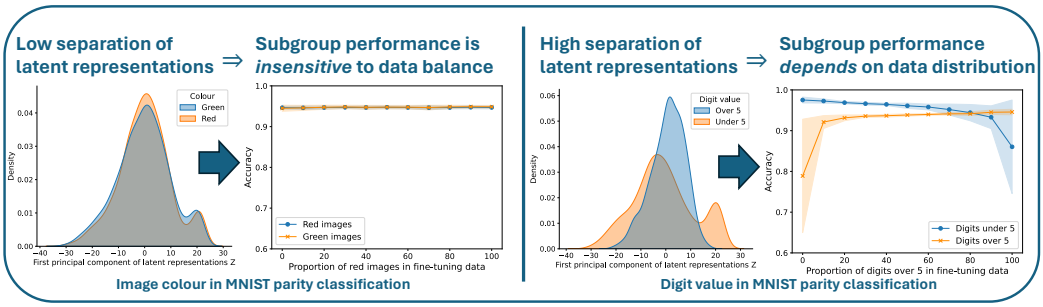

Figure 1: **Model sensitivity to data balance depends on latent separation of subgroups.** Left plots show PCA projections of latent representations of MNIST parity classifiers. Right plots show subgroup accuracy as training data allocation changes.

fine-tuning on a larger set of general data (Hulkund et al., 2025). More broadly, given the cost of data collection and annotation, knowing when subgroup representation matters can guide whether to prioritise general high-quality data or group-specific data.

In this work, we aim to understand how the allocation of data across subgroups affects subgroup performance at inference time, given a fixed training budget. Through extensive experiments in vision and language tasks, we find that subgroup sensitivity to allocation varies dramatically across datasets, models, and attributes. We probe *why* these differences arise and put forward a novel hypothesis: the degree to which redistributing subgroup data improves subgroup performance is determined by how strongly the set of subgroups are separated in the pre-trained model's latent representations.

Our contributions. In §3, we show that widely held explanations for sensitivity to subgroup allocation fail to match empirical behaviour. In §4, we derive a theoretical upper bound on this sensitivity based on subgroup separation in the pre-trained model's latent space. In §5, we demonstrate empirically that sensitivity correlates strongly with the distance between two subgroups in the pre-trained model's representations. Finally, in §J, we show how our findings can guide data selection decisions to improve fairness in a case-study fine-tuning a vision-language foundation model.

2 Problem setting

To study the impact of subgroup allocation on subgroup performance, we consider supervised fine-tuning of a pre-trained model on a dataset of input–label pairs $(x, y) \in \mathcal{X} \times \mathcal{Y}$. The data are drawn from an underlying distribution $\mathcal{P}$, which we randomly split into three disjoint subsets: $\mathcal{P}_{\text{pre-train}}$, $\mathcal{P}_{\text{fine-tune}}$, and $\mathcal{P}_{\text{test}}$. We study settings where the training and test distributions are annotated with m binary attributes $\{A^{(1)}, \ldots, A^{(m)}\}$ which can represent demographic or other sample-level characteristics. Each attribute $A^{(j)}$ induces a binary partition of the data into two subgroups, $a_0^{(j)} = \{(x, y) \mid A^{(j)} = 0\}$ and $a_1^{(j)} = \{(x, y) \mid A^{(j)} = 1\}$. Examples of attributes include gender (male/female), imaging view (frontal/lateral), or dataset source (scanner A/scanner B).

For each subgroup, we record its base population prevalence under $\mathcal{P}$: $\gamma_k^{(j)} = \Pr_{(X,Y,A^{(j)})\sim\mathcal{P}}[A^{(j)} = k], \quad k \in \{0, 1\}$. During fine-tuning, we investigate the impact of manipulating the prevalence of each subgroup, which we refer to as **subgroup allocation**. We assume there is a fixed fine-tuning budget of K examples $\{(x_i, y_i, A_i^{(j)})\}_{i=1}^K$. For each subgroup $a_k^{(j)}$, following Rolf et al. (2021), we define its allocation $\alpha_k^{(j)}$ as the fraction of the fine-tuning dataset coming from subgroup $a_k^{(j)}$.

Our objective is to characterise how subgroup-specific test performance (for instance the loss $\ell_k^{(j)}$) depends on these allocations $\alpha_k^{(j)}$, and to understand why this varies across subgroups.

3 Current explanations are unreliable

Experimental setup. We begin by systematically comparing how subgroup allocation affects subgroup performance across various empirical settings. We explore whether existing hypotheses can

explain the patterns we observe. We fine-tune models on datasets of varying subgroup allocations and assess performance sensitivity to subgroup allocation. We conduct these experiments in four image and text datasets with a range of model architectures for binary classification tasks. This includes **MNIST** (Lecun et al., 1998), **MIMIC-CXR** (chest-X rays) (Johnson et al., 2019), **HAM10000** (skin images) (Tschandl et al., 2018), and **Civil_comments** (Borkan et al., 2019). Additional details are provided in Section C.

Sensitivity to subgroup allocation is highly variable. Across our three real-world datasets, we find substantial variability in subgroup performance sensitivity to allocation. Certain subgroups show no benefit from increased allocation and achieve equivalent performance whether the model is fine-tuned only on that subgroup or entirely without it (e.g., age in MIMIC). In contrast, other subgroups, like dataset of origin in HAM10000, are sensitive to their allocation. This results an estimated slopes of the accuracy change which range from 0 (no effect) to 0.05 (strong sensitivity), as shown in Figure D3. In other words, while the MIMIC model performs equivalently on old individuals whether or not it has been trained on such images, the HAM model is almost 10% less accurate on one dataset source if it has not been trained on any images from it. We summarise all slopes obtained in Tables D3 and D4. These results mirror other recent work which showed that while subgroup performance is sometimes maximised when a dataset is balanced with respect to a certain attribute, it can also be maximised at skewed allocations, or in other cases it can be equally maximised across allocations (Rolf et al., 2021; Claucich et al., 2025; Čevora et al., 2025; Weng et al., 2023).

Current hypotheses do not explain differences in allocation sensitivity. We explore common explanations for sensitivity to subgroup allocation including whether it could be linked to certain subgroups being under-represented in the initial pre-training dataset, certain subgroups being disadvantaged by the pre-trained model's performance, or certain subgroups having substantial class imbalances (Figures E4 and 5 respectively). However, none of these three explanations appear to be consistently correlated with sensitivity to subgroup allocation. For instance, we see certain subgroups which are extremely under-represented in the pre-training set (e.g., less than 20% of the pre-training data) which show no reduction in loss as fine-tuning data allocation increases (Figure E4 top row). This suggests that over-representing under-represented subgroups does not necessarily yield performance improvements, in line with other recent work (Schrouff et al., 2023; Claucich et al., 2025; Roh et al., 2021) and contradicting many other pieces of research (Idrissi et al., 2022; Wang & Russakovsky, 2023; Alabdulmohsin et al., 2024). Similarly, we see surprising examples where certain subgroups which are initially amongst the lowest performing by the pre-trained model also show almost no decrease in loss as allocation increases (Figure E4 bottom row).

4 Bounding sensitivity to subgroup allocation with model latent representations

Given the lack of a coherent explanation for differences in sensitivity to subgroup balance, we introduce the follwoing hypothesis: sensitivity to subgroup allocation may be explained by whether a model learns distinct representations for subgroups a_0 and a_1, and thus needs to be trained on sufficiently high proportion of samples from, say a_0, in order to achieve good performance on a_0.

Through theoretical analysis and with certain assumptions, we are able to validate this hypothesis. We show that low latent separation with respect to an attribute A implies low sensitivity to A allocation in the fine-tuning dataset, i.e., fine-tuned model performance on a_0 and on a_1 does not significantly vary for different allocations $\alpha^{(A)}$. We formalise this idea by proving that small total variation distance (TV) between class-conditioned subgroup representations Z in a pre-trained model, implies that last-layer fine-tuning on any dataset which differs in the allocation of A, but not in its proportion of Y, cannot result in models which differ significantly specifically in their subgroup accuracies (Theorem 4.1). To the best of our knowledge, this is the first result that links subgroup allocation sensitivity to class-conditional representation separation. For readability, we provide only proof sketches here, deferring the full derivations to Appendix F.

Lemma 4.1. *Let $f_{\theta,\eta}(x) = g_\theta(h_\eta(x))$ with representation $Z = h_\eta(X)$ and predictor $\hat{Y} = g_\theta(Z)$, where g_θ is the last layer. Assume that*

$$\mathsf{TV}\big(\mathbb{P}_\theta[Z \mid Y = y, A = 1],\, \mathbb{P}_\theta[Z \mid Y = y, A = 0]\big) \leq \varepsilon,$$

for $y \in \{0,1\}$*. Then, it holds* $\left|\mathbb{P}_\theta(Z \mid Y=y, A=a) - \mathbb{P}_\theta(Z \mid Y=y)\right| \leq \varepsilon$ *for all* $a \in \{0,1\}$*.*

This theorem tells us that if the representation Z is similar between groups (i.e., $A=0$ and $A=1$) for a given label $Y=y$, then each group's representation is also close to the overall representation for that label, meaning group membership doesn't significantly affect the representation once the label is fixed. We can use this lemma to prove the main result. The proof is shown in F.1.

Assumptions. We assume that (i) fine-tuning datasets $\mathtt{D}'$, $\mathtt{D}''$ differ only in subgroup allocations $\alpha^{(A)}$, and (ii) the marginal label distribution $P(Y)$ and conditional distribution $P(Y \mid A)$ remain unchanged across datasets.

Theorem 4.1 (Group accuracy parity)**.** *Let* $f_{\theta,\eta}(x) = g_\theta(h_\eta(x))$ *with representation* $Z = h_\eta(X)$ *and predictor* $\hat{Y} = g_\theta(Z)$*, where* g_θ *is the last layer. For a set of parameters* θ*, define the quantity*

$$\mathsf{TV}(\theta) := \mathbb{E}_y\left[\mathsf{TV}\big(\mathbb{P}_\theta[Z \mid Y=y, A=1], \mathbb{P}_\theta[Z \mid Y=y, A=0]\big)\right].$$

Suppose that the model is fine-tuned on two datasets $\mathtt{D}'$*,* $\mathtt{D}''$ *which differ only in* $\alpha^{(A)}$*, yielding two models with parameters* θ' *and* θ''*. If* $\mathsf{TV}(\theta') \leq \varepsilon$ *and* $\mathsf{TV}(\theta'') \leq \varepsilon$*, then for all* $a \in \{0,1\}$*:*

$$|\textsc{Acc}_{\theta'}(A=a) - \textsc{Acc}_{\theta''}(A=a)| \leq 4\varepsilon + |\textsc{Acc}_{\theta'} - \textsc{Acc}_{\theta''}|.$$

This theorem tells us that if two models are fine-tuned on datasets that differ only in the group proportions (i.e., the distribution of A), and both models learn approximately group-invariant representations (i.e., $\mathsf{TV}(\theta) \leq \varepsilon$), then the accuracy on any group $A=a$ will not differ much between the models.

5 Subgroup separation predicts sensitivity to allocation in real-world experiments

We showed that if subgroup representations are nearly indistinguishable (as measured by TV), then modifying fine-tuning dataset subgroup allocation (assuming that $P(Y)$ and $P(Y|A)$ are unchanged) has little effect on downstream accuracies. We now turn to empirical analyses to test how well this theoretical upper bound captures real-world behaviour, and to investigate whether finer-grained patterns, such as correlations between representation separation and allocation sensitivity, emerge beyond what the bound alone reveals.

Assessment of representation invariance. We keep the same setup as previously, where models are pre-trained on a random subset of each of the four datasets, and their last layer is then fine-tuned with varying subgroup allocations. To cover a broad range of subgroups, we relax the theorem's assumption that $P(Y)$ is fixed across fine-tuning distributions. Notably, many attributes do satisfy (or closely approximate) this assumption as they have equal class prevalences. This includes gender, marital status, language, and race in MIMIC; localisation in HAM10000; year and race in Civil Comments; and random groups across all datasets.

We quantify subgroup separation by extracting penultimate-layer embeddings, projecting them to a lower-dimensional space using PCA (retaining $\geq 70\%$ variance), and computing the mean **total variation** distance (TV) between subgroup distributions conditioned on Y. For completeness, we explore additional distance metrics, the **Wasserstein distance** (WD) and the **Fréchet distance** (FD), which emphasise distinct representation differences. We present more details and ablations in §G.

Intuition for our results in MNIST. In our synthetic MNIST set-up, we *know* that a good parity classification model should not rely on colour ($A^{(0)}$) to make a prediction, so learnt representations $\boldsymbol{z}$ should be invariant to $A^{(0)}$, in other words $P(\boldsymbol{z} \mid Y, a_0^{(0)}) = P(\boldsymbol{z} \mid Y, a_1^{(0)})$. In contrast, a model must encode some notion of which digit is represented in order to classify it into even vs. odd, and therefore its learned representation *should* depend on whether the digit is over 5 or under 5 ($A^{(1)}$). We test this by training a 2-layer CNN. As expected, the penultimate layer model embeddings do cluster by digit $A^{(1)}$ but not by colour $A^{(0)}$ (Figure 1). Quantitatively, the average TV between the images representing digits over 5 and those under 5 is approximately twice the average distance between the red and green images (Figure G6).

Our hypothesis predicts that fine-tuning this model on datasets with different proportions of the same subgroups will show that the model is only sensitive to the allocation of the under 5/over 5 groups

but not to the red/green groups. Indeed, we find that the subgroup accuracy on under 5 and over 5 images drops sharply as their fine-tuning dataset allocation drops, while the subgroup accuracy in the red/green images is roughly independent of the fine-tuning dataset allocation (Figure 1, bottom row). The model can effectively generalise "zero-shot" to new colours, but not to unseen digit groups. Looking back at common explanations in the literature, we note that both red/green and over 5/under 5 groups have equal $P(Y)$, equal base accuracy, and equal base allocation in the pre-training dataset, highlighting the inadequacy of current explanations.

Subgroup separation is highly correlated to allocation sensitivity. We next explore how our hypothesis holds in real-world datasets with more subgroups and larger models, where there is no clear-cut separation between attributes a model should be invariant to or not. We first note that there is wide variation in subgroup separation in penultimate-layer representations (as shown in Figure G6). Across all our experiments, we find a significant correlation between subgroup separation in pre-trained representations and subgroup sensitivity to allocation during fine-tuning (Figure 2). This correlation holds across the three distance metrics (TV, WD, FD). Intuitively, when two subgroups have similar representation of the target Y (i.e., $P(\boldsymbol{z} \mid Y, a_0^{(0)}) = P(\boldsymbol{z} \mid Y, a_1^{(0)})$), then additional subgroup specific data provides little added value. Conversely, when subgroup representations are separated, allocation has a large effect.

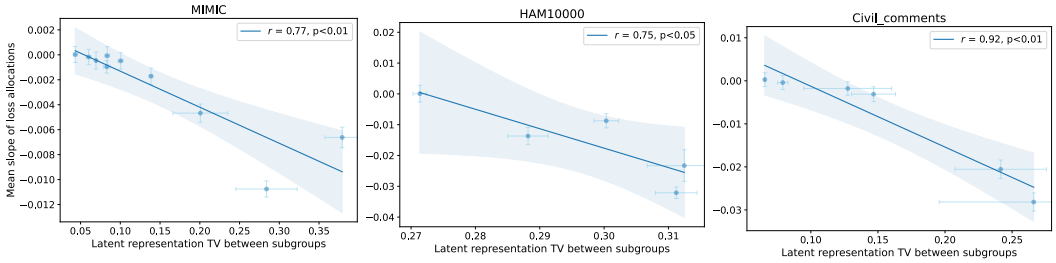

Figure 2: **Sensitivity to subgroup allocation is highly correlated with separation** in the pre-trained model's representation space (as measured by total variation distance, TV) across the three datasets. Each dot represents mean TV and loss slope for one subgroup, averaged across 9 fine-tuning runs.

We also observe that zero-shot generalisation is directly related to subgroup separation. As shown in Figure H10, subgroup AUC is constant between 100% and 0% training data allocation only when representations are (approximately) invariant across them. This echoes work in the domain adaptation literature, which stipulate that invariant representations are more robust as they generalise across environments (Arjovsky et al., 2020). Here, we extend this principle to subgroup balancing by using a measurable property of the *pre-trained* model, class-conditional latent separation, as a predictive diagnostic for when allocation will matter. Altogether, these empirical results support our theoretical finding of an upper bound to performance differences across allocations, and the consistent correlation suggests there may even be a stronger phenomenon at play.

While we first experiment with last-layer fine-tuning for consistency with our theory, we test whether this correlation extends to settings where the fine-tuning occurs on the full network (and therefore separations of representations Z could change). We find equally significant correlations, with effects of stronger magnitudes, across all three datasets (Figure H12). We also show the practical value of our results in fine-tuning a vision-language foundation model in J. We present limitations of this work and opportunities for further investigation in Section L.

Conclusions. We propose, prove (under certain assumptions), and give extensive empirical evidence for, a novel hypothesis explaining why in some cases subgroup training data allocation does not matter for subgroup performance, and in other cases, it is crucial. Unlike standard explanations, our hypothesis consistently matches empirical results across diverse datasets and models. By predicting subgroup allocation sensitivity through latent representation analysis, we provide a new way to inform crucial training data decisions, maximising fairness, accuracy, and efficiency.

6 Reproducibility Statement

We provide extensive details in the appendix to ensure reproducibility of our results. For each dataset, we describe pre-processing steps, model architectures, hyper-parameter optimisation procedures, and training details. We pre-train three models with different random seeds per dataset and conduct fine-tuning across three more random data splits, yielding nine runs per allocation. We further conduct ablations on different representation distance metrics, full vs. last-layer fine-tuning, and fine-tuning budget K. We release code and scripts (upon acceptance) to replicate all figures and analyses.

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A Appendix structure

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- § E: Supplementary results on common hypotheses not holding
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- § G: Supplementary results on latent representation distances
- § H: Supplementary results on the correlation between representation distance and subgroup allocation sensitivity
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B Related work

Dataset scaling is not straightforward when the train and deployment settings are not i.i.d. We are broadly interested in the relationship between model training data and model performance. Research in this area has investigated different aspects of data (e.g., size, composition, or individual points) and different performance metrics (e.g., overall loss, fairness, or domain-specific accuracy) Hashimoto (2021). While dataset scaling laws have shown that performance improvements follow predictable power law trends (Rosenfeld et al., 2020; Kaplan et al., 2020), this relationship becomes more complex when the training and test data are drawn from different distributions. In such cases more data is not necessarily better. For instance, Hulkund et al. (2025) and Shen et al. (2024) both show that when optimising for a specific deployment setting, a subset of the data can yield better performance than the full dataset. Similarly, Diaz & Madaio (2025) argue that scaled training data can have a negative impact depending on the evaluation metrics and subpopulations considered.

Subgroup data scaling through the lens of fairness This problem has also been studied indirectly in the field of fairness, where data are grouped into subgroups (e.g., based on demographic attributes), and one investigates how training data composition (i.e., number/proportion of samples from certain subgroups) affects fairness (i.e., some metric based on model performance on certain subgroups). The prevailing assumption is again that more subgroup-specific training data leads to improved performance on that subgroup (Raji & Buolamwini, 2019; Chen et al., 2018). When more data cannot be collected, the standard intervention is to rebalance the model training data by under- or over-sampling samples from certain subgroups (Idrissi et al., 2022). Many works show that this simple method can yield remarkable fairness improvements both when training from scratch (Idrissi et al., 2022) and when fine-tuning (potentially biased) pre-trained models (Kirichenko et al., 2023; Wang & Russakovsky, 2023; Alabdulmohsin et al., 2024).

Inconsistencies in subgroup balancing results However, a growing body of work argues that balancing data does not necessarily improve fairness, and that it can even be detrimental. Schrouff et al. (2024) use a causal framework to show conditions under which data balancing will *not* improve model fairness. Similarly, Claucich et al. (2025) and Roh et al. (2021) show that fairness is not necessarily maximized at 50/50 group ratios, and argue that this is due to unequal task difficulty across groups. Weng et al. (2023) and Čevora et al. (2025) even show cases where a model's performance on female medical images remains constant (and sometimes decreases) as the proportion of female images in the training set increases. Loss-reweighting based approaches such as group distributionally robust optimisation (Sagawa et al., 2020) are based on a related principle: rather than balancing group size, they reweight groups with higher losses. However, loss-based methods also exhibit failure modes (Zong et al., 2023), and it remains unclear whether up-weighting data from poor-performing groups necessarily improves performance on those groups. Together, these studies reflect an emerging trend in fairness research, that different subgroups have distinct properties and causes of under-performance, and therefore respond differently to interventions like data balancing or loss re-weighting (Wang, 2025; Alloula et al., 2025; Jones et al., 2024; Yang et al., 2023). However, without a causal framework (which is difficult to apply in practice) (Jones et al., 2024; Schrouff et al., 2024) or direct experimentation, it is difficult to determine *a priori* whether balancing will improve fairness.

Impact of subgroup allocation on subgroup performance Our work differs from fairness-oriented studies in that we address a more fundamental question: how does subgroup allocation affect subgroup performance? We argue that understanding this is prerequisite for tackling fairness concerns and implementing any bias mitigation methods. Despite its importance, this question has received little direct attention, and as discussed above, it is not clear whether redistributing data from under-represented or poorly performing groups reliably improves performance for those groups. Rolf et al. (2021) take a first step by fitting a per-group power-law scaling model describing the impact of subgroup and total training data size on subgroup performance. Similarly to the fairness papers, they show that the optimal allocation varies across datasets and tasks, and is not necessarily balanced. Our work builds on theirs in several ways, but differs crucially in that we propose a (theoretically-grounded) explanation for why subgroup allocation impacts subgroup performance so variably. This enables us to determine, for a given model and subgroup, whether subgroup allocation is likely to matter, and can help guide fine-tuning strategies to maximise subgroup performance.

C Supplementary experimental details

We present additional details on the four datasets and models used. We do not use the full datasets for fine-tuning because to keep all fine-tuning allocation datasets the same size we set them to the size of the smallest subgroup in each dataset. For each dataset, we pre-train three models with different random seeds. For each subgroup allocation experiment, we fine-tune the final classification layer of each pre-trained model on the three randomly generated data splits, resulting in a total of nine fine-tuned models per subgroup allocation. This is then repeated across all the subgroups we consider. This reflects one of the strengths of our work, that compared to the typical data balancing paper, we compare many different subgroups within a dataset (5 to 11), allowing us to gain new insights on why they may have different properties.

Table 1 summarises the implementation choices for each dataset. We conduct hyperparameter tuning both for pre-training and fine-tuning. We optimise hyperparameters within the following ranges:

- **Batch size**: 64, 128, 256
- **Learning rate**: [1e-6:1e-3]
- **Weight decay**: [1e-6:1e-2]

Table 1: Implementation details for all models.

Training strategy	**MNIST**	**MIMIC-CXR**	**HAM10000**	**Civil_comments**
Y	Even/odd digit	Pleural effusion	Malignant/benign lesion	Comment toxicity
Backbone	2-layer CNN	DenseNet121 Huang et al. (2016)	ViTB16 Dosovitskiy et al. (2021)	BERTClassifier (uncased) Devlin et al. (2018)
Initialisation	None	ImageNet Deng et al. (2009)	ImageNet Deng et al. (2009)	Bookcorpus, Wikipedia (English)
$\mathbf{N_{pretrain}}$	5,000	33,237	2,000	17,920
$\mathbf{N_{finetune}}$	24,000	22,353	3,184	24,000
$\mathbf{N_{test}}$	4,000	26,590	1,000	12,243
Image size	3x28x28	3x256x256	3x256x256	NA
Augmentation	Flip, rotation, Gaussian blur	Flip, rotation, affine transformation, crop	Flip, rotation, color jitter, affine transformation, crop	None
Optimiser	Adam	Adam	AdamW	AdamW
Loss	Binary cross-entropy	Binary cross-entropy	Binary cross-entropy	Binary cross-entropy

We create natural subgroups based on the metadata available, such as based on sex, age, ethnicity, or image-related attributes like the position of the X-ray. subgroups are generated based on whether the comment mentions specific attributes, like gender, religion, or sexual orientation.

Table 2: Subgroups used for each dataset and their base population prevalence and class prevalence.

Dataset	**Attribute**	**Group 0 / 1**	**Pop prev.**	**Class prev.**
MNIST	Colour	Red / Green	0.50	0.50
	Digit value	Below 5 / Over	0.50	0.50
	Random	/	0.50	0.50
MIMIC-CXR	View Position	Lateral / frontal	0.65	0.26
	Patient Orientation	Recumbent / erect	0.77	0.20
	Procedure	Portable / fixed scanner	0.46	0.11
	Support_Devices	Absent / Present	0.23	0.40
	Gender	Female / male	0.51	0.25
	Insurance	Not-private / private	0.27	0.19
	Language	English / non-English	0.88	0.23
	Marital Status	Married / unmarried	0.44	0.25
	Race	White / non-White	0.66	0.25
	Age	Under 60 / Above 60	0.56	0.28
	Random	/	0.50	0.23
HAM10000	Sex	Male / Female	0.54	0.22
	Age	under 50 / above 50	0.46	0.27
	Dataset of origin	Rosendahl or Vidir_molemax / Neither	0.62	0.13
	Localisation of lesion	Central / Extremity	0.38	0.17
	Random	/	0.53	0.17
Civil_comments	Gender	No mention / mention	0.20	0.15
	Orientation	No mention / mention	0.03	0.27
	Religion	No mention / mention	0.16	0.12
	Race	No mention / mention	0.09	0.11
	Year of comment	Pre-2017 / Post-2017	0.76	0.11
	Random	/	0.50	0.11

D Supplementary results on subgroup allocation sensitivity

D.1 Sensitivity of all attributes

We report full results for subgroup allocation sensitivity across all attributes for two full fine-tuning and last-layer fine-tuning (Tables 3 and 4 respectively).

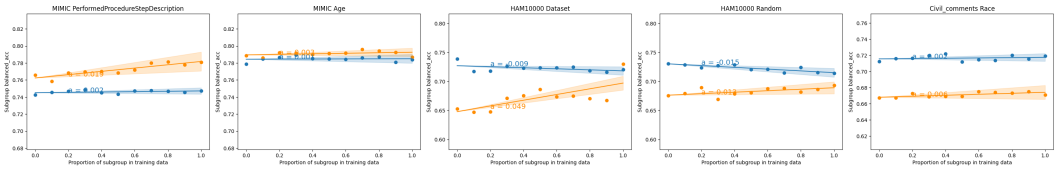

Figure 3: **While some subgroups' accuracy increases with increased representation in training data, others' performance is independent of their training data representation**. Mean subgroup performance across 9 fine-tuning runs is shown along side estimated linear regression slopes a.

Table 3: Sensitivity of subgroup performance to fine-tuning allocation across datasets and attributes in full model fine-tuning. Reported values are esimated regression slopes (mean $\pm$ standard deviation across nine fine-tuning runs) for subgroup loss, balanced accuracy, and AUC as allocation varies from 0% to 100%. Larger absolute slopes indicate higher sensitivity.

Dataset	Attribute	Loss slope	Balanced Acc slope	AUC slope
MIMIC	View Position	-0.027 (0.001)	0.034 (0.003)	0.012 (0.001)
MIMIC	Patient Orientation	-0.009 (0.001)	0.013 (0.002)	0.003 (0.000)
MIMIC	Procedure	-0.026 (0.001)	0.026 (0.002)	0.016 (0.001)
MIMIC	Support_Devices	-0.009 (0.001)	0.010 (0.001)	0.006 (0.000)
MIMIC	Gender	-0.003 (0.001)	0.004 (0.002)	0.002 (0.000)
MIMIC	Insurance	-0.002 (0.001)	0.003 (0.002)	0.001 (0.000)
MIMIC	Language	-0.002 (0.001)	0.001 (0.002)	0.001 (0.000)
MIMIC	Marital_Status	-0.001 (0.001)	-0.002 (0.001)	0.000 (0.000)
MIMIC	Race_cat	-0.002 (0.001)	-0.003 (0.002)	0.001 (0.000)
MIMIC	Age	-0.006 (0.001)	0.009 (0.002)	0.003 (0.000)
MIMIC	Random	-0.000 (0.001)	0.000 (0.002)	0.000 (0.000)
HAM10000	Sex	-0.090 (0.008)	0.058 (0.003)	0.032 (0.001)
HAM10000	Age	-0.157 (0.011)	0.049 (0.003)	0.044 (0.001)
HAM10000	Dataset	-0.230 (0.011)	0.129 (0.003)	0.077 (0.002)
HAM10000	Localization	-0.173 (0.010)	0.082 (0.003)	0.049 (0.001)
HAM10000	Random	0.018 (0.008)	-0.010 (0.003)	-0.002 (0.001)
Civil_comments	Gender	-0.034 (0.011)	0.023 (0.003)	0.017 (0.003)
Civil_comments	Orientation	-0.243 (0.012)	0.094 (0.004)	0.089 (0.003)
Civil_comments	Religion	-0.048 (0.012)	0.031 (0.003)	0.025 (0.003)
Civil_comments	Race	-0.146 (0.014)	0.037 (0.002)	0.041 (0.002)
Civil_comments	Year	-0.022 (0.013)	0.013 (0.003)	0.011 (0.004)
Civil_comments	Random	-0.014 (0.011)	0.008 (0.003)	0.007 (0.003)

D.2 Experimenting with more complex fits

We consider more complex functional forms for fitting subgroup loss vs. allocation (e.g., power-law models as in Rolf et al. (2021)), but we find them unstable: fits vary substantially with small changes in data, and standard deviations are large (Table C5, an issue also reported in Rolf et al. (2021)). We attribute this to small sample sizes and higher heterogeneity across subgroups in our setting. In contrast, linear regression provides robust and interpretable summaries, and subgroup losses appear roughly linear across allocations. We therefore adopt linear fits as a first-order sensitivity measure.

We report results from fitting the subgroup allocation scaling law model introduced by Rolf et al. (2021) below. This model describes subgroup loss as a function of group size and total sample size:

$$\ell_g(n_g, n) \;=\; \sigma_g^2\, n_g^{-p_g} \;+\; \tau_g^2\, n^{-q_g} \;+\; \delta_g,$$

Table 4: Sensitivity of subgroup performance to fine-tuning allocation across datasets and attributes in last-layer fine-tuning. Reported values are esimated regression slopes (mean $\pm$ standard deviation across nine fine-tuning runs) for subgroup loss, balanced accuracy, and AUC as allocation varies from 0% to 100%. Larger absolute slopes indicate higher sensitivity.

Dataset	Attribute	Loss slope	Balanced Acc slope	AUC slope
MIMIC	View Position	-0.007 (0.001)	0.001 (0.002)	-0.000 (0.000)
MIMIC	Patient Orientation	-0.008 (0.001)	-0.002 (0.001)	0.000 (0.000)
MIMIC	Procedure	-0.011 (0.001)	0.014 (0.002)	0.003 (0.000)
MIMIC	Support_Devices	-0.005 (0.001)	0.000 (0.001)	0.000 (0.001)
MIMIC	Gender	-0.000 (0.001)	0.001 (0.002)	0.000 (0.000)
MIMIC	Insurance	-0.000 (0.001)	0.000 (0.002)	-0.000 (0.000)
MIMIC	Language	-0.001 (0.001)	-0.000 (0.001)	0.001 (0.000)
MIMIC	Marital_Status	-0.000 (0.001)	-0.000 (0.002)	-0.000 (0.000)
MIMIC	Race_cat	-0.000 (0.001)	-0.002 (0.002)	-0.000 (0.000)
MIMIC	Age	-0.002 (0.001)	0.002 (0.002)	0.001 (0.000)
MIMIC	Random	0.000 (0.001)	0.001 (0.002)	-0.000 (0.000)
HAM10000	Sex	-0.009 (0.002)	0.004 (0.003)	0.005 (0.001)
HAM10000	Age	-0.032 (0.002)	-0.011 (0.003)	0.004 (0.001)
HAM10000	Dataset	-0.023 (0.005)	0.020 (0.003)	0.010 (0.001)
HAM10000	Localization	-0.014 (0.003)	0.010 (0.003)	0.004 (0.001)
HAM10000	Random	0.000 (0.003)	-0.001 (0.003)	-0.000 (0.001)
Civil_comments	Gender	-0.002 (0.002)	0.001 (0.002)	0.000 (0.002)
Civil_comments	Orientation	-0.026 (0.002)	-0.000 (0.002)	0.002 (0.002)
Civil_comments	Religion	-0.003 (0.002)	-0.000 (0.002)	0.000 (0.002)
Civil_comments	Race	-0.022 (0.002)	0.004 (0.002)	0.000 (0.002)
Civil_comments	Year	-0.000 (0.002)	0.000 (0.003)	0.000 (0.002)
Civil_comments	Random	-0.000 (0.002)	-0.001 (0.002)	-0.000 (0.002)

where n_g is the number of samples in subgroup g, n is the total number of samples, and $\sigma_g, \tau_g > 0$, $p_g, q_g > 0$. Following their setup, we conduct additional experiments where we vary the fine-tuning dataset size K (to obtain more data points) and fit this model under the same parameter constraints.

In our experiments, however, we find that the fitted parameters often come with very large standard deviations, suggesting instability in the estimates. An example for two groups in MIMIC is given below:

Table 5: Estimated power-law scaling model parameter fits and standard deviations.

Attribute	Group	σ_g	p_g	τ_g	q_g	δ_g
Gender	0	0.000 ± 0.000	1.298 ± 0.000	3136.744 ± 13745.833	2.000 ± 0.964	0.302 ± 0.020
Gender	1	278.269 ± 647.492	2.000 ± 0.650	3002.106 ± 13853.293	2.000 ± 1.015	0.348 ± 0.019
ViewPosition	0	4.843 ± 6.518	0.850 ± 0.387	1.523 ± 4.728	0.225 ± 1.188	0.000 ± 1.388
ViewPosition	1	1.035 ± 0.806	0.260 ± 0.336	2.175 ± 8.930	0.276 ± 1.359	0.000 ± 1.609

E Supplementary results on common hypotheses not holding

We additionally test the hypothesis that subgroups with a high class imbalance may lead to greater model sensitivity to their allocation by modifying $\mathcal{P}(Y)$. However, we see no consistent correlation between the two variables.

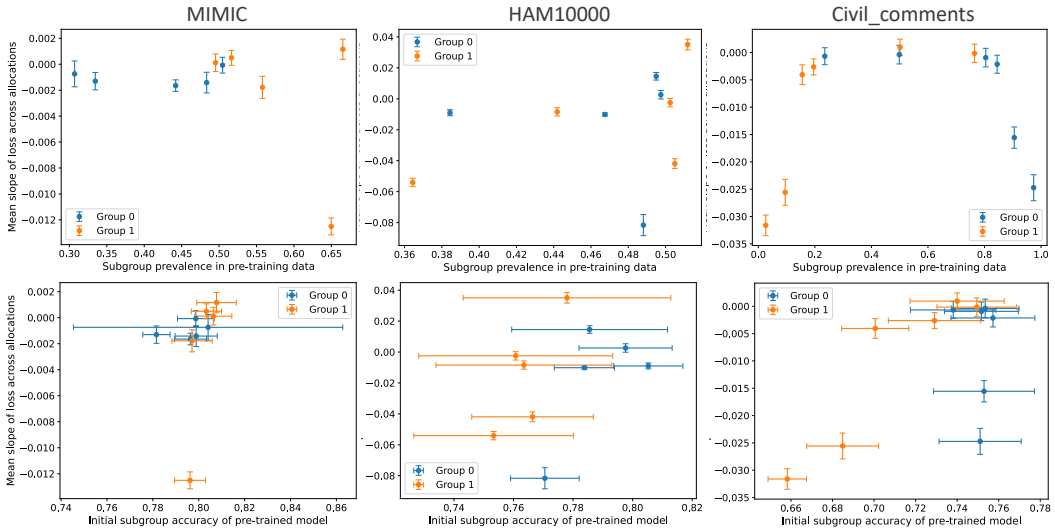

Figure 4: **Performance on subgroups under-represented** during pre-training (top) and performance on **disadvantaged subgroups** (bottom) do not necessarily improve with increasing dataset allocation. Each dot represents a subgroup with bars indicating variation across 9 fine-tuning runs.

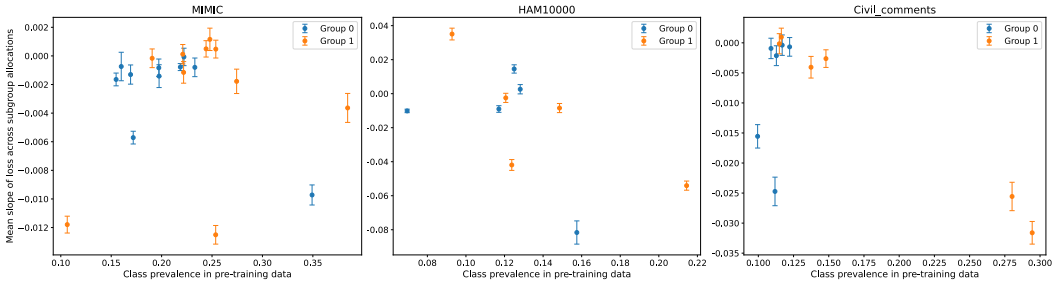

Figure 5: Performance on subgroups with high class imbalance during pre-training does not necessarily improve with increasing training data allocation. Each dot represents one subgroup with bars indicating variation across 9 fine-tuning runs.

F Full theory

Assumptions F.1. *Assumptions for theory Throughout this section, we impose the following conditions:*

1. *Fine-tuning datasets* $\mathtt{D}$ *differ in subgroup allocations* $\alpha^{(A)}$*, but not in their overall label distribution* $P(Y)$ *or conditional distribution* $P(Y \mid A)$*.*
2. *Fine-tuning is restricted to the last-layer classifier* g_θ*, with representation* Z *held fixed.*
3. *We assume that all classifiers are Bayes optimal under realisability.*

Lemma F.1. *Let* $f_{\theta,\eta}(x) = g_\theta(h_\eta(x))$ *with representation* $Z = h_\eta(X)$ *and predictor* $\hat{Y} = g_\theta(Z)$*, where* g_θ *is the last layer. Assume that*

$$\mathsf{TV}\big(\mathbb{P}_\theta[Z \mid Y = y, A = 1],\, \mathbb{P}_\theta[Z \mid Y = y, A = 0]\big) \le \varepsilon,$$

for $y \in \{0, 1\}$*. Then, it holds* $\big|\mathbb{P}_\theta(Z \mid Y = y, A = a) - \mathbb{P}_\theta(Z \mid Y = y)\big| \le \varepsilon$ *for all* $a \in \{0, 1\}$*.*

Proof. We only prove the case for $A = 1$, since the case for $A = 0$ is analogous. For simplicity, write

$$\begin{aligned}
\mu_a(\cdot \mid y) &:= \mathbb{P}_\theta\big(Z \,\big|\, Y = y, A = a\big) \\
\mu(\cdot \mid y :) &= \mathbb{P}_\theta\big(Z \,\big|\, Y = y\big) \\
\pi_y &:= \mathbb{P}_\theta(A = 1 \mid Y = y)
\end{aligned}$$

By the law of total probability, we have that

$$\mu(\cdot \mid y) = \pi_y\, \mu_1(\cdot \mid y) + (1 - \pi_y)\, \mu_0(\cdot \mid y). \tag{1}$$

View $\mu_0(\cdot \mid y)$ and $\mu_1(\cdot \mid y)$ as probability measures on the same measurable space. By using equation 1, we have that

$$\mu_1(\cdot \mid y) - \mu(\cdot \mid y) = \mu_1(\cdot \mid y) - \big(\pi_y \mu_1(\cdot \mid y) + (1 - \pi_y)\mu_0(\cdot \mid y)\big) = (1 - \pi_y)\, \big(\mu_1(\cdot \mid y) - \mu_0(\cdot \mid y)\big).$$

Taking total variation norms and using homogeneity of total variation for signed measures,

$$\mathrm{TV}\big(\mu_1(\cdot \mid y), \mu(\cdot \mid y)\big) = (1 - \pi_y)\, \mathrm{TV}\big(\mu_1(\cdot \mid y), \mu_0(\cdot \mid y)\big).$$

By the defining property of total variation,

$$\sup_z \big|\mu_1(z \mid y) - \mu(z \mid y)\big| = \mathrm{TV}\big(\mu_1(\cdot \mid y), \mu(\cdot \mid y)\big) \le (1 - \pi_y)\, \varepsilon \le \varepsilon,$$

where we have used that $1 - \pi_y \le 1$. Hence, it holds $\big|\mathbb{P}_\theta(Z \mid Y = y, A = 1) - \mathbb{P}_\theta(Z \mid Y = y)\big| \le \varepsilon$, as claimed. □

We can use this lemma to prove the main result.

Theorem F.1 (Group accuracy parity)**.** *Let* $f_{\theta,\eta}(x) = g_\theta(h_\eta(x))$ *with representation* $Z = h_\eta(X)$ *and predictor* $\hat{Y} = g_\theta(Z)$*, where* g_θ *is the last layer. For a set of parameters* θ*, define the quantity*

$$\mathsf{TV}(\theta) := \mathbb{E}_y \left[\mathsf{TV}\big(\mathbb{P}_\theta[Z \mid Y = y, A = 1],\, \mathbb{P}_\theta[Z \mid Y = y, A = 0]\big)\right].$$

Suppose that the model is fine-tuned on two balanced datasets $\mathtt{D}'$*,* $\mathtt{D}''$ *with different proportions of the attribute* A*, yielding parameters* θ' *and* θ''*. If* $\mathsf{TV}(\theta') \le \varepsilon$ *and* $\mathsf{TV}(\theta'') \le \varepsilon$*, then*

$$|\textsc{Acc}_{\theta'}(A = a) - \textsc{Acc}_{\theta''}(A = a)| \le 4\varepsilon + |\textsc{Acc}_{\theta'} - \textsc{Acc}_{\theta''}|$$

for all $a \in \{0, 1\}$*.*

Proof. We can write the accuracy via conditioning on the true label as

$$\textsc{Acc}_{\theta'}(A = a) = \mathbb{P}_{\theta'}(\hat{Y} = Y \mid A = a) = \sum_y \mathbb{P}_{\theta'}(\hat{Y} = y \mid Y = y, A = a)\, \mathbb{P}_{\theta'}(Y = y \mid A = a) \tag{2}$$

and

$$\textsc{Acc}_{\theta''}(A=a) = \mathbb{P}_{\theta''}(\hat{Y}=Y \mid A=a) = \sum_y \mathbb{P}_{\theta''}(\hat{Y}=y \mid Y=y, A=a)\,\mathbb{P}_{\theta''}(Y=y \mid A=a) \tag{3}$$

We know by construction that class-conditional label distributions match, i.e.,

$$\mathbb{P}_{\theta'}(Y=y \mid A=a) = \mathbb{P}_{\theta''}(Y=y \mid A=a) =: p_{a,y} \tag{4}$$

Combining equation 2-equation 4 it holds

$$\begin{aligned}
|\textsc{Acc}_{\theta'}(A=a) - \textsc{Acc}_{\theta''}(A=a)| &= \left| \sum_y \mathbb{P}_{\theta'}(\hat{Y}=y \mid Y=y, A=a)\, p_{a,y} - \sum_y \mathbb{P}_{\theta''}(\hat{Y}=y \mid Y=y, A=a)\, p_{a,y} \right| \\
&\leq \sum_y \left| \mathbb{P}_{\theta'}(\hat{Y}=y \mid Y=y, A=a) - \mathbb{P}_{\theta''}(\hat{Y}=y \mid Y=y, A=a) \right| |p_{a,y}| \\
&\leq \sum_y \left| \mathbb{P}_{\theta'}(\hat{Y}=y \mid Y=y, A=a) - \mathbb{P}_{\theta''}(\hat{Y}=y \mid Y=y, A=a) \right|,
\end{aligned} \tag{5}$$

where we have used the triangle inequality, together with the fact that $0 \leq p_{a,y} \leq 1$. Furthermore, using the triangle inequality and Lemma 4.1, it holds

$$\begin{aligned}
&\sum_y \left| \mathbb{P}_{\theta'}(\hat{Y}=y \mid Y=y, A=a) - \mathbb{P}_{\theta''}(\hat{Y}=y \mid Y=y, A=a) \right| \\
&\quad \leq \sum_y \left| \mathbb{P}_{\theta'}(\hat{Y}=y \mid Y=y, A=a) - \mathbb{P}_{\theta'}(\hat{Y}=y \mid Y=y) \right| \\
&\qquad + \left| \sum_y \mathbb{P}_{\theta'}(\hat{Y}=y \mid Y=y) - \mathbb{P}_{\theta''}(\hat{Y}=y \mid Y=y) \right| \\
&\qquad + \sum_y \left| \mathbb{P}_{\theta''}(\hat{Y}=y \mid Y=y, A=a) - \mathbb{P}_{\theta''}(\hat{Y}=y \mid Y=y) \right| \\
&\quad \leq 4\varepsilon + \left| \sum_y \mathbb{P}_{\theta'}(\hat{Y}=y \mid Y=y) - \mathbb{P}_{\theta''}(\hat{Y}=y \mid Y=y) \right|
\end{aligned} \tag{6}$$

The claim follows by combining equation 5 with equation 6, and noting that since the parameters θ' and θ'' are Bayes optimal under realisability, it holds

$$\left| \sum_y \mathbb{P}_{\theta'}(\hat{Y}=y \mid Y=y) - \mathbb{P}_{\theta''}(\hat{Y}=y \mid Y=y) \right| = |\textsc{Acc}_{\theta'} - \textsc{Acc}_{\theta''}|$$

□

The proof uses a result that bounds how much the model's class-conditioned predictions can differ across groups using total variation distance (Lemma 4.1). Assuming that the label distribution $\mathbb{P}(Y)$ and the conditional distribution $\mathbb{P}(Y \mid A)$ remain unchanged across the datasets, this implies that the subgroup accuracy difference is bounded by a term depending on ε and the difference in overall accuracies between the models.

Remark In practice, for a fixed fine-tuning budget K, the overall accuracy difference is typically negligible relative to subgroup accuracy differences, i.e. $|\Delta\textsc{Acc}| \ll |\Delta\textsc{Acc}_a|$. We observe this in our experiments, implying that TV differences are the dominant driver in the upper bound.

G Supplementary results on representation invariance

G.1 Additional details on distance metrics

We analyse the pre-trained models' representations by extracting the penultimate layer embeddings of the test inputs (e.g., 1024-dimensional vector $\boldsymbol{z}$ for DenseNet121) and projecting them to a lower dimensional space via principal component analysis. To reduce noise, we retain the top-k principal components that explain at least 70% of the variance (in practice $k \in [2, 81]$ depending on the dataset and model) and measure the mean **total variation distance** (TV) between the representations of examples in subgroups $a_0^{(j)}$ and $a_1^{(j)}$ in this lower dimensional space. We condition on Y to control for class imbalances.

Given subgroup embeddings $\boldsymbol{z}_{a_0^{(j)}}$ and $\boldsymbol{z}_{a_1^{(j)}}$, we estimate their distributions by constructing normalized histograms along each principal component dimension. For a given component, let p and q denote the resulting discrete probability mass functions over bins b. The TV is then $\mathrm{TV}(p, q) = \frac{1}{2}\sum_b |p(b) - q(b)|$, where the sum is over histogram bins. In practice, we compute TV for each principal component dimension separately over 50 bins and report the average across dimensions. TV provides a bounded ($[0, 1]$) measure of separation. Values near 0 indicate nearly identical marginal distributions, while values near 1 indicate almost complete disjointness.

For completeness, we also explore additional distance metrics including the **Wasserstein distance** (WD) and the **Fréchet distance** (FD) which emphasise distinct representation differences.

For WD, we compute the 1D WD along each principal component vector of the embeddings and report the mean across components:

$$\mathrm{WD}\big(\boldsymbol{z}_{a_0^{(j)}}, \boldsymbol{z}_{a_1^{(j)}}\big) = \frac{1}{k}\sum_{i=1}^{k} W_1\big(\boldsymbol{z}^{(i)}_{a_0^{(j)}}, \boldsymbol{z}^{(i)}_{a_1^{(j)}}\big),$$

where $\boldsymbol{z}^{(i)}_{a_0^{(j)}}$ and $\boldsymbol{z}^{(i)}_{a_1^{(j)}}$ denote the projections of the embeddings from subgroups $a_0^{(j)}$ and $a_1^{(j)}$ onto the i-th principal component, and W_1 is the univariate Wasserstein-1 distance.

The FD approximates the subgroup feature distributions as multivariate Gaussians. Let $(\mu_{a_0^{(j)}}, \Sigma_{a_0^{(j)}})$ and $(\mu_{a_1^{(j)}}, \Sigma_{a_1^{(j)}})$ be the empirical means and covariances of the embeddings for the two subgroups. Then

$$\mathrm{FD}\big(\boldsymbol{z}_{a_0^{(j)}}, \boldsymbol{z}_{a_1^{(j)}}\big) = \|\mu_{a_0^{(j)}} - \mu_{a_1^{(j)}}\|_2^2 + \mathrm{Tr}\left(\Sigma_{a_0^{(j)}} + \Sigma_{a_1^{(j)}} - 2(\Sigma^{1/2}_{a_0^{(j)}}\Sigma_{a_1^{(j)}}\Sigma^{1/2}_{a_0^{(j)}})^{1/2}\right).$$

Together, these measures provide different estimates of on invariance: TVD emphasizes the largest discrepancies in probability mass between subgroups, WD gives a precise estimate of marginal distribution shift (including differences in distributional shape), and FD captures coarse differences in means and covariances.

G.2 Representation distances obtained across the four datasets

We highlight the variation in TV across subgroups in the four datasets in Figure 6.

G.3 Metrics are robust

All three distance metrics are correlated and appear robust to whether the calculation is done on the dimension reduced space or full feature space. We show results in MIMIC as an example because it is the dataset with the largest number of subgroups.

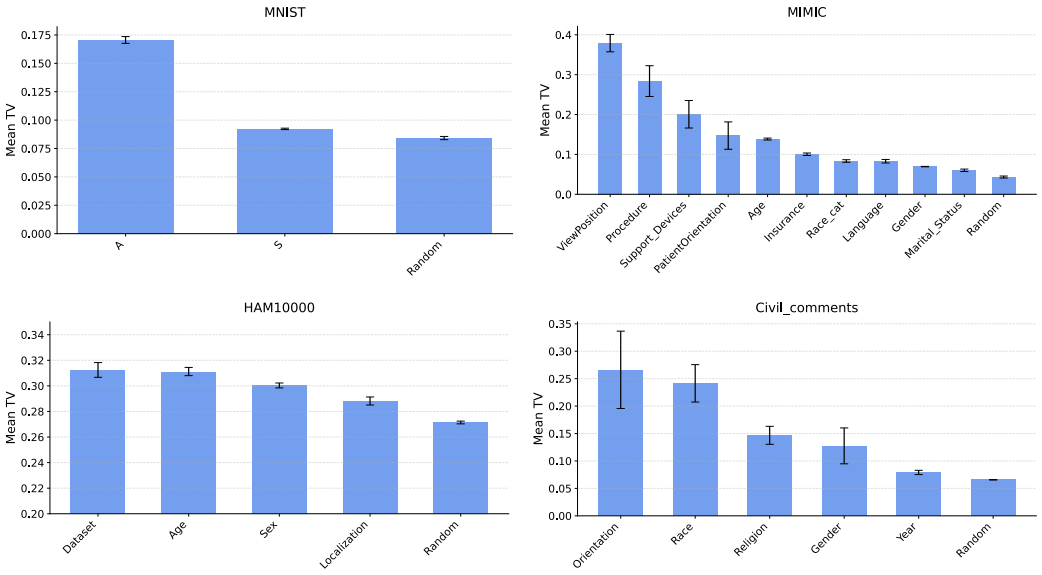

Figure 6: Total variation (TV) distances across subgroups in the pre-trained model's penultimate-layer representation space. We report mean and standard deviation across three random seeds.

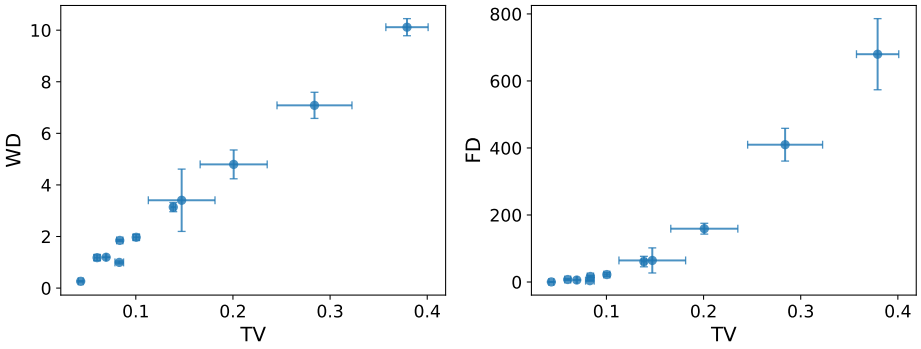

Figure 7: Total variation distance (TV), wasserstein distance (WD), and Fréchet distance (FD) across subgroups appear correlated in MIMIC. Each dot represents a subgroup with error bars representing standard deviation across three pre-training runs.

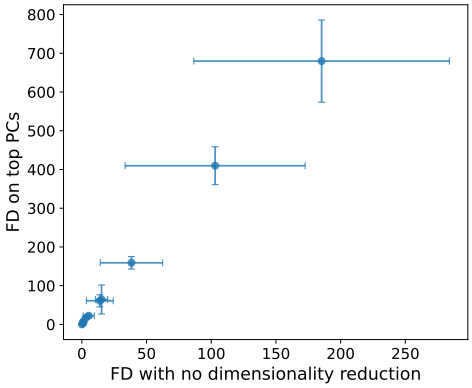

Figure 8: Representation distance metrics appear robust to whether they are calculated on the full latent space or on the lower dimensional space (after PCA). Each dot represents a subgroup in MIMIC with error bars representing standard deviation across three pre-training runs.

H Supplementary results on the correlation between representation distance and subgroup allocation sensitivity

Figure 9 extends the main text by showing subgroup allocation sensitivity against additional distance metrics across our datasets. The correlation is strong across metrics, supporting the latent separation hypothesis.

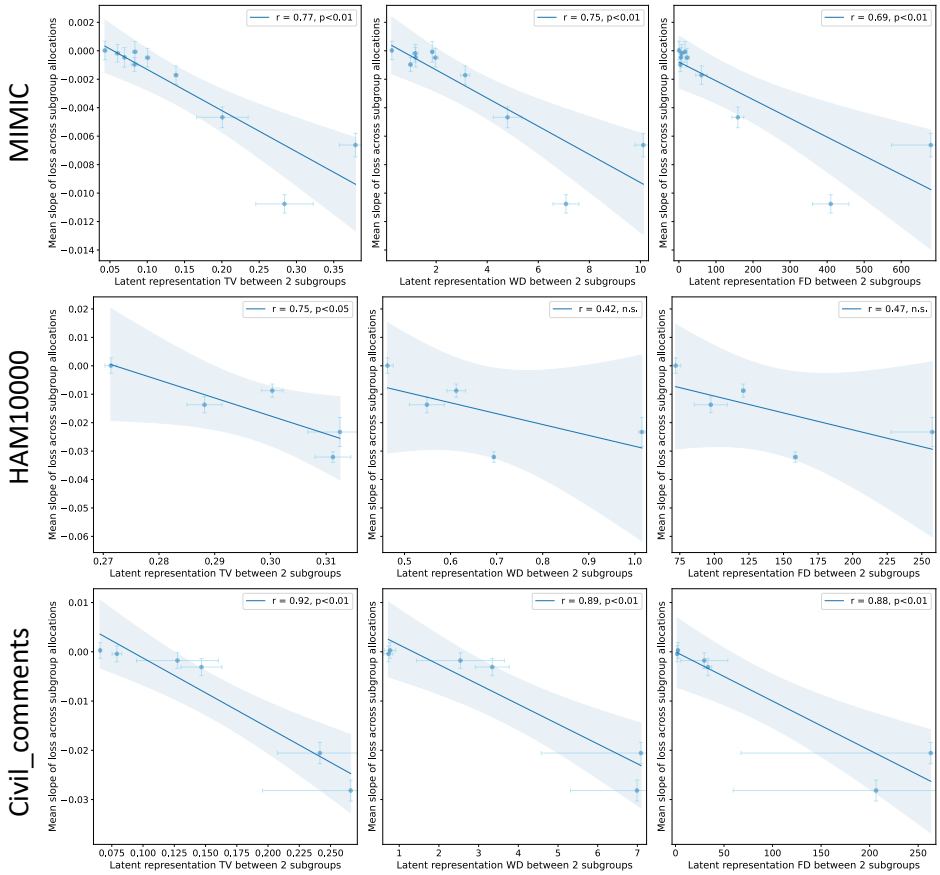

Figure 9: Sensitivity to subgroup allocation is correlated with separation in the pre-trained model's representation space (as measured by total variation distance, wasserstein distance, and Fréchet distance) across the three datasets. Each dot represents mean distance and loss slope for one subgroup, averaged across 9 fine-tuning runs, with bars corresponding to standard deviations, and Pearson correlation coefficients also shown.

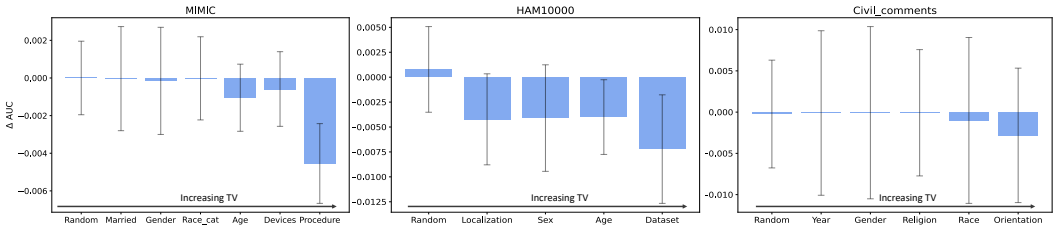

Figure 10: **AUC difference when a model has not been trained on a subgroup increases with the separation of the latent representations of the subgroups.** Mean Δ AUC across 9 fine-tuning runs.

I Extending setup to full fine-tuning

We additionally extend our setup from last-layer fine-tuning to full model fine-tuning. Overall, the correlations are equally significant, and the magnitude of the effects larger (Figures 12 and 13. We attribute this to the fact that in practice, separation of subgroup representations does not change with subgroup allocations (Figure 11). Therefore, while some of our theoretical assumptions still hold, the effect increases as full fine-tuning allows for greater modification of network parameters.

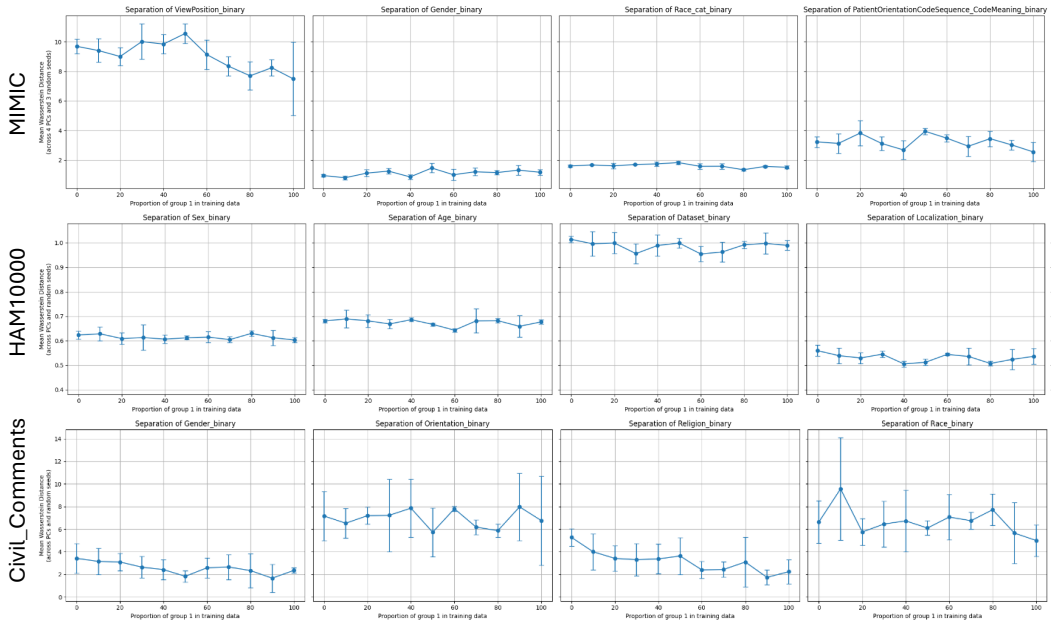

Figure 11: Examples showing latent representation separations are stable across subgroup allocations under full fine-tuning.

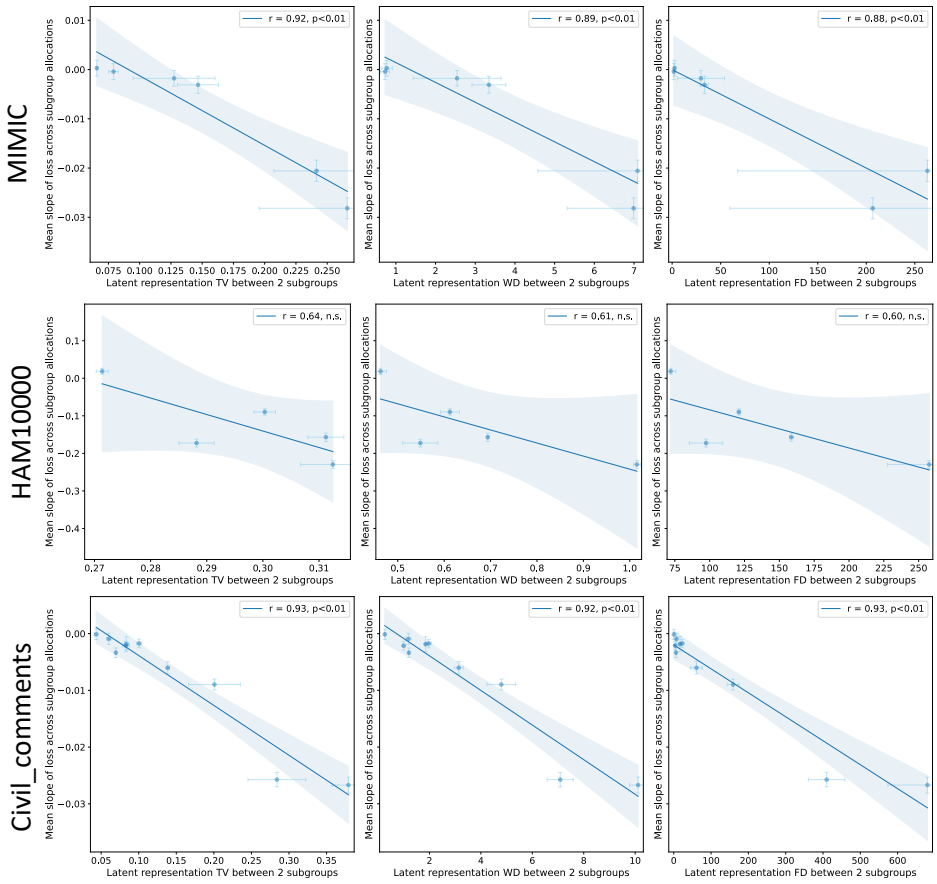

Figure 12: Correlation between loss slope and distance metrics remains strong under full fine-tuning. Each dot represents mean distance (total variation distance, wasserstein distance, or Fréchet distance) and loss slope for one subgroup, averaged across 9 fine-tuning runs, with bars corresponding to standard deviations, and Pearson correlation coefficients also shown.

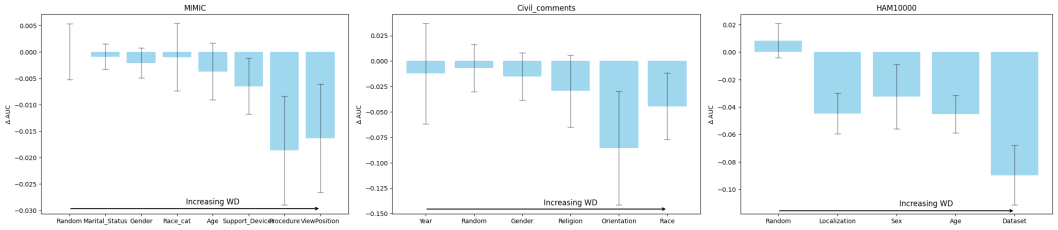

Figure 13: Representation distances continue to predict AUC gaps at extreme subgroup allocations under full fine-tuning at 100% allocation vs 0%. Bars represent mean and standard deviation of the AUC across 9 fine-tuning runs.

J Results on VLM fine-tuning

J.1 Experimental details

Beyond our findings' analytical value in explaining previously observed discrepancies in subgroup allocation sensitivity, we also examine its practical utility. Specifically, we consider a more realistic setup where a foundation model (FM) is used to generate informative embeddings of a dataset, on top of which a simple classification layer is trained. For this, we use CheXagent, a radiology vision-language FM, trained on over 8.5 million image and text samples (Chen et al., 2024), to embed images in the MIMIC-CXR test set. This differs from our traditional setup as the pre-training and fine-tuning datasets are drawn from different distributions and the fine-tuning task does not directly overlap with the pre-training tasks.

We first obtain 1024-dimensional embeddings of MIMIC-CXR images by passing them through the CheXagent `XraySigLIP__vit-l-16-siglip-384__webli`. We train a single classification layer on 16,000 MIMIC-CXR training embeddings, varying allocation in the same way as previous experiments. We test the classification model on the MIMIC-CXR test set (which CheXagent has not been trained on).

J.2 Results

We again measure distances between subgroup representations and find substantial variability (Figure I14). Interestingly, demographic attributes such as gender and age are more separated in the FM than in our task-specific model, likely because a general-purpose model encodes broad features rather than only task-specific ones. The most separated groups remain imaging-related, followed by age, gender, and ethnicity. Based on our hypothesis, we predict that sensitivity to subgroup allocation should follow this ordering, and that if we are concerned about maximising subgroup performance across each of these attributes, we should prioritise balancing the dataset with respect to imaging variables. Indeed, our experiments confirm this: balancing random subgroups has no effect, balancing by race or age has little effect, but balancing by chest X-ray procedure or view position significantly increases mean subgroup accuracy when using a 50/50 allocation compared to 100/0 (Figure I15). Full analysis of subgroup allocation sensitivity and latent separation shows strong correlations (with correlation values ranging from 0.63 to 0.95, Figure I16), supporting our hypothesis under much weaker assumptions and highlighting its potential practical implications for data curation and model fine-tuning.

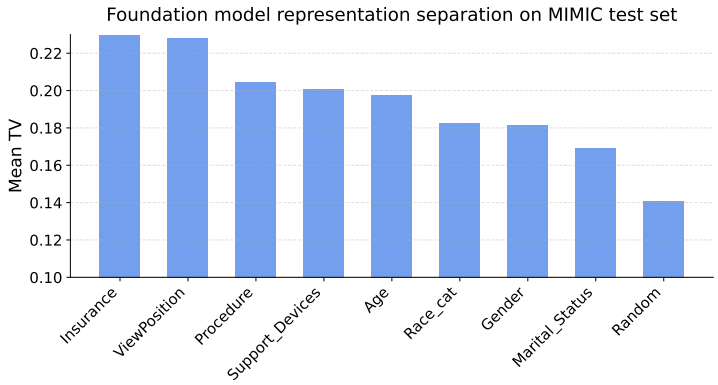

Figure 14: Total variation distances of CheXagent Vision-Language Model embeddings of MIMIC images across subgroups.

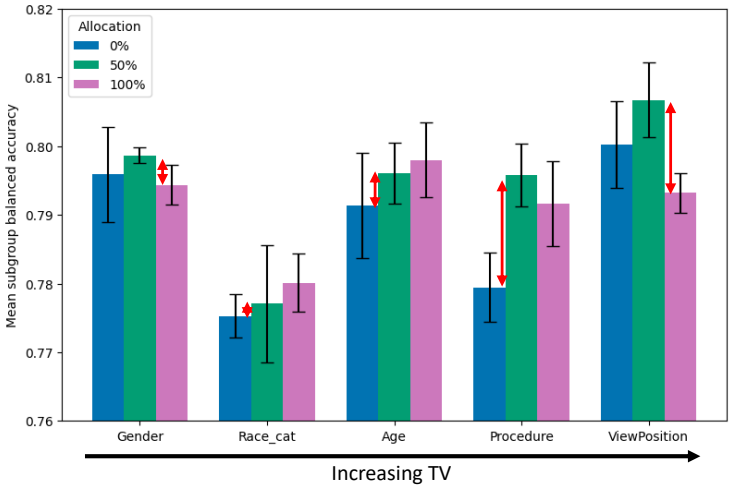

Figure 15: **In foundation model fine-tuning**, selecting a balanced allocation for imaging subgroups increases subgroup accuracy by over 0.02, while it has less importance for demographic subgroups, as predicted by their reduced total variation distance. We report mean accuracy and standard deviations for 3 fine-tuning runs, with red arrows for potential gains from balanced allocations.

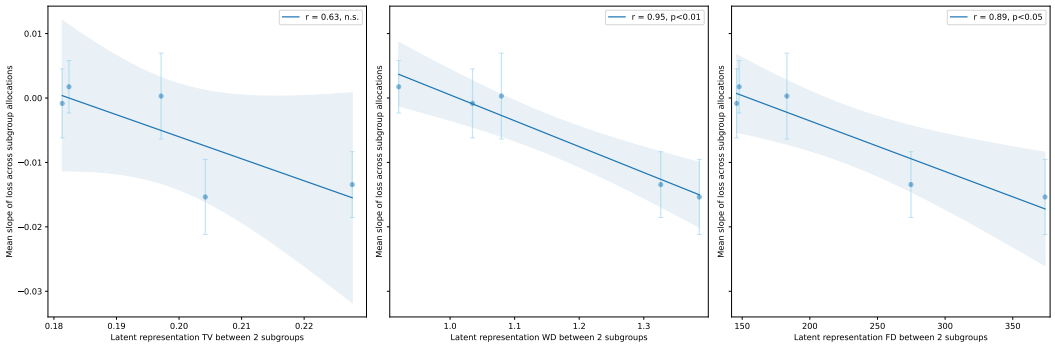

Figure 16: Correlation between latent representation separation and subgroup allocation sensitivity in the vision-language model setting. Results confirm that our hypothesis generalizes beyond task-specific models. Each dot represents mean distance and loss slope for one subgroup, averaged across 9 fine-tuning runs, with bars corresponding to standard deviations, and Pearson correlation coefficients also shown.

K Ablations on fine-tuning budget K

We repeat the MIMIC experiments with a smaller fine-tuning budget K to explore whether changing sample size modifies our results. We find that smaller budgets show slightly higher absolute magnitudes of allocation sensitivity. This is most likely due to larger sample sizes allowing the model to learn more robust disease representations which generalise better across allocations. However, correlations are equally significant across fine-tuning budgets, suggesting our latent representation hypothesis is valid across dataset sizes.

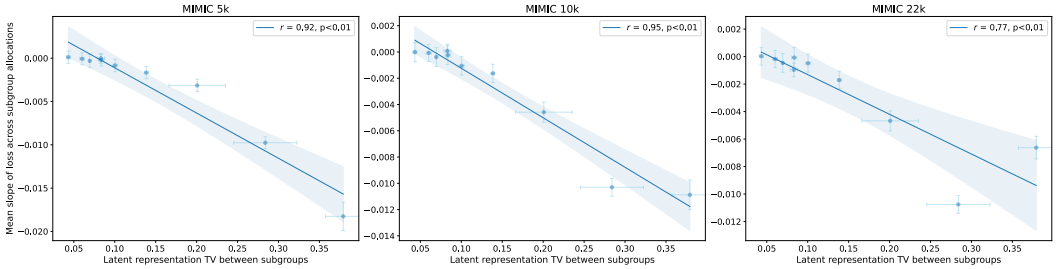

Figure 17: Correlation between representation separation and subgroup allocation sensitivity is strong across fine-tuning dataset sizes in MIMIC. Each dot represents mean distance and loss slope for one subgroup, averaged across 9 fine-tuning runs, with bars corresponding to standard deviations, and Pearson correlation coefficients also shown.

L Limitations and future work

Our study is restricted to a pre-training–then–fine-tuning setup, as this is necessary to first explore representation separation before deciding how to allocate data. One consequence is that the effects we observe are often modest in magnitude, but they remain consistent and statistically significant across the different random seeds, data splits, and pre-trained checkpoints we use. We also do not provide a method for determining the optimal allocation strategy to maximise accuracy across multiple subgroups. We caution that precise recommendations (e.g., targeting 40/60 female/male or 20/80 young/old ratios) would likely be unreliable due to dataset confounders, and in any case difficult to operationalise. Instead, we argue that a broader understanding of *whether* and *how strongly* subgroup allocation influences performance is more relevant to current concerns in fairness and domain generalisation. Moreover, future work should explore how our findings apply to more complex settings, including tasks other than binary classification, with multi-valued or even continuous subgroups. Finally, our work is limited, as in all fairness research, by our use of subgroups. For example, finding that a model is insensitive to the allocation of white vs. non-white samples does not imply that allocation across ethnic groups as a broad concept is irrelevant, simply that, when described by our chosen coarse groupings, it does not matter.