An Empirical Study of Data Ability Boundary in LLMs' Math Reasoning

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Abstract

Large language models (LLMs) exhibit emerging capabilities in math reasoning tasks. Recent efforts focus on supervised fine-tuning (SFT) of small-parameter open-source models to match the math abilities of closed-source models. However, current data augmentation methods lack an exploration of their ability boundaries, including unclear relationships between capabilities and data volume, as well as the generalization and stacking of different abilities. In this paper, we explore these two issues under the framework of Data Ability Boundary, establishing a data strategy that balances sampling efficiency with ability enhancement. Specifically, we first identify the ability boundaries of inference path augmentation through determining the minimal optimal set of inference paths. Subsequently, we validate that different abilities of the model can be cumulatively enhanced by the **M**ix of **M**inimal **O**ptimal **S**ets (MMOS[1]) of corresponding types of data. This demonstrates the effectiveness of MMOS with lower construction costs and interpretable optimal solutions, while also showcasing its generalizability to tackle mathematical problems of varying difficulties. Finally, we address two special math abilities: numerical robustness and automated theorem proving. We show that GSM-HARD is not genuinely hard and that current LLMs possess numerical robustness. Furthermore, the MMOS method enhances the precision in tactic sampling more than overall accuracy in automated theorem proving.

1 Introduction

In the context of significant emergent abilities demonstrated by large language models (LLMs) (Wei et al., 2022a; OpenAI, 2023), the focus on math reasoning tasks, particularly Math Word Problems (MWP) (Kushman et al., 2014, Upadhyay and Chang, 2017, Miao et al., 2020a), is paramount. The current approach to activate these abilities in LLMs involves carefully engineered prompting (Brown et al., 2020, Chen et al., 2022b) or supervised fine-tuning (SFT). Particularly due to computational costs and stability concerns (Yuan et al., 2023), there is growing attention on enhancing the abilities of open-source small-parameter LLMs (Rozière et al., 2023) through SFT. These methods typically employ the knowledge distillation paradigm (Xu et al., 2024), using models with strong math abilities, such as GPT-4 (OpenAI, 2023), and design their own prompts to create supervised datasets.

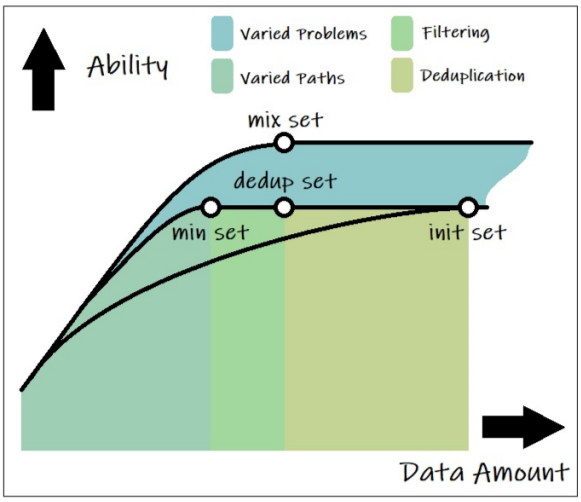

Figure 1: Conceptual figure of the ability boundary

However, most of these methods focus on intuitively producing more and better data, without analyzing the boundaries of their methods' capabilities. This inefficiency complicates the estimation of required data volume and costs for new seed datasets. Additionally, the generality of these methods is limited by the types of problems they address. For example, (Mitra et al., 2024) employs multiple agents to enhance problem understanding, making it effective for the GSM8K dataset

[1]The code and data are provided in the attachment.

(Cobbe et al., 2021), whereas the MATH dataset (Hendrycks et al., 2021) demands prompt methods that optimize multi-step reasoning accuracy.

In this paper, we explore a efficient and general data augmentation strategy to help optimize and expand math reasoning ability. We primarily investigate the following research questions (RQs):

- RQ1: What is the ability boundary of reasoning paths, and how to select paths optimally?
- RQ2: Can different math abilities be enhanced through the simple aggregation of augmented datasets?

RQ1 addresses a common challenge in response augmentation methods: determin the optimal subset of the training set should cover to balance data amount and effectiveness. RQ2 addresses the generalizability of methods to varying difficulties math problems and validates the effectiveness of simple aggregation of datasets for different abilities.

The overall data strategy is illustrated in Figure 1. Based on the initial set obtained from n-sampling on open-source models, we determine the ability boundary of reasoning paths augmentation and then achieve optimization by identifying the Minimal Optimal Set (MOS) for individual datasets through deduplication. Furthermore, we facilitate expansion by creating **M**ix of **M**inimal **O**ptimal **S**ets (MMOS). In summary, we have developed an efficient and general data augmentation strategy. MMOS can produce data at a low cost, verifying that it reaches the data capability boundary while also demonstrating generality in handling mathematical problems of varying difficulties.

Finally, we address two special math abilities: numerical robustness and automated theorem proving. Numerical robustness, which refers to LLMs' difficulty in handling large numbers and multi-digit decimals, shows lower performance on GSM-HARD (Gao et al., 2023) compared to its seed dataset GSM8K. However, we found that the differences came from the labeling standards. After unifying them, we conducted comparative experiments and found that GSM-HARD is not truly hard and that current LLMs possess numerical robustness. Automated theorem proving (ATP) task has gained significant attention recently. We believe there is no methodological difference between this task and using program aid methods to solve math word problems, and we hope our method can also enhance the automated theorem proving task. However, we found that due to differences in supervision objectives, MMOS mainly enhances the precision in tactic sampling more than overall accuracy.

2 Ability Boundary of Reasoning Paths

2.1 Overview

In this section, for RQ1, we aim to determine the ability boundary of reasoning paths on a single dataset. We hypothesize that a minimal set capable of maximizing math reasoning ability consists of varied and deduplicated reasoning paths.

In following Section 2.2, we discuss about the datasets. In Section 2.3, we identify this minimal optimal set and determine the benefits of removing duplicates and keeping varied reasoning paths within a certain range. In Section 2.4, we conduct ablation experiments to assess the necessity of the clustering method as a duplication filter and the impact of ensuring the correctness of the reasoning paths. All detailed experiment settings are in E.

2.2 Dataset Comparation

Six datasets are involved in this study. Detailed information about their origins, example analyses, and a preliminary estimation of their difficulty levels can be found in the Appendix B.

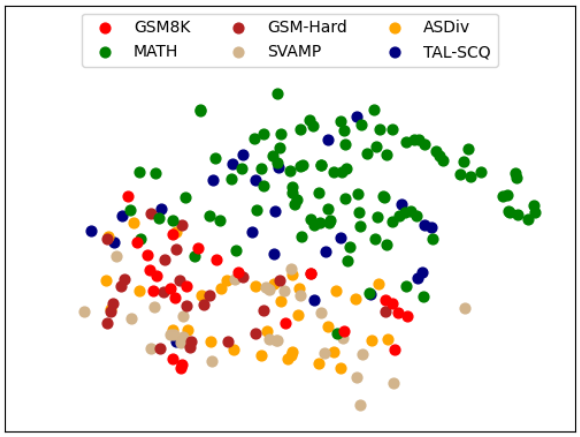

Figure 2: Visualization of query embedding distribution through t-SNE across six distinct datasets.

To better understand the problems' difference across these datasets, we visualize the hidden representations of problems using t-SNE. This visualization as Figure 2 reveals a notable separation in the distribution of problems from the GSM8K and MATH datasets into two distinct clusters. This divergence emphasizes the contrast in question styles:

GSM8K being text-intensive, while MATH is more focused on math expressions.

For the experiments presented in this section, we exclusively use GSM8K without bootstrapping its questions. Consequently, GSM8K is categorized as our IND data. Conversely, the MATH dataset, with its significant stylistic and content differences, is classified as OOD data. Additionally, two other datasets, SVAMP and ASDiV, although different in origin from GSM8K, show similarities in both question types and spatial representations. Therefore, we consider these to be Similar-Domain Datasets. And we denote SVAMP and ASDiV as S&A in the subsequent analysis.

2.3 Identify the Minimal Optimal Set

To identify the minimal optimal set, we follow these steps: 1) Sample a sufficient number of correct reasoning paths to form initial set. 2) Implement a deduplication algorithm to obtain its deduplicated subset. 3) Conduct a statistical analysis on the upper limit of reasoning paths per question k with the subset data amount N to analyze the effectiveness of the deduplication method and perform further processing. 4) Perform SFT on several subsets to identify the minimal optimal set.

Initial set created by various original methods face API and learning costs, and there is a scarcity of training data being open-sourced. Therefore, we attempt to directly use open-source models. Specifically, we opt for SFT models ToRA (Gou et al., 2023) that combine programs and rationales, and apply rejection sampling (Yuan et al., 2023) to build initial set. (Additional experiments have found that sampling through few-shot prompting is equally effective on pre-trained models such as deepseek-math.) Specifically, we employ ToRA-CODE 7B/13B/34B and ToRA 70B. For every question in the GSM8K dataset, these models sample 100 reasoning paths each with temperature 0.9. We then merge 400 reasoning paths and extract those whose code can be executed and have correct answers to obtain the initial training set D_{init}.

Deduplication Algorithm 1 aim to extract the deduplicated subset D_{dedup} from D_{init} by codes which share the same calculation process.

We iterate all n data with following steps:

1) Extract the code block c_i from data d_i, which includes query q_i, completion a_i and source s_i .

2) Employ the Abstract Syntax Tree (AST) method to parse the code into the tree t_i.

3) Normalize the tree by replacing variable names v with lowercase letters and function names f with uppercase letters, resulting in t'_i.

4) Convert the normalized tree back into normalized code, denoted as c'_i.

After completing the iteration, the normalized codes are duplicated through plain text matching. Finally deduplicated data d' is recovered with the deduplicated code, query, completion and source.

Algorithm 1 Deduplicate Data by Codes

Require: data d, extract $\xi(\cdot)$, recovery $\widetilde{\xi}(\cdot)$, astparse $\mathbb{P}(\cdot)$, astunparse $\widetilde{\mathbb{P}}(\cdot)$, deduplicate $\widetilde{\mathbb{D}}(\cdot)$

for $i = 1$ **to** n **do**
  $c_i \leftarrow \xi(d_i|q_i \oplus a_i \oplus s_i)$ ▷ Code Extraction
  $t_i \sim \mathbb{P}(c_i)$ ▷ Code Astparse
  $t'_i \leftarrow \pi(t_i|v \oplus f)$ ▷ Code Substitution
  $c'_i \sim \widetilde{\mathbb{P}}(t'_i)$ ▷ Code Astunparse
end for
$c' \leftarrow \mathbb{D}(c')$ ▷ Code Deduplication
$d' \leftarrow \widetilde{\xi}(c' \oplus q \oplus a \oplus s)$ ▷ Data Recovery

The k-N relation can be regarded as an estimation of the relationship between the number of reasoning paths per question k and the corresponding subset data amount N. This relation is obtained by implementing an upper limit on the reasoning paths per question in the initial set. Since the number of common solutions for math problems falls within a reasonable range, we aim to use this analysis to determine the effectiveness of our deduplication method and to perform further filtering as shown in Figure 1.

As shown in Appendix C, the k-N curve demonstrates a linear relationship on initial training set D_{init} with a median of $k = 400$ and a mean of $k = 392.14$. In contrast, on deduplication subset D_{dedup}, it exhibits a log-linear relationship with a median of 7 and a mean of 12.01. This indicates that the deduplication method is effective but still leaves room for improvement. Therefore, we perform further filtering using a simple method of randomly selecting k paths, followed by comparative experiments on a series of subsets D^k_{dedup}.

Experiment includes two aspects. Firstly, to identify the minimal optimal set, we conduct random selection of k paths for each question within D_{dedup} to obtain twelve D^k_{dedup} subsets with k from 1 to ∞ and N from 7.5K to 90K. Secondly, to better assess the impact of duplicate removal, we maintain a consistent order of magnitude in terms

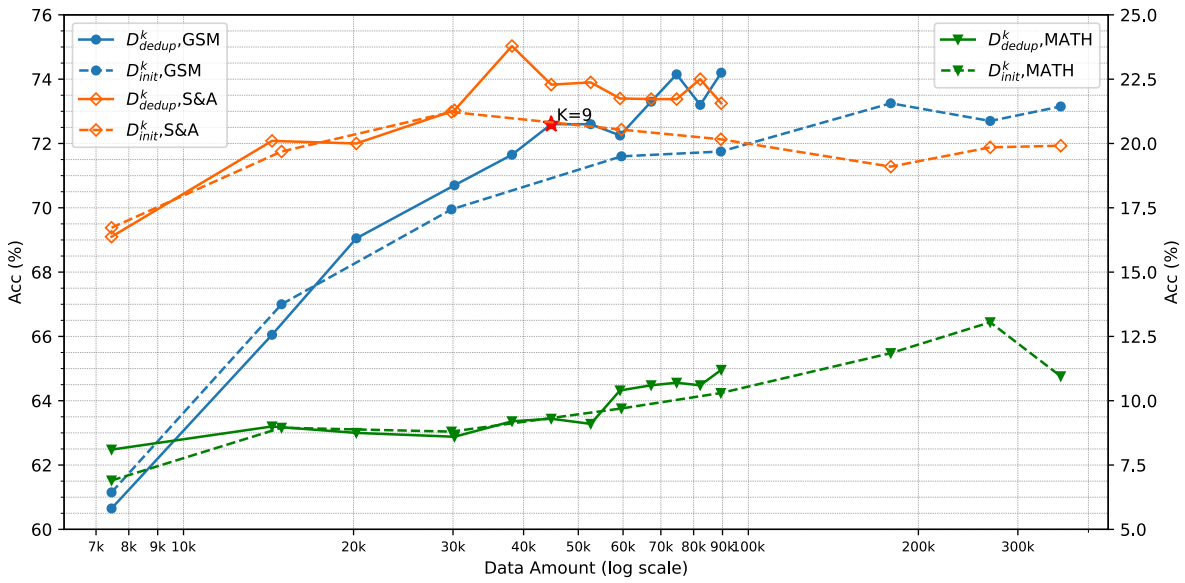

Figure 3: Comparison of test set accuracy on GSM8K, S&A and MATH for models after SFT on Code LLaMA 7B using series subsets of D^k_{dedup} and E^k_{u400} with different data amount.

	k	5	7	9	15	27	-
$D^{cluster,k}_{dedup}$	GSM8K	71.4(+0.7)	70.9(-0.7)	72.6(-1.2)	73.4(+0.8)	74(-0.1)	-
	S&A	73.4(+0.5)	73.4(-0.9)	73.1(-0.9)	74.2(+0.6)	73.4(+0.0)	-
	k	2	4	8	12	24	36
$D^{cluster,k}_{init}$	GSM8K	67.6(+0.6)	70.5(+0.6)	72.1(+0.5)	74.0(+2.3)	73.2(+0.0)	73.5(+0.8)
	S&A	72.0(+0.3)	71.8(-1.1)	74.4(+2.0)	72.3(+0.2)	73.0(+2.0)	73.3(+1.4)

Table 1: Comparison of test set accuracy on GSM8K and S&A for models after SFT on Code LLaMA 7B using series subsets of D^k_{dedup} and E^k_{u400} through clustering.

Dataset	k	N	GSM8K	S&A
D^k_{dedup}	9	44771	71.4	73.6
$D^{+wrong,k}_{dedup}$	9	46740	69.8(-1.6)	73.7(+0.1)
D^k_{dedup}	∞	89530	74.2	73.3
$D^{+wrong,k}_{dedup}$	∞	126391	71.7(-2.5)	73.0(-0.3)

Table 2: Comparison of test set accuracy on GSM8K and S&A for models after SFT on Code LLaMA 7B using D_{dedup} and D^{total}_{u400}.

of data amount on D_{init} and obtain D^k_{init} with k from 1 to 48 and N from 7.5K to 360K. Detail settings are in Appendix C.

We conduct SFT on Code LLaMA 7B using D^k_{dedup} and D^k_{init}, and then inference on the test split of GSM8K, S&A, and MATH.

Results are shown in Figure 3. To better understand the experimental results, we refer to Figure 1. The rightmost point of the blue solid line to the rightmost point of the blue dashed line represents the transition from the initial set to the deduplicated set. The rightmost point of the blue solid line to other points on the blue solid line represents the exploration from the deduplicated set to the minimal (optimal) set.

On the IND dataset GSM8K, as indicated by the blue solid line, the model's ability maintains a linear relationship with the logarithm of data amount before $k = 9$, $N = 45K$. In contrast, the blue dashed line representing the initial set data aligns with this trend only when k is small and duplicate paths are less likely to be selected. Beyond this point, further increasing the data amount sharply diminishes the marginal improvement in model ability. This suggests that enhancing the model's ability stems from adding varied reasoning paths, rather than merely increasing the data amount. We also observe that with the same data amount, beyond $N = 30K$, the performance on D_{dedup} consistently surpasses that on D_{init}. This reflects that removing duplicates can not only diminish the training duration but also enhance the model's ability.

On the Similar-Domain Datasets S&A, potentially due to the inherently easier nature of the questions, the models achieve high effectiveness even at k=1. The other conclusions are similar to those

observed on GSM8K.

However, on the OOD dataset MATH, the models consistently exhibit weaker ability. This may be, as shown in Section 2.2, due to the differing types of questions presented in the dataset. Thus far, we have essentially reached the conclusion that providing varied and deduplicated reasoning paths can improve math reasoning ability in both IND and Similar-Domain data but the assistance for OOD data is limited.

Finally, we conduct a case study, as shown in Appendix F, where our example problem has 10 different solutions which is similar to the previously inflection point of k=9. Therefore, we consider $D_{dedup}^{k=9}$ as the minimal optimal set. Subsequently, we conducted the same experiment on Codellama 34B to validate our conclusions, with the results presented in the Appendix D.1.

From this, we draw another conclusion: the ability boundary is reached, that is, we identify the minimal optimal set (MOS), when the number of reasoning paths is similar to the number of potential problem solutions.

2.4 Ablation Experiments

Clustering as a Filter. We first aim to explore whether there are better methods to improve upon random selection as a complement to the deduplication method. The example problem shown in Appendix F has only 10 distinct solutions. However, in D_{dedup}, 43 paths are still retained. When we implement random selection to obtain $D_{dedup}^{k=9}$, it only includes 6 distinct solutions.

We attempt to use clustering as a filter, replacing random selection, in order to ensure that the resulting subset contains a greater number of distinct solutions. Specifically, we first obtain the embedding vectors of the codes. Then, we apply Latent Semantic Analysis (LSA) for dimensionality reduction, followed by k-means clustering. We extract and retain the central data points from these clusters. On the same example problem, the new subset contains 7 distinct solutions.

In the comparative experiment, we replace random selection with clustering to obtain new subsets, $D_{dedup}^{cluster,k}$ and $D_{init}^{cluster,k}$. We then conduct SFT on Code LLaMA 7B using these subsets.

As shown in Table 1, the results on $D_{init}^{cluster,k}$ exhibited a consistent improvement, suggesting that using clustering as a filter is viable. However, this is not the case for $D_{dedup}^{cluster,k}$. We speculate that the remaining similar paths after deduplication have only a minor impact.

The Correctness of reasoning paths. While ensuring the correctness of paths is intuitively sound, we also observe that some methods, despite not guaranteeing correct answers for created problems, still yield reasonably good results. Therefore, we aim to ablate the effect of ensuring the correctness of paths. During the acquisition of the initial set D_{init}, we retain all data, including those with incorrect answers, resulting in $D_{init}^{+wrong,k}$. After deduplicating this set, we obtain $D_{dedup}^{+wrong,k}$. Subsequently, we generated subsets for $k = 9$ and $k = \infty$ through random selection from these sets and conduct comparative experiments with these subsets. As illustrated in Table 2, on GSM8K, not filtering out incorrect paths leads to a noticeable decline in performance. However, this effect is not observed on S&A, which could be attributed to the lower difficulty level of S&A.

3 Expand Different Math Abilities

In this section, for RQ2, based on the effectiveness of the minimal optimal set (MOS) found using our method on GSM8K, we aim to further validate the generalizability of our method across datasets representing different mathematical abilities and to enhance mathematical abilities through the simple aggregation of augmented datasets. Then, we attempted to further extend the model's existing abilities by continuing to introduce data. Ultimately, we obtained mix of minimal optimal sets (MMOS) and conducted a comparative analysis to evaluate its effectiveness.

3.1 Enhance Weak Ability on MATH Datasets

To address the issue of the weak ability of models trained with the minimal optimal set of GSM8K when applied to OOD set MATH, as shown in Figure 3, a straightforward solution is to provide corresponding data with the same method.

Initially, following the same method described in Section 2.2, we obtain a series of deduplicated subsets D_{MATH}^{k} constructed using the MATH dataset and subsequently conduct SFT on them. And, as indicated by the green dashed line in Figure 4, we identify the minimal optimal set $D_{MATH}^{k=9}$ on MATH. As expected, compared to the models trained on D_{dedup}^{k} originating from GSM8K, which we now refer to as D_{GSM}^{k}, there is a significant improvement in ability on MATH, and the abilities on GSM8K and S&A, represented by the blue and

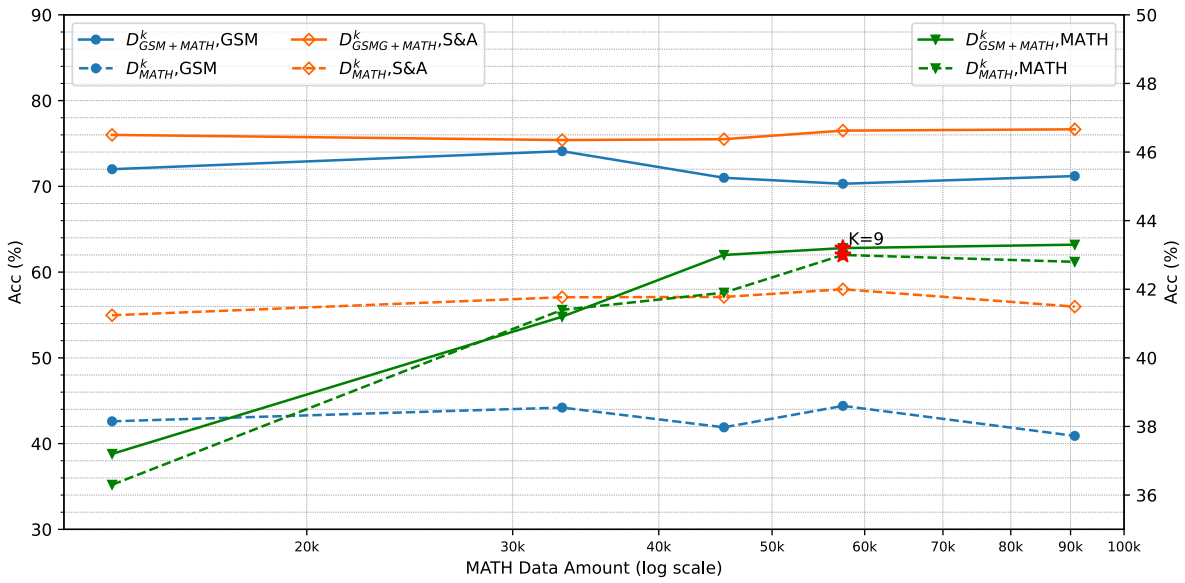

Figure 4: Comparison of test set accuracy on GSM8K, S&A and MATH for models after SFT on Code LLaMA 7B using series subsets of $D^k_{GSM+MATH}$ and D^k_{MATH} with different MATH data amount.

yellow dashed lines, are weaker. At this point, we have validated our first objective: our method can be generlized to datasets of varying difficulties.

Subsequently, we merge the subsets D^k_{MATH} from the MATH dataset with the minimal optimal set of GSM8K $D^{k=9}_{GSM}$, denoted as $D^k_{GSM+MATH}$. The experimental results on $D^k_{GSM+MATH}$, as shown by the various solid lines, indicate that compared to D^k_{MATH}, which provides the same amount of data from MATH, there is a slight improvement in performance on MATH, and a significant improvement on GSM8K and S&A. Additionally, the local optimum point for $D^k_{GSM+MATH}$, similar to D^k_{MATH}, is also achieved at k=9. Similarly, compared to $D^{k=9}_{GSM}$, $D^k_{GSM+MATH}$ shows a slight decrease in performance on GSM8K and a marginal decline on S&A. However, there is a significant improvement on MATH, with a rise from 10.4% to 43.2%.

Overall, $D^{k=9}_{GSM+MATH}$ (102K) effectively combines the strengths of $D^{k=9}_{GSM}$ (45K) and $D^{k=9}_{MATH}$ (57K), showcasing enhanced abilities on GSM8K, S&A, and MATH datasets. We arrive at the other conclusion: different abilities of the model can be cumulatively enhanced by mixing minimal optimal sets of corresponding types of data. This finding provides a simple yet effective method for enhancing the model's weak abilities by acquiring the corresponding datasets.

3.2 Expand Existing Ability

After utilizing all data from GSM8K and MATH, we try to further expand existing ability in the absence of corresponding data, As shown in Figure 2, dataset TAL-SCQ displays query embeddings that overlap with GSM8K and MATH. We generate its minimal optimal set and merge it with $D^{k=9}_{GSM+MAT}$, denoted as $D_{GSM+MATH+TAL}$. Similarly, we conduct SFT on Code LLaMA 7B and achieve an accuracy of 73.9(+3.6)% on GSM8K, 77.5(+1.0)% on S&A, and 44.3(+1.1)% on MATH. We conclude that an overlapping dataset can continue to enhance the model's existing ability in the absence of corresponding data.

3.3 MMOS' Advantage

Our data strategy MMOS is general and efficient, with its efficiency demonstrated by higher performance and lower construction costs. 1) The results, as shown in Table 3, indicate that our model using MMOS achieves consistently high performance across various base models. 2) When constructing the initial set, n-sampling on GPT-4 is costly. Sampling 20 reasoning paths for each seed question of D_{G+M+T} will exceed a cost of $10,000, and additional learning costs are required for post-processing using various methods. In contrast, MMOS can directly utilize corresponding method models for sampling, avoiding these issues. The sampled data will possess higher quality and lower diversity. Furthermore, we also attempt to signifi-

Model (Size)	GSM8K	SVAMP	ASDiv	MATH
WizardMath-7B	54.9	57.3	59.1	10.7
WizardMath-13B	63.9	64.3	65.8	14.0
WizardMath-70B	81.6	82.7	86.8	49.7
MetaMath-7B	66.5	-	-	19.8
MetaMath-13B	72.3	-	-	22.4
MetaMath-70B	82.3	-	-	26.6
TORA-CODE-7B	72.6	70.4	78.7	44.6
TORA-CODE-13B	75.8	75.7	81.4	48.1
TORA-CODE-34B	80.7	80.5	84.2	50.8
MMOS-CODE-7B	73.9	76.4	78.6	44.3
MMOS-CODE-13B	77.1	77.5	81.9	48.1
MMOS-CODE-34B	81.7	81.9	82.8	48.8
MMOS-Min-CODE-7B	70.3	72.5	76.7	44.6
MMOS-LLEMMA-7B	76.5	77.7	81.4	48.8
MMOS-LLEMMA-34B	82.8	81.8	84.8	51.3
MMOS-DeepSeekMath-7B	80.5	79.3	87.6	55.0
MMOS-DeepSeekMath-7B (SC, k=50)	87.2	-	-	63.7

Table 3: Comparison of test set accuracy on 4 datasets for different SFT models. CODE stands for CodeLlama.

cantly reduce computational costs by sampling 100 solutions for 19k seed questions using only a 7B model. This can be completed within 12 hours on 8 A100 40G GPUs. This approach yields about 30% amount of the GSM8K reasoning paths and 90% for MATH, possibly because simpler problems are more prone to repetition. The model obtained, MMOS-Min-CODE, also demonstrates satisfactory performance.

4 Numerical Robustness

4.1 Is GSM-HARD Really Hard?

Another 'weak ability' of the $D^{k=9}_{GSM+MATH}$ model is demonstrated on GSM-HARD (54.8% vs 70.3% on GSM8K). This dataset is created by replacing the numbers in GSM8K with larger ones (Gao et al., 2023). Given that only the numerical values are altered, the distribution of problems in Figure 2 remains almost identical. Based on the conclusions from Section 2.3, such a significant discrepancy should not occur, whether we consider it as IND or Similar-Domain data. This leads us to questions: Is GSM-HARD really hard? Is the model's numerical robustness indeed weak?

The first source of discrepancy arises from the standards of ground truth. Due to the lack of meticulous design in the numerical values of the questions, some answers are not impractical, such as receiving answers with decimals when asking about quantities, or negative numbers when asking about the amount decreased. In practice, these initial calculation results should be rounded or converted to absolute values when providing answers, but GSM-HARD directly annotates these initial calculation results as the ground truth. We do not consider this to be indicative of a gap in ability. Therefore, using the standards of GSM-HARD, and evaluating based on the initial calculation results, the accuracy rate increases to 63.3(+8.5)%.

The second source of the discrepancy is due to errors in the ground truth annotation, stemming from an imperfect automated annotation process in GSM-HARD after modifying the problems. The corresponding values in the code are not updated in line with the changes in the numerical values in the problems, thus leading to execution with retained incorrect results as ground truth. We review the first 50 samples where the $D^{k=9}_{GSM+MATH}$ model make incorrect inferences and discover 25 errors in the ground truth annotations.

Finally, based on the above two points, we believe that GSM-HARD is not difficult, and compared to GSM8K, the model does not show significant differences in performance.

4.2 Auto Problem Generator

Considering this, developing an Auto Problem Generator capable of reliably producing data similar to GSM-HARD is meaningful. Such a generator can be used to test the numerical robustness of models. Additionally, it can also be utilized in educational applications to assess students' abilities.

Auto Problem Generator follows these steps:

1) Generate the deduplicated subset $D_{test,u400}$ from the seed dataset, the test split of GSM8K, following the method in Section 2.3.

2) For each question, extract the reasoning path with the highest repetition as the main path and separate the remaining path as the remain paths.

3) Extract numbers from questions using template matching and modify them with function $f(\cdot)$.

4) Modify the corresponding numbers in the code of the main path and execute it to obtain the answer A_{main}.

5) If the code execution fails or $A_{main} < 0$, modify the numbers again with 50 times limit.

6) Repeat step 4 on the remaining paths and obtain the answer set $A_{remains}$.

7) If all elements in $A_{remains}$ are identical to A_{main}, then we believe A_{main} is correct.

8) Combine the correct A_{main} with the modified questions to form the generated dataset P.

We apply the Distribution Perturbation (Xu et al., 2022) on numerical values with the following function $f(n)$ with μ=5, σ=1 and μ=1000, σ=300 to

create datasets P_5 and P_{1000},

$$f(n) = n + \lfloor X \rfloor, X \sim \mathcal{N}(\mu, \sigma^2)$$

that $\mathcal{N}$ represents normal distribution. We manually review the first 100 QA pairs in P_5 and achieve a 98% accuracy rate, with only two questions having incorrectly annotated answers. A detailed analysis of these errors and their reasons can be found in the Appendix G.

Thus, we have successfully developed a high-quality Auto Problem Generator, which can be used for testing the numerical robustness of models as well as for educational application.

Numerical Robustness represents a model's consistent ability to handle different types of numerical values. Distribution Perturbation, as applied in GSM-HARD, P_5, and P_{1000}, is one such example. We evaluate P_5 and P_{1000} with the model trained on $D_{GSM}^{k=9}$ with only GSM8K data. The experimental results show 73.8% on GSM8K, 72.1(-1.7)% on P_5 and 70.1(-3.7)% on P_{1000}.

Then, employing the same approach, we produce $P_{1000}^{'}$ using the train split of GSM8K and include it in our training data. However, the results show tiny improvement, achieving 73.2% on GSM8K, 72.6(-0.6)% on P_5 and 70.4(-2.8)% on P_{1000}. Considering the results of both sets of experiments, since providing corresponding data does not enhance ability, we infer that the discrepancies in P_{1000} are more likely due to annotation issues caused by the inclusion of large numbers.

We also experiment with other numerical perturbation approaches including Language Perturbation and Noise Perturbation. Language Perturbation does not entail changes to the answers and simply involves converting numerical values into their English word representations. This has led to a slight improvement in the model's performance. Noise Perturbation introduces noise by adding decimal parts to the numerical values. The conclusions drawn from this method are similar to those from Distribution Perturbation. Overall, we conclude that current LLMs no longer face significant issues with numerical robustness.

5 Automated Theorem Proving

Automated Theorem Proving (ATP) holds significant importance in mathematics by enhancing the ability to verify and discover proofs with minimal human intervention. ATP systems use formal logic to represent math statements and algorithms to systematically search for proofs. We found that formal languages like Lean and commonly used programming languages such as Python do not have fundamental differences and even feature less redundancy and more concise expressions. Therefore, we believe our method should also possess general applicability in ATP tasks. We tested the formal2formal task on the MiniF2F (Zheng et al., 2021) dataset, where the main difference is that typically LLMs are used to generate the next tactic. Therefore, we used MMOS method, but after obtaining the correct paths, we created data based on generating the next tactic for each statement.

Model	TSA	OA
Llemma-7B	22.0	27.8
Llemma-34B	23.2	27.5
DeepSeekMath-7B	29.4	27.8
MMOS-Llemma-7B	32.5	29.1
MMOS-Llemma-34B	30.8	28.7
MMOS-DeepSeekMath-7B	32.7	28.3

Table 4: Comparison of tactic sample accuracy (TSA) and overall accuracy (OA).

As shown in the table 4, we achieved some improvements, particularly in generating the next tactic. However, the overall accuracy improvement is limit. We believe this is constrained by two factors. First, in current ATP systems, LLMs are only responsible for generating the next tactic, making it difficult to learn preferences for generating complete proofs. Second, due to limitations in search algorithms, generating a complete proof still faces a vast search space.

6 Conclusion

In this paper, we explore the data ability boundary of data augmentation methods and determine a general and efficient data strategy. Firstly, we ascertain the ability boundary related to the augmentation of reasoning paths by identifying the minimal optimal set of these paths, with a focus on maximizing the data's potential. Secondly, we corroborate the premise that different abilities of the model can be collectively enhanced by amalgamating minimal optimal sets of data, each corresponding to specific types of information. In addition, we also explored two special math abilities: numerical robustness and automated theory provision. We discussed the related work in the Appendix A.

Limitations

The limitations of our paper include the following aspects:

Datasets and Models. In our research, we use only three datasets to create a mix of minimal optimal sets as training data. However, we are uncertain whether the two conclusions drawn in Section 4 – that different abilities of the model can be cumulatively enhanced by mixing minimal optimal sets of corresponding types of data, and that an overlapping dataset can continue to enhance the model's ability in the absence of corresponding data – would still hold true with the introduction of more and larger datasets. Additionally, we are also unsure if these conclusions would apply to larger-scale models, such as the 70B model.

Sampling Bias. Our conclusions regarding the numerical robustness of the model, the GSM-HARD dataset and the Auto Problem Generator are based on our numerical analysis of accuracy and results from sample checks. This approach may introduce bias.

Ethical Statements

We claim from various aspects that our work is free of ethical risks:

1) Our research utilizes open-source models like LLaMA-2 and Code LLaMA and open datasets, and we strictly adhere to their licensing protocols.

2) Despite providing a new auto problem generator, its functionality is confined to numerical perturbation derived from open-source datasets. We endeavour to prevent the generation of illogical problems and the dissemination of inappropriate information resulting from numerical perturbations.

3) During the writing process, we used GPT4 to translate and correct grammatical errors, and the text was human-checked and rewritten to ensure that there were no ethical issues.

4) Our experiments are designed to be resource-efficient, requiring minimal compute time and power.

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A Related Work

A.1 LLM for Math Reasoning

Prompt based methods activate the emergent abilities without training. A significant breakthrough comes from Chain-of-thought prompting (CoT) (Wei et al., 2022b), which enhances the ability of LLMs to tackle complex reasoning by using explicit reasoning steps. The least-to-most prompting strategy (Zhou et al., 2023) deconstructs complex problems into a series of simpler sub-problems, which are then solved sequentially. Program of thoughts prompting (Chen et al., 2022a) and program-aided language models (Gao et al., 2023) address the limited numerical abilities of LLMs and utilize LLMs solely for understanding problems and generating programs, while offloading computation to an external Python interpreter.

Decoding related methods focus on enhancing performance by replacing the greedy decoding strategy during the inference stage. (Wang et al., 2023b) samples a diverse set of reasoning paths and selects the most consistent answer, while (Xie et al., 2023) proposes a decoding algorithm that integrates self-evaluation guidance through the use of stochastic beam search.

Supervised Fine-tuning (SFT) based methods are designed to enhance the math reasoning abilities of open-source models such as LLaMA (Touvron et al., 2023a), LLaMA2 (Touvron et al., 2023b), and Code LLaMA (Rozière et al., 2023), while ensuring transparency. Current methods (Yu et al., 2023; Wang et al., 2023a) largely utilize various prompt-based approaches, employing GPT-4 (OpenAI, 2023) or other open-source models, to generate reasoning steps as training datasets based on original QA in various datasets like GSM8k (Cobbe et al., 2021), MATH (Hendrycks et al., 2021). These generated reasoning steps can either be in natural language (rationales) (Zelikman et al., 2022) or a combination with program (Yue et al., 2023; Gou et al., 2023).

A.2 Supervised Data Augmentation

Response augmentation approaches (Luo et al., 2023; Gou et al., 2023) involve employing techniques such as nucleus sampling (top-p sampling) (Holtzman et al., 2020) and combining inferences from models of varying sizes, with the aim of enlarging the amount of generated reasoning steps (Zhu et al., 2023). These methods generally adhere to an intuitive understanding (Ni et al., 2022) that fine-tuned models are prone to biases towards a limited set of reference solutions.

Query augmentation methods focus on modifying existing questions to generate new ones. Li et al. (2023) finds that the diversity and complexity of problems contribute positively to performance, and Yu et al. (2023) believes that bootstrapping questions can provide multiple perspectives of meta-knowledge, crucial for covering more unseen scenarios and enabling stronger generalization. Earlier researches applied Named Entity Recognition or Regular Expression matching to build templates for augmenting questions (Li et al., 2022). Xu et al. (2022) focused on categorizing questions based on numerical abilities and designing numerical perturbations.

B Datasets

In this paper, we have used 6 datasets, including: GSM8K (Cobbe et al., 2021), MATH (Hendrycks et al., 2021), GSM-HARD (Gao et al., 2023), SVAMP (Patel et al., 2021), ASDiv (Miao et al., 2020b) and TAL-SCQ5K.
In terms of difficulty, by rough estimation:

$$\text{SVAMP} \approx \text{ASDiV} < \text{GSM8K} \approx \text{GSM-HARD} < \text{TAL-SCQ5k} < \text{MATH}$$

with ASDiV as a diversed dataset covering problem types taught in elementary school; SVAMP as a structural modified version of a subset of ASDiv; GSM8K being an immense dataset covering grade school problems, with 2-8 steps; GSM-HARD built upon GSM8K, replacing numbers with less-common large numbers; TAL-SCQ5K containing primary, junior high and high school level mathematical topics; MATH full of challenging competition mathematics problems which requires a strong mathematical background to perform well on. Among which, MATH dataset and TAL-SCQ5K dataset further process notations of difficulty levels.

Dataset	Num	Example Q&A
GSM8K	Train: 7473 Test: 1319	**question:** In a dance class of 20 students, 20% enrolled in contemporary dance, 25% of the remaining enrolled in jazz dance, and the rest enrolled in hip-hop dance. What percentage of the entire students enrolled in hip-hop dance? **answer:** There are 20 x 20/100 = «20*20/100=4»4 students who enrolled in contemporary dance. So, 20 - 4 = «20-4=16»16 students are enrolled in either jazz or hip-hop dance. There are 16 x 25/100 = «16*25/100=4»4 students who enrolled in jazz dance. Hence, 16 - 4 = «16-4=12»12 students enrolled in hip-hop dance. This is 12/20 x 100% = 60% of the entire students. #### 60
MATH	Train: 7500 Test: 5000	**question:** How many vertical asymptotes does the graph of $ y=\frac {2}{x^2+x-6}$ have? **answer:** The denominator of the rational function factors into $x^2+x-6=(x-2)(x+3)$. Since the numerator is always nonzero, there is a vertical asymptote whenever the denominator is 0, which occurs for $x = 2$ and $x = -3$. Therefore, the graph has $\boxed{2}$ vertical asymptotes.
GSM-HARD	Test: 1319	**input:** A robe takes 2287720 bolts of blue fiber and half that much white fiber. How many bolts in total does it take? **code:** def solution(): """A robe takes 2 bolts of blue fiber and half that much white fiber. How many bolts in total does it take?""" blue_fiber = 2287720 white_fiber = blue_fiber / 2 total_fiber = blue_fiber + white_fiber result = total_fiber return result **target:** 3431580.0

Dataset	Num	Example Q&A
SVAMP	Test: 1000	**Body:** The Razorback t-shirt shop makes $ 78 dollars off each t-shirt sold. During the Arkansas game and the Texas tech game they sold a total of 186 t-shirts. If they sold 172 t-shirts during the Arkansas game **Question:** How much money did they make from selling the t-shirts during the Texas tech game? **Equation:** (78.0 * (186.0 - 172.0)) **Answer:** 1092.0
ASDiv	Test: 2215	**body:** Robert wants to practice goal kicks for soccer. He decides to have 98 kicks before going home from the park. He takes 43 kicks before taking a break to get a drink of water. He then takes another 36 kicks. **question:** How many more kicks does he need to make before he goes home? **equation:** 98-43-36=19 **answer:** 19 (kicks)
TAL-SCQ	5000	**problem:** If n is an even positive integer, the double factorial notation $n!!$ represents the product of all the even integers from 2 to n. For example, $8!!=2\\cdot4\\cdot6\\cdot8$. What is the units digit of the following sum? $2!!+4!!+6!!+\\cdot\\cdot\\cdot+2018!!+2020!!+2022!!$ **solution:** Answer: $$2$$

Table 5: Examples of datasets in their original format.

C Relationships of k & N

Figure 5 illustrates the relationships of the number of reasoning paths and the data amounts of the respective D_{dedup} and D_{init}.

We select multiple points from D_{dedup} at regular intervals based on data amount. Simultaneously, we choose corresponding points from D_{init} with similar data amounts to ensure consistence. The statistic that the relationships of the number of reasoning paths and the data amount is detailed in Table 6 and 7.

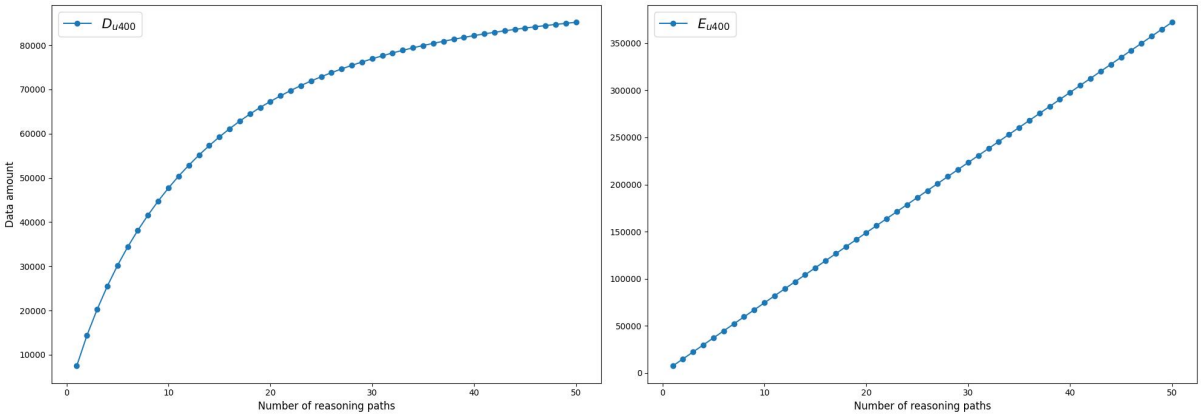

Figure 5: The relationships of k & N.

k	1	2	3	5	7	9	12	15	20	27	40	∞
N	7457	14344	20225	30179	38150	44771	52857	59261	67281	74643	82180	89530

Table 6: Extract subsets from relationships of k & N of D_{dedup} for experiments.

k	1	2	4	8	12	24	36	48
N	7457	14911	29810	59603	89386	178707	268003	357295

Table 7: Extract subsets from relationships of k & N of D_{init} for experiments.

D Supplementary experiments

D.1 Supplementary experiments to identify the minimal optimal set

Base Model	Upper Limit	#Data	GSM8K	SVAMP	ASDiv	MATH
Codellama 34B	k = 1	7457	74.7	80.0	77.3	19.0
Seed 0	k = 3	20225	78.4	82.1	77.3	17.8
	k = 9	44771	80.2	83.0	79.7	19.9
	k = 20	67281	81.0	82.3	79.5	19.9
	k = ∞	89530	81.3	82.7	79.1	20.5
Codellama 7B	k = 1	7457	60.65	69.15	69.05	8.10
Data in Figure 3	k = 3	20225	69.05	71.85	72.15	8.75
Seed 0 and 42	k = 9	44771	72.60	73.85	73.80	9.30
	k = 20	67281	73.30	73.25	73.50	10.6
	k = ∞	89530	74.20	73.00	73.50	11.2

Table 8: Performance of Codellama 34B and Codellama 7B

E Detailed Experiment Setting

Generate Deduplicated Datasets

We spent 4 days generating both D_{dedup}, D_M and the deduplicated dataset of TAL-SCQ in Section 2.3, 3.1 and 3.2 which is formed by employing four pre-trained models: ToRA-CODE 7B/13B/34B and ToRA 70B on the GSM8K, MATH and TAL-SCQ seperately, these models sample 100 reasoning paths each with temperature 0.9.

Training Models

We conducted SFT on Code LLaMA 7B using various deduplicated dataset and their subsets in Section 2.4, **??**, 4.1 and 4.2. Addtionally we conducted SFT on LLaMA-2 7B/13B for a horizontal comparison in Section 3.2.

We used a learning rate of 2e-5 with a 3% warm-up period for 1 epoch and a global batch size of 128 on NVIDIA A100 40G GPUs. We trained all models with DeepSpeed ZeRO Stage3 and Flash-Attention 2.

Apart from validating the effectiveness of the deduplication algorithm, where the random selection process with seeds set to 0 and 42 and then averaging the inference results, all other training and inference processes used a seed of 0.

The training sessions were completed within 1 day, with an average training duration of approximately 5 hours. The average evaluation time is less than 10 minutes.

F Case Study: Actual Distinct Solutions

To validate the effectiveness of deduplication and using clustering as a filter, we conduct a case study focusing on the relationship of reasoning paths and their problems' actual distinct solutions.

In the deduplicated subset D_{dedup} of the GSM8K dataset, we select the first question that has more than 15 reasoning paths, which has 43 reasoning paths for this problem in fact. Next, we utilize random selection and clustering as a filter to derive the subsets $D_{u400}^{k=15}$ and $D_{u400}^{cluster,k=15}$. We then separately analyze the 15 reasoning paths in these two subsets for the corresponding problem to categorize their actual distinct solutions on Table 9 and 10.

The question is formulated as follows:

Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?

Upon human analysis of this question, 10 distinct solutions have been summarized. These solutions are categorized based on whether the default daily salary is the same, whether the default working hours exceed the regular working hours, and how the total salary is calculated.

A Assuming that the daily wage is different, calculate the total wage by iterating over each day's wage and summing them in a loop.

B Assuming that the daily wage remains the same and that the working hours exceed the regular hours. Calculate the regular wage and the overtime wage for five days separately, then sum them up.

C Assuming that the daily wage remains the same and that the working hours exceed the regular hours. Calculate the regular hours worked and the overtime hours worked for five days separately, then sum the total wages.

D Assuming that the daily wage remains the same and that the working hours exceed the regular hours. Calculate daily wages, then sum them up.

E Assuming that the daily wage remains the same and that the Min and Max functions are used to avoid situations where the working hours are smaller than the regular hours. Calculate the regular wage and the overtime wage for five days separately, then sum them up.

F Assuming that the daily wage remains the same and that the Min and Max functions are used to avoid situations where the working hours are smaller than the regular hours. Calculate the regular hours worked and the overtime hours worked for five days separately, then sum the total wages.

G Assuming that the daily wage remains the same and that the Min and Max functions are used to avoid situations where the working hours are smaller than the regular hours. Calculate daily wages, then sum them up.

H Assuming that the daily wage remains the same and determine if the working hours exceed the regular hours. Calculate the regular wage and the overtime wage for five days separately, then sum them up.

I Assuming that the daily wage remains the same and determine if the working hours exceed the regular hours. Calculate the regular hours worked and the overtime hours worked for five days separately, then sum the total wages.

J Assuming that the daily wage remains the same and determine if the working hours exceed the regular hours. Calculate daily wages, then sum them up.

Table 9: Examples for Clustering as a Filter's Actual Solutions.

ID	Completion	Type

1	```python
	def total_salary():
	"""Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
	hourly_wage = 18.00
	hours_per_shift = 10
	num_days = 5
	overtime_hours = hours_per_shift - 8
	overtime_wage = hourly_wage + 0.5 * hourly_wage
	salary_per_day = hourly_wage * 8 + overtime_wage * overtime_hours
	total_salary = salary_per_day * num_days
	return total_salary
	total_salary = total_salary()
	print(total_salary)
	``
	`` output
	990.0
	``
	Tina makes 990.0 dollars.
		J

2	```python
	def total_pay():
	"""Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
	hourly_wage = 18.00
	hours_per_shift = 10
	num_shifts = 5
	overtime_hours = hours_per_shift - 8
	overtime_wage = hourly_wage + 0.5 * hourly_wage
	pay_for_overtime = overtime_hours * overtime_wage * num_shifts
	pay_for_regular_hours = (8 * hourly_wage) * num_shifts
	total_pay = pay_for_overtime + pay_for_regular_hours
	return total_pay
	total_pay = total_pay()
	print(total_pay)
	``
	`` output
	990.0
	``
	Tina makes 990.0 dollars.
		B

	``python
	def calculate_earnings():
	"""Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""	

3	```python
	hourly_wage = 18.00
	overtime_hours = 10 - 8 # hours worked in excess of 8 hours
	overtime_wage = hourly_wage + 0.5 * hourly_wage # overtime wage = hourly wage + half hourly wage
	daily_pay = 8 * hourly_wage + overtime_hours * overtime_wage
	total_pay = daily_pay * 5 # total pay for 5 days
	return total_pay
	total_pay = calculate_earnings()
	print(total_pay)
	``
	`` output
	990.0
	``
	Tina makes 990.0 dollars.
		D

4	```python
	def total_pay():
	"""Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
	hourly_wage = 18.00
	hours_per_shift = 10
	overtime_threshold = 8
	num_shifts = 5
	total_pay = 0
	for i in range(num_shifts):
	if hours_per_shift <= overtime_threshold:
	total_pay += hourly_wage * hours_per_shift
	else:
	overtime_hours = hours_per_shift - overtime_threshold
	regular_hours = overtime_threshold
	total_pay += (hourly_wage * regular_hours) + (hourly_wage * overtime_hours * 1.5)
	return total_pay
	total_pay = total_pay()
	print(total_pay)
	``
	`` output
	990.0
	``
	Tina makes 990.0 dollars.
		A

	``python
	def calculate_pay():
	"""Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
	hourly_wage = 18.00
	hours_per_shift = 10
	num_days = 5
	overtime_hours = hours_per_shift - 8	

5 D

```
    overitime_pay = hourly_wage + 0.5 * hourly_wage
    regular_pay = hourly_wage * 8
    total_pay = (overtime_hours * overtime_pay + regular_pay) * num_days
    return total_pay
total_pay = calculate_pay()
print(total_pay)
```
```output
990.0
```
Tina makes 990.0 dollars.

---

6 J

```python
def total_earnings():
    """Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
    hourly_wage = 18.00
    hours_per_shift = 10
    num_days = 5
    overtime_hours = hours_per_shift - 8 if hours_per_shift > 8 else 0
    earnings = (hourly_wage * 8) + (hourly_wage * overtime_hours * 1.5)
    total_earnings = earnings * num_days
    return total_earnings
total_earnings = total_earnings()
print(total_earnings)
```
```output
990.0
```
Tina makes 990.0 dollars.

---

7 C

```python
total_earnings():
    """Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
    hourly_wage = 18.00
    overtime_wage = hourly_wage + 0.5 * hourly_wage
    hours_per_shift = 10
    num_days = 5
    total_hours = hours_per_shift * num_days
    overtime_hours = total_hours - 8 * num_days
    earnings = (8 * num_days * hourly_wage) + (overtime_hours * overtime_wage)
    return earnings
total_earnings = total_earnings()
print(total_earnings)
```
```output
990.0
```

	`` Tina makes 990.0 dollars.	

8	```python
	def total_pay():
	"""Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
	hourly_wage = 18.00
	hours_per_day = 10
	num_days = 5
	overtime_hours = hours_per_day - 8 if hours_per_day > 8 else 0
	overtime_pay = overtime_hours * (hourly_wage + 0.5 * hourly_wage)
	regular_pay = (8 * hourly_wage) + overtime_pay
	total_pay = regular_pay * num_days
	return total_pay
	total_pay = total_pay()
	print(total_pay)
	``
	`` output
	990.0
	``
	Tina makes 990.0 dollars.
		J

9	```python
	def total_earnings():
	"""Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
	hourly_wage = 18
	hours_per_shift = 10
	num_days = 5
	overtime_hours = hours_per_shift - 8
	overtime_wage = hourly_wage + 0.5 * hourly_wage
	earnings = (8 * hourly_wage + overtime_hours * overtime_wage) * num_days
	return earnings
	total_earnings = total_earnings()
	print(total_earnings)
	``
	`` output
	990.0
	``
	Tina makes 990.0 dollars.
		D

	``python
	def calculate_pay():
	"""Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
	hourly_wage = 18	

```
    hours_per_shift = 10
    days_worked = 5
    overtime_hours = hours_per_shift - 8
    overtime_wage = hourly_wage + 0.5 * hourly_wage
    regular_pay = hourly_wage * 8
10  overtime_pay = overtime_wage * overtime_hours                    D
    total_pay = (regular_pay + overtime_pay) * days_worked
    return total_pay
total_pay = calculate_pay()
print(total_pay)
```
```output
990.0
```
Tina makes 990.0 dollars.

```python
def total_earnings():
    """Tina makes $18.00 an hour. If she works more than 8 hours per shift, she
is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly
wage. If she works 10 hours every day for 5 days, how much money does she
make?"""
    hourly_wage = 18.00
    overtime_wage = hourly_wage + 0.5 * hourly_wage
    hours_per_shift = 10
    num_shifts = 5
11  earnings_straight_time = hourly_wage * 8 * num_shifts            B
    earnings_overtime = overtime_wage * (hours_per_shift - 8) * num_shifts
    total_earnings = earnings_straight_time + earnings_overtime
    return total_earnings
total_earnings = total_earnings()
print(total_earnings)
```
```output
990.0
```
Tina makes 990.0 dollars.

```python
def calculate_salary():
    """Tina makes $18.00 an hour. If she works more than 8 hours per shift, she
is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly
wage. If she works 10 hours every day for 5 days, how much money does she
make?"""
    hourly_wage = 18.00
    overtime_wage = hourly_wage + 0.5 * hourly_wage
    hours_per_shift = 10
    days = 5
    regular_hours = min(hours_per_shift, 8)
12  overtime_hours = max(hours_per_shift - 8, 0)                     E
    regular_pay = regular_hours * hourly_wage * days
    overtime_pay = overtime_hours * overtime_wage * days
    total_pay = regular_pay + overtime_pay
```

```
    return total_pay
total_pay = calculate_salary()
print(total_pay)
```
```output
990.0
```
Tina makes 990.0 dollars.

```python
def total_pay():
    """Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
    hourly_wage = 18.00
    overtime_wage = hourly_wage + 0.5 * hourly_wage
    hours_per_day = 10
    num_days = 5
    overtime_hours = max(hours_per_day - 8, 0)
13  regular_hours = min(hours_per_day, 8)
    total_pay = (regular_hours * hourly_wage + overtime_hours * overtime_wage) * num_days  G
    return total_pay
total_pay = total_pay()
print(total_pay)
```
```output
990.0
```
Tina makes 990.0 dollars.

```python
def total_earnings():
    """Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
    hourly_wage = 18.00
    overtime_hours = 10
    days = 5
    regular_hours = 8
    regular_earnings = hourly_wage * regular_hours * days
14  overtime_earnings = (hourly_wage + 0.5 * hourly_wage) * (overtime_hours - regular_hours) * days  B
    total_earnings = regular_earnings + overtime_earnings
    return total_earnings
total_earnings = total_earnings()
print(total_earnings)
```
```output
990.0
```

	Tina makes 990.0 dollars.	

15	```python
	def calculate_pay():
	"""Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
	hourly_wage = 18.00
	hours_per_shift = 10
	num_shifts = 5
	overtime_hours = hours_per_shift - 8
	overtime_wage = hourly_wage + 0.5 * hourly_wage
	pay = (8 * hourly_wage + overtime_hours * overtime_wage) * num_shifts
	return pay
	total_pay = calculate_pay()
	print(total_pay)
	``
	`` output
	990.0
	``
	Tina makes 990.0 dollars.
		D

Table 10: Examples for Random Selection's Actual Solutions.

ID	Completion	Type

1	```python
	def total_salary():
	"""Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
	hourly_wage = 18.00
	hours_per_shift = 10
	num_days = 5
	overtime_hours = hours_per_shift - 8
	overtime_wage = hourly_wage + 0.5 * hourly_wage
	salary_per_day = hourly_wage * 8 + overtime_wage * overtime_hours
	total_salary = salary_per_day * num_days
	return total_salary
	total_salary = total_salary()
	print(total_salary)
	``
	`` output
	990.0
	``
	Tina makes 990.0 dollars.
		D

	``python def total_pay():	

2	```python
	def calculate_pay():
	"""Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
	hourly_wage = 18.00
	hours_per_shift = 10
	overtime_threshold = 8
	num_shifts = 5
	total_pay = 0
	for i in range(num_shifts):
	if hours_per_shift <= overtime_threshold:
	total_pay += hourly_wage * hours_per_shift
	else:
	overtime_hours = hours_per_shift - overtime_threshold
	regular_hours = overtime_threshold
	total_pay += (hourly_wage * regular_hours) +
	(hourly_wage * overtime_hours * 1.5)
	return total_pay
	total_pay = total_pay()
	print(total_pay)
	``
	``output
	990.0
	``
	Tina makes 990.0 dollars.
		A

3	```python
	def calculate_pay():
	"""Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
	hourly_wage = 18.00
	overtime_wage = 0.5 * hourly_wage
	hours_per_shift = 10
	num_shifts = 5
	overtime_hours = max(hours_per_shift - 8, 0)
	regular_pay = hourly_wage * 8
	overtime_pay = (hours_per_shift - 8) * (hourly_wage + overtime_wage)
	total_pay = (regular_pay + overtime_pay) * num_shifts
	return total_pay
	total_pay = calculate_pay()
	print(total_pay)
	``
	``output
	990.0
	``
	Tina makes 990.0 dollars.
		G

	``python
	def calculate_pay():	

4	```python
	def calculate_pay():
	"""Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
	hourly_wage = 18.00
	hours_per_shift = 10
	num_shifts = 5
	overtime_hours = hours_per_shift - 8
	overtime_wage = hourly_wage + 0.5 * hourly_wage
	pay_for_shifts = (8 * hourly_wage) + (overtime_hours * overtime_wage)
	total_pay = num_shifts * pay_for_shifts
	return total_pay
	total_pay = calculate_pay()
	print(total_pay)
	``
	`` output
	990.0
	``
	Tina makes 990.0 dollars.
		D

5	```python
	def total_earnings():
	"""Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
	hourly_wage = 18.00
	hours_per_shift = 10
	num_days = 5
	overtime_hours = hours_per_shift - 8
	if overtime_hours > 0:
	overtime_wage = hourly_wage + 0.5 * hourly_wage
	overtime_earnings = overtime_hours * overtime_wage * num_days
	else:
	overtime_earnings = 0
	regular_hours = 8
	regular_wage = hourly_wage
	regular_earnings = regular_hours * regular_wage * num_days
	total_earnings = regular_earnings + overtime_earnings
	return total_earnings
	total_earnings = total_earnings()
	print(total_earnings)
	``
	`` output
	990.0
	``
	Tina makes 990.0 dollars.
		H

	``python
	def calculate_earnings():	

6	"""Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?""" hourly_wage = 18.00 overtime_wage = hourly_wage + 0.5 * hourly_wage hours_per_day = 10 days_per_week = 5 overtime_hours = hours_per_day - 8 # overtime is any hours over 8 base_earnings = hourly_wage * 8 * days_per_week overtime_earnings = overtime_wage * overtime_hours * days_per_week total_earnings = base_earnings + overtime_earnings return total_earnings total_earnings = calculate_earnings() print(total_earnings) ` ` ` ` ` ` output 990.0 ` ` ` Tina makes 990.0 dollars.	B
7	` ` `python def total_pay(): """Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?""" hourly_wage = 18.00 hours_per_day = 10 num_days = 5 overtime_hours = hours_per_day - 8 if hours_per_day > 8 else 0 overtime_pay = overtime_hours * (hourly_wage + 0.5 * hourly_wage) regular_pay = (8 * hourly_wage) + overtime_pay total_pay = regular_pay * num_days return total_pay total_pay = total_pay() print(total_pay) ` ` ` ` ` ` output 990.0 ` ` ` Tina makes 990.0 dollars.	J
	` ` `python def calculate_earnings(): """Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?""" hourly_wage = 18.00 hours_per_shift = 10 num_days = 5 overtime_hours = hours_per_shift - 8	

8

```
    overtime_wage = hourly_wage + 0.5 * hourly_wage
    earnings_regular = 8 * hourly_wage * num_days
    earnings_overtime = overtime_hours * overtime_wage * num_days
    total_earnings = earnings_regular + earnings_overtime
    return total_earnings
total_earnings = calculate_earnings()
print(total_earnings)
```
```output
990.0
```
Tina makes 990.0 dollars.

B

9

```python
def calculate_pay():
    """Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
    hourly_wage = 18.00
    hours_per_shift = 10
    num_shifts = 5
    overtime_hours = hours_per_shift - 8
    overtime_wage = hourly_wage + 0.5 * hourly_wage
    pay = (8 * hourly_wage + overtime_hours * overtime_wage) * num_shifts
    return pay
total_pay = calculate_pay()
print(total_pay)
```
```output
990.0
```
Tina makes 990.0 dollars.

D

10

```python
def total_earnings():
    """Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
    hourly_wage = 18.00
    hours_per_shift = 10
    overtime_hours = hours_per_shift - 8
    num_shifts = 5
    overtime_wage = hourly_wage + 0.5 * hourly_wage
    regular_earnings = 8 * hourly_wage * num_shifts
    overtime_earnings = overtime_hours * overtime_wage * num_shifts
    total_earnings = regular_earnings + overtime_earnings
    return total_earnings
total_earnings = total_earnings()
print(total_earnings)
```
```output
```

B

990.0
```
Tina makes 990.0 dollars.

---

11 D

```python
def total_pay():
    """Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
    hourly_wage = 18.00
    hours_per_shift = 10
    num_shifts = 5
    overtime_hours = hours_per_shift - 8
    overtime_wage = hourly_wage + 0.5 * hourly_wage
    regular_pay = hourly_wage * 8
    overtime_pay = overtime_wage * overtime_hours
    total_pay = (regular_pay + overtime_pay) * num_shifts
    return total_pay
total_pay = total_pay()
print(total_pay)
```
```output
990.0
```
Tina makes 990.0 dollars.

---

12 D

```python
def calculate_earnings():
    """Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
    hourly_wage = 18.00
    hours_per_shift = 10
    num_shifts = 5
    overtime_hours = hours_per_shift - 8
    overtime_wage = hourly_wage + 0.5 * hourly_wage
    earnings = (8 * hourly_wage) + (overtime_hours * overtime_wage)
    return earnings * num_shifts
total_earnings = calculate_earnings()
print(total_earnings)
```
```output
990.0
```
Tina makes 990.0 dollars."

```python
def calculate_salary():
    """Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""

13 J

```
    hourly_wage = 18.00
    overtime_wage = hourly_wage + 0.5 * hourly_wage
    hours_per_shift = 10
    num_shifts = 5
    overtime_hours = hours_per_shift - 8 if hours_per_shift > 8 else 0
    salary = (8 * hourly_wage + overtime_hours * overtime_wage) * num_shifts
    return salary
salary = calculate_salary()
print(salary)
```
```output
990.0
```
Tina makes 990.0 dollars.

---

14 D

```python
def calculate_pay():
    """Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
    hourly_wage = 18.00
    overtime_wage = hourly_wage + 0.5 * hourly_wage
    hours_per_shift = 10
    num_shifts = 5
    overtime_hours = hours_per_shift - 8
    regular_hours = 8
    total_pay = (regular_hours * hourly_wage + overtime_hours * overtime_wage) * num_shifts
    return total_pay

total_pay = calculate_pay()
print(total_pay)
```
```output
990.0
```
Tina makes 990.0 dollars.

---

15 H

```python
def total_earnings():
    """Tina makes $18.00 an hour. If she works more than 8 hours per shift, she is eligible for overtime, which is paid by your hourly wage + 1/2 your hourly wage. If she works 10 hours every day for 5 days, how much money does she make?"""
    hourly_wage = 18.00
    hours_per_day = 10
    num_days = 5
    overtime_hours = hours_per_day - 8 if hours_per_day > 8 else 0
    overtime_wage = hourly_wage + 0.5 * hourly_wage
    regular_earnings = hourly_wage * 8 * num_days
    overtime_earnings = overtime_wage * overtime_hours * num_days
    total_earnings = regular_earnings + overtime_earnings
```

```
    return total_earnings
total_earnings = total_earnings()
print(total_earnings)
```
```output
990.0
```
Tina makes 990.0 dollars.

G Error Analysis: Numerical Perturb to create P_5

Table 11: Wrong case for P_5: Semantic Ambiguity

Problem:
Grandma Jones baked 5 apple pies for the fireman's luncheon. She cut each pie into 8 pieces and set the five pies out on the buffet table for the guests to serve themselves. At the end of the evening, after the guests had taken and eaten their pieces of pie, there were 14 pieces of pie remaining. How many pieces were taken by the guests?
Completion:

```python
def pie_pieces_taken():
    """Grandma Jones baked 5 apple pies for the fireman's luncheon. She cut each pie into 8 pieces and set the five pies out on the buffet table for the guests to serve themselves. At the end of the evening, after the guests had taken and eaten their pieces of pie, there were 14 pieces of pie remaining. How many pieces were taken by the guests?"""
    pies_initial = 5
    pieces_per_pie = 8
    total_pie_pieces = pies_initial * pieces_per_pie
    remaining_pie_pieces = 14
    taken_pie_pieces = total_pie_pieces - remaining_pie_pieces
    return taken_pie_pieces

taken_pie_pieces = pie_pieces_taken()
print(taken_pie_pieces)
```

```output
26
```

The guests took $\boxed{26}$ pieces of pie.

Perturbed Problem:
Grandma Jones baked 10 apple pies for the fireman's luncheon. She cut each pie into 13 pieces and set the five pies out on the buffet table for the guests to serve themselves. At the end of the evening, after the guests had taken and eaten their pieces of pie, there were 19 pieces of pie remaining. How many pieces were taken by the guests?
Perturbed Completion:

```python
def pie_pieces_taken():
    """Grandma Jones baked 10 apple pies for the fireman's luncheon. She cut each pie into 13 pieces and set the five pies out on the buffet table for the guests to serve themselves. At the end of the evening, after the guests had taken and eaten their pieces of pie, there were 19 pieces of pie remaining. How many pieces were taken by the guests?"""
```

```
    pies_initial = 10
    pieces_per_pie = 13
    total_pie_pieces = pies_initial * pieces_per_pie
    remaining_pie_pieces = 19
    taken_pie_pieces = total_pie_pieces - remaining_pie_pieces
    return taken_pie_pieces

taken_pie_pieces = pie_pieces_taken()
print(taken_pie_pieces)
```
```output
111
```
The guests took $\boxed{26}$ pieces of pie.

Table 12: Wrong case for P_5: Illogical Numerical Perturb

Problem:
A mechanic charges different rates to repair the tires of trucks and cars. For each truck tire that is repaired, the mechanic will charge $60 and for each car tire that is repaired, the mechanic will charge $40. On Thursday, the mechanic repairs 6 truck tires and 4 car tires. On Friday, the mechanic repairs 12 car tries and doesn't repair any truck tires. How much more revenue did the mechanic earn on the day with higher revenue?
Completion:
```python
def revenue]_difference():
    """A mechanic charges different rates to repair the tires of trucks and cars. For each truck tire that is repaired, the mechanic will charge $60 and for each car tire that is repaired, the mechanic will charge $40. On Thursday, the mechanic repairs 6 truck tires and 4 car tires. On Friday, the mechanic repairs 12 car tries and doesn't repair any truck tires. How much more revenue did the mechanic earn on the day with higher revenue?"""
    truck_tire_cost = 60
    car_tire_cost = 40
    truck_tires_repaired_thursday = 6
    car_tires_repaired_thursday = 4
    car_tires_repaired_friday = 12
    truck_tires_repaired_friday = 0
    revenue_thursday = (truck_tire_cost * truck_tires_repaired_thursday) + (car_tire_cost * car_tires_repaired_thursday)
    revenue_friday = (truck_tire_cost * truck_tires_repaired_friday) + (car_tire_cost * car_tires_repaired_friday)
revenue_difference = revenue_friday - revenue_thursday
    return revenue_difference
revenue_difference = revenue_difference()
print(revenue_difference)
```
```output
-40
```
The mechanic earned $\boxed{40}$ dollars less on Friday than on Thursday.

Perturbed Problem:
A mechanic charges different rates to repair the tires of trucks and cars. For each truck tire that is repaired, the mechanic will charge $65 and for each car tire that is repaired, the mechanic will charge $45. On Thursday, the mechanic repairs 11 truck tires and 9 car tires. On Friday, the mechanic repairs 17 car tries and doesn't repair any truck tires. How much more revenue did the mechanic earn on the day with higher revenue?
Perturbed Completion:
```python
def revenue]_difference():
    """A mechanic charges different rates to repair the tires of trucks and cars. For each truck tire that is repaired, the mechanic will charge $65 and for each car tire that is repaired, the mechanic will charge $45. On Thursday, the mechanic repairs 11 truck tires and 9 car tires. On Friday, the mechanic repairs 17 car tries and doesn't repair any truck tires. How much more revenue did the mechanic earn on the day with higher revenue?"""
    truck_tire_cost = 65
    car_tire_cost = 45
    truck_tires_repaired_thursday = 11
    car_tires_repaired_thursday = 9
    car_tires_repaired_friday = 17
    truck_tires_repaired_friday = 0
    revenue_thursday = (truck_tire_cost * truck_tires_repaired_thursday) + (car_tire_cost * car_tires_repaired_thursday)
    revenue_friday = (truck_tire_cost * truck_tires_repaired_friday) + (car_tire_cost * car_tires_repaired_friday)
revenue_difference = revenue_friday - revenue_thursday
    return revenue_difference
revenue_difference = revenue_difference()
print(revenue_difference)
```
```output
355
```
The mechanic earned $\boxed{-355}$ dollars less on Friday than on Thursday.