How Do Large Language Models Learn Concepts During Continual Pre-Training?

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Abstract

Human beings primarily understand the world through concepts (e.g., *dog*), abstract mental representations that structure perception, reasoning, and learning. However, how large language models (LLMs) acquire, retain, and forget such concepts during continual pretraining remains poorly understood. In this work, we study how individual concepts are acquired and forgotten, as well as how multiple concepts interact through interference and synergy. We link these behavioral dynamics to LLMs' internal **concept circuits**, computational subgraphs associated with specific concepts, and incorporate graph metrics to characterize **circuit topology**. Our analysis reveals: (1) LLMs concept circuits provide a non-trivial, consistent signal of concept learning and forgetting; (2) concept circuits exhibit a stage-wise temporal pattern during continual pretraining, with an early increase followed by gradual decrease and stabilization; (3) concepts with larger learning gains tend to exhibit greater forgetting under subsequent training; (4) semantically similar concepts induce stronger interference than weakly related ones; (5) conceptual knowledge differs in their transferability, with some significantly facilitating the learning of others. Together, our findings provide a circuit-level view of concept learning dynamics and motivate concept-aware training strategies, such as Circuit-aware Experience Replay, which uses circuit topology to prioritize concepts vulnerable to forgetting.

1 Introduction

Humans organize perception and reasoning around **concepts**—abstract mental categories (e.g., *dog*) that enable generalization from individual observations to shared properties, relations, and actions (Harpaintner et al., 2018). Similarly, a defining characteristic of large language models (LLMs) is their ability to abstract conceptual knowledge from vast corpora, encoding it within internal *concept-level* representations that facilitate downstream reasoning and generation(Aljaafari et al., 2024; Wang et al., 2024b; Park et al., 2023). As new concepts continually emerge and models are updated through continual pre-training, it becomes essential to understand how reliably new concepts are acquired and prior concepts are retained, and how to make concept learning more efficient in the presence of interactions among the many concepts and types of conceptual knowledge.

Prior studies have examined concept learning in LLMs from two angles. One line of work uses prompt-based **knowledge probing** to test whether specific conceptual properties or relations (e.g., *commonsense facts*) can be elicited from a pretrained model (Gu et al., 2023b; Liao et al., 2023; Shani et al., 2023a; Zheng et al., 2024; Peng et al., 2022; Xu et al., 2024a), while another leverages **mechanistic interpretability** tools to localize internal structures associated with conceptual information (Aljaafari et al., 2024; Wang et al., 2024c). However, these efforts provide only a partial view of concept learning: they typically probe isolated pieces of conceptual knowledge that models may already possess, rather than systematically characterizing how concepts are acquired, consolidated, and forgotten during continual pre-training.

In this work, we take a step toward filling this gap by asking two more research questions: (1) *how internal concept representations correlate with concept acquisition and forgetting*, and (2) *how they relate to interference and synergy among multiple concepts and types of knowledge during joint training*. Answering these questions matters not only for interpretability, but also for practice: they can inform concept-aware continual pre-training decisions, such as how much training to allocate to previously learned concepts to mitigate forgetting and to newly introduced concepts to facilitate efficient acquisition, how to schedule and reorder training data, and how to mitigate destructive inter-

ference among semantically related concepts.

To enable a controlled study, we introduce the FICO dataset, built from ConceptNet-derived conceptual relations but mapped to synthetic, non-existent concept names, preserving realistic knowledge structure while making the concepts novel to the model. We then conduct controlled continual pre-training experiments and link behavioral changes to internal mechanisms by extracting **Concept Circuits**, which are computational subgraphs associated with each concept, using circuit identification methods and characterizing their topology over time with graph metrics. This joint analysis allows us to trace how concept-level behavior (e.g., learning and forgetting measures) co-evolves with circuit topology, and how cross-concept relatedness and knowledge-type ordering shape interference and transfer, as shown in Figure 1.

Our results reveal several meaningful and systematic patterns in concept learning dynamics in LLMs: **(1)** circuit topology is non-trivially correlated with both concept learning and forgetting, suggesting a trade-off between acquisition strength and subsequent stability: more centralized and redundant circuit structures are associated with stronger acquisition, but also with greater forgetting under continual training. This trade-off is also reflected behaviorally, as concepts with larger learning gains tend to exhibit greater forgetting during subsequent training. **(2)** We observe that during continued training on new domains, concept circuits corresponding to previously learned concepts follow a stable stage-wise temporal trajectory across multiple graph metrics, with an early increase followed by gradual decrease and stabilization, suggesting distinct phases of concept consolidation. **(3)** At the level of multi-concept learning, semantically similar concepts interfere more strongly than weakly related concepts, and different types of conceptual knowledge differ substantially in their transferability; e.g., pretraining on Hyponym & Hypernym knowledge improves subsequent learning performance on Synonym & Antonym knowledge by 63.74%. Together, these findings offer a circuit-level view of concept learning dynamics and show that the resulting circuit-based signals can be translated into actionable training strategies, such as targeted replay, to improve continual pre-training stability.

Our contributions are summarized as follows:

- We present the first systematic study of concept acquisition and forgetting in LLMs during continual pretraining, revealing consistent correlations between concept circuits, learning dynamics, and forgetting behavior, suggesting these signals as indicators of future learning and forgetting.
- We present a relational perspective on knowledge learning in LLMs, revealing interference and synergy patterns across concepts and knowledge types during joint training, motivating interference-aware data scheduling.
- Our analysis informs actionable concept-aware training strategies for continual pre-training. As a proof of value, we instantiate a simple circuit-aware replay strategy for improving knowledge retention.

2 Dataset Construction

We follow previous studies (Wang et al., 2024b) and define a **concept** (e.g., *dog*) as an abstraction over the world that captures the shared features and essential characteristics of a class of entities. **Conceptual knowledge** (e.g., *a dog has four legs*) refers to factual or relational information associated with a concept, reflecting familiarity with and understanding of its properties and relations (Wang et al., 2024a). Each concept is therefore linked to a set of conceptual knowledge that collectively describe its semantic content. To study concept learning across different levels of concreteness, we consider both **concrete concepts** that are grounded in direct sensory experience (e.g., *dog*), and **abstract concepts** which are not directly perceptible (e.g., *love*). We sample 500 concrete concepts from the THINGS dataset (Hebart et al., 2023) and 500 abstract concepts from the Concreteness Ratings dataset (Brysbaert et al., 2014). For each concept, we retrieve associated conceptual knowledge from ConceptNet (Speer et al., 2017), a large-scale knowledge graph of millions of concepts and 34 typed relations.

To mitigate influence of pre-existing knowledge encoded in LLMs, we replace each real concept name with a fictional name generated by GPT-5 (Hurst et al., 2024) and perform relation filtering and canonicalization, as detailed in Appendix A.2. This process forms our FICO, **FI**ctional **CO**ncept dataset, illustrated in Figure 1(A), with statistics in Appendix B. Following prior work (Zucchet et al., 2025b), we use GPT-5 to generate multiple natural-language templates for each relation

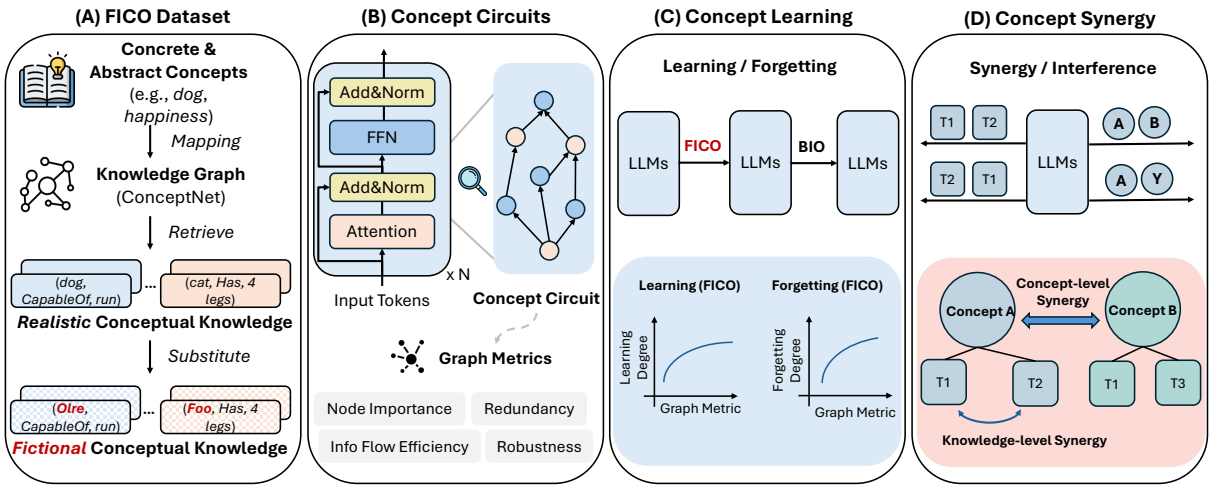


Figure 1: (A) Construct the FICO dataset based on ConceptNet. (B) Extract **Concept Circuits**, LLM computational subgraph associated with individual concepts, and characterize their structure using graph metrics. (C) Analyze **concept learning and forgetting dynamics** in two-stage continual pretraining. (D) Study **synergy and interference** across concepts (e.g., A, B, Y) and knowledge type (e.g., $T1$ and $T2$).

type (e.g., "*{concept} has the ability to*" for *CapableOf*), converting each knowledge triple (e.g., (dog, CapableOf, run)) into prefix-target examples. For evaluation, we sample 500 concepts and use a *disjoint* template pool to construct test instances with unseen surface forms, enabling assessment of whether LLMs learn underlying relations rather than memorizing specific templates.

3 How Internal Concept Representations Correlate with LLM Learning and Forgetting Dynamics?

3.1 Concept Circuit as Internal Concept Representation

Given a knowledge triple $k_{ij} = (c_i, r_{ij}, o_{ij})$, where c_i is a subject concept (e.g., *dog*), r_{ij} is a relation type (e.g., *HasProperty*), and o_{ij} is an object which can be another concept or a descriptive phrase (e.g., *fleas* or *four legs*), a **Knowledge Circuit** is defined as the minimal computational subgraph in a pretrained LLM that can faithfully predict the target object o_{ij} conditioned on a textual prefix converted from the subject–relation pair (c_i, r_{ij}) (Yao et al., 2025). We adapt this notion to concept-level analysis and define a **Concept Circuit** for concept c_i as the computational subgraph that jointly and faithfully predict all conceptual knowledge $\mathcal{K}_i = \{k_{i0}, k_{i1}, \ldots, k_{ij}, \ldots\}$ associated with c_i. Unlike a union of independently extracted circuits for individual facts, a Concept Circuit is identified at the level of $\mathcal{K}_i$ and captures the shared computation underlying the conceptual knowledge.

To extract concept circuits at different checkpoints during continual training, we use EAP-IG (Hanna et al., 2024) as our circuit identification method. EAP-IG assigns an importance score to each edge while balancing computational efficiency with attribution faithfulness[1]. Given the edge importance scores, we construct a circuit by selecting the top-scoring edges such that the resulting subgraph preserves at least 70%[2] of the full model's performance on the corresponding concept.

Topology of Circuit Graph Previous work has shown that the geometric and topological properties of LLM circuits and features can provide insight into model behaviors such as grokking, reasoning, and learning. (Humayun et al., 2024; Zhou et al., 2025; Raj; Li et al., 2025; Ou et al., 2025). Motivated by these findings, we compute four families of standard graph metrics to characterize the topology of concept circuits and their relationship to concept learning dynamics in LLMs: **(1) Density**, defined as the ratio of existing edges to the maximum possible number of edges. Higher density reflects more redundant connections (Humayun et al., 2024; Newman, 2010). **(2) Eigenvector Centrality**, whose standard deviation quantifies how unevenly structural influence is distributed across nodes. Higher variance indicates a more concentrated hub structure (Newman, 2010). **(3) Global Efficiency**, i.e., the average inverse shortest-path distance between node pairs. Higher global efficiency indicates that signals can propagate more efficiently across the graph (Latora and Marchiori, 2001). **(4) k-core**, whose average number captures the depth of a graph's densely connected core and

[1] More details for the implementation of EAP-IG can be found in Appendix C.

[2] Following prior circuit-level interpretability work (Ou et al., 2025; Yao et al., 2025), we use 70% as a heuristic preservation threshold and provide an 80% ablation in Section 3.3. Thresholds $\geq$ 90% often yield overly large, less interpretable circuits.

is used as a proxy for resilience to disruption in graph (Laishram et al., 2018; Seidman, 1983). We select these four metrics because, in the graph analysis literature, they are standard measures of graph redundancy, node importance, information flow efficiency, and graph robustness, respectively.

3.2 Experiment Design

To quantify how internal concept representations (i.e., concept circuits) relate to concept acquisition and forgetting, given a pre-trained LLM π_0, we conduct a two-stage [3] continual pretraining setup. In **Stage 1 (Concept Acquisition)**, we continually train on the FICO training set to learn novel concepts, yielding π_1, which is used to analyze concept learning dynamics. In **Stage 2 (Forgetting Induction)**, we further train π_1 on the BIO dataset (Allen-Zhu and Li, 2023), an standard pretraining dataset used in previous LLMs knowledge acquisition studies (Allen-Zhu and Li, 2023; Zucchet et al., 2025a), yielding a new model π_2, which is used to analyze concept forgetting dynamics. We experiment with GPT-2 Large (Radford et al., 2019), LLaMA-3.2-1B, and LLaMA-3.1-8B (Dubey et al., 2024).[4] In both stages, we concatenate the prefix and target phrase, as described in Section 2, and optimize the model using next-token prediction.

For evaluation, we provide only the prefix on the FICO test set and compute concept learning/forgetting degrees from the model's gold-target logit, as defined below. We use **Spearman's correlation coefficient** (Spearman, 1961) to analyze the correlation between concept learning/forgetting dynamics and concept circuit topology.[5]

Definition 1 (Knowledge Learning/Forgetting Degree)**.** *For a knowledge triple $k = (c, r, o)$ with prefix $x = T(c, r)$, let $s_\pi(k) = \text{logit}_\pi(o \mid x)$ denote the gold-target logit.*[6] *The* **knowledge learning degree** *and* **knowledge forgetting degree** *of k are defined as $\Delta_{\text{learn}}(k) = s_{\pi_1}(k) - s_{\pi_0}(k)$ and $\Delta_{\text{forget}}(k) = s_{\pi_1}(k) - s_{\pi_2}(k)$, respectively.*

[3] We further evaluate longer continual updates and observe consistent trends; see Appendix D.

[4] We observe similar trends across models. Due to space constraints, we present GPT-2 results in the main paper and defer LLaMA results to Appendix E.

[5] See Appendix F for additional experiment details.

[6] Following prior work (Heimersheim and Nanda, 2024; Hanna et al., 2024), we use logits as a fine-grained indicator of LLM learning and report log-probability results in Appendix G.

Definition 2 (Concept Learning/Forgetting Degree)**.** *For a concept c with associated triples $\mathcal{K}_c$, its* **concept learning degree** *and* **concept forgetting degree** *are defined as $\Delta_{\text{learn}}(c) = \frac{1}{|\mathcal{K}_c|}\sum_{k\in\mathcal{K}_c}\Delta_{\text{learn}}(k)$ and $\Delta_{\text{forget}}(c) = \frac{1}{|\mathcal{K}_c|}\sum_{k\in\mathcal{K}_c}\Delta_{\text{forget}}(k)$, respectively.*

3.3 Experiment Finding

Finding 1. *Concepts exhibit substantial cross-concept variation in both learning degree and forgetting degree, indicating that LLMs acquire and forget different concepts to markedly different extents under the same training regime.*

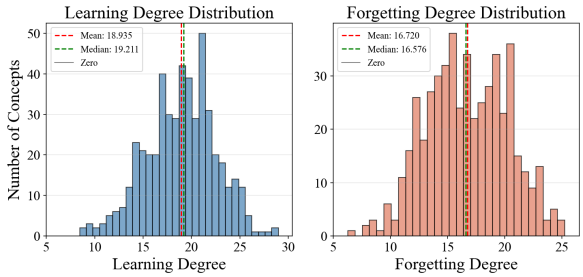


Figure 2: Distribution of learning and forgetting degree across concepts.

Concept Learning/Forgetting Degree Distribution. Figure 2 shows the distribution of concept learning (logit increase) and forgetting (logit decrease) degrees across 500 concepts on the FICO test set. The distribution is unimodal but widely spread, indicating substantial variability in how concepts are learned and forgotten. Practically, some concepts are learned more readily and robustly, while others require greater effort to learn and retain. This observation motivates our subsequent analysis, which seeks to identify indicators that can account for the differing learning and forgetting behaviors across concepts.

Finding 2. *Concept learning degree and forgetting degree shows non-trivial, consistent correlations with multiple circuit graph metrics, suggesting that circuit topology can serve as an informative indicator of concept learning and forgetting dynamics.*

Correlation between Concept Learning/Forgetting and Concept Circuits. As shown in Table 1, our circuit topology metrics exhibit substantially

Type	Metric	Learning	Forgetting
Circuit Topology (70%)	Density	0.3201	0.3815
Circuit Topology (70%)	k-core	0.3176	0.3783
Circuit Topology (70%)	Eigenvector Centrality	0.3173	0.3720
Circuit Topology (70%)	Global Efficiency	0.2934	0.3653
Circuit Topology (80%)	Density	0.3198	0.3746
Circuit Topology (80%)	k-core	0.2990	0.3553
Circuit Topology (80%)	Eigenvector Centrality	0.2745	0.3306
Circuit Topology (80%)	Global Efficiency	0.3153	0.3703
Activation	Mean EAP-IG	0.1907	0.2648
Activation	STD EAP-IG	0.1802	0.2547
Activation	Max EAP-IG	0.1185	0.1483
Random Topology	Density	-0.0205	-0.0252
Random Topology	k-core	-0.0550	-0.0433
Random Topology	Eigenvector Centrality	0.0734	0.0735
Random Topology	Global Efficiency	-0.0497	-0.0764

Table 1: Correlation between learning/forgetting degree and circuit metrics.

stronger correlations [7] with concept learning degree than either edge-level EAP-IG statistics (Hanna et al., 2024) or random topology metrics [8], indicating that circuit topology may be systematically associated with how concepts are acquired. For example, positive correlations with eigenvector centrality and k-core suggest that circuits with centralized bottlenecks and stable structural cores tend to achieve larger logit gains during learning. Notably, these same topological properties are also associated with greater vulnerability under continual training. In particular, concepts whose circuits are dominated by a small set of influential hub nodes, reflected by high eigenvector centrality, tend to exhibit larger forgetting degrees, potentially because these centralized components are more susceptible to perturbation during subsequent training.

This structural trade-off is also reflected behaviorally: concepts that are learned more strongly also tend to be forgotten more strongly under subsequent training, revealing a positive correlation between learning degree and forgetting degree, as detailed in Appendix J. Furthermore, these correlations remain comparable when increasing the circuit performance-preservation threshold from 70% to 80%, suggesting that the observed circuit-behavior associations are robust to the threshold choice. Taken together, these results indicate that circuit topology captures informative signals of both concept learning and forgetting dynamics in LLMs. Such signals may provide practical guidance for concept-aware continual pretraining decisions, such as how to allocate training effort between newly introduced concepts and previously learned concepts, as further explored in Section 5.

[7] p-values are provided in Appendix H

[8] See baseline metrics in Appendix I.

Finding 3. *During continued training on unrelated data, LLMs exhibit a consistent stage-wise temporal pattern across multiple circuit graph metrics, characterized by an early-phase change followed by gradual relaxation and stabilization.*

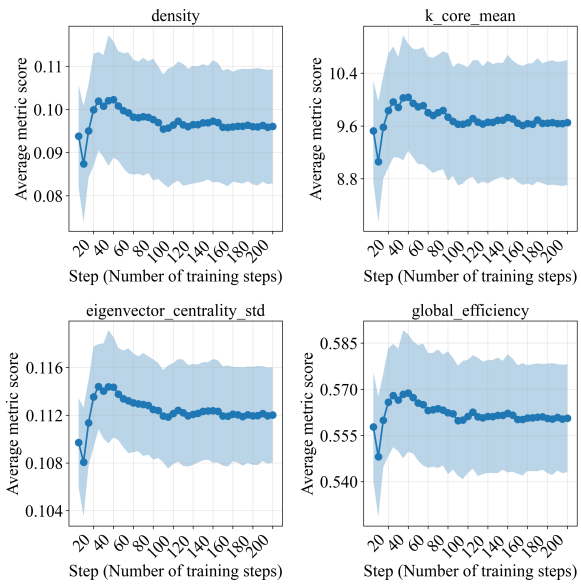
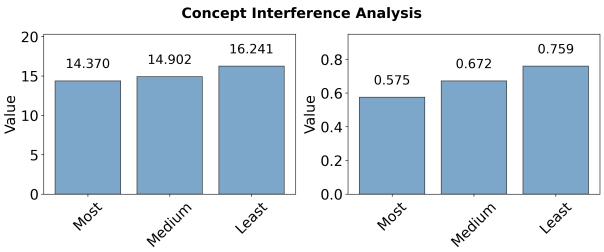


Figure 3: Graph Metric over training steps

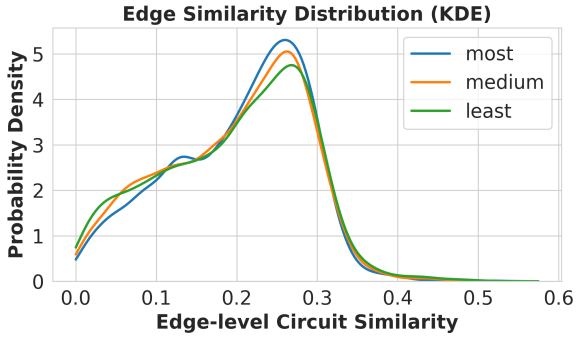
Evolution of Circuit Topology. Figure 3 shows the evolution of circuit graph metrics during the forgetting stage. LLMs exhibit a consistent stage-wise temporal pattern across most graph metrics: an initial increase followed by a gradual decrease and eventual stabilization. This behavior is observed across all structural aspects of concept circuits, suggesting that forgetting is accompanied by systematic circuit reorganization rather than monotonic structural decay. The early increase may indicates transient entanglement of previously learned concept circuits with newly introduced knowledge during continued training, while the subsequent decrease and stabilization reflect structural relaxation and convergence to a weaker representation under interference.

4 How Do Interference and Synergy Arise Across Concepts and Conceptual Knowledge During Joint Training?

The analyses above suggest a trade-off between acquisition strength and vulnerability under continual training. This raises a natural question: is such vulnerability intrinsic to individual concepts, or is it also shaped by interactions among concurrently learned concepts?



(a) Concept interference under joint training.



(b) Jaccard similarity across circuits

Figure 4: Concept interference under joint training.

4.1 Interference and Synergy across Concepts

4.1.1 Experiment Design

To examine cross-concept interactions during joint training, we construct relatedness-based groups for each target concept. Relatedness is defined as the cosine similarity between embeddings of the concatenated conceptual knowledge associated with each concept, computed using Qwen3-Embedding-4B (Yang et al., 2025). For each concept c, we rank all other concepts by similarity and select the top-K, middle-K, and bottom-K concepts as the highly, moderately, and weakly related groups, respectively, with $K = 100$. We then perform joint training for each concept c under three settings, where c is trained together with concepts from one of the three relatedness groups. Performance is evaluated using (1) the average logit assigned to target objects and (2) the corresponding average probability. This procedure is repeated across all concepts in the FICO test set to characterize cross-concept interactions.

4.1.2 Experiment Finding

Finding 4. *Training with highly related concepts yields lower performance than training with weakly related concepts, indicating stronger interference among semantically similar concepts during joint learning.*

Figure 4a shows a clear and consistent dependence on semantic relatedness. Across both evaluation metrics (average logit and average probability), training with weakly related concepts achieves the highest performance (75.9%), substantially outperforming training the highly related (57.5%) and moderately related (67.2%) concepts. These results demonstrate that cross-concept interactions meaningfully affect how effectively a target concept can be acquired in a multi-concept setting.

To understand the underlying mechanism driving this effect, we analyze the degree of overlap between the internal representations of co-trained concepts. For each target concept c, we pair it with each of its related concepts drawn from three semantic similarity groups: highly related, moderately related, and weakly related. For each pair (c, c'), we obtain their concept circuits and compute the Jaccard similarity between their edge sets. Figure 4b visualizes the resulting similarity distributions using KDE plots. The highly related setting exhibits a sharper, higher-density peak at moderate circuit similarity, indicating consistent reuse of overlapping computational pathways between target and auxiliary concepts, which likely induces sustained representational competition during joint training and results in stronger interference. Instead, the moderately and weakly related settings show lower and more dispersed similarity distributions, reflecting reduced circuit overlap and correspondingly weaker interference. This observation motivates interference-aware data scheduling that reduces circuit similarity among co-trained concepts within the same batch.

4.2 Interference and Synergy across Knowledge

4.2.1 Experiment Design

Beyond concepts, we ask whether *different types of conceptual knowledge* can also exhibit interference or synergy, even when they describe the *same* concept. We consider five high-level semantic knowledge categories: (1) Hyponym & Hypernym (HAH), (2) Synonym & Antonym (SAA), (3) Meronym & Holonym (MAH), (4) Property & Affordance (PAA), and (5) Spatial Relation (SR), as detailed in Appendix A. To study interactions across knowledge types, we adopt a pairwise continual-training setup. In Stage 1, the model is trained on either a source knowledge category R_i or the BIO dataset (Allen-Zhu and Li, 2023),

which does not contain conceptual knowledge, for the same training steps. In Stage 2, it is continually trained on a target knowledge category R_j. We evaluate performance on the target category R_j to quantify which type transitions induce synergy or interference. We define **paired transferability** from R_i to R_j as

$$T(R_i \rightarrow R_j) = \frac{\text{logit}(R_j \mid R_i) - \text{logit}(R_j \mid \text{BIO})}{|\text{logit}(R_j \mid \text{BIO})|}. \quad (1)$$

Positive $T(R_i \rightarrow R_j)$ indicates *synergy*, where learning R_i facilitates subsequent learning of R_j, while negative values indicate *interference*.

4.2.2 Experiment Finding

Finding 5. *Substantial and asymmetric transfer effects emerge across knowledge types, where pretraining on one type can facilitate learning of another, with highly directional and uneven benefits across ordered pairs.*

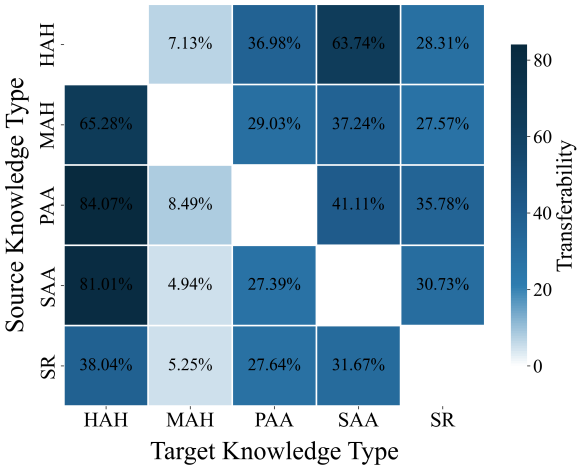


Figure 5: Paired transferability across knowledge

Figure 5 provides a fine-grained view of pairwise synergy among the different knowledge types, where each cell shows the paired transferability $T(R_i \rightarrow R_j)$ when a *source* R_i type is trained earlier before a *target* R_j knowledge type. While all five categories encode concept-related information, they capture distinct semantic facets, leading to non-redundant and uneven transfer behaviors. The heatmap reveals heterogeneous and directional effects: for example, pretraining on Property & Affordance (PAA) yields substantial gains when transferring to Hyponym & Hypernym (HAH) and Synonym & Antonym (SAA), suggesting that learning functional and attribute-level regularities can effectively scaffold the acquisition of more abstract relational structures. In contrast, the reverse directions exhibit markedly weaker transfer, highlighting the asymmetry of these interactions. Moreover, some target types, such as Meronym & Holonym (MAH), show relatively small improvements across most sources, indicating higher intrinsic learnability and reduced sensitivity to prior knowledge-type pretraining. Overall, these findings suggest practical guidance for future training curricula, such as reordering training data to place knowledge types with strong positive transfer earlier, thereby encouraging synergy and efficiency of downstream concept learning.

Method	Replay Ratio	Accuracy(%)	Hit@5(%)	Logit
w/o Replay	0	1.93	7.23	3.93
Random Replay	0.01	11.76	19.96	6.77
Circuit Replay	0.01	22.49	38.78	8.85
Random Replay	0.05	40.39	61.11	10.88
Circuit Replay	0.05	51.58	71.06	12.72
Random Replay	0.10	60.94	77.36	12.98
Circuit Replay	0.10	69.15	82.74	14.26
Random Replay	0.50	82.48	91.42	16.34
Circuit Replay	0.50	86.48	93.04	17.42

Table 2: Retention performance on the AntiLeak dataset under continual pretraining.

5 How Circuit Analysis Can Inform Better Continual Pretraining?

As shown in Finding 2, circuit topology metrics provide informative signals of concept forgetting during continual pre-training of LLMs. Leveraging this insight, we derive a simple yet actionable training strategy, **Circuit-aware Experience Replay**, that prioritizes concepts predicted to be more susceptible to forgetting and allocates additional training effort to them (e.g., via experience replay (Isele and Cosgun, 2018)).

Method Given a model π_{learn} that has learned a sequence of concepts $\{c_0, c_1, \ldots\}$, our goal is to further train the model on a new dataset while preserving performance on previously learned concepts. To this end, we extract concept circuits for all learned concepts and compute their corresponding circuit topology metrics. Based on topology metric[9], we select the top $p\%$ of concepts with the largest topology metric values and replay their associated training samples during continual pretraining on the new dataset.

[9] As shown in Table 1, the Density metric exhibits the strongest correlation with forgetting, and is therefore used to rank concepts.

Experiment To evaluate this strategy in a realistic setting, we conduct experiments on the AntiLeak dataset (Wu et al., 2025), which contains newly introduced real-world knowledge not previously learned by existing LLMs. We first train GPT-2 Large on AntiLeak to obtain the model π_{learn}, then continue training it on the BIO dataset (Allen-Zhu and Li, 2023) with **Circuit-aware Experience Replay**. We evaluate retention on AntiLeak using **Logit**, **Accuracy**, and **Hit@5** (see Appendix K for definitions). As shown in Table 2, our strategy consistently outperforms a random replay baseline across all replay ratios and metrics, demonstrating the practical utility of circuit topology for identifying forgetting-prone concepts and guiding targeted replay.

6 Robustness and Ablations

We assess the stability of our findings along two additional axes: model scale and continual-training horizon. First, we repeat both RQ1 and RQ2 on LLaMA-3.2-1B and LLaMA-3.1-8B. Across LLaMA models, circuit topology achieve around $\rho \approx 0.3$ correlations with concept learning and forgetting degree, as detailed in Appendix E. Second, we extend the two-stage FICO→BIO setup to a four-stage FICO→BIO→FICO→BIO training sequence. This longer horizon tests whether our conclusions persist under repeated acquisition and interference phases. As shown in Appendix D, graph metrics exhibit even stronger correlations with learning degree in this setting, reaching $\rho = 0.425$. This may indicate that repeated training encourages the reuse or consolidation of previously learned computational structures for concept acquisition.

7 Related Work

LLM Knowledge Acquisition. A growing body of work studies learning mechanism behinds LLM knowledge acquisition. One line uses synthetic or fictional corpora, e.g., biographies of fabricated individuals (Allen-Zhu and Li, 2023; Zucchet et al., 2025a; Ou et al., 2025; Zhu et al., 2025; Feng et al., 2024), Wikipedia-style entries for fictional entities (Chang et al., 2024), or **post-cutoff data** (Huang et al., 2024), to examine how data properties, training choices, and curricula (e.g., ordering dependencies between facts and implications) affect learning. Another line (Chang et al., 2024; Xu et al., 2024b; Leybzon and Kervadec, 2024; Im and Li, 2024; Qian et al., 2024; Tigges et al., 2024) leverages mechanistic interpretability to analyze LLM learning by tracing how internal representations evolve across training stages. The third line (Ren and Sutherland, 2024; Chen et al., 2023; Jain et al., 2023) identify phase transitions that reveal discrete, objective-dependent shifts in model behavior during training. In contrast, we target **concept-level** knowledge rather than isolated factual triples, enabling the study of higher-order phenomena such as concept learning, forgetting, interference, and synergy, while also opening the door to future work on concept generalization and hierarchical interactions (Zaeem and Komeili, 2021).

LLM Concept Probing and Editing. Prior work studies conceptual knowledge in LLMs mainly through: (i) **prompt-based probing** of concept properties and relations (Gu et al., 2023a; Liao et al., 2023; Shani et al., 2023b; Zheng et al., 2024; Peng et al., 2022); (ii) **definition-name alignment** tests (e.g., reverse-dictionary probes) that assess mapping between descriptions and names (Xu et al., 2024a); and (iii) **compositional binding and consistency** evaluations that test correct attribution of concept knowledge to instances and consistency across hierarchies (He et al., 2023; Sosa et al., 2024; Sahu et al., 2022). Complementing these behavioral probes, mechanistic interpretability aims to localize internal components responsible for concept-related behavior (Aljaafari et al., 2024; Wang et al., 2024b). Unlike these largely static evaluations, we focus on the **dynamics of concept learning**: we connect internal *concept circuits* to acquisition, forgetting, and cross-concept interactions, providing a mechanistic, time-resolved complement to existing probing approaches.

8 Conclusion

In this work, we study how large language models acquire, retain, and forget concepts during continual pretraining by linking output behavior with circuit-level interpretability. We introduce the FICO dataset, define **Concept Circuits** as internal representations of concepts, and use graph metrics to characterize **Circuit Topology**. Our experiments reveal consistent patterns in concept learning, forgetting, interference, and synergy, showing that circuit topology provides meaningful signals of concept dynamics. Our findings offer a step toward more concept-aware, interpretable, and robust continual pretraining of LLMs.

Limitations

Despite conducting extensive analyses of concept acquisition, forgetting, and cross-concept interactions in LLMs, our study has several limitations. (1) **Limited Model Scale**. Due to computational constraints, our experiments focus on GPT-2 Large (0.7B), and use LLaMA-3.2-1B and LLaMA-3.1-8B for ablation study. Extending our analysis to larger LLMs remains an important direction for future work. (2) **Limited Exploration of Actionable Training Strategies**. Our primary focus is analytical: characterizing how internal concept circuits relate to learning, forgetting, interference, and synergy. We instantiate **Circuit-aware Experience Replay** only as a proof-of-concept rather than a complete continual-learning method. Its evaluation is limited to GPT-2 Large and AntiLeak. Future work should validate circuit-aware replay across broader models, datasets, while also exploring other concept-aware training strategies such as adaptive training effort allocation, interference-aware data scheduling, or curriculum design.

Ethical considerations

We carefully follow the ACM Code of Ethics and have not found potential societal impacts or risks so far. To the best of our knowledge, this work has no notable harmful effects and uses, environmental impact, fairness considerations, privacy considerations, security considerations, or other potential risks. We will release the constructed FICO dataset to support reproducibility and future research. Since FICO is derived from ConceptNet, which is distributed under the Creative Commons Attribution-ShareAlike 4.0 license, our released dataset will follow the same license and include appropriate attribution to ConceptNet. Users of the dataset should comply with the corresponding attribution and share-alike terms. The dataset does not contain information that names or uniquely identifies individual people, and we manually check that it does not include offensive content. We have not identified notable societal, fairness, privacy, security, environmental, or other risks beyond those associated with standard use of commonsense knowledge resources. The dataset is in English and focuses on concept-centric commonsense knowledge. It covers broad everyday domains represented in ConceptNet, including objects, animals, locations, properties, affordances, part-whole relations, spatial relations, taxonomic relations, and synonym/antonym relations.

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A Knowledge Type Grouping and Knowledge Filtering

A.1 Knowledge Type Grouping

Our dataset is constructed from the ConceptNet knowledge graph, where each knowledge instance is represented as a triplet $\langle$subject, relation, object$\rangle$. Since ConceptNet contains a large number of fine-grained relation types, we group them into a smaller set of semantically coherent, high-level knowledge categories to facilitate concept-level analysis. Each relation is assigned to a category according to its primary semantic function.

As shown in Table 3, taxonomic and definitional relations (*IsA*, *DefinedAs*, *FormOf*, *InstanceOf*) are grouped into **Hyponym and Hypernym**. Relations expressing similarity or contrast (*Synonym*, *SimilarTo*, *Antonym*, *DistinctFrom*) are grouped into **Synonym and Antonym**. Part-whole and compositional relations (*PartOf*, *HasA*, *MadeOf*) are grouped into **Meronym and Holonym**. Relations describing properties, affordances, or functional roles (*HasProperty*, *UsedFor*, *CapableOf*, *ReceivesAction*) are grouped into **Property and Affordance**. Spatial relations (*AtLocation*, *LocatedNear*) are grouped into **Spatial Relation**.

A.2 Knowledge Filtering

We retain the above five concept-centric knowledge types in our experiments because they capture relatively stable semantic properties of real-world concepts. In contrast, we filter out the **Causality & Event**, **Lexical/Etymological**, and **Other** categories, since they primarily encode event-level associations, linguistic form, or noisy relations that are less suitable for modeling concept-level knowledge.

Based on these filtered relations, we construct fictional concepts from real concepts. Following previous work (Allen-Zhu and Li, 2023; Zucchet et al., 2025a; Ou et al., 2025; Zhu et al., 2025; Feng et al., 2024), for each concept-relation pair, we enforce a single canonical object and remove competing candidates. For example, if *dog CapableOf* is associated with multiple possible objects such as *run* and *guard your home*, we retain only one canonical object. This procedure yields a controlled and constrained concept-relation mapping, ensuring that fictional concepts are not merely renamed versions of existing real-world concepts.

B Dataset Statistics

As shown in Table 4, following dataset construction, we leverage 1,000 concepts for training and 500 concepts for testing. The training set contains 3,075 knowledge triples, which are instantiated into 92,250 training samples with different templates, comprising approximately 1.04M tokens in total. We train LLMs on our dataset for 10 epochs, leading to total training tokens of 10.4M, similar as previous continual-pretraining work (Ou et al., 2025). The test set consists of 1,586 knowledge triples and corresponding evaluation samples, totaling 13,530 tokens.

C Circuit Discovery

We instantiate concept circuits using EAP-IG Hanna et al. (2024), an efficient edge-level circuit discovery method that approximates activation patching with integrated gradients. We choose EAP-IG because our analysis requires extracting circuits for many concepts across multiple continual-pretraining checkpoints, making exhaustive causal intervention methods computationally prohibitive. Following prior circuit-discovery work, we view a circuit as a sparse computational subgraph whose standalone computation preserves the model behavior on a target task.

For each concept c_i, we construct a circuit-discovery set from its associated held-out knowledge triples $\{(c_i, r_{ij}, o_{ij})\}_j$. Each clean input is the prefix generated from the subject–relation pair (c_i, r_{ij}), with the gold object o_{ij} as the correct target. To construct a corrupted counterpart, we replace the gold object with an incorrect object sampled from another concept, yielding clean–corrupted pairs for attribution. EAP-IG assigns an importance score to each edge by integrating the gradient of the task loss along the path from corrupted activations to clean activations. The task metric is the logit difference between the correct and corrupted objects:

$$M(x) = \mathrm{logit}(o_{ij} \mid x) - \mathrm{logit}(\tilde{o}_{ij} \mid x),$$

where $\tilde{o}_{ij}$ denotes the corrupted object. Following EAP-IG, we use $L(x) = -M(x)$ as the attribution loss.

After scoring all edges, we rank them by the absolute EAP-IG score and select the smallest set of top-ranked edges whose resulting subgraph preserves at least τ of the full model's performance

High-level Knowledge Type	Relation Types
Hyponym and Hypernym	*IsA*, *DefinedAs*, *FormOf*, *InstanceOf*
Synonym and Antonym	*Synonym*, *SimilarTo*, *Antonym*, *DistinctFrom*
Meronym and Holonym	*PartOf*, *HasA*, *MadeOf*
Property and Affordance	*HasProperty*, *UsedFor*, *CapableOf*, *ReceivesAction*
Spatial Relation	*AtLocation*, *LocatedNear*
Excluded Relation Categories	
Causality & Event	*Causes*, *MotivatedByGoal*, *HasPrerequisite*, *HasSubevent*, *HasFirstSubevent*, *HasLastSubevent*, *CreatedBy*
Desire	*Desires*, *CausesDesire*
Lexical / Etymological	*DerivedFrom*, *EtymologicallyDerivedFrom*, *EtymologicallyRelatedTo*
Other	*RelatedTo*, *HasContext*, *ExternalURL*, *SymbolOf*

Table 3: Mapping from fine-grained relation types to high-level knowledge categories. We retain five concept-centric categories for experiments and exclude relation types that primarily encode events, lexical form, or noisy contextual associations.

Data	Train	Test
# Concepts	1,000	500
# Knowledges	3,075	1,586
# Samples	92,250	1,586
# Tokens	1,039,784	13,530

Table 4: Dataset statistics of FICO.

on the corresponding concept. Performance preservation is measured by the average logit-difference metric $M(x)$ over the concept's triples. In our main experiments, we set $\tau = 70\%$, following prior circuit-level interpretability work (Ou et al., 2025; Yao et al., 2025) as a practical faithfulness–sparsity trade-off. Lower thresholds may omit important edges, while higher thresholds yield denser circuits that are less suitable for topology analysis. To verify that our conclusions are not tied to this heuristic, we additionally evaluate $\tau = 80\%$ and observe similar circuit–behavior correlations, as shown in Table 1.

D Robustness Under Longer or Repeated Continual Training

To assess whether our findings persist beyond the two-stage continual pretraining setup in the main paper, we conduct an additional **repeated continual training** experiment with GPT-2 Large. Specifically, after obtaining checkpoint π_2 from Stage 2 (Forgetting Induction), as discussed in Section 3.2, we continue training the model for two more stages using the same protocol: we further train π_2 on FICO dataset, followed by BIO dataset, resulting in a **four-stage continual training setup**. Across this extended setting, we observe that the main trends remain qualitatively consistent: concept learning and forgetting continue to show substantial cross-concept variability, maintain non-trivial correlations with circuit graph metrics, and exhibit a positive association between stronger learning and greater forgetting. These results suggest that our findings are robust beyond a single two-stage continual training cycle.

We show results in Figure 6, Figure 7, Figure 8, and Figure 9.

E LLaMA Results

E.1 LLaMA-3.2-1B Results

E.1.1 RQ1 Results

Figure 10 shows the distribution of concept learning and forgetting degree for LLaMA-3.2-1B, again revealing substantial variability across concepts. Figure 11 and Figure 12 show that circuit topology

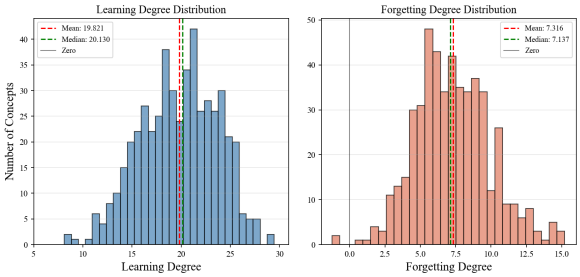


Figure 6: Distribution of learning and forgetting degree across concepts for LLaMA.

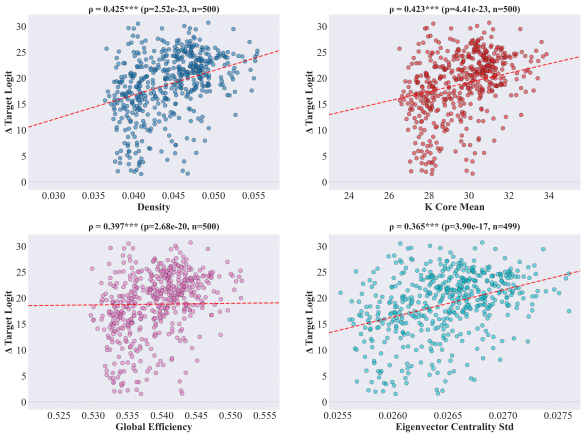


Figure 7: Correlation between learning degree and LLM circuit graph scores.

metrics remain correlated with concept learning and forgetting degree, respectively, reproducing the same qualitative trend observed for GPT-2 Large in the main paper. Figure 13 further shows the temporal evolution of circuit topology during continual forgetting, exhibiting a similar pattern of early increase followed by later decrease and stabilization. Finally, Figure 14 shows a positive correlation between learning degree and forgetting degree, confirming that concepts learned more strongly also tend to be forgotten more strongly under subsequent training.

E.1.2 RQ2 Results

Figure 15 shows concept interference under joint training for LLaMA-3.2-1B, where semantically related concepts induce stronger interference than weakly related concepts. Figure 16 further shows that transferability across knowledge types remains highly heterogeneous and directional, indicating that concept synergy and interference patterns generalize beyond GPT-2 Large.

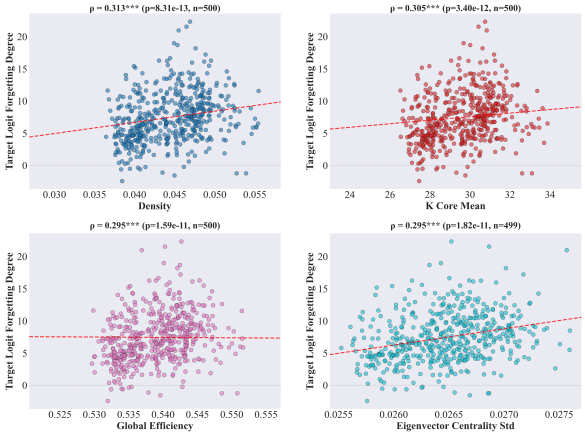


Figure 8: Correlation between forgetting degree and LLM circuit graph scores.

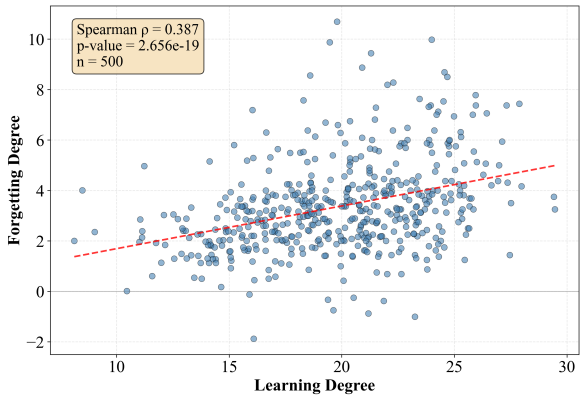


Figure 9: Spearman Correlations between learning and forgetting of concepts

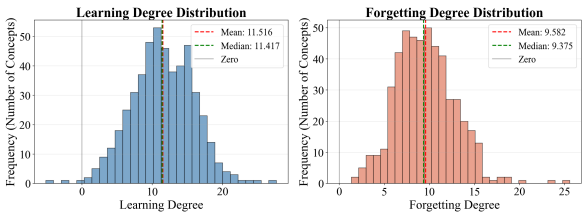


Figure 10: Distribution of learning and forgetting degree across concepts for LLaMA.

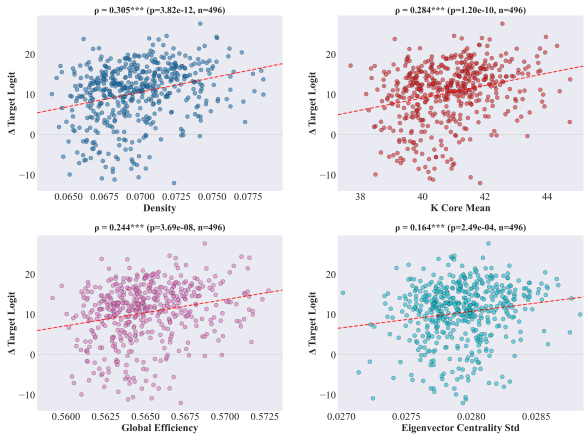


Figure 11: Correlation between learning degree and LLM circuit graph scores.

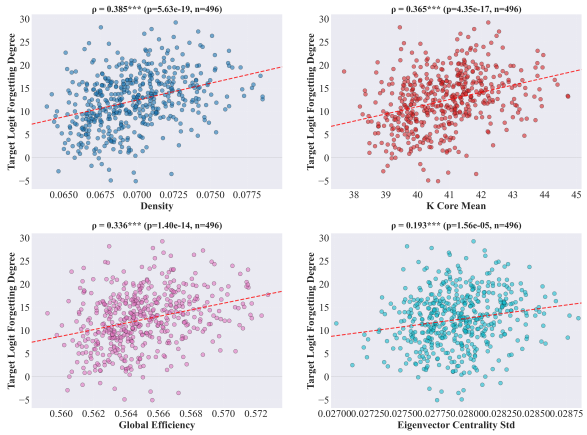


Figure 12: Correlation between forgetting degree and LLM circuit graph scores.

E.2 LLaMA-3.1-8B Results

E.2.1 RQ1 Results

Figure 17 shows the distribution of concept learning and forgetting degree for LLaMA-3.1-8B, again indicating substantial cross-concept variability. Figure 18 and Figure 19 show that circuit topology metrics remain meaningfully associated with concept learning and forgetting degree, respectively, reproducing the same overall trend observed in smaller models and in the GPT-2 Large results.

E.2.2 RQ2 Results

Figure 20 shows that semantically related concepts continue to induce stronger interference during joint training in LLaMA-3.1-8B. Figure 21 further shows that transferability across knowledge types remains asymmetric and uneven, suggesting that concept-level synergy and interference patterns are robust across model scale.

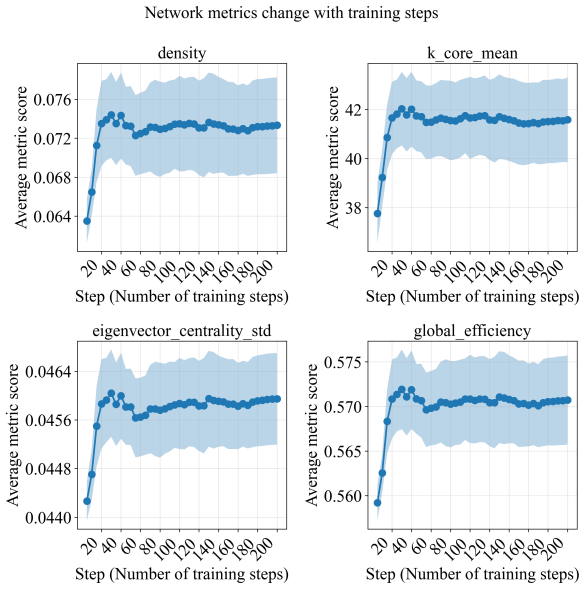


Figure 13: Graph Metric over training steps

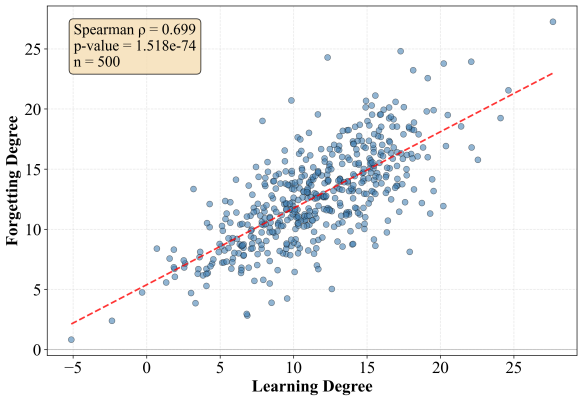


Figure 14: Correlations between learning and forgetting of concepts

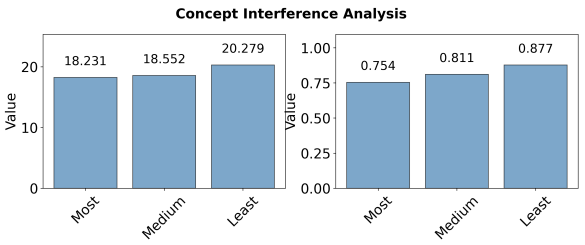


Figure 15: Concept-level interference under joint training.

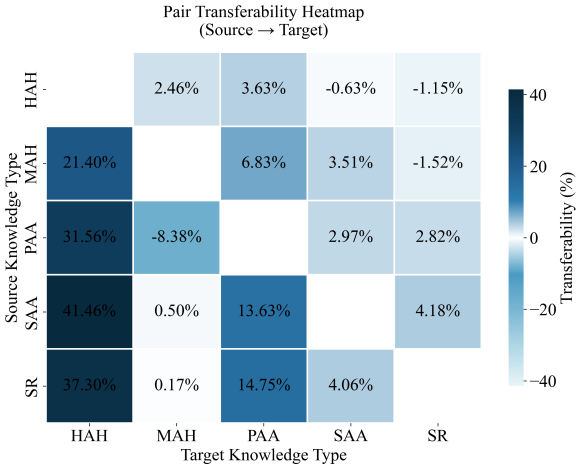


Figure 16: Pairwise dependency between knowledge types

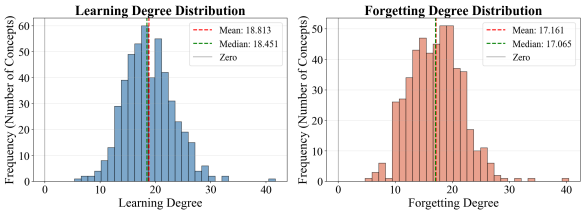


Figure 17: Distribution of learning and forgetting degree across concepts for LLaMA.

F Experiment Details

We conduct experiments on 4×40GB A40 GPUs. We use learning rate as 5×10^{-5} and batch size as 128. In our experiments for RQ1, the training data is automatically reshuffled before each training epoch. Since each experiment runs for multiple epochs, this ensures that our analysis is independent of the order in which the concepts are presented to the model, addressing any sensitivity to concept presentation order.

G GPT-2 Large Results for Log Probability

To verify that our findings are not specific to logit-based evaluation, we repeat the RQ1 analyses using **log probability** (Heimersheim and Nanda, 2024; Hanna et al., 2024) as the behavioral metric.

Figure 22 shows the distribution of concept learning and forgetting degree under log probability. Similar to the logit-based results in the main paper, we observe substantial variability across concepts, indicating that both acquisition and forgetting remain highly heterogeneous.

Figure 23a and Figure 23b show that circuit topology metrics remain positively correlated with

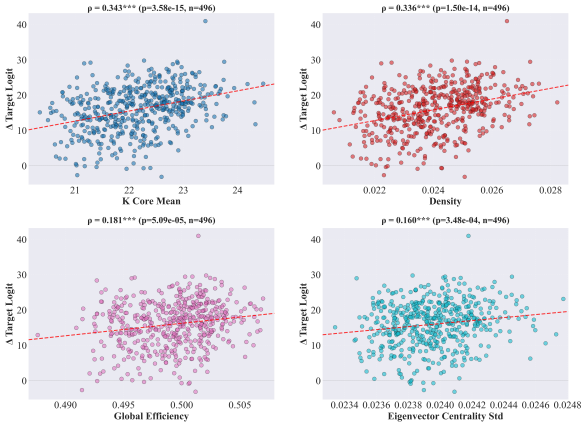


Figure 18: Correlation between learning degree and LLM circuit graph scores.

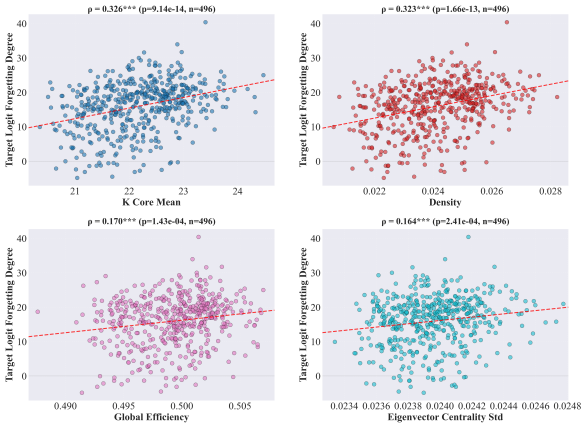


Figure 19: Correlation between forgetting degree and LLM circuit graph scores.

concept learning and forgetting degree, respectively. In particular, metrics such as eigenvector centrality and k-core continue to exhibit consistent associations, suggesting that the relationship between circuit structure and learning/forgetting dynamics is robust to the choice of behavioral metric.

Finally, Figure 24 shows that concepts with larger learning gains also tend to exhibit greater forgetting under subsequent training, confirming

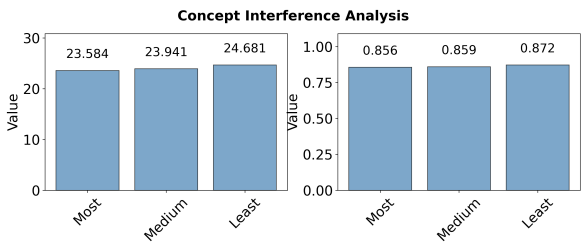


Figure 20: Concept-level interference under joint training.

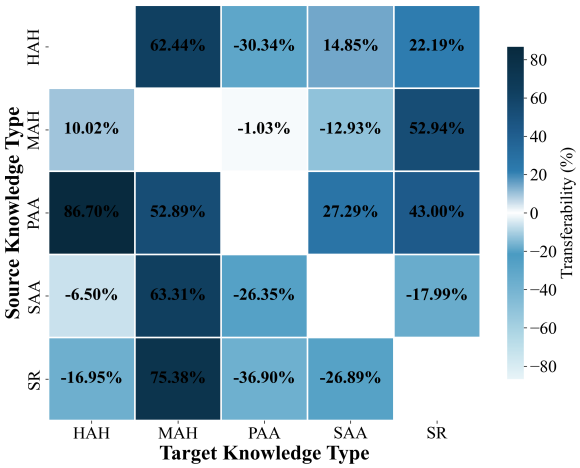


Figure 21: Pairwise dependency between knowledge types

Type	Metric	Learning	Learning p	Forgetting	Forgetting p
Circuit Topology (70%)	Density	0.3201	2.38e-13	0.3815	9.75e-19
Circuit Topology (70%)	k-core	0.3176	3.69e-13	0.3783	2.00e-18
Circuit Topology (70%)	Eigenvector Centrality	0.3173	3.91e-13	0.3720	7.95e-18
Circuit Topology (70%)	Global Efficiency	0.2934	2.30e-11	0.3653	3.37e-17
Circuit Topology (80%)	Density	0.3198	2.38e-13	0.3746	4.19e-18
Circuit Topology (80%)	k-core	0.2990	8.71e-12	0.3553	2.55e-16
Circuit Topology (80%)	Eigenvector Centrality	0.2745	4.45e-10	0.3306	3.42e-14
Circuit Topology (80%)	Global Efficiency	0.3153	5.25e-13	0.3703	1.07e-17
Activation	Mean EAP-IG	0.1907	2.12e-13	0.2648	3.74e-17
Activation	STD EAP-IG	0.1802	1.37e-12	0.2547	1.19e-15
Activation	Max EAP-IG	0.1185	1.86e-08	0.1483	1.58e-09
Random Topology	Density	-0.0205	0.65	-0.0252	0.72
Random Topology	k-core	-0.0550	0.22	-0.0433	0.16
Random Topology	Eigenvector Centrality	0.0734	0.10	0.0735	0.12
Random Topology	Global Efficiency	-0.0497	0.27	-0.0764	0.16

Table 5: Correlation between learning/forgetting degree and circuit metrics. p is the corresponding p-value testing the null hypothesis of zero correlation.

the learn-forget trade-off observed in the main paper. Overall, these results demonstrate that our key findings remain consistent when measured using log probability.

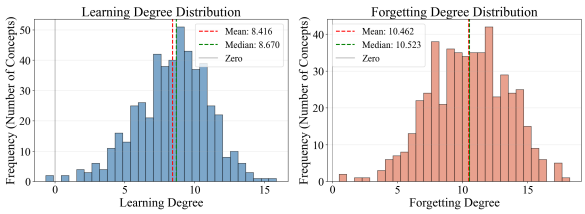


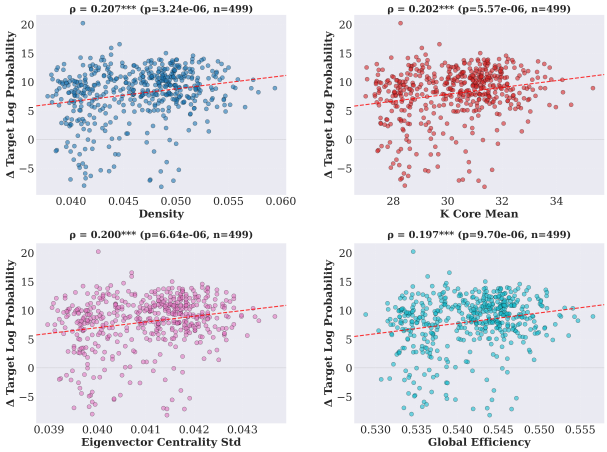
Figure 22: Distribution of learning and forgetting degree across concepts.

H Complete Results

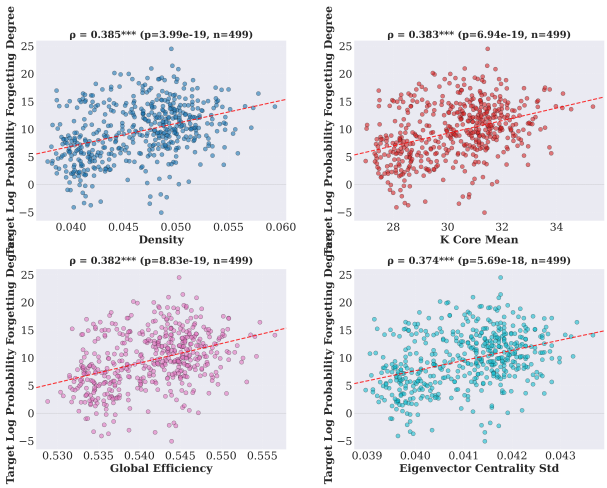
Table 5 shows complete correlation results with p-values, where p is the corresponding p-value testing the null hypothesis of zero correlation.

I Baselines Against Circuit Topology

To further evaluate the indicative power of circuit graph topology, we compare our method



(a) Correlation between learning degree and LLM circuit pattern.



(b) Correlation between forgetting degree and LLM circuit pattern.

Figure 23: Correlation between learning dynamics and LLM circuit pattern.

against two baselines: (1) **Random graph topology**, where we randomly sample the same number of edges to form a graph and compute its topology metrics; and (2) **edge-level EAP-IG statistics** (Hanna et al., 2024), including the mean, maximum, and standard deviation of EAP-IG scores. EAP-IG is a first-order approximation of each edge's indirect effect and serves as a proxy for both gradient norm and activation magnitude at the circuit level.

As shown in Table 1, circuit graph metrics exhibit consistently stronger correlations with concept learning degree (Spearman $\rho \approx 0.29$–0.32) than either edge-level EAP-IG statistics ($\rho \approx 0.12$–0.19) or random graph metrics (near zero). This suggests that the observed associations are not explained solely by edge importance magnitudes or by generic graph statistics independent of circuit structure.

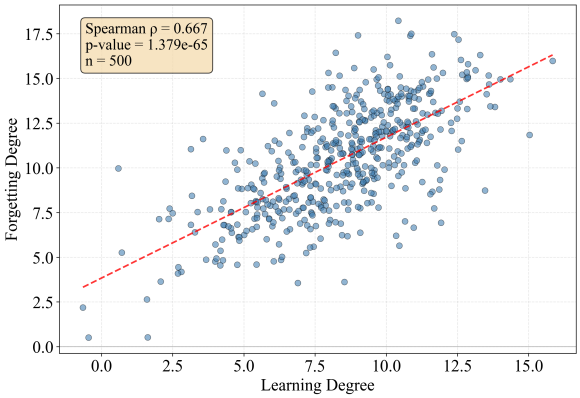


Figure 24: Spearman Correlations between learning and forgetting of concepts

In the random graph baseline, topology metrics are nearly constant by construction (e.g., k-core mean and standard deviation: 38.69 and 2.42), reflecting the relatively uniform structure induced by random edge placement. In contrast, Concept Circuits exhibit substantially higher variability in these metrics (e.g., k-core mean and standard deviation: 30.27 and 16.78), indicating more heterogeneous and non-random graph organization. Compared with edge-level EAP-IG summary statistics, graph metrics may better capture global structural patterns, whereas aggregating edge-level scores into simple statistics can obscure higher-order information.

J Correlation between Learning and Forgetting

Finding 6. *Concepts with larger learning degree tend to exhibit larger forgetting degree during subsequent training, suggesting a trade-off between acquisition strength and subsequent stability.*

Correlation between Concept Learning and Forgetting. Figure 25 shows a positive correlation between learning degree and forgetting degree under continual training, indicating that concepts learned more strongly also tend to be forgotten more strongly. Together with the circuit correlation results in Section 2, this pattern is consistent with a trade-off between acquisition strength and subsequent stability.

To shed light on the nature of the learning–forgetting relationship, we further analyze the 20% of concepts with the highest learning degrees. Figure 26 and Figure 27 show the distributions of their final learning logit and forgetting degree. Two observations are notable. First, while these high-learning-degree concepts do tend to reach elevated final logit values (mean = 21.27), their final logits span a wide range (18–24), indicating that high learning degree does not simply equate to a uniformly high logit ceiling. Second, and more importantly, these concepts also exhibit substantially higher forgetting degrees (mean = 20.95) but span a wide range (14–26). Together, these distributions suggest that the correlation between learning degree and forgetting degree reflects a genuine property of how concepts are encoded during training.

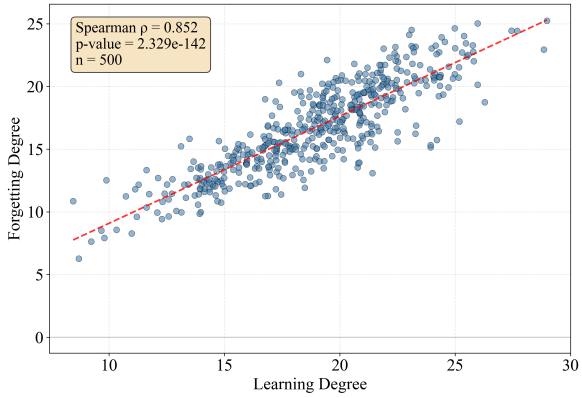


Figure 25: Spearman Correlations between learning and forgetting of concepts

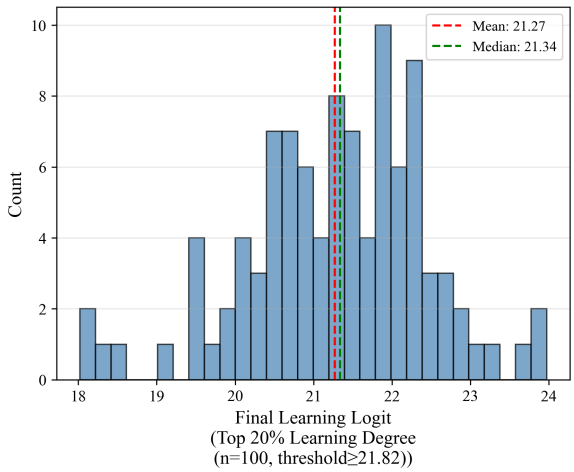


Figure 26: Distribution of final learning logit.

K Metrics for AntiLeak Experiments

We report three evaluation metrics on the AntiLeak test set: (1) **Logit**, the average logit assigned to the target object; (2) **Accuracy**, the proportion of examples for which the top-1 predicted token matches

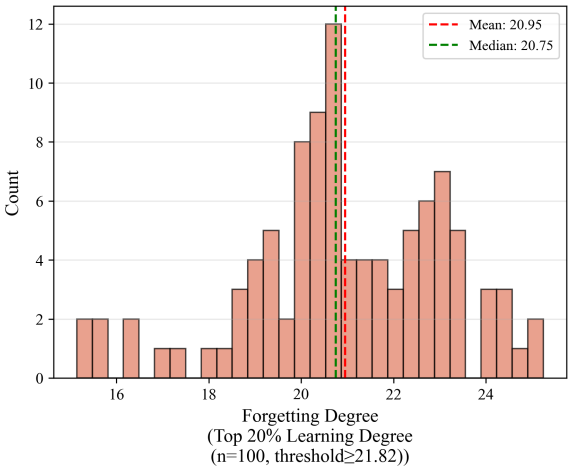


Figure 27: Distribution of forgetting degree

the ground-truth object; and (3) **Hit@5**, the proportion of examples for which the ground-truth object appears in the top-5 predicted tokens.

L Use Of AI Assistants

We used AI assistants to help with writing and editing the manuscript. The assistants were used to refine wording, improve clarity, and enhance overall presentation.