REFUSAL-ORTHOGONAL GATED EDITING FOR SAFER LOCALIZED LLM ADAPTATION

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ABSTRACT

As LLMs are increasingly adapted post-training under tight data and compute budgets, standard parameter-efficient fine-tuning (PEFT) can inadvertently degrade safety-critical refusal behavior. Prior work suggests refusal is concentrated in a small number of directions in representation space, yet most safety interventions either operate in weight space or rely on post-hoc steering, leaving unclear how to adapt models while avoiding interference with refusal-sensitive representations. We propose **Safety-Aware Gated Editing (SAGE)**, a two-stage method that (i) estimates a refusal subspace and (ii) learns sparse, gated additive and multiplicative activation edits for task performance while constraining all learned edits to the refusal-orthogonal subspace. The base model remains frozen and the learned edits are applied at inference time. Across low-resource adaptation on ARC and BoolQ with Llama-3.1-8B-Instruct, SAGE maintains competitive task accuracy while consistently reducing attack success rates on StrongREJECT, AdvBench, and JailbreakBench relative to standard PEFT and unconstrained activation editing. These results suggest that lightweight geometry-based constraints can mitigate safety degradation without additional safety supervision or full-model fine-tuning.

1 INTRODUCTION

Parameter-efficient fine-tuning (PEFT) methods such as LoRA (Hu et al., 2022) enable large language models (LLMs) to adapt to downstream tasks with limited data and compute. However, even benign task adaptation can unintentionally interfere with safety-critical refusal behavior, increasing susceptibility to jailbreaks or harmful outputs (Qi et al., 2023; Yang et al., 2023; Soligo et al., 2025; Pandey et al., 2025). Existing approaches to safe fine-tuning largely operate in weight space via safety losses (Zhang et al., 2024; Zhao et al., 2025a), safety-shaped updates (Lu et al., 2025; Wu et al., 2025), or post hoc layer merging (Djuhera et al., 2025). While effective, these methods do not directly address how task adaptation interacts with the internal representations that mediate refusal.

Recent work suggests that refusal behavior is mediated by a small number of directions spanning a low-dimensional subspace within the model's representation space (Arditi et al., 2024; Wollschläger et al., 2025; Pan et al., 2025). This motivates a representation-level perspective: safety degradation under fine-tuning may arise when task-specific updates interfere with refusal-sensitive directions. Rather than modifying the task objective or introducing additional safety supervision, we ask whether downstream adaptation can be constrained to avoid modifying these representations altogether.

Despite growing evidence for refusal geometry, existing safety interventions primarily use these directions for post hoc steering or ablation at inference time (Siu et al., 2025; Vu & Nguyen, 2025). At the same time, recent activation- and representation-based fine-tuning methods enable lightweight task adaptation but do not account for refusal-sensitive directions during optimization (Lai et al., 2025; Yin et al., 2024; Zou et al., 2023a; Wu et al., 2024b). This leaves a critical gap: current methods either manipulate refusal-related representations without optimizing a downstream task, or optimize a task without explicitly constraining updates away from refusal-sensitive directions.

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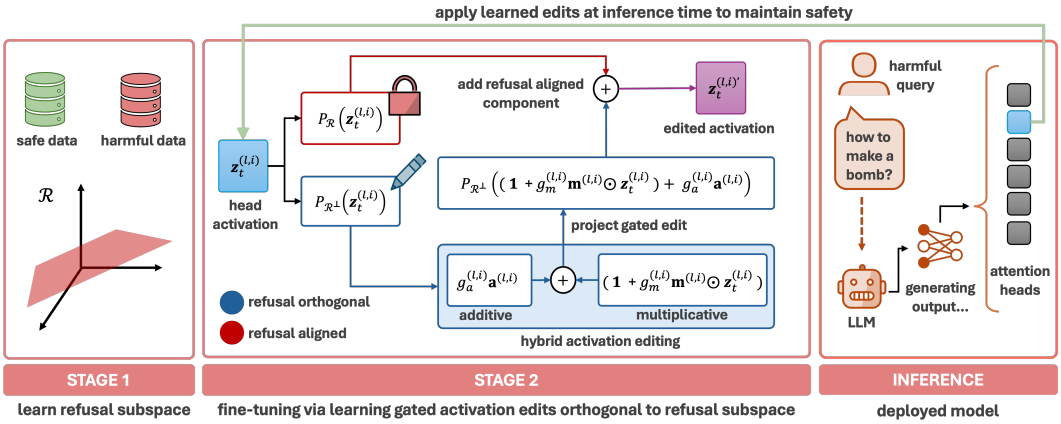

Figure 1: Overview of SAGE. **Stage 1:** Estimate refusal subspace $\mathcal{R}$. **Stage 2:** Adapt for the downstream task using activation edits, while constraining edits to lie in $\mathcal{R}^{\perp}$.

We propose **Safety-Aware Gated Editing (SAGE)**, a two-stage method that addresses this gap by treating refusal geometry as a protected region of representation space that bounds where downstream adaptation is allowed to occur. The base model parameters remain frozen, refusal geometry is estimated once offline, and task-specific updates are learned as sparse, gated activation edits that are executed at inference time.

At each layer l, we index editable components (e.g., attention heads) by i and denote their activation at token position t as $\mathbf{z}_t^{(l,i)} \in \mathbb{R}^d$. Following prior evidence (Arditi et al., 2024; Wollschläger et al., 2025), we model refusal as a low-dimensional linear subspace $\mathcal{R} \subset \mathbb{R}^d$. This induces the decomposition

$$\mathbf{z}_t^{(l,i)} = P_{\mathcal{R}}\, \mathbf{z}_t^{(l,i)} + P_{\mathcal{R}^{\perp}}\, \mathbf{z}_t^{(l,i)}, \tag{1}$$

where $P_{\mathcal{R}}$ and $P_{\mathcal{R}^{\perp}}$ denote projection onto the refusal subspace and its orthogonal complement, respectively. Intuitively, $P_{\mathcal{R}}\, \mathbf{z}_t^{(l,i)}$ captures refusal-relevant information that should remain stable, while $P_{\mathcal{R}^{\perp}}\, \mathbf{z}_t^{(l,i)}$ represents degrees of freedom available for downstream adaptation.

To parameterize controlled adaptation, we adopt gated additive and multiplicative activation edits following JoLA (Lai et al., 2025). For each editable component (l, i), we associate additive and multiplicative edit vectors $\mathbf{a}^{(l,i)}, \mathbf{m}^{(l,i)} \in \mathbb{R}^d$, together with scalar gates $g_a^{(l,i)}, g_m^{(l,i)} \in [0, 1]$ that encourage sparse, component-wise editing. Gates are learned jointly with the edit parameters and replaced by their expected values at inference time.

In Stage 1, SAGE estimates a basis for the refusal subspace $\mathcal{R}$. In Stage 2, we perform gated activation editing for task adaptation while constraining all learned edits to the refusal-orthogonal subspace $\mathcal{R}^{\perp}$. Edit parameters are projected onto $\mathcal{R}^{\perp}$, and post-edit activations are re-projected to reduce interference with refusal-aligned representations as shown in Figure 1.

2 Related Work

Activation editing for parameter-efficient adaptation. Recent work explores parameter-efficient adaptation via activation- or representation-space edits rather than full weight updates. JoLA (Lai et al., 2025) introduces sparse, gated additive and multiplicative activation edits optimized under a task loss with an L_0 sparsity penalty. Related approaches include bias-only or scaling-based edits (e.g., BitFit (Ben Zaken et al., 2022), RED (Wu et al., 2024a)), low-rank or head-selective interventions (LoFIT (Yin et al., 2024)), and subspace-restricted activation editing (ReFT (Wu et al., 2024b)). These methods focus on efficiency and localization, but do not explicitly address safety preservation during downstream adaptation. SAGE builds on gated activation editing and introduces a geometric constraint to control how adaptation interacts with safety-relevant representations.

Refusal geometry and representation-level safety. Early work identifies single refusal directions (Arditi et al., 2024), while subsequent work uncovers higher-dimensional refusal subspaces and proposes Refusal Direction Optimization (RDO) to learn layer-wise refusal vectors (Wollschläger et al., 2025; Pan et al., 2025). Most prior methods leverage these directions for post hoc steering, ablation, or inference-time intervention to control refusal behavior (Li et al., 2025; Oozeer et al., 2025; Vu & Nguyen, 2025; Siu et al., 2025). In contrast, SAGE uses refusal geometry as a constraint during downstream adaptation, optimizing task-specific updates while restricting learned activation edits to avoid interference with refusal-aligned directions.

3 BACKGROUND

Activations and refusal subspace. Let $\mathbf{z}_t^{(l,i)} \in \mathbb{R}^d$ denote the activation of component (l, i) at token position t. Following prior work, we model refusal behavior as being mediated by a low-dimensional subspace $\mathcal{R} \subset \mathbb{R}^d$. This induces the orthogonal decomposition in Eq. 1, where $P_{\mathcal{R}}$ and $P_{\mathcal{R}^\perp}$ denote projection onto the refusal subspace and its orthogonal complement.

Gated activation editing. We adopt gated additive and multiplicative activation edits following JoLA (Lai et al., 2025). For each editable component (l, i), we associate additive and multiplicative edit vectors $\mathbf{a}^{(l,i)}, \mathbf{m}^{(l,i)} \in \mathbb{R}^d$ with scalar gates $g_a^{(l,i)}, g_m^{(l,i)} \in [0, 1]$. The gated hybrid edit is

$$\mathbf{z}_t^{(l,i)\prime} = \left(\mathbf{1} + g_m^{(l,i)}\mathbf{m}^{(l,i)}\right) \odot \mathbf{z}_t^{(l,i)} + g_a^{(l,i)}\mathbf{a}^{(l,i)}, \tag{2}$$

where the gates encourage sparse, localized adaptation. Full details are provided in Appendix B.

Estimating refusal geometry. Refusal directions are estimated using prior methods such as difference-in-means or Refusal Direction Optimization (RDO) (Wollschläger et al., 2025), which identify directions along which additive or ablative interventions reliably toggle refusal behavior. We estimate a low-dimensional refusal subspace spanned by a small set of such directions. Full details of refusal geometry estimation are provided in Appendix B.4.

4 METHOD

We propose SAGE , a two-stage activation-editing procedure for safe downstream adaptation. First, we estimate a refusal subspace on a frozen base model. Second, we fine-tune sparse, gated activation edits for a downstream task while constraining all edits to lie in the orthogonal complement of this subspace. See Appendices B and C for details.

Stage 1: Estimating the refusal subspace. Let f_θ be a frozen base model. Following prior work on refusal geometry (Wollschläger et al., 2025), we estimate a low-dimensional refusal basis $B \in \mathbb{R}^{d\times N}$ using Refusal Cone Optimization (RCO). We define the induced refusal subspace and associated projectors as

$$\mathcal{R} := \operatorname{span}(B), \qquad P_{\mathcal{R}} := BB^\top, \qquad P_{\mathcal{R}^\perp} := I - BB^\top. \tag{3}$$

Stage 2: Refusal-orthogonal gated editing. Let $\mathbf{z}_t^{(l,i)} \in \mathbb{R}^d$ denote the activation of component (l, i) at token position t. With the refusal subspace fixed, we decompose each activation as described in Eq. 1. We apply gated additive and multiplicative edits only to the refusal-orthogonal component. Before each forward pass, unconstrained edit parameters are projected into $\mathcal{R}^\perp$, and the post-edit activation is re-projected to prevent reintroducing refusal-aligned components. The resulting edited activation is

$$\mathbf{z}_t^{(l,i)\prime} = P_{\mathcal{R}}\mathbf{z}_t^{(l,i)} + P_{\mathcal{R}^\perp}\Big((\mathbf{1} + g_m^{(l,i)}\mathbf{m}^{(l,i)}) \odot P_{\mathcal{R}^\perp}\mathbf{z}_t^{(l,i)} + g_a^{(l,i)}\mathbf{a}^{(l,i)}\Big). \tag{4}$$

Thus, refusal-related activations are preserved, while downstream adaptation occurs only in orthogonal directions.

Table 1: **Main Results**. Task accuracy (↑), MMLU-Pro retention (↑), and and safety robustness (↓) reported as SR/Adv/JB (StrongREJECT score; AdvBench and JailbreakBench ASR).

ARC-c adaptation + retention

Method	ARC-c↑	ARC-e↑	MMLU-Pro↑	SR / Adv / JB↓
zero-shot	0.765	0.883	0.377	0.220 / 0.231 / 0.320
LoRA	0.802	0.914	0.341	0.207 / 0.229 / 0.330
JoLA	0.801	0.925	0.302	0.156 / 0.225 / 0.251
SAGE-1D (Ours)	0.780	0.921	0.343	0.151 / 0.218 / 0.245
SAGE-5D (Ours)	0.780	0.920	0.345	0.152 / 0.219 / 0.251

BoolQ adaptation

Method	BoolQ↑	SR / Adv / JB↓
zero-shot	0.800	0.220 / 0.231 / 0.320
LoRA	0.830	0.230 / 0.246 / 0.314
JoLA	0.995	0.224 / 0.204 / 0.286
SAGE-1D (Ours)	0.910	0.212 / 0.216 / 0.269
SAGE-5D (Ours)	0.915	0.224 / 0.204 / 0.287

5 Experimental Setup

We evaluate on Llama-3.1-8B-Instruct; each run uses a single NVIDIA A100-SXM4-80GB GPU. We estimate the refusal subspace using the RCO procedure of Wollschläger et al. (2025). We adapt the model using 200 training examples from ARC-Challenge (ARC-c) (Clark et al., 2018) or BoolQ Clark et al. (2019), and compare zero-shot, LoRA, JoLA (Lai et al., 2025), and SAGE (ours); full training details in Appendix D.We report task accuracy on ARC-c/ARC-e and BoolQ, and evaluate capability retention on MMLU-Pro (Wang et al., 2024). We include capability retention to ensure safety is not achieved by degrading general model capabilities. Safety robustness is evaluated on StrongREJECT (Souly et al., 2024), AdvBench (Zou et al., 2023b), and JailbreakBench (Chao et al., 2024). Safety is measured using the StrongREJECT score and attack success rates (ASR) on AdvBench and JailbreakBench, with the StrongREJECT evaluator used across benchmarks.

6 Results

On ARC-c, LoRA and JoLA achieve the largest in-domain gains, while SAGE-1D incurs a modest reduction relative to JoLA but remains competitive on ARC-e and retains higher MMLU-Pro performance than JoLA. On BoolQ, SAGE-1D improves accuracy substantially over zero-shot and outperforms LoRA. Across both tasks, safety metrics reveal a consistent trade-off between task adaptation and robustness (Figure 2).

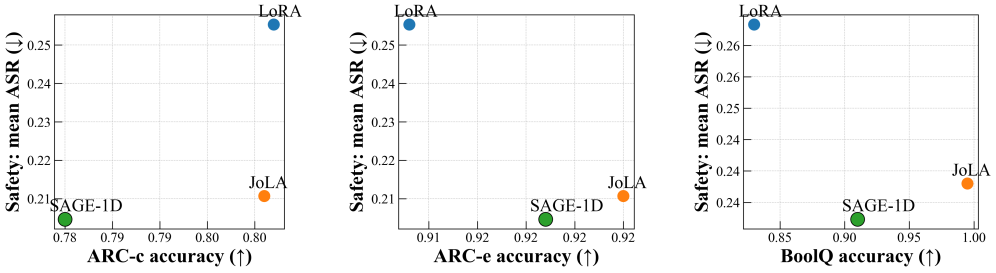

Figure 2: Safety–performance trade-off. Safety is the mean attack success rate (ASR) across StrongREJECT, AdvBench, and JailbreakBench, shown for ARC-c, ARC-e, and BoolQ adaptation.

SAGE-1D attains lower attack success rates than JoLA on all three safety benchmarks for both ARC and BoolQ. While some fine-tuned models improve on individual safety benchmarks relative to zero-shot, they may remain vulnerable to jailbreak and refusal-bypass attacks. Consistent with prior observations (Wu et al., 2025; Shah et al., 2025), we find that standard LoRA slightly improves some safety scores but remains susceptible to JailbreakBench attacks.

7 Discussion and Conclusion

SAGE enables safe downstream adaptation as enforcing geometric constraints in representation space during activation editing. In our experiments, SAGE maintains competitive task performance while consistently reducing attack success rates relative to standard PEFT and unconstrained activation editing. We also find that expanding the protected refusal subspace from 1D to 5D does

not yield additional safety gains, consistent with the possibility that higher-dimensional estimates introduce weaker or noisier directions (Wollschläger et al., 2025). Future work should test broader tasks and model scales/families and study how improved refusal-subspace estimation affects this trade-off.

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A Related Work

Scope and taxonomy. Methods for safe fine-tuning differ along three key axes: (i) *where* the intervention is applied (weight/parameter space vs. activation/representation space), (ii) *when* safety information is incorporated (e.g., offline parameter selection, during fine-tuning, or post-hoc), and (iii) *how* safety signals influence adaptation (e.g., explicit loss term, parameter selection, post-hoc layer merging, or geometric constraints). Methods also vary in the degree of localization or sparsity of the intervention, ranging from full-model updates to sparse, component-level modifications, as well as the mechanism used to induce sparsity (e.g., low-rank parameterization, gating, or layer/head selection). Our method, SAGE, is a training-time, activation-space approach that applies sparse, localized edits gated by Hard–Concrete distributions and constrains these edits using a refusal-orthogonality constraint.

Activation editing for parameter-efficient fine-tuning. Lai et al. (2025) propose joint localization and activation editing (JoLA): lightweight, training-time edits applied to selected components (e.g., attention heads) with Hard–Concrete gating to induce sparsity, optimized under task cross-entropy plus an expected L_0 penalty. Adjacent training-time activation-editing approaches include BitFit (bias-only tuning) (Ben Zaken et al., 2022), RED (layer-wise scaling and bias vectors) (Wu et al., 2024a), ReFT direct hidden-state intervention that edits only a learned low-rank subspace via an orthonormal projector (Wu et al., 2024b), and LoFIT (head selection + bias tuning) (Yin et al., 2024). Inference-time activation modification has also been explored, e.g., RePE (Zou et al., 2023a). Relative to these works, our proposed method retains JoLA's low-resource, gated edit parameterization but introduces a geometric safety constraint.

Refusal geometry in the representation space. Several works suggest that refusal behavior can be captured by low-dimensional directions in representation space. Arditi et al. (2024) argue for a single refusal direction, while Wollschläger et al. (2025) discover refusal cones and introduce Refusal Direction Optimization (RDO) to learn layer-specific refusal vectors using CE losses for additive/ablation interventions and a KL retain term. Pan et al. (2025) further study multi-dimensional (orthogonal) safety subspaces. Our method directly builds on this line of work: we estimate layer-wise refusal directions using RDO and use the best one to constrain downstream activation edits, rather than steering or patching representations directly.

Activation-space approaches to safety interventions. A growing line of work manipulates internal activations to improve safety, either by patching specific neurons/circuits or by steering along control directions. Neuron-level safety realignment and identification of "safety neurons" have been studied in Yi et al. (2025); Zhao et al. (2025b); Chen et al. (2024). Activation steering methods control attributes by adding learned vectors during inference or training, including multi-attribute steering and dynamic (context-dependent) debiasing (Nguyen et al., 2025; Li et al., 2025; Oozeer et al., 2025). Our method differs in that it learns a refusal geometry (via RDO) and then trains sparse edits that are orthogonal to that geometry at the edit site, aiming to preserve refusal while improving benign utility.

Weight-space approaches to safe fine-tuning. These methods preserve safety by modifying or selecting parameter updates, through explicit safety losses during training, gradient-based update steering using safety probes, or post-hoc layer merging.

The first class of methods incorporate safety information directly in the fine-tuning objective by adding explicit safety terms or preference-based supervision. For example, SaRFT introduces a safety regularizer (e.g., KL-based) during role-play fine-tuning to reduce unsafe behaviors (Zhao et al., 2025a), and joint safety–helpfulness preference optimization optimizes a preference objective that explicitly trades off helpfulness and safety (Zhang et al., 2024). Dynamic Safety Shaping (DSS) further refines this approach by using token-level guardrail feedback to modulate the loss within a single response, reinforcing safe segments while suppressing unsafe ones during training (Peng et al., 2025). Other methods also incorporate safety/alignment signals during training to improve robustness to harmful fine-tuning, such as LISA (Huang et al., 2024) and refusal-feature adversarial training (Yu et al., 2024).

The second class uses safety supervision to determine which updates are safe to apply, without augmenting the task loss with an explicit safety penalty. SafeDelta uses a safety dataset to estimate parameter sensitivity to a safety objective via a second-order (Hessian-informed) approximation, then selects and compensates parameter deltas to bound safety loss change under downstream fine-tuning (Lu et al., 2025). In contrast, safety-aware probing methods estimate harmful *gradient* directions using safety supervision and use these probes to steer or constrain the fine-tuning trajectory in a bi-level or constrained optimization procedure (Wu et al., 2025). SafeLoRA provides a related weight-space constraint by projecting LoRA updates toward a safety-aligned subspace computed from aligned and unaligned model weights (Hsu et al., 2024).

The third class preserves safety via post-hoc operations on weights. SafeMerge fine-tunes a task model and a safety-aligned model separately, then selectively merges layers guided by projection-based similarity to retain safety-critical parameters (Djuhera et al., 2025). In this setting, safety information is used after training to decide which layers or parameters to keep from the aligned reference.

A related class acts before optimization by curating or filtering the fine-tuning data to prevent unsafe gradients from being induced in the first place. Data curation/filtering approaches (e.g., SAFT, CTRL) aim to reduce harmful gradients by selecting safe training instances or removing unsafe segments from the fine-tuning distribution (Choi et al., 2024; Liu et al., 2024).These methods incorporate safety information at the data level rather than through update constraints or modified objectives.

B BACKGROUND

B.1 NOTATION

Let $\mathbf{s} = (s_1, \ldots, s_T)$ be an input token sequence of length T over a vocabulary. We consider a decoder-only Transformer with L layers and hidden size d. For each layer $l \in \{1, \ldots, L\}$ and token position $t \in \{1, \ldots, T\}$ we denote by $\mathbf{x}_t^{(l)} \in \mathbb{R}^d$ the residual stream entering layer l at position t. Each layer consists of an attention block followed by an MLP block, both with residual connections:

$$\tilde{\mathbf{x}}_t^{(l)} = \mathbf{x}_t^{(l)} + \mathrm{ATTN}^{(l)}\left(\mathbf{x}_{1:t}^{(l)}\right), \tag{5}$$

$$\mathbf{x}_t^{(l+1)} = \tilde{\mathbf{x}}_t^{(l)} + \mathrm{MLP}^{(l)}\left(\tilde{\mathbf{x}}_t^{(l)}\right). \tag{6}$$

As in prior work, we omit layer normalization and other architectural details.

At each layer l we index individual components (e.g. attention heads, MLP blocks) by i. We write the activation of component (l, i) at position t as

$$\mathbf{z}_t^{(l,i)} \in \mathbb{R}^d.$$

These $\mathbf{z}_t^{(l,i)}$ are the intermediate activations we will edit.

We write $\|\cdot\|_2$ for the Euclidean norm. A refusal direction is a vector $\mathbf{r} \in \mathbb{R}^d$ with unit vector $\hat{\mathbf{r}} = \mathbf{r}/\|\mathbf{r}\|_2$. This gives the projection operators $P_{\hat{\mathbf{r}}} = \hat{\mathbf{r}}\hat{\mathbf{r}}^\top$ and $P_{\hat{\mathbf{r}}^\perp} = I - \hat{\mathbf{r}}\hat{\mathbf{r}}^\top$. Any component activation then decomposes as

$$\mathbf{z}_t^{(l,i)} = P_{\hat{\mathbf{r}}}\, \mathbf{z}_t^{(l,i)} + P_{\hat{\mathbf{r}}^\perp}\, \mathbf{z}_t^{(l,i)}.$$

We write f_θ for the base model with parameters θ. When we introduce additional lightweight edit parameters we write $f_{\theta,\phi}$, keeping ϕ (edit parameters) separate from the refusal geometry (e.g. $\hat{\mathbf{r}}$ or a basis B).

B.2 INTERVENTION TYPES

Activation intervention types. Activation editing modifies intermediate activations $\mathbf{z}_t^{(l,i)} \in \mathbb{R}^d$ for component (l, i) at token position t. Following Lai et al. (2025), we consider three classical edit types. Let $\mathbf{a}^{(l,i)}, \mathbf{m}^{(l,i)} \in \mathbb{R}^d$ denote additive and multiplicative edit parameters, and let $\odot$ denote element-wise multiplication.

- **Additive:** add a bias vector
$$\mathbf{z}_t^{(l,i)\prime} = \mathbf{z}_t^{(l,i)} + \mathbf{a}^{(l,i)}.$$
- **Multiplicative:** element-wise rescaling
$$\mathbf{z}_t^{(l,i)\prime} = \mathbf{m}^{(l,i)} \odot \mathbf{z}_t^{(l,i)}.$$
- **Hybrid:** combine both transformations
$$\mathbf{z}_t^{(l,i)\prime} = \mathbf{m}^{(l,i)} \odot \mathbf{z}_t^{(l,i)} + \mathbf{a}^{(l,i)}.$$

Refusal intervention in activation space. Let $\hat{\mathbf{r}} := \mathbf{r}/\|\mathbf{r}\|_2$ denote a unit refusal direction. Following Wollschläger et al. (2025), we consider two interventions applied to residual activations $\mathbf{z}_t^{(l,i)} \in \mathbb{R}^d$ (at selected layers/components and token positions):

$$\text{Add:} \quad \mathbf{z}_t^{(l,i)\prime} = \mathbf{z}_t^{(l,i)} + \alpha\,\hat{\mathbf{r}}, \tag{7}$$

$$\text{Ablate:} \quad \mathbf{z}_t^{(l,i)\prime} = P_{\hat{\mathbf{r}}^\perp}\,\mathbf{z}_t^{(l,i)} = (I - \hat{\mathbf{r}}\hat{\mathbf{r}}^\top)\,\mathbf{z}_t^{(l,i)}. \tag{8}$$

Choosing $\alpha < 0$ in equation 7 corresponds to subtraction along $\hat{\mathbf{r}}$.

B.3 Gated Activation Editing

Lai et al. (2025) extend the hybrid edit by introducing stochastic gates $g_a^{(l,i)}, g_m^{(l,i)} \in [0,1]$, which independently enable additive and multiplicative edits. Let $\mathbf{a}^{(l,i)}, \mathbf{m}^{(l,i)} \in \mathbb{R}^d$ denote additive and multiplicative edit parameters. The gated hybrid edit is

$$\mathbf{z}_t^{(l,i)\prime} = \left(\mathbf{1} + g_m^{(l,i)}\mathbf{m}^{(l,i)}\right) \odot \mathbf{z}_t^{(l,i)} + g_a^{(l,i)}\mathbf{a}^{(l,i)}, \tag{9}$$

where $\mathbf{1} \in \mathbb{R}^d$ is a vector of ones. When both gates are zero ($g_a^{(l,i)} = g_m^{(l,i)} = 0$), the edit reduces to the identity, i.e., $\mathbf{z}_t^{(l,i)\prime} = \mathbf{z}_t^{(l,i)}$. Each gate is drawn from a Hard–Concrete distribution (Louizos et al., 2018), a mixed discrete–continuous distribution on $[0,1]$ with point masses at $\{0,1\}$ and a continuous density on $(0,1)$. At inference time, we replace each stochastic gate by its mean, setting $g_a^{(l,i)} = \mathbb{E}[g_a^{(l,i)}]$ and $g_m^{(l,i)} = \mathbb{E}[g_m^{(l,i)}]$ to ensure determinism.

Hard–Concrete sparsity penalty. Following Lai et al. (2025), each gate $g_a^{(l,i)}, g_m^{(l,i)}$ is parameterized by Hard–Concrete parameters $\psi_a^{(l,i)}, \psi_m^{(l,i)}$. The expected L_0 penalty is

$$\Omega_{L_0}(\psi) = \sum_{l,i} \Big(\Pr(g_a^{(l,i)} \neq 0 \mid \psi_a^{(l,i)}) + \Pr(g_m^{(l,i)} \neq 0 \mid \psi_m^{(l,i)}) \Big). \tag{10}$$

B.4 Refusal Geometry Discovery

Difference-in-Means (DIM). The difference-in-means (DIM) method (Arditi et al., 2024; Marks & Tegmark, 2023; Tigges et al., 2023) estimates a linear refusal direction from mean activations of harmful and harmless prompts.

Let D_{harm} and D_{safe} be datasets of harmful and harmless prompts, respectively. For a prompt p, let $\mathbf{z}_t^{(l,i)}(p) \in \mathbb{R}^d$ denote the activation at layer l, component (l,i), and token position t when running f_θ on p, where t is a fixed extraction position used consistently across prompts. The class-conditional mean activations are

$$\begin{aligned} \boldsymbol{\mu}^{(l,i)} &= \frac{1}{|D_{\text{harm}}|} \sum_{p \in D_{\text{harm}}} \mathbf{z}_t^{(l,i)}(p), \\ \boldsymbol{\nu}^{(l,i)} &= \frac{1}{|D_{\text{safe}}|} \sum_{p \in D_{\text{safe}}} \mathbf{z}_t^{(l,i)}(p). \end{aligned} \tag{11}$$

The resulting difference-in-means vector is

$$\mathbf{r}^{(l,i)} = \boldsymbol{\mu}^{(l,i)} - \boldsymbol{\nu}^{(l,i)}. \tag{12}$$

For interventions we use the unit direction.

Refusal Direction Optimization (RDO). DIM finds a direction that separates harmful from harmless activations, but it does not guarantee that editing along that direction actually produces the behaviors we want. Refusal Direction Optimization (RDO) addresses this by directly optimizing, via a gradient-based loss, a unit vector $\hat{\mathbf{r}} \in \mathbb{R}^d$ so that the two edits we defined in §B.2 (additive steering and directional ablation) reliably toggle refusal.

Let D_{harm} and D_{safe} be datasets of harmful and benign prompts, respectively. RDO defines two modified forward passes of the base model f_θ: $f_{\text{add}}(p; \hat{\mathbf{r}})$, which runs f_θ but adds $\alpha \cdot \hat{\mathbf{r}}$ at a chosen layer l, and $f_{\text{abl}}(p; \hat{\mathbf{r}})$, which runs f_θ but ablates the activation along $\hat{\mathbf{r}}$ using $I - \hat{\mathbf{r}}\hat{\mathbf{r}}^\top$. RDO trains the add pass on benign prompts toward a refusal target, trains the ablate pass on harmful prompts toward an answering target, and adds a retain KL on benign prompts to keep outputs close to f_θ.

$$\mathcal{L}_{\text{add}}(\hat{\mathbf{r}}) = \frac{1}{|D_{\text{safe}}|} \sum_{p \in D_{\text{safe}}} \text{CE}\big(f_{\text{add}}(p; \hat{\mathbf{r}}),\, y_{\text{refuse}}\big), \tag{13}$$

$$\mathcal{L}_{\text{abl}}(\hat{\mathbf{r}}) = \frac{1}{|D_{\text{harm}}|} \sum_{p \in D_{\text{harm}}} \text{CE}\big(f_{\text{abl}}(p; \hat{\mathbf{r}}),\, y_{\text{answer}}\big), \tag{14}$$

$$\mathcal{L}_{\text{retain}}(\hat{\mathbf{r}}) = \frac{1}{|D_{\text{safe}}|} \sum_{p \in D_{\text{safe}}} \text{KL}\big(f_{\text{abl}}(p; \hat{\mathbf{r}}) \,\|\, f_\theta(p)\big). \tag{15}$$

The total RDO loss objective is

$$\mathcal{L}_{\text{RDO}}(\hat{\mathbf{r}}) = \lambda_{\text{add}}\mathcal{L}_{\text{add}}(\hat{\mathbf{r}}) + \lambda_{\text{abl}}\mathcal{L}_{\text{abl}}(\hat{\mathbf{r}}) + \lambda_{\text{ret}}\mathcal{L}_{\text{retain}}(\hat{\mathbf{r}}). \tag{16}$$

RDO optimizes $\mathbf{r}$ by gradient descent through these modified passes and re-normalizes after each update, so the learned object is a direction.

Multi-directional refusal geometry: Refusal Cone Optimization (RCO). The single-direction RDO objective learns one vector, but refusal-related activations may not be captured by a single axis (Wollschläger et al., 2025; Pan et al., 2025). Refusal Cone Optimization (RCO) extends RDO by learning a region of directions in activation space for which the same intervention losses hold.

RCO maintains an orthonormal basis

$$B = [\mathbf{b}_1, \ldots, \mathbf{b}_N] \in \mathbb{R}^{d \times N}, \qquad B^\top B = I_N, \tag{17}$$

where each column $\mathbf{b}_i \in \mathbb{R}^d$ is a trainable direction. This basis defines an N-dimensional cone

$$\mathcal{R}_N = \Big\{ \sum_{i=1}^{N} \lambda_i \mathbf{b}_i \;\Big|\; \lambda_i \geq 0 \Big\} \setminus \{\mathbf{0}\}. \tag{18}$$

For any $\boldsymbol{\lambda} \in \mathbb{R}^N_{\geq 0}$, RCO forms the direction

$$\mathbf{r}_{\boldsymbol{\lambda}} = B\boldsymbol{\lambda}, \qquad \hat{\mathbf{r}}_{\boldsymbol{\lambda}} = \frac{\mathbf{r}_{\boldsymbol{\lambda}}}{\|\mathbf{r}_{\boldsymbol{\lambda}}\|_2}. \tag{19}$$

RCO reuses the RDO loss $\mathcal{L}_{\text{RDO}}(\cdot)$, now evaluated on sampled directions. Given sampled coefficients $\{\boldsymbol{\lambda}^{(m)}\}_{m=1}^M \subset \mathbb{R}^N_{\geq 0}$, define

$$\mathcal{L}_{\text{sample}}(B) = \frac{1}{M} \sum_{m=1}^{M} \mathcal{L}_{\text{RDO}}\big(\hat{\mathbf{r}}_{\boldsymbol{\lambda}^{(m)}}\big). \tag{20}$$

To ensure that the basis vectors themselves satisfy the refusal properties, RCO additionally applies the same loss to each column of B:

$$\mathcal{L}_{\text{basis}}(B) = \frac{1}{N} \sum_{i=1}^{N} \mathcal{L}_{\text{RDO}}(\mathbf{b}_i). \tag{21}$$

The full objective is

$$\mathcal{L}_{\text{RCO}}(B) = \mathcal{L}_{\text{sample}}(B) + \mathcal{L}_{\text{basis}}(B). \tag{22}$$

After every gradient update, the columns of B are orthonormalized (by Gram–Schmidt) to restore equation 17. This prevents co-linear directions and keeps the cone from degenerating into a single line.

C METHOD

C.1 STAGE 1: DISCOVERING THE REFUSAL GEOMETRY

Let f_θ be the base decoder-only Transformer, and let $\mathcal{D}_{\text{harm}}$ and $\mathcal{D}_{\text{safe}}$ be harmful and benign prompt sets. We run Refusal Cone Optimization (RCO) (Wollschläger et al., 2025) on the frozen model f_θ, using the additive and ablation interventions from Section B.2 applied at a fixed intervention site (a chosen layer and token position). RCO maintains an orthonormal basis

$$B = [\mathbf{b}_1, \ldots, \mathbf{b}_N] \in \mathbb{R}^{d \times N}, \qquad B^\top B = I_N,$$

and optimizes B so that for nonnegative coefficient vectors $\boldsymbol{\lambda} \in \mathbb{R}^N_{\geq 0}$, the normalized combination

$$\hat{\mathbf{r}}_{\boldsymbol{\lambda}} = \frac{B\boldsymbol{\lambda}}{\|B\boldsymbol{\lambda}\|_2}$$

satisfies the same three RDO losses (add, ablate, retain) as in equation 13–equation 15. While RCO defines a cone of directions via nonnegative combinations, our constraint in Stage 2 will use the associated linear span of the learned basis,

$$\mathcal{R} := \operatorname{span}(B), \qquad P_{\mathcal{R}} := BB^\top, \qquad P_{\mathcal{R}^\perp} := I - BB^\top. \tag{23}$$

We refer to $\mathcal{R}$ as the refusal subspace induced by the learned cone basis.

C.2 STAGE 2: GATED EDITING IN $\mathcal{R}^\perp$

We focus on edits applied to attention-head components. Let $\mathbf{z}_t^{(l,i)} \in \mathbb{R}^d$ denote the activation of head i at layer l and token position t, where d is the head dimension. Following prior refusal-geometry interventions Wollschläger et al. (2025), we apply the decomposition at every token position, and restrict it to an early-layer prefix to avoid over-constraining representations near the unembedding: for a model with L layers, we intervene only for $l \in 0, \ldots, \lfloor 0.9L \rfloor - 1$. With the frozen cone, we decompose

$$\mathbf{z}_t^{(l,i)} = P_{\mathcal{R}} \mathbf{z}_t^{(l,i)} + P_{\mathcal{R}^\perp} \mathbf{z}_t^{(l,i)}.$$

We then apply JoLA-style gated hybrid edits (Lai et al., 2025) only to the orthogonal component. Let $\tilde{\mathbf{m}}^{(l,i)}, \tilde{\mathbf{a}}^{(l,i)} \in \mathbb{R}^d$ be unconstrained multiplicative and additive edit vectors and let $g_m^{(l,i)}, g_a^{(l,i)} \in [0, 1]$ be Hard–Concrete gates. Before the forward pass we project the parameters:

$$\mathbf{m}^{(l,i)} := P_{\mathcal{R}^\perp} \tilde{\mathbf{m}}^{(l,i)}, \qquad \mathbf{a}^{(l,i)} := P_{\mathcal{R}^\perp} \tilde{\mathbf{a}}^{(l,i)}. \tag{24}$$

To ensure the learned edit cannot reintroduce components along $\mathcal{R}$ (e.g., due to elementwise scaling), we additionally re-project the post-edit result back into $\mathcal{R}^\perp$. The edited activation is

$$\mathbf{z}_t^{(l,i)\prime} = P_{\mathcal{R}} \mathbf{z}_t^{(l,i)} + P_{\mathcal{R}^\perp} \Big(\big(\mathbf{1} + g_m^{(l,i)} \, \mathbf{m}^{(l,i)} \big) \odot \big(P_{\mathcal{R}^\perp} \mathbf{z}_t^{(l,i)} \big) + g_a^{(l,i)} \, \mathbf{a}^{(l,i)} \Big). \tag{25}$$

Thus, the refusal part $P_{\mathcal{R}} \mathbf{z}_t^{(l,i)}$ is always copied through unchanged, while the task-specific edit is constrained to lie in $\mathcal{R}^\perp$.

C.3 OVERALL TRAINING OBJECTIVE

Let $\mathcal{D} = \{(\mathbf{s}^{(n)}, \mathbf{y}^{(n)})\}_{n=1}^N$ be a supervised dataset, where $\mathbf{s}$ is the input token sequence and $\mathbf{y}$ are the target next-token labels. We train the edited model $f_{\theta,\phi}$ by minimizing the sum of the task loss and a sparsity regularizer on the Hard–Concrete gate parameters ψ:

$$\min_{\phi} \; \mathcal{L}_{\text{CE}}(\theta, \phi; \mathcal{D}) \; + \; \rho \, \Omega_{L_0}(\psi), \tag{26}$$

where $\mathcal{L}_{\text{CE}}$ is the standard next-token cross-entropy under the edited forward pass (i.e., using $\mathbf{z}_t^{(l,i)\prime}$ from equation 25), $\Omega_{L_0}(\psi)$ and $\rho \geq 0$ controls the sparsity–performance trade-off. We optimize only the intervention parameters $\phi = \{\mathbf{a}^{(l,i)}, \mathbf{m}^{(l,i)}, \psi_a^{(l,i)}, \psi_m^{(l,i)}\}$, keeping the base model parameters θ fixed.

C.4 SAGE ALGORITHM OVERVIEW

Algorithm 1 SAGE: Safety-Aware Gated Editing

Require: Base model f_θ; refusal datasets $\mathcal{D}_{\text{harm}}, \mathcal{D}_{\text{safe}}$ (Stage 1); task data $\mathcal{D}_{\text{ben}}$ (Stage 2); refusal subspace dimension N; samples per iteration M; sparsity weight ρ

Ensure: Refusal basis $B \in \mathbb{R}^{d\times N}$ and projectors $P_{\mathcal{R}}, P_{\mathcal{R}^\perp}$; edit parameters $\phi = \{\tilde{\mathbf{m}}^{(l,i)}, \tilde{\mathbf{a}}^{(l,i)}\}$ and gate parameters $\psi = \{\psi_m^{(l,i)}, \psi_a^{(l,i)}\}$.

// STAGE 1: LEARN REFUSAL BASIS

Initialize $B \in \mathbb{R}^{d\times N}$ with $B^\top B = I$
for iter $= 1 \dots K$ **do**
  Sample $\{\boldsymbol{\lambda}^{(m)}\}_{m=1}^M$ with $\boldsymbol{\lambda}^{(m)} \in \mathbb{R}^N_{\geq 0}$ {M candidate directions sampled from the cone.}
  for $m = 1 \dots M$ **do**
    $\mathbf{r}_m \leftarrow B\boldsymbol{\lambda}^{(m)}$; $\hat{\mathbf{r}}_m \leftarrow \mathbf{r}_m / \|\mathbf{r}_m\|_2$
    $\mathcal{L}_m \leftarrow \mathcal{L}_{\text{RDO}}(\hat{\mathbf{r}}_m; \mathcal{D}_{\text{harm}}, \mathcal{D}_{\text{safe}})$ {Refusal direction optimization (RDO) (Appendix B.4, Eq. 16).}
  end for
  $\mathcal{L}_{\text{RCO}} \leftarrow \frac{1}{M}\sum_{m=1}^M \mathcal{L}_m$ {Refusal cone optimization (RCO) (Appendix B.4, Eq. 22).}
  $B \leftarrow B - \eta \nabla_B \mathcal{L}_{\text{RCO}}$
  Orthonormalize columns of B
end for
Freeze B; define the refusal subspace $\mathcal{R} := \operatorname{span}(B)$;
define projectors $P_{\mathcal{R}} = BB^\top$, $P_{\mathcal{R}^\perp} = I - BB^\top$

// STAGE 2: REFUSAL-ORTHOGONAL GATED ADAPTATION

Choose editable components $C = \{(l,i)\}$ {e.g., selected attention heads.}
for each $(l,i) \in C$ **do**
  Initialize $\tilde{\mathbf{m}}^{(l,i)}, \tilde{\mathbf{a}}^{(l,i)} \in \mathbb{R}^d$, $\psi_m^{(l,i)}, \psi_a^{(l,i)}$
end for
for iter $= 1 \dots K'$ **do**
  Sample $(x, y) \sim \mathcal{D}_{\text{ben}}$
  Forward f_θ on x
  for each $(l,i) \in C$ **do**
    $\mathbf{m}^{(l,i)} \leftarrow P_{\mathcal{R}^\perp}\tilde{\mathbf{m}}^{(l,i)}$; $\mathbf{a}^{(l,i)} \leftarrow P_{\mathcal{R}^\perp}\tilde{\mathbf{a}}^{(l,i)}$
    $g_m^{(l,i)}, g_a^{(l,i)} \leftarrow \text{HC}(\psi_m^{(l,i)}, \psi_a^{(l,i)})$ {Hard–Concrete gates in $[0,1]$; mean used at inference.}
    $\mathbf{z}_{\mathcal{R}} \leftarrow P_{\mathcal{R}}(\mathbf{z}_t^{(l,i)})$; $\mathbf{z}_\perp \leftarrow P_{\mathcal{R}^\perp}(\mathbf{z}_t^{(l,i)})$
    $\mathbf{z}_t^{(l,i)\prime} \leftarrow \mathbf{z}_{\mathcal{R}} + P_{\mathcal{R}^\perp}\Big((\mathbf{1} + g_m^{(l,i)}\mathbf{m}^{(l,i)}) \odot \mathbf{z}_\perp + g_a^{(l,i)}\mathbf{a}^{(l,i)}\Big)$
  end for
  $\mathcal{L} \leftarrow \text{CE}(f_{\theta,\phi}(x; B), y) + \rho\,\Omega_{L_0}(\psi)$
  Freeze θ, B; update (ϕ, ψ) with learning rate η'
end for

D EXPERIMENT CONFIGURATIONS

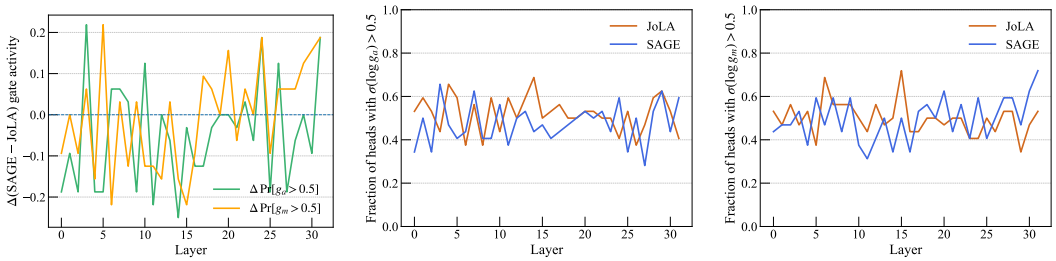
We fine-tune Llama-3.1-8B-Instruct in a low-resource regime with 200 training examples. Unless otherwise stated, we train for up to 20 epochs with batch size 16 using. We apply early stopping based on validation performance with patience 3. Table 2 lists method-specific hyperparameters.

Table 2: Method-specific training hyperparameters.

Method	Hyperparameter	Values
LoRA	learning rate	5×10^{-6}
LoRA	rank r	16
LoRA	α	32
LoRA	target modules	{q_proj, k_proj, v_proj, o_proj, up_proj, gate_proj, down_proj}
LoRA	LR scheduler	cosine
JoLA	applied module	attention
JoLA	learning rate	5×10^{-5}
JoLA	LR scheduler	cosine
JoLA	ρ	4×10^{-5}
JoLA	gate scheduler	expon
SAGE	cone dim N	{1,5}
SAGE	applied module	attention
SAGE	learning rate	5×10^{-5}
SAGE	LR scheduler	cosine
SAGE	ρ	4×10^{-5}
SAGE	gate scheduler	expon

E GATE ACTIVITY WITH AND WITHOUT THE REFUSAL-ORTHOGONALITY CONSTRAINT

We analyze how the refusal-orthogonality constraint affects the utilization of JoLA-style activation edits by examining layer-wise gate statistics. Figure 3 reports both absolute gate activity and relative changes induced by the constraint. Both methods are trained using the same settings except the refusal-orthogonality constraint.



(a) Change in gate activity (b) Active additive gates (c) Active multiplicative gates

Figure 3: **Gate-level effects of the refusal-orthogonality constraint. (a)** Difference in active-gate fraction per layer (SAGE − JoLA), showing a structured, sign-changing redistribution of gate activity across depth rather than uniform suppression. **(b–c)** Absolute fraction of heads with active additive (g_a) and multiplicative (g_m) gates, respectively. Across both gate types, SAGE retains substantial gate activity at most layers, comparable in magnitude to JoLA. Together, these plots show that safety improvements under SAGE arise from a depth-dependent reallocation of editing capacity, not from disabling the editor.

Insight 1: The refusal-orthogonality constraint does not suppress gate activity.
Relation to main result: rules out the interpretation that SAGE behaves as a weaker variant of unconstrained editing.

Across both additive and multiplicative edits, SAGE retains substantial gate activity throughout the network (Figure 3b– 3c). The fraction of heads with active gates remains comparable in magnitude to JoLA across most layers. Thus, SAGE preserves non-trivial adaptation capacity while achieving lower attack success rates in Table 1.

Insight 2: The refusal-orthogonality constraint induces non-uniform, layer-dependent changes in gate activity.
Relation to main result: shows that SAGE's differences from JoLA are not explained by a uniform, global weakening of the editor.

Figure 3a shows that the difference in active-gate fraction (SAGE − JoLA) varies in sign across layers, with some layers exhibiting increased gate usage and others decreased usage. This behavior is inconsistent with a uniform, depth-independent reduction in gate activity relative to JoLA, which would instead produce a near-constant negative shift across layers.

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