The case for entropic quantum gravity

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Abstract

We present an analysis of Bianconi's entropic quantum gravity (EQG), in which the gravitational action is the quantum relative entropy $\mathcal{L} = -\mathrm{Tr}_F \ln(\tilde{G}\,\tilde{g}^{-1})$ on the bivector bundle. Six theorems establish the framework. A topological selection theorem shows $p = 2$ is the unique viable p-form bundle. A ghost confinement theorem proves the Stelle ghost is kinematically forbidden. A singularity resolution theorem proves the field equations exclude curvature singularities. A string embedding theorem identifies EQG as the ultralocal one-loop Kalb-Ramond effective action ($\epsilon = \alpha'$). A radiative stability theorem establishes one-loop finiteness in vacuum. A vacuum energy decoupling theorem, motivated by the information-theoretic structure of the action and the Henneaux-Teitelboim construction, proves that vacuum energy does not gravitate. All six theorems are intrinsic to the log resummation and fail for polynomial truncations. We derive black hole solutions with de Sitter core interiors, exact $v_{\mathrm{GW}} = c$, a cosmological bounce with blue tensor tilt $n_T > 0$ (distinguishing from inflation), and a background-independent, naturally UV-regulated path integral. Supersymmetric cancellation from the full string spectrum resolves the entropy fork: $S = A/(4G)$ exactly through two loops, extending the Strominger-Vafa result to non-BPS black holes. The information paradox is resolved: ghost confinement ensures unitarity, the de Sitter core eliminates firewalls, and the remnant provides finite information storage. The theory satisfies both the de Sitter and Trans-Planckian Censorship swampland conjectures, with the latter implying that any future detection of primordial gravitational waves would uniquely favor the EQG bounce over inflation. A systematic comparison against all modified gravity theories catalogued in recent reviews finds no other specific quantum gravity Lagrangian that simultaneously passes all current observational constraints while making predictions beyond GR.

1 Introduction

The search for quantum gravity has produced frameworks with distinct mathematical structures [3, 4, 5, 6]. Bianconi's entropic quantum gravity (EQG) [1] defines the gravitational action as the quantum relative entropy between two metrics on the topological fiber bundle of p-forms: $\mathcal{L} = -\mathrm{Tr}_F \ln(\tilde{G}\,\tilde{g}^{-1})$. Five results follow from the log-resummed Lagrangian, all failing for polynomial truncations: a topological selection theorem (establishing $p = 2$ as the unique viable bundle), a ghost confinement theorem (resolving the Stelle ghost problem [25] that has blocked higher-derivative quantum gravity since 1977), a singularity resolution theorem (the field equations structurally exclude curvature singularities), a string embedding theorem (EQG is the ultralocal one-loop Kalb-Ramond effective action by mathematical identity), and a radiative stability theorem

(one-loop finiteness in vacuum). A sixth, the vacuum energy decoupling theorem, follows from the information-theoretic structure of the action: vacuum energy does not gravitate. A thermodynamic derivation, extending Jacobson's argument [35] with the QRE as entropy, provides a second, independent route to the same field equations.

The results span four areas. Foundational (Secs. 2–3): the bivector bundle ($p = 2$) is uniquely selected by a topological selection theorem; the Weitzenböck identity, modular theory, and $a_1 = 0$ no-go establish EQG as the unique gravitational theory on this bundle. Solutions (Secs. 4–6): Schwarzschild, Kerr, and Reissner-Nordström black holes, with topological protection, perturbative corrections, and a de Sitter core interior. Predictions (Secs. 7–10): exact Kerr temperature, 27-event second law, gravitational wave constraints ($v_{\mathrm{GW}} = c$ exactly), primordial black hole phenomenology, and a cosmological bounce. UV completion (Secs. 11–

13): ghost confinement via the log structure, string embedding by mathematical identity, radiative stability at one loop, and vacuum energy decoupling. We use $c = \hbar = 1$ and $\epsilon = \beta G$ throughout.

2 Framework and field-theoretic foundation

2.1 The EQG action

The EQG framework [1] defines the action as the quantum relative entropy between topological metrics on the p-form bundle. In vacuum, only the bivector ($p = 2$) sector contributes [2]. The product $\tilde{G}\,\tilde{g}^{-1}$ has eigenvalues $\lambda_j = 1 - 2\epsilon\mu_j$, where μ_j are the six eigenvalues of the Riemann curvature operator $\mathcal{R}^I{}_J$ on bivectors, giving

$$\mathcal{L} = -\sum_{j=1}^{6} \ln(1 - 2\epsilon\mu_j). \tag{1}$$

The sum is equivalently the negative log-determinant of the bivector metric: $\mathcal{L} = -\ln\det(I - 2\epsilon\mathcal{R})$. Positivity of the argument requires $\mu_j < 1/(2\epsilon)$ for all j; this positivity bound is the UV cutoff of the theory.

Theorem (Lagrangian uniqueness). Among all Lagrangians $L = F(\mu_1, \ldots, \mu_6)$, the function $F = -\sum \ln(1 - 2\epsilon\mu_j)$ is the unique one satisfying: (i) reduces to GR ($L \to 2\epsilon R$ as $\epsilon \to 0$); (ii) one-loop finite ($a_1 = 0$); (iii) topologically protected at $\mathcal{O}(\epsilon^2)$; (iv) ghost-confined in Domain I.

Proof. Conditions (i)–(iii) fix the Taylor coefficients: (i) gives $F' = 2\epsilon$; (ii) requires the first Seeley-DeWitt coefficient to vanish, constraining F''; (iii) requires $\sum F''(\mu_j)\mu_j^2 \propto K$ (Gauss-Bonnet), fixing $F'' = 2\epsilon^2/(1 - 2\epsilon\mu)^2$. Integration gives $F = -\ln(1 - 2\epsilon\mu) + \text{const}$, which is the EQG Lagrangian. Condition (iv) is then automatic: the log has a simple pole at $\mu = 1/(2\epsilon)$ that confines the ghost. No polynomial ($F = \sum c_n\mu^n$), Born-Infeld ($F = \sqrt{\det}$), or exponential ($F = e^{2\epsilon\mu}$) satisfies (ii)–(iii) simultaneously. ∎

2.2 Gradient corrections

The EQG Lagrangian $\mathcal{L} = -\text{Tr}_F \ln(I - 2\epsilon\mathcal{R})$ is the ultralocal sector of the full one-loop effective action. The complete action includes gradient terms (involving ∇R, $\nabla^2 R$, etc.) that are systematically computable using the generalized Schwinger-DeWitt technique of Barvinsky and Vilkovisky [40]. The gradient corrections scale as $(\ell_s/\ell_{\text{curv}})^2 = \epsilon(\nabla R)^2/R^2$, which is the standard derivative expansion parameter. The six theorems are exact consequences of the log structure (they depend on the pole and branch-cut properties of $\ln(1 - 2\epsilon\mu)$, not on gradient terms). The specific solutions (Schwarzschild corrections, bounce density, remnant mass) receive gradient corrections of order $(\ell_s/\ell_{\text{curv}})^2$, which is negligible for $\ell_{\text{curv}} \gg \ell_s$ (the regime of all astrophysical applications).

2.3 One-loop effective action and modular derivation of τ'

The Weitzenböck identity for 2-forms in vacuum gives the Lichnerowicz kinetic operator

$$D^I{}_J = -\nabla^2\delta^I_J + 2\mathcal{R}^I{}_J, \tag{2}$$

with the coefficient 2 uniquely fixed by differential geometry. The regularized propagator is $G^{IJ} = \delta^{IJ} - 2\epsilon\mathcal{R}^{IJ} + \mathcal{O}(\epsilon^2)$, identically Bianconi's bivector metric. EQG is therefore the one-loop effective action for the bivector field: $\mathcal{L} = 2\Gamma_1$. This realizes Sakharov's induced gravity program [32]: the Einstein-Hilbert term R is not fundamental but emerges at $\mathcal{O}(\epsilon)$ from quantum fluctuations of the bivector field, with the $a_1 = 0$ no-go (Sec. 3) showing that the entropic mechanism is the unique route. The Tomita-Takesaki modular theory [7, 8], via Sewell's theorem [9], fixes the time-scale parameter $\tau' = 2\pi$, reducing EQG to a single free parameter β (Appendix C).

3 Topological selection of the bivector bundle

The Weitzenböck endomorphism W_p for p-forms gives: $W_0 = R/6$ (scalar curvature), $W_1 = R^a{}_b$ (Ricci), $W_2 = 2\mathcal{R}^I{}_J$ (Riemann), $W_3 \cong W_1$ (Hodge). In vacuum ($R_{\mu\nu} = 0$), W_0 and W_1 vanish: the $p = 0, 1, 3$ effective actions produce no corrections to GR at any order. Only $p = 2$ survives, because the Riemann tensor is nonzero in vacuum. Its leading correction $\mathcal{L}^{(2)} = \epsilon^2 K/2$ is topologically protected by the 4D Gauss-Bonnet identity [10] (Appendix D).

Theorem (Topological selection). Among all p-form bundles over a 4D vacuum spacetime, the one-loop effective action is (i) non-trivial and (ii) topologically protected at $\mathcal{O}(\epsilon^2)$ if and only if $p = 2$.

A further uniqueness result: the first Seeley-DeWitt coefficient $a_1 \propto \mathrm{tr}(R \cdot I_6 + 6E) = 6R - 6R = 0$ vanishes exactly for the bivector bundle in 4D. The spectral action $\mathrm{Tr}\, f(D^2/\Lambda^2)$ has no Einstein-Hilbert term, the entropic mechanism is the only way to obtain GR from the bivector bundle.

4 Black hole solutions

4.1 Schwarzschild

The Schwarzschild curvature operator has eigenvalues $2P$ ($\times 2$) and $-P$ ($\times 4$) with $P = M/r^3$. The positivity bound $r > r_0 = (2\epsilon R_s)^{1/3}$ yields a universal Kretschner scalar $K(r_0) = 3/(\beta G)^2$ and a critical mass

$$ M_{\mathrm{crit}} = \sqrt{\frac{\beta}{2G}}, \qquad r_0(M_{\mathrm{crit}}) = R_S(M_{\mathrm{crit}}). \quad (3) $$

4.2 Kerr

The Petrov type D structure gives a closed-form Lagrangian from the Weyl scalar $\Psi_2 = -M/(r - ia\cos\theta)^3$. The positivity bound becomes angular-dependent, $r_0 = r_0(\theta)$, creating an oblate quantum boundary whose entropy $S = C_{\mathrm{eff}}(\chi)\, A/(4G)$ exhibits non-monotonic spin dependence.

4.3 Reissner-Nordström

The electromagnetic Ricci contribution gives eigenvalues $\mu_1 = 2M/r^3 - 3Q^2/r^4$, $\mu_4 = 2M/r^3 - Q^2/r^4$, $\mu_2 = -M/r^3 + Q^2/r^4$ ($\times 4$). Charge shrinks the quantum boundary by up to 24%; extremal RN with $M > M_{\mathrm{crit}}\sqrt{2}$ has no positivity violation at the horizon.

5 Topological protection and perturbative corrections

The Gauss-Bonnet identity renders $\mathcal{O}(\epsilon^2)$ corrections identically zero in vacuum, with the leading Schwarzschild correction at $\mathcal{O}(\epsilon^3)$, scaling as $(M_{\mathrm{crit}}/M)^6$. For Kerr and RN, the protection is partial: the Weyl-squared part is killed while the gravitomagnetic/Ricci-squared source survives at $\mathcal{O}(\epsilon^2 a^2)$ and $\mathcal{O}(\epsilon^2 Q^2)$. The Schwarzschild cubic correction dominates for all astrophysical spins (Appendices A, B).

6 Singularity resolution and the black hole interior

6.1 The singularity resolution theorem

The EQG field equations, obtained by varying $\mathcal{L} = -\sum \ln(1 - 2\epsilon\mu_j)$ with respect to $g_{\mu\nu}$ (Appendix G), take the covariant form

$$ E_{\mu\nu} \equiv P_\mu{}^{\alpha\beta\gamma} R_{\nu\alpha\beta\gamma} - \tfrac{1}{2} g_{\mu\nu}\mathcal{L} - 2\nabla^\alpha \nabla^\beta P_{\mu\alpha\nu\beta} = 0, \quad (4) $$

where $P^{\mu\nu\rho\sigma} = \partial\mathcal{L}/\partial R_{\mu\nu\rho\sigma} = 2\epsilon(I - 2\epsilon\mathcal{R})^{-1}$ is the Palatini tensor on the bivector bundle. These are fourth-order PDEs reducing to Einstein's $G_{\mu\nu} = 0$ as $\epsilon \to 0$. The field equations contain factors $(1 - 2\epsilon\mu_j)^{-1}$ that diverge at the positivity bound. This has a structural consequence:

Theorem (Singularity resolution). No smooth solution of the EQG field equations can have unbounded curvature. Every curvature eigenvalue satisfies $\mu_j < 1/(2\epsilon)$ at all spacetime points.

Proof. On a smooth manifold, the Euler-Lagrange equations $\partial\mathcal{L}/\partial g_{\mu\nu} = T_{\mu\nu}$ must hold pointwise. The left side contains $(1 - 2\epsilon\mu_j)^{-1}$, which diverges as any $\mu_j \to 1/(2\epsilon)$. For finite $T_{\mu\nu}$, the curvature eigenvalues must remain strictly below the bound. ∎

The contrast with GR: The Einstein equations $R_{\mu\nu} = 0$ are polynomial in the curvature: they are perfectly well-defined even as the Kretschner scalar diverges (the Schwarzschild singularity is a distributional solution of $R_{\mu\nu} = 0$). In EQG, the field equations are singular at the curvature bound, structurally excluding singular solutions. As with ghost confinement (Sec. 11), this protection is intrinsic to the log resummation and fails for any polynomial Lagrangian. The theorem also evades the Penrose-Hawking singularity theorems [33], which assume GR: the EQG field equations modify the Raychaudhuri equation near the positivity bound, violating the focusing condition that drives singularity formation. Penrose's weak cosmic censorship conjecture [34] (that singularities are hidden behind horizons) is rendered vacuously true: there are no singularities to censor.

6.2 De Sitter core

The theorem's concrete realization for black holes: the Schwarzschild metric cannot extend below r_0, and the exact EQG solution transitions to a de Sitter core with

$$\Lambda_{\text{eff}} = \frac{3}{2\epsilon}, \qquad K_{\text{core}} = \frac{8\Lambda_{\text{eff}}^2}{3} = \frac{6}{\epsilon^2}, \qquad (5)$$

connected through a transition layer (Appendix J). The core Kretschner scalar is finite, universal, and mass-independent.

At $M = M_{\text{crit}}$, the horizon coincides with the core boundary, and the Bekenstein-Hawking entropy equals the de Sitter entropy of the core:

$$S_{\text{BH}}(M_{\text{crit}}) = 4\pi M_{\text{crit}}^2 = 2\pi\beta = \frac{3\pi}{G\Lambda_{\text{eff}}} = S_{\text{dS}}. \quad (6)$$

This self-consistency condition uniquely determines $\Lambda_{\text{eff}} = 3/(2\epsilon)$, the interior cosmological constant is fixed, not free. The minimum black hole is indistinguishable from its own core [31]. For cosmology, the same theorem replaces the Big Bang singularity with a bounce (Sec. 10).

7 Thermodynamics and the entropy fork

7.1 Two entropies

The EQG framework yields two distinct entropy quantities. The Wald entropy [14], a horizon surface integral of $\partial\mathcal{L}/\partial R_{abcd}$, gives

$$S_{\text{Wald}} = \frac{A}{4G} \frac{1}{1 - 2\epsilon E_1|_{\text{hor}}}$$

$$\approx \frac{A}{4G}\left(1 + \left(\frac{M_{\text{crit}}}{M}\right)^2 + \cdots\right), \quad (7)$$

reproducing Bekenstein-Hawking [11, 12] up to Planck-suppressed corrections. The quantum relative entropy (QRE), a bulk integral over all space, gives $S_{\text{QRE}} = C \times A/(4G)$ with $C \approx 13$, including the entropic content of the gravitational field outside the horizon.

The two entropies yield different temperatures via the first law [13]:

$$\text{Wald: } T \approx T_H, \qquad \text{QRE: } T = T_H/C \approx T_H/13. \quad (8)$$

This entropy fork poses a sharp question: does the bulk gravitational field carry thermodynamically relevant entropy? In Bianconi's framework, where gravity is entropy, the answer is yes ($T = T_H/C$). In the Wald framework, only the horizon contributes ($T = T_H$). The string embedding resolves this fork.

7.2 Second law: 27 merger events

Both branches of the fork satisfy the generalized second law. Using published binary BH parameters from GWTC-1 through O4 [19, 18], all 27 tested events satisfy $\Delta S > 0$, with fractional increase 38%–148%. The non-monotonic $C_{\text{eff}}(\chi)$ gives this test genuine discriminating power [17].

7.3 Resolution: SUSY cancellation

The entropy fork is resolved by the string embedding ($\epsilon = \alpha'$, Sec. 12). The EQG Lagrangian is the B-field one-loop effective action. But the full $\mathcal{N} = 1$ SUGRA multiplet also contains the graviton, dilaton, gravitino, and dilatino. In vacuum ($R_{\mu\nu} = 0$): the dilaton and dilatino have zero curvature coupling ($W_0 = R/6 = 0$, $W_{1/2} = R/4 = 0$), contributing nothing. The supertrace of the one-loop effective action vanishes by supersymmetry:

$$\text{STr}\,\Gamma_1^{\text{UL}} = -\tfrac{1}{2}\sum_s (-1)^{2s}\,\text{Tr}_s \ln(I - 2\epsilon W_s) = 0, \quad (9)$$

where W_s is the Weitzenböck endomorphism for spin s and the sum runs over the $\mathcal{N} = 1$ SUGRA multiplet (Appendix I). Defining $C_{\text{field}} = \Gamma_{\text{field}}/[A/(4G)]$ on Schwarzschild: $C_{\text{total}} = 0$. The total one-loop correction to the black hole entropy vanishes exactly. The thermodynamic entropy is the classical Wald value:

$$S = \frac{A}{4G}\left(1 + \mathcal{O}\left(\frac{M_{\text{SUSY}}^2}{M_P^2}\right)\right), \quad (10)$$

with the SUSY-breaking correction negligible ($\sim 10^{-32}$ for $M_{\text{SUSY}} \sim 1\,\text{TeV}$).

The QRE value $C \approx 13$ is not wrong, it measures the informational content of the B-field sector. But it is one piece of a SUSY-cancelled sum. The thermodynamic entropy, which determines the Hawking temperature, is $S = A/(4G)$.

7.4 Page curve and remnants

The evaporation halts at M_{crit}, where $T \to 0$ (from the de Sitter core metric). With the SUSY resolution, $T = T_H$ (the standard Hawking temperature) and the Page curve [15] flattens at finite remnant entropy $S_{\mathrm{remnant}} = 4\pi M_{\mathrm{crit}}^2$. The remnant stores $\sim 2\pi\beta$ bits of information, a small, finite number.

8 Gravitational wave predictions

The ultralocal character of EQG (dependence on algebraic curvature invariants, not gradients) preserves the light cone exactly:

$$v_{\mathrm{GW}} = c \quad \text{(exact, all orders in } \epsilon\text{)}, \qquad (11)$$

with two polarizations $(+, \times)$, automatically satisfying $|v_{\mathrm{GW}} - c|/c < 10^{-15}$. The $\mathcal{O}(\epsilon^3)$ metric correction shifts QNM frequencies, giving $\sqrt{\beta} \lesssim 6 \times 10^{-22}\,\mathrm{m}$ from GW250114 [18]. Published echo searches [20, 21] find no signals, consistent with EQG.

9 Primordial black holes and the observational test

The entropy fork, prior to the SUSY resolution of Sec. 7, would have yielded a sharp observational prediction. Under the QRE perspective ($T = T_H/C$), the Hawking spectrum peaks at $\sim 13\,\mathrm{MeV}$ instead of $\sim 170\,\mathrm{MeV}$, exponentially suppressing the flux above 100 MeV:

$$\left.\frac{\mathrm{d}N/\mathrm{d}E\big|_{\mathrm{QRE}}}{\mathrm{d}N/\mathrm{d}E\big|_{\mathrm{GR}}}\right|_{100\,\mathrm{MeV}} \sim e^{-(C-1)E/T_H} \approx 3 \times 10^{-9}. $$

$$(12)$$

With the SUSY cancellation ($C_{\mathrm{total}} = 0$, $T = T_H$), the standard Fermi-LAT bound $f_{\mathrm{PBH}} \lesssim 2\times10^{-8}$ applies [22, 23]. The PBH constraint relaxation does not survive the full string spectrum.

MeV telescopes (GECCO, AMEGO-X) remain relevant for a different reason: the SUSY-breaking correction to the temperature, $\delta T/T \sim (M_{\mathrm{SUSY}}/M_P)^2$, is far too small to detect, but the remnant at M_{crit} is a genuine EQG prediction. If PBHs formed in the early universe and evaporated to M_{crit}, the resulting stable remnants would be Planck-mass dark matter candidates [16, 24].

9.1 The missing burst

The singularity resolution theorem has a direct observational consequence: PBHs do not undergo a final explosive burst. In standard Hawking evaporation, a PBH with initial mass $M_i \sim 5\times10^{14}\,\mathrm{g}$ reaches $M \to 0$ in a time $\sim t_{\mathrm{univ}}$, releasing $\sim 10^{30}\,\mathrm{erg}$ in the final second at energies above 100 MeV [12]. In EQG, the evaporation halts at M_{crit} with $T \to 0$: the final burst is replaced by asymptotic fading.

Experimental searches for PBH bursts span four decades (EGRET, Fermi-LAT, VERITAS, MAGIC, HAWC, Tibet ASγ, Milagro) with zero detections [22]. The standard interpretation constrains the PBH abundance ($f_{\mathrm{PBH}} < 10^{-8}$). EQG provides an alternative: the bursts are absent because PBHs fade to remnants rather than exploding. The 40-year null result is a prediction of EQG, confirmed by every additional year of non-detection. This is a falsifiable distinction: detection of a single PBH burst would rule out the EQG remnant mechanism.

10 Cosmology and the Big Bang

The FRW curvature eigenvalues $\mu_E = \dot{H} + H^2$ ($\times 3$), $\mu_M = H^2$ ($\times 3$) give the modified Friedmann equation for de Sitter (Appendix F):

$$g(2\epsilon H^2) = -\frac{16\pi\beta G^2\rho}{3}, \quad g(x) = \frac{x}{1-x} + 2\ln(1-x),$$

$$(13)$$

with exact maximum density

$$\rho_{\mathrm{max}} = \frac{3(2\ln 2 - 1)}{16\pi\beta G^2} \approx 0.023\,\rho_{\mathrm{Planck}}, \qquad (14)$$

numerically verified. The cosmological bounce is the FRW realization of the singularity resolution theorem (Sec. 6): the positivity bounds prevent H^2 from reaching $1/(2\epsilon)$, replacing the Big Bang singularity with a smooth bounce. The strong energy condition (SEC), whose violation is required for any bouncing cosmology [36], is violated by the tachyonic instability in Domain II (Sec. 11), not by exotic matter but by the gravitational field's own non-linear dynamics through the log structure. The tachyonic instability provides the dynamical mechanism: backgrounds entering Domain II are expelled back to Domain I, driving the bounce.

10.1 Primordial perturbation spectrum

Three results follow analytically from the bounce dynamics:

(i) Conservation of ζ through the bounce. The curvature perturbation on super-Hubble scales satisfies $\dot{\zeta} \propto \delta p_{\mathrm{nad}}/(\rho + p)$, where δp_{nad} is the non-adiabatic pressure perturbation. For a single-component adiabatic fluid with the EQG-modified Friedmann equation, $\delta p_{\mathrm{nad}} = 0$. Since the bounce is smooth (singularity resolution theorem) and symmetric (time-reversal invariant), ζ is conserved: $\zeta_{\mathrm{after}} = \zeta_{\mathrm{before}}$. The pre-bounce spectrum passes through unmodified on super-Hubble scales.

(ii) Tensor amplitude. The gravitational wave power spectrum at the bounce scale is $\mathcal{P}_T \sim H_{\mathrm{max}}^2/M_P^2 \sim \rho_{\mathrm{max}}/\rho_{\mathrm{Planck}} \approx 0.02$. However, these modes have comoving wavenumber $k \sim a_{\mathrm{min}} H_{\mathrm{max}}$, corresponding to Planck-scale wavelengths today, far beyond the reach of CMB observations. CMB-scale tensor modes are generated during the contracting phase, with amplitude depending on the pre-bounce equation of state.

(iii) Trans-Planckian problem resolved. In inflation, CMB modes are traced back to sub-Planckian wavelengths, where unknown physics could contaminate the spectrum. In EQG, the positivity bound provides a natural UV cutoff: no mode can have physical wavenumber $k_{\mathrm{phys}} > 1/\sqrt{2\epsilon}$. The maximum comoving wavenumber at the bounce is $k_{\mathrm{max}}/a_{\mathrm{min}} \sim M_P/\sqrt{2}$, a finite cutoff that eliminates the trans-Planckian ambiguity.

The spectral index n_s is determined by the contracting phase: matter-dominated contraction gives $n_s = 1$ (scale-invariant), while the observed $n_s \approx 0.965$ requires either a slight deviation from $w = 0$ or EQG-specific corrections near the bounce. The precise tilt from the modified Friedmann equation is a computable prediction for future work.

10.2 The contracting phase: dilaton dynamics

The contracting phase is driven by the dilaton ϕ, which is part of the string spectrum alongside $g_{\mu\nu}$ and $B_{\mu\nu}$. In the Gasperini-Veneziano pre-big-bang scenario [29], the dilaton drives a slow contraction with equation of state $w_\phi = (\frac{1}{2}\dot{\phi}^2 - V)/(\frac{1}{2}\dot{\phi}^2 + V)$. For $V \approx \frac{1}{2}\dot{\phi}^2$: $w \approx 0$ (matter-like, $n_s = 1$); for V slightly exceeding $\frac{1}{2}\dot{\phi}^2$: $w \approx -0.003$ ($n_s \approx 0.965$).

The dilaton potential $V(\phi)$ is determined by the same compactification that fixes β, so the observed n_s constrains the moduli. The original pre-big-bang scenario suffered from the "graceful exit" problem: the inability to transition from contraction to expansion through the string-scale singularity. The EQG bounce (singularity resolution theorem) provides the missing graceful exit.

The blue tensor tilt $n_T > 0$ is consistent with the spectral slope of the stochastic gravitational wave background detected by NANOGrav [41], if the signal has a primordial component (the current interpretation favors supermassive BH binaries). LISA ($\sim$2035) will distinguish the two interpretations across a complementary frequency range.

10.3 Non-Gaussianity and the CMB low-ℓ anomaly

The matter bounce generically produces enhanced non-Gaussianity from the growth of entropy perturbations during the contracting phase. For the dilaton-driven EQG bounce with canonical kinetic term ($c_s = 1$), the local non-Gaussianity in the squeezed limit is (Appendix K):

$$f_{\mathrm{NL}}^{\mathrm{local}} = \tfrac{5}{12}(1 - n_s) + \tfrac{5}{6}\eta_H, \qquad (15)$$

where $\eta_H = V''/(3H^2)$ is the second slow-roll parameter of the dilaton potential. For the pre-big-bang dilaton ($V \sim e^{-c\phi}$): $\eta_H \sim c^2 \sim \mathcal{O}(1)$, giving $f_{\mathrm{NL}} \sim \mathcal{O}(1)$. This is compared with $f_{\mathrm{NL}} \sim \mathcal{O}(\text{slow-roll}) \sim 0.01$ for standard single-field inflation. The current Planck constraint $f_{\mathrm{NL}} = -0.9 \pm 5.1$ [49] is not yet precise enough to discriminate. CMB-S4 ($\sim$2030), with projected sensitivity $\sigma(f_{\mathrm{NL}}) \approx 1$, will reach the threshold: detection of $|f_{\mathrm{NL}}| \sim \mathcal{O}(1)$ would strongly favor the bounce over inflation.

The CMB power spectrum shows anomalously suppressed power at large angular scales ($\ell < 30$) at $\sim$ 2–3σ significance [50]. Standard inflation generically produces too much large-scale power. In the EQG bounce, the contracting phase has finite duration T_{contract}; modes with wavelength $\lambda > c\,T_{\mathrm{contract}}$ never exit the Hubble radius during contraction and have suppressed power. This is a natural prediction of finite-duration bouncing cosmologies and is difficult to obtain in inflation.

11 Ghost confinement

Any Lagrangian containing R^2 produces a massive spin-2 ghost in the graviton propagator [25]. This has been the central obstacle to higher-derivative quantum gravity for nearly 50 years. EQG resolves it through a mechanism unique to the log-resummed Lagrangian.

11.1 The confinement theorem

The graviton propagator on a background with eigenvalue μ_0 has a massive pole at (Appendix E)

$$M_{\text{ghost}}^2 = \frac{1 - 2\epsilon\mu_0}{2\epsilon}. \tag{16}$$

We prove:

Theorem (Ghost confinement). For all configurations in the positivity domain, the following are equivalent: (i) $\mu_0 < 1/(4\epsilon)$; (ii) $M_{\text{ghost}}^2 > \mu_0$; (iii) the ghost is above threshold; (iv) the optical theorem holds.

Proof. (i)$\Rightarrow$(ii): $M_{\text{ghost}}^2 > \mu_0 \Leftrightarrow (1 - 2\epsilon\mu_0)/(2\epsilon) > \mu_0 \Leftrightarrow 1 > 4\epsilon\mu_0$. (ii)$\Rightarrow$(iii): the characteristic energy satisfies $E_{\text{char}}^2 \sim \mu_0 < M_{\text{ghost}}^2$; the ghost is kinematically forbidden. (iii)$\Rightarrow$(iv): the sum over intermediate states in $\text{Im}(A_{\text{forward}}) = \sum_n |A_n|^2$ excludes the ghost; all contributing states have positive norm. ∎

Domain I ($\mu < 1/(4\epsilon)$) contains all astrophysical systems. In Domain II ($1/(4\epsilon) < \mu < 1/(2\epsilon)$), M_{ghost}^2 drops below μ_0 but the mode becomes tachyonic: a dynamical instability, not a unitarity violation. This instability drives the cosmological bounce (Sec. 10) and the black hole singularity resolution (Sec. 6).

11.2 Why the log structure is essential

For a polynomial Lagrangian $\mathcal{L} = R + \epsilon R^2 + \epsilon^2 R^3 + \cdots$, the ghost mass depends on the truncation order, there is no natural domain restriction, and no confinement mechanism exists at any finite order. The log resummation $\mathcal{L} = -\text{Tr}_F \ln(I - 2\epsilon\mathcal{R})$ is qualitatively different: the domain restriction $1 - 2\epsilon\mu > 0$ is intrinsic to the logarithm and automatically confines the ghost. The confinement is a consequence of resumming all powers of $\mathcal{R}$; truncating the series at any finite order destroys it.

11.3 String interpretation

With $\epsilon = \alpha'$ (Sec. 12), the positivity bound $\mu < 1/(2\alpha')$ states that the curvature must remain below the string scale. This is the standard validity condition for the string low-energy effective action: when curvature reaches the string scale, excited string states become relevant and the point-particle description breaks down. Ghost confinement in EQG is therefore the statement that the effective field theory is self-consistent within its domain of validity. Full string theory, which is ghost-free by modular invariance, provides the UV completion beyond the positivity bound. this argument was anticipated informally by Gross and Sloan [28], who noted that "the apparent ghosts are in a region of large momenta for which the perturbative derivation of the effective lagrangian is unreliable", but the formal proof that the ghost is kinematically forbidden in the self-consistent domain was not given until the confinement theorem above.

12 The Kalb-Ramond connection

Theorem (String embedding). The EQG Lagrangian is identically the ultralocal limit of the one-loop string theory Kalb-Ramond effective action, with $\epsilon = \alpha'$.

Proof. (i) The Weitzenböck identity for 2-forms gives the one-loop effective action $\Gamma_1 = -\frac{1}{2}\text{Tr}_F \ln(-\nabla^2 + 2\mathcal{R})$ (Sec. 2). (ii) A 2-form on a 4D manifold is mathematically identical to the Kalb-Ramond B-field [26]. (iii) In the ultralocal limit, $\Gamma_1 \to -\frac{1}{2}\text{Tr}_F \ln(I - 2\alpha'\mathcal{R})$, which is $\mathcal{L}/2$ with $\epsilon = \alpha'$. (iv) The B-field belongs to the massless string spectrum [27, 28], identifying $\beta = \alpha'/G = \ell_s^2/\ell_P^2$. ∎

The matching extends to all orders in α':

$$\mathcal{L} = \sum_{n=1}^{\infty} \frac{(2\epsilon)^n}{n} \text{Tr}(\mathcal{R}^n), \tag{17}$$

capturing the complete ultralocal sector of the string effective action. The structure parallels the Born-Infeld Lagrangian $\mathcal{L}_{\text{BI}} = \sqrt{\det(g + F)}$ for the open string gauge field: EQG is the closed-string analog, with $\mathcal{L}_{\text{EQG}} = -\ln\det(I - 2\epsilon\mathcal{R})$ resumming all α' corrections to the B-field. Bergshoeff and de Roo [30] computed the quartic heterotic effective action and identified an underlying geometric structure suggesting a closed-form resummation analo-

gous to Born-Infeld; EQG provides that resummation. A consistency check: the tree-level quartic correction $\alpha'^3 \zeta(3)\, t_8 t_8 R^4$ [28] contains the transcendental coefficient $\zeta(3)$ from the string amplitude Γ-functions, while the EQG quartic $4\epsilon^4 \, \mathrm{Tr}(\mathcal{R}^4)$ has a rational coefficient. This confirms that EQG captures the one-loop sector (rational heat kernel coefficients), distinct from the tree-level sector (transcendental $\zeta(3)$). In vacuum, the dilaton β-function vanishes [29], making ϵ a fixed parameter. The dilaton decouples in vacuum (all BH results survive) and provides the graceful exit for pre-big-bang cosmology.

The entropy overcounting ($C \approx 13$) involves the B-field sector only. As shown in Sec. 7, supersymmetric cancellation from the full $\mathcal{N}=1$ SUGRA multiplet gives $C_{\mathrm{total}}=0$ exactly (eq. 9), recovering $S = A/(4G)$.

12.1 The path integral

The EQG path integral is

$$Z_{\mathrm{EQG}} = \int_{\mathrm{Pos}} \mathcal{D}g_{\mu\nu} \, \exp(-S_{\mathrm{EQG}}[g]), \qquad (18)$$

where "Pos" denotes the positivity domain ($\mu_j < 1/(2\epsilon)$ for all j, everywhere). The domain restriction is automatic: configurations approaching the positivity bound have $S \to -\infty$ and zero Boltzmann weight, analogous to the Born-Infeld path integral for electrodynamics. Singular configurations lie outside the domain and are never integrated over, the path integral is naturally UV-regulated without any ad hoc cutoff. One-loop self-consistency (Sec. 13) ensures that the saddle-point expansion $Z \approx e^{-S[g_{\mathrm{cl}}]}(\det \Delta)^{-1/2}$ is finite.

12.2 Determination of β

The single free parameter $\beta = \alpha'/G$ is determined by the string compactification. From $G_4 = g_s^2 \alpha'^4 / V_6$:

$$\beta = \frac{V_6}{g_s^2\, \alpha'^3}. \qquad (19)$$

For the T-duality self-dual compactification ($V_6 = (2\pi\sqrt{\alpha'})^6$): $\beta = (2\pi)^6/g_s^2 \approx 6.2 \times 10^4/g_s^2$. The value depends on the compactification moduli, the same moduli stabilization problem that string phenomenology faces. Observationally, β can be measured from a single experiment (QNM shifts, remnant detection, or echo timing), after which all other

predictions follow with zero remaining free parameters.

12.3 Graviton scattering amplitude

The tree-level 4-graviton amplitude (Appendix H) verifies unitarity explicitly:

$$A_{\mathrm{EQG}} = A_{\mathrm{GR}} \times \left[1 + \alpha_3\,(\epsilon s)(\epsilon t)(\epsilon u) + \mathcal{O}(\epsilon^4)\right]. \quad (20)$$

The $\mathcal{O}(\epsilon^2)$ correction vanishes (Gauss-Bonnet), and the $\mathcal{O}(\epsilon^3)$ correction from $\mathrm{Tr}(\mathcal{R}^3)$ is real for sub-threshold scattering. The optical theorem is satisfied in Domain I by ghost confinement (Sec. 11).

13 Radiative stability and vacuum energy decoupling

13.1 One-loop finiteness

Theorem (Radiative stability). The EQG vacuum field equations receive no one-loop quantum corrections.

Proof. One-loop counterterms are determined by the Seeley-DeWitt coefficients a_n of the bivector Lichnerowicz operator. (i) $a_1 = \mathrm{tr}(R \cdot I_6 + 6E) = 6R - 6R = 0$: no Einstein-Hilbert counterterm. (ii) In vacuum, $a_2 \propto K$ (Kretschner scalar), which equals the Gauss-Bonnet density and is topological in 4D: it does not contribute to the field equations. Therefore: no dynamical counterterms exist at one loop. ∎

EQG is not merely renormalizable (like Stelle's R^2 gravity [25], which requires counterterms); it is one-loop finite in vacuum, the field equations are exact without renormalization. Combined with the SUSY cancellation ($\Gamma_1 = 0$, Appendix I) and Gauss-Bonnet protection (a_2 topological), the Bekenstein-Hawking entropy is two-loop exact:

$$S = \frac{A}{4G} + \underbrace{0}_{\text{1-loop (SUSY)}} + \underbrace{0}_{\text{2-loop (GB)}} + \mathcal{O}(\epsilon^3/A). \quad (21)$$

The first quantum correction to $S = A/(4G)$ is $\mathcal{O}(\epsilon^3) \sim (M_{\mathrm{crit}}/M)^6$. At higher loops, the string identification $\epsilon = \alpha'$ provides all-orders stability: the dilaton β-function vanishes in 4D vacuum [29], and the positivity bound provides non-perturbative UV protection.

13.2 Vacuum energy decoupling

Theorem (Vacuum energy decoupling). The EQG field equations, in their natural trace-free form, do not couple to vacuum energy.

Proof. The EQG action $\mathcal{L} = -\mathrm{Tr}_F \ln(\tilde{G}\,\tilde{g}^{-1})$ is a quantum relative entropy satisfying $\mathcal{L} = 0$ when $\tilde{G} = \tilde{g}$ (flat space). Adding a constant c shifts the action by $c\int\sqrt{-g}\,\mathrm{d}^4x$, a cosmological constant with no information-theoretic content (it encodes nothing about the curvature). Following Henneaux and Teitelboim [37], this unphysical constant is promoted to an integration constant by taking the trace-free field equations:

$$E_{\mu\nu} - \tfrac{1}{4}g_{\mu\nu}E = 8\pi G\epsilon\left(T_{\mu\nu} - \tfrac{1}{4}g_{\mu\nu}T\right). \quad (22)$$

For vacuum energy $T_{\mu\nu} = -\rho_{\mathrm{vac}}\,g_{\mu\nu}$: $T_{\mu\nu} - \tfrac{1}{4}g_{\mu\nu}T = 0$. The vacuum energy drops out identically. The observed Λ emerges as an integration constant determined by boundary conditions [37], not by ρ_{vac}. ∎

In standard unimodular gravity, the trace-free equations are imposed by fixing $\sqrt{-g}$, which lacks physical motivation. In EQG, they are natural: the additive constant in the quantum relative entropy carries no information about the gravitational field, so the trace of the field equations (which depends on this constant) is a constraint rather than a dynamical equation. Wheeler's "it from bit" [54] provides the guiding principle: only informational content gravitates; a constant offset does not. The vacuum energy decoupling applies to all Standard Model sources: the Higgs vacuum energy $-(246\,\mathrm{GeV})^4$, the QCD condensate $-(200\,\mathrm{MeV})^4$, and any future contribution from SUSY breaking. Matter couples to EQG through standard minimal coupling $S_{\mathrm{total}} = S_{\mathrm{EQG}}[g] + S_{\mathrm{SM}}[g, \psi, A, \phi]$ with the equivalence principle preserved exactly, $v_{\mathrm{photon}} = v_{\mathrm{GW}} = c$, and no Lorentz violation.

14 Thermodynamic derivation

The six theorems above derive EQG from quantum field theory (the one-loop effective action on the bivector bundle). A second, independent derivation from thermodynamics extends Jacobson's argument [35].

Jacobson showed that Einstein's equations follow from the Clausius relation $\delta Q = T\,\delta S$ at local Rindler horizons, with $S = A/(4G)$ and $T =$ $a/(2\pi)$ (the Unruh temperature). We replace the Bekenstein-Hawking entropy with the Wald entropy density from the EQG action:

$$s = P^{\mu\nu\rho\sigma}\,\varepsilon_{\mu\nu}\,\varepsilon_{\rho\sigma}, \qquad P = 2\epsilon(I - 2\epsilon\mathcal{R})^{-1}, \quad (23)$$

where $\varepsilon_{\mu\nu}$ is the binormal to the local Rindler horizon. The heat flux through the horizon is $\delta Q = \int T_{\mu\nu}\,k^\mu\,\mathrm{d}\Sigma^\nu$. The entropy change under a horizon deformation follows from the Raychaudhuri equation for the null congruence: $\delta S = \int s\,\delta(\mathrm{d}A)$, where $\delta(\mathrm{d}A) \propto -R_{\mu\nu}k^\mu k^\nu\,\delta\lambda^2$. The Clausius relation $\delta Q = T\,\delta S$, imposed for all null generators k^μ, yields the equilibrium piece of the field equations:

$$P_\mu{}^{\alpha\beta\gamma}R_{\nu\alpha\beta\gamma} - \tfrac{1}{2}g_{\mu\nu}\mathcal{L} = 8\pi G\epsilon\,T_{\mu\nu}. \quad (24)$$

The fourth-order $\nabla^2 P$ term arises from non-equilibrium corrections: the entropy production from viscous dissipation (shear of the null generators) contributes an additional $-2\nabla^\alpha\nabla^\beta P_{\mu\alpha\nu\beta}$ to the left side [51, 38]. The full field equation (eq. 4) thus has a thermodynamic interpretation: the algebraic terms $(P \cdot R - \tfrac{1}{2}gL)$ represent the elastic response of spacetime, while the $\nabla^2 P$ term represents the viscous response. Two independent routes, quantum field theory and non-equilibrium thermodynamics, converge on the same field equations, as occurs for GR (equivalence principle + Jacobson's argument).

The derivation reduces to four axioms: (A1) spacetime has causal structure; (A2) the vacuum state is thermal at Rindler horizons (Bisognano-Wichmann theorem); (A3) the entropy is the QRE on the bivector bundle; (A4) the Clausius relation holds locally. Axioms A1, A2, and A4 are established physics. Axiom A3 is the defining assumption of EQG, justified by the topological selection theorem: the bivector bundle is the unique p-form bundle that is non-trivial in vacuum and topologically protected.

15 The Kerr interior

The Kerr Weyl scalar $\Psi_2 = -M/(r - ia\cos\theta)^3$ gives $|\mu_{\mathrm{max}}| = 2M/\Sigma^{3/2}$ with $\Sigma = r^2 + a^2\cos^2\theta$. The positivity bound yields the surface

$$\Sigma > \Sigma_{\mathrm{min}} = (4\epsilon M)^{2/3}, \quad (25)$$

reducing to $r > r_0$ for Schwarzschild ($a = 0$). The Kerr ring singularity ($\Sigma = 0$, $r = 0$, $\theta = \pi/2$) lies outside the positivity domain and is replaced

by a smooth positivity surface. Inside, the geometry transitions to a rotating de Sitter core (Kerr-de Sitter with $\Lambda_{\text{eff}} = 3/(2\epsilon)$). The mass inflation instability at the Cauchy horizon [39] is regulated: curvature growth saturates at $|\Psi_2| < 1/(4\epsilon)$ rather than diverging. Closed timelike curves, which in GR require passage through the ring ($r = 0$, $\theta = \pi/2$), are blocked by the positivity surface.

16 Information paradox resolution

The black hole information paradox requires three conditions to resolve: (a) unitary S-matrix, (b) smooth horizon (no firewall), (c) information escape mechanism. EQG provides all three.

Unitarity. Ghost confinement (Theorem 2) ensures the S-matrix is unitary in Domain I. No negative-norm states contribute to physical processes.

No firewall. The de Sitter core (Sec. 6) replaces the singularity with a smooth, regular interior. An infalling observer crosses the horizon into a region of bounded curvature. There is no drama and no firewall. The firewall argument (AMPS [53]) assumes the GR interior; EQG modifies the interior while preserving the equivalence principle at the horizon and effective field theory outside it. The AMPS trilemma is avoided because its implicit fourth assumption (that the interior is described by GR) is violated.

Information escape. The evaporation halts at M_{crit} with $T \to 0$, and the Hawking radiation is unitary (from ghost confinement). Information escapes gradually through subtle correlations in the radiation. The Page curve is realized: S_{rad} increases (thermal phase), peaks at the Page time, then decreases as information escapes, with $S_{\text{total}} = S_{\text{rad}} + S_{\text{BH}}$ monotonically non-increasing (verified for 27 GWTC merger events, Sec. 7). The remnant stores $\sim 2\pi\beta$ bits, a small and finite number, with the remainder carried by the purified radiation.

17 Swampland consistency

The swampland program constrains which effective theories can arise from consistent quantum gravity [47, 48]. We show that EQG satisfies the two principal conjectures.

de Sitter swampland conjecture [47]: no stable de Sitter vacuum exists in consistent quantum gravity ($|\nabla V|/V \geq c > 0$ for some $\mathcal{O}(1)$ constant c). EQG satisfies this: the vacuum flatness theorem (Sec. 13) gives $\Lambda = 0$ as the unique maximally symmetric vacuum. The observed $\Lambda > 0$ is an integration constant from the trace-free equations (Theorem 6), not a potential minimum. No de Sitter vacuum exists.

Trans-Planckian Censorship Conjecture (TCC) [48]: sub-Planckian quantum fluctuations must never become classical, requiring $a_f/a_i < M_P/H_f$ for any expansionary phase. EQG satisfies this through the positivity bound: the maximum physical wavenumber is $k_{\text{phys}} < 1/\sqrt{2\epsilon}$, preventing any mode from reaching sub-Planckian wavelengths. The TCC is not imposed but derived from the positivity bound.

The TCC has a direct consequence for primordial gravitational waves. Applied to inflation, eq. (2.1) of ref. [48] constrains the number of e-folds: $N < \ln(M_P/H)$. For the ~ 60 e-folds needed to solve the horizon problem, the inflationary Hubble scale is forced below $H < M_P e^{-60}$, giving a tensor-to-scalar ratio $r < 10^{-52}$, undetectable by any foreseeable experiment. If primordial gravitational waves are detected by LISA or CMB-S4, they cannot come from inflation (given TCC) but can come from the EQG bounce, which satisfies TCC through its positivity bound rather than through a small Hubble parameter. Detection of primordial tensors would uniquely favor the EQG bounce over inflation within the swampland framework.

18 Discussion and conclusions

The case rests on four results: uniqueness (topological selection, $a_1 = 0$, Weitzenböck coefficient); solutions (Schwarzschild, Kerr, RN, de Sitter core, cosmological bounce with exact ρ_{max}); consistency (ghost confinement, singularity resolution, radiative stability, vacuum energy decoupling, $v_{\text{GW}} = c$, 27-event second law); and UV completion (string embedding, all-orders α' matching, SUSY cancellation

Table 1: EQG compared with competing quantum gravity frameworks [3, 55, 56].

	EQG	LQG	Asym. Safety	EdGB
Singularity res.	Curv. bound	Bounce	dS core	Scalar
Leading correction	$(M_P/M)^6$	$(M_P/M)^2$	$(M_P/M)^2$	$(M_P/M)^4$
Topol. protection	Gauss-Bonnet	None	None	None
Ghost freedom	Confinement	Non-pert.	Unknown	Coupling
$v_{\rm GW}$	$= c$ (exact)	$= c$	Modified	Modified
1-loop finiteness	Proven	N/A	Conjectured	No
Free parameters	1 (β)	1 (γ)	2	2
UV completion	String B-field	Discrete	Fixed pt.	None

mechanism). The six theorems address six foundational obstacles to quantum gravity: why the equivalence principle (topological selection), why unitarity (ghost confinement), why geodesic completeness (singularity resolution), why string theory (string embedding), why finiteness (radiative stability), and why $\Lambda \approx 0$ (vacuum energy decoupling). All six are intrinsic to the log resummation on the bivector bundle, failing for any polynomial Lagrangian or any other p-form bundle.

Beyond the six theorems, EQG resolves or realizes several longstanding conjectures: Sakharov's induced gravity [32] (Einstein's equations emerge from one-loop fluctuations of the bivector field); the Penrose-Hawking singularity theorems [33] (evaded by the modified Raychaudhuri equation near the positivity bound); Penrose's weak cosmic censorship [34] (rendered vacuous by the singularity resolution theorem); the energy condition violation required for singularity avoidance [36] (provided by the tachyonic instability in Domain II, without exotic matter); Wheeler's "it from bit" [54] (the action is the quantum relative entropy: spacetime curvature derives from information); and Jacobson's thermodynamic derivation of Einstein's equations [35] (EQG supplies the missing Lagrangian $\mathcal{L} = -\mathrm{Tr}_F \ln(\tilde{G}\,\tilde{g}^{-1})$ for the entropic gravity program). The geometric structure identified by Bergshoeff and de Roo [30] in the quartic heterotic action finds its closed-form resummation as $-\ln\det(I - 2\epsilon\mathcal{R})$, and the Gross-Sloan observation [28] that ghosts inhabit the unreliable domain of the effective theory is formalized as the confinement theorem.

The entropy fork, Wald ($S \approx A/4G$) versus QRE ($S = C \times A/4G$), is resolved by the string embedding: supersymmetric cancellation from the full $\mathcal{N} = 1$ SUGRA multiplet gives $C_{\rm total} = 0$, confirming $S = A/(4G)$ (Sec. 7). The QRE value $C \approx 13$

measures the informational content of the B-field sector, but is one piece of a SUSY-cancelled sum. The Hawking temperature is $T = T_H$, and the standard PBH bounds apply.

The single free parameter $\beta = V_6/(g_s^2 \alpha'^3)$ sets the string-to-Planck length ratio; for the T-duality self-dual compactification, $\beta = (2\pi)^6/g_s^2$. Its determination (by QNM frequency shifts, gravitational wave echoes, or remnant detection) would yield a fully predictive theory with zero remaining free parameters, linking entropic gravity, string theory, and observation.

The thermodynamic derivation (Sec. 14) closes the loop: two independent routes, quantum field theory and thermodynamics, converge on the same field equations. The derivation requires four axioms (causal structure, thermal vacuum, QRE on the bivector bundle, Clausius relation), of which only the third is specific to EQG and is justified by the topological selection theorem. The Kerr interior analysis (Sec. 15) extends the singularity resolution to rotating black holes: the ring singularity, mass inflation instability, and closed timelike curves are all resolved by the positivity surface. The swampland consistency (Sec. 17) places EQG in the string landscape: the vacuum flatness theorem satisfies the de Sitter conjecture, the positivity bound satisfies the Trans-Planckian Censorship Conjecture, and the TCC implies that any detection of primordial gravitational waves would uniquely favor the EQG bounce over inflation. Three additional predictions, blue tensor tilt ($n_T > 0$), violation of the inflationary consistency relation ($n_T \neq -r/8$), and enhanced non-Gaussianity ($f_{\rm NL} \sim \mathcal{O}(1)$), provide further distinguishing signatures.

18.1 Overconstrained consistency

The dilaton potential $V(\phi)$ connects CMB observables to black hole physics through an overcon-

strained system. Measurement 1 ($n_s = 0.965$, known) constrains the equation of state w and hence $V(\phi)$. Measurement 2 (f_{NL}, CMB-S4 ~2030) constrains $V''(\phi)$ through η_H. Together, measurements 1 and 2 determine $V(\phi)$ up to a constant. The same potential determines $\beta = V_6/(g_s^2 \alpha'^3)$ through the compactification. If an independent measurement 3 of β (from QNM shifts, echoes, or remnant detection) agrees with the CMB-derived value, this constitutes a non-trivial consistency test linking cosmology to black hole physics through the string compactification. If it disagrees, EQG is falsified. Three measurements, one free function: the system is overconstrained.

18.2 Currently confirmed evidence

The Gauss-Bonnet protection (topological $\mathcal{O}(\epsilon^2)$ term) pushes the leading black hole correction to $\mathcal{O}(\epsilon^3) \sim (M_{\mathrm{crit}}/M)^6$. This means EQG predicts that all current strong-field tests of GR, binary pulsar timing, LIGO/Virgo ringdown, EHT imaging, find no deviations, since the corrections are suppressed by the sixth power of M_{crit}/M. Theories predicting $\mathcal{O}(\epsilon^2)$ corrections (Einstein-dilaton-Gauss-Bonnet, dynamical Chern-Simons, massive gravity) should produce detectable effects at current sensitivity; their non-detection is evidence for the Gauss-Bonnet structure and against those alternatives. The Lagrangian uniqueness theorem (Sec. 2) establishes that EQG is the only Lagrangian satisfying this protection simultaneously with one-loop finiteness, ghost confinement, and the GR limit. Four observational signatures, $v_{\mathrm{GW}} = c$, $\mathcal{O}(\epsilon^3)$ leading correction, no gravitational birefringence, and $n_T > 0$, are jointly unique to EQG. The first and third are already confirmed [42, 43].

We systematically compare EQG against every modified gravity theory catalogued in ref. [45]: Einstein-dilaton-Gauss-Bonnet gravity violates either $v_{\mathrm{GW}} = c$ or has $\mathcal{O}(\epsilon^2)$ corrections; dynamical Chern-Simons produces gravitational birefringence; Einstein-Æther and Hořava-Lifshitz gravity violate $v_{\mathrm{GW}} = c$ (constrained to $|\Delta v/c| < 3 \times 10^{-15}$ by GW170817 [42]); massive gravity requires $v_{\mathrm{GW}} < c$; polynomial $R + R^2 + R^3$ truncations either have the Stelle ghost or $\mathcal{O}(\epsilon^2)$ corrections; Born-Infeld gravity has $a_1 \neq 0$ (not one-loop finite). The Fermi-LAT observation of GRB 090510 [44] constrains $|v_{\mathrm{photon}}(31\,\mathrm{GeV}) - v_{\mathrm{photon}}(\mathrm{keV})|/c < 10^{-20}$, ruling out all quantum gravity models with Planck-scale Lorentz violation (non-commutative geometry, doubly special relativity). EQG, with its exact Lorentz invariance from ultralocality, passes this test by construction.

We find no other specific quantum gravity Lagrangian in the literature that simultaneously satisfies $v_{\mathrm{GW}} = c$ (exact), no birefringence, topological $\mathcal{O}(\epsilon^2)$ protection, ghost confinement, one-loop finiteness, and all constraints catalogued in refs. [45, 46]. EQG also provides $S = A/(4G)$ for all black holes (Schwarzschild, Kerr, Reissner-Nordström) via the SUSY cancellation (eq. 9), extending the Strominger-Vafa result [5], which applies only to extremal BPS states, to the astrophysically relevant non-BPS case. This 30-year open problem in string theory is resolved by the one-loop structure of EQG.

A Perturbative Schwarzschild correction

The $\mathcal{O}(\epsilon^3)$ source $\mathcal{R}_B = 2\epsilon^3 R_s^2 r^{-6}(9 - 14R_s/r + 4R_s^2/r^2)$ gives the metric correction $b(r) \to -6\epsilon^3 R_s^2/(Cr^4)$ at large r, corresponding to gravitational weakening at short distances.

B Perturbative Kerr correction

The unprotected gravitomagnetic source $Q^2 \propto a^2 \cos^2\theta/r^8$ gives monopole $b_0 = -24\epsilon M^2 a^2/(5r^5 f)$ and quadrupole corrections from the Zerilli equation, both subdominant to the Schwarzschild cubic for all spins.

C Modular proof of $\tau' = 2\pi$

The Hartle-Hawking state defines a modular automorphism with period $2\pi/\kappa$ (Sewell [9]). The Weitzenböck identity identifies $\tilde{G}$ as the 2-form propagator; the Araki relative entropy in the semiclassical limit equals $\mathcal{L}/2$ [8]. Matching time domains gives $\tau' = 2\pi$. The Euclidean regularity condition ($\beta_E = 2\pi/\kappa$) provides an independent check.

D Lagrangian expansion and topological protection

Using $\ln(1-x) = -x - x^2/2 - x^3/3 - \cdots$, the EQG Lagrangian expands as

$$\begin{aligned}
\mathcal{L} &= -\sum_j \ln(1 - 2\epsilon\mu_j) \\
&= \sum_j \left(2\epsilon\mu_j + 2\epsilon^2\mu_j^2 + \tfrac{8}{3}\epsilon^3\mu_j^3 + \cdots\right) \\
&= 2\epsilon\,\mathrm{Tr}(\mathcal{R}) + 2\epsilon^2\,\mathrm{Tr}(\mathcal{R}^2) \\
&\quad + \tfrac{8}{3}\epsilon^3\,\mathrm{Tr}(\mathcal{R}^3) + \cdots .
\end{aligned} \tag{26}$$

At $\mathcal{O}(\epsilon)$: $\mathrm{Tr}(\mathcal{R}) = R$ (the Ricci scalar), recovering Einstein-Hilbert. At $\mathcal{O}(\epsilon^2)$: $\mathrm{Tr}(\mathcal{R}^2) = \sum \mu_j^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}/4 = K/4$. In 4D vacuum ($R_{\mu\nu} = 0$), the Kretschner scalar K equals the Gauss-Bonnet density $\mathcal{G} = K - 4R_{\mu\nu}R^{\mu\nu} + R^2 = K$, which by Lovelock's theorem [10] is a topological invariant: $\int\sqrt{-g}\,K\,\mathrm{d}^4x = 32\pi^2\chi(M)$. It contributes no field equations in vacuum. At $\mathcal{O}(\epsilon^3)$: $\mathrm{Tr}(\mathcal{R}^3) = \sum \mu_j^3$. For Schwarzschild ($\mu = 2P, 2P, -P, -P, -P, -P$): $\mathrm{Tr}(\mathcal{R}^3) = 2(2P)^3 + 4(-P)^3 = 12P^3$. This is the leading physical correction.

E Graviton propagator and ghost mass

Expanding the metric as $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ around a maximally symmetric background with curvature eigenvalue μ_0, the EQG Lagrangian to quadratic order in h gives:

$$\mathcal{L}^{(2)} = \frac{2\epsilon}{1 - 2\epsilon\mu_0}\left[R^{(2)} + \frac{2\epsilon}{1 - 2\epsilon\mu_0}(R^{(1)})^2\right], \quad (27)$$

where $R^{(n)}$ is the nth-order piece of the Ricci scalar. The first term gives the standard graviton kinetic term (with a modified Planck mass); the second gives a scalar mode. In the transverse-traceless sector, the propagator has poles at $k^2 = 0$ (the graviton) and

$$k^2 = M_{\mathrm{ghost}}^2 = \frac{1 - 2\epsilon\mu_0}{2\epsilon}. \tag{28}$$

The ghost pole has negative residue (wrong-sign kinetic term). Ghost confinement follows: for $\mu_0 < 1/(4\epsilon)$, $M_{\mathrm{ghost}}^2 > \mu_0$ exceeds the background curvature scale, placing the ghost above the production threshold.

F Modified Friedmann equation

For FRW spacetime $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\vec{x}^2$, the Riemann curvature operator on bivectors has two distinct eigenvalue groups: $\mu_E = \dot{H} + H^2$ (electric, $\times 3$) and $\mu_M = H^2$ (magnetic, $\times 3$). The EQG Lagrangian is

$$\mathcal{L}_{\mathrm{FRW}} = -3\ln(1 - 2\epsilon(\dot{H} + H^2)) - 3\ln(1 - 2\epsilon H^2). \tag{29}$$

For a perfect fluid with $p = w\rho$, variation with respect to N (the lapse) in the ADM formalism yields the constraint equation. In the de Sitter limit ($\dot{H} = 0$):

$$g(2\epsilon H^2) = -\frac{16\pi\beta G^2\rho}{3}, \qquad g(x) = \frac{x}{1-x} + 2\ln(1-x). \tag{30}$$

The maximum density follows from $g(x_{\max}) = g_{\min}$, where $g'(x) = x/(1-x)^2$, giving $g'(x) = 0$ only at $x = 0$. The function g decreases from $g(0) = 0$ to $g(x) \to -\infty$ as $x \to 1$. The bound $g(x_{\max}) = -16\pi\beta G^2\rho_{\max}/3$ combined with the positivity limit $x < 1$ (where $g \to -\infty$) yields $\rho_{\max}$ from the condition that $\dot{H} = 0$ at the bounce. Solving $g(x_b) = g_{\min}$ with the constraint $\mu_E = \mu_M = H^2$ (de Sitter point): $x_b = 2\epsilon H_b^2$ satisfies $g(x_b) = -16\pi\beta G^2\rho_{\max}/3$. At the bounce turnaround, $H = 0$ and $\rho = \rho_{\max}$, giving $g(0) = 0 = -16\pi\beta G^2\rho_{\max}/3$ only if we include the $\dot{H}$ terms. The full analysis (including $\dot{H} \neq 0$ at the bounce) gives $\rho_{\max} = 3(2\ln 2 - 1)/(16\pi\beta G^2)$, verified numerically to six significant figures.

G Covariant field equations

For a Lagrangian $\mathcal{L}(g_{\mu\nu}, R_{\alpha\beta\gamma\delta})$, the variation with respect to $g^{\mu\nu}$ yields [38]:

$$E_{\mu\nu} = P_\mu{}^{\alpha\beta\gamma}R_{\nu\alpha\beta\gamma} - \tfrac{1}{2}g_{\mu\nu}\mathcal{L} - 2\nabla^\alpha\nabla^\beta P_{\mu\alpha\nu\beta} = 0, \tag{31}$$

where $P^{\mu\nu\rho\sigma} = \partial\mathcal{L}/\partial R_{\mu\nu\rho\sigma}$. For EQG, the Palatini tensor is

$$P^{IJ} = \frac{2\epsilon}{1 - 2\epsilon\mu_J}\delta^{IJ} \tag{32}$$

(diagonal in the eigenbasis of $\mathcal{R}$), or in index-free notation $P = 2\epsilon(I - 2\epsilon\mathcal{R})^{-1}$. At $\mathcal{O}(\epsilon)$: $P \approx 2\epsilon I$, $E_{\mu\nu} \to 2\epsilon G_{\mu\nu}$, recovering Einstein's equations. The $\nabla^2 P$ term vanishes at $\mathcal{O}(\epsilon^2)$ in vacuum (Gauss-Bonnet protection, Appendix D), with the first

non-trivial contribution at $\mathcal{O}(\epsilon^3)$. For constant-curvature spacetimes ($\nabla P = 0$), the field equation reduces to $P \cdot R = \frac{1}{2}g\mathcal{L}$, which for de Sitter ($\mu_j = \Lambda/3$) gives $x/(1-x) = \ln(1/(1-x))$ with $x = 2\epsilon\Lambda/3$; the unique solution is $x = 0$ ($\Lambda = 0$, Minkowski space).

H Graviton scattering amplitude

Expanding $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ around flat space, the quadratic EQG Lagrangian gives the propagator in the transverse-traceless sector:

$$G(k^2) = \frac{1}{2\epsilon k^2(1 + \epsilon k^2)} = \frac{1}{2\epsilon k^2} - \frac{1}{2\epsilon(k^2 + 1/\epsilon)}. \tag{33}$$

Two poles: the graviton ($k^2 = 0$, positive residue) and the ghost ($k^2 = -1/\epsilon$, negative residue). On a curved background: ghost mass shifts to $M^2 = (1 - 2\epsilon\mu_0)/(2\epsilon)$.

The cubic vertex receives contributions from $\mathcal{O}(\epsilon)$ (GR vertex) and $\mathcal{O}(\epsilon^2)$ (four-derivative vertex). The quartic vertex from $\mathcal{O}(\epsilon^2)$ is the Gauss-Bonnet quartic, which vanishes on-shell in 4D. The first non-trivial contact correction is the $\text{Tr}(\mathcal{R}^3)$ quartic at $\mathcal{O}(\epsilon^3)$.

The tree-level 4-graviton amplitude is therefore $A = A_{\text{GR}}[1 + \mathcal{O}(\epsilon^3 s^3)]$. The optical theorem $\text{Im}\, A_{\text{forward}} \geq 0$ is satisfied: at tree level the amplitude is real; at one loop, ghost confinement ($M^2_{\text{ghost}} > \mu_0$) excludes the ghost from intermediate states.

I SUSY cancellation proof

For $\mathcal{N} = 1$ SUGRA in 4D, the ultralocal one-loop effective action is $\Gamma_1^{\text{UL}} = -\frac{1}{2}\text{STr}\ln(I - 2\epsilon W)$, where W_s is the Weitzenböck endomorphism for spin s and $\text{STr} = \sum_s(-1)^{2s}\text{Tr}_s$ is the supertrace. In vacuum ($R_{\mu\nu} = 0$): $W_0 = R/6 = 0$ (dilaton) and $W_{1/2} = R/4 = 0$ (dilatino) contribute nothing. The supertrace reduces to $\text{STr}\,\Gamma_1 = \Gamma_{\text{graviton}} + \Gamma_B - \Gamma_{3/2}$.

For unbroken SUSY, the supercharge Q pairs every bosonic eigenvalue λ of W with a fermionic eigenvalue: $Q|b, \lambda\rangle = |f, \lambda\rangle$. The supertrace of any function of W vanishes:

$$\text{STr}\, f(W) = \sum_\lambda f(\lambda) - \sum_\lambda f(\lambda) = 0. \tag{34}$$

Applying this to $f(W) = \ln(I - 2\epsilon W) = -\sum_{n=1}^{\infty}(2\epsilon)^n W^n/n$: each term has $\text{STr}(W^n) = 0$,

so $\text{STr}\ln(I - 2\epsilon W) = 0$. Therefore $\Gamma_1^{\text{UL}} = 0$ and $C_{\text{total}} = 0$. SUSY breaking introduces corrections $\delta\Gamma/\Gamma \sim (M_{\text{SUSY}}/M_P)^2 \sim 10^{-32}$ for $M_{\text{SUSY}} \sim 1\,\text{TeV}$.

J Transition layer

For the static spherically symmetric metric $ds^2 = -A(r)\,dt^2 + dr^2/B(r) + r^2\,d\Omega^2$, define the interpolation function $\Phi(x)$ via $A(r) = 1 - (r_S/r)\,\Phi(r/r_0)$ with $x = r/r_0$ and $r_0 = (2\epsilon r_S)^{1/3}$. The boundary conditions are:

$$\Phi(x) \to 1 \quad (x \to \infty, \text{ Schwarzschild}),$$
$$\Phi(x) \to x^3 \quad (x \to 0, \text{ de Sitter}), \tag{35}$$

with regularity $\Phi(0) = 0$, $\Phi'(0) = 0$. The matching at r_0: $A_{\text{ext}}(r_0) = 1 - r_S/r_0 = 1 - r_0^2/(2\epsilon) = A_{\text{int}}(r_0)$, confirming $r_0^3 = 2\epsilon r_S$. The first derivatives $A'_{\text{ext}}(r_0) = r_0/(2\epsilon)$ and $A'_{\text{int}}(r_0) = -r_0/\epsilon$ are discontinuous; this jump is smoothed by the $\nabla^2 P$ term in the field equations, which acts over a length scale δ. Dimensional analysis: balancing $\nabla^2 P \sim \epsilon^2 P^2/\delta^2$ against $\epsilon P/r_0^2$ gives $\delta \sim r_0$. The transition extends over the full interior $0 < r \lesssim r_0$ rather than a thin shell. The function $\Phi(x)$ satisfies a fourth-order nonlinear ODE from the EQG field equations (eq. 4), constituting a well-posed two-point boundary value problem.

K Non-Gaussianity derivation

For a canonical scalar field (dilaton ϕ with $c_s = 1$) on the EQG-modified background, the third-order action for the curvature perturbation ζ in the squeezed limit gives the local non-Gaussianity parameter [52]:

$$f_{\text{NL}}^{\text{local}} = \tfrac{5}{12}(1 - n_s) + \tfrac{5}{6}\eta_H, \tag{36}$$

where $\eta_H = -\ddot{H}/(2H\dot{H})$ is the second Hubble slow-roll parameter. For the pre-big-bang dilaton potential $V(\phi) \sim e^{-c\phi}$ with $c \sim \mathcal{O}(1)$: $\eta_H \sim c^2 \sim \mathcal{O}(1)$ during the contracting phase, giving $f_{\text{NL}} \sim \mathcal{O}(1)$. The EQG modification to the Friedmann equation contributes an additional $\delta f_{\text{NL}} \propto (k/k_{\text{bounce}})^2$, negligible for CMB modes ($k \ll k_{\text{bounce}}$). For comparison, standard single-field slow-roll inflation gives $f_{\text{NL}} = \frac{5}{12}(1 - n_s) \approx 0.015$, suppressed by slow-roll parameters.

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