LLM-Driven Algorithm Design for Quantum Circuit Synthesis based on Binary Decision Diagrams

Abstract

Quantum circuits are central to implementing quantum algorithms on quantum devices, where quantum gates must be reversible. Many quantum algorithms rely on Boolean functions, which must therefore be implemented reversibly within quantum circuits. Reversible circuit synthesis provides a way to translate such Boolean functions into reversible circuits. Binary decision diagrams (BDDs) offer a scalable approach to this task, but the resulting BDDs and circuits depend heavily on variable ordering. Existing ordering heuristics commonly minimize BDD size because it is closely tied to the circuit size. However, BDD size is an imperfect proxy for the quantum cost of the synthesized circuit (QCC). We propose `QuantumEvo`, an evolutionary framework that uses an LLM as a heuristic generator for QCC-aware BDD variable ordering. Instead of predicting orderings directly, `QuantumEvo` searches over ordering heuristics initialized from multiple heuristic families. Candidate heuristics directly manipulate variable orderings using standard BDD operations and are selected by downstream QCC. The discovered heuristic, HGA-QE, modifies the sifting step inside a genetic algorithm so that the procedure is better aligned with QCC. Across the benchmark set, HGA-QE is strictly best on 13.5% of functions and achieves a 70.9% tie-or-win rate against the per-function best baseline.

1 Introduction

Quantum computing has attracted increasing attention with advances in both quantum algorithms and quantum hardware (Fedorov et al., 2022; Blekos et al., 2024; Bluvstein et al., 2024). To run a quantum algorithm on a quantum device, its operations must be expressed as a quantum circuit, i.e., a cascade of quantum gates. Since quantum operations are unitary, quantum circuits are *reversible* in nature. Many quantum algorithms contain Boolean components, such as oracle functions (Grover, 1996; Simon, 1997) and arithmetic subroutines (Vedral et al., 1996). To implement such components in a quantum circuit, a Boolean function must be embedded into a reversible transformation. Reversible circuit synthesis addresses this task by translating Boolean functions into reversible circuits that can be mapped to a target gate library (Shende et al., 2003; Wille et al., 2016). The quality of a synthesized circuit is often evaluated by its quantum cost, which reflects the cost of implementing the circuit under a quantum gate library. We refer to this metric as the quantum cost of the synthesized circuit (QCC).

Several approaches have been developed for reversible circuit synthesis, including transformation-based (Miller et al., 2003) and cycle-based (Saeedi et al., 2010). Such methods can produce high-quality circuits for small functions, but their scalability is limited as the number of inputs grows. A BDD-based synthesis aims to improve scalability by representing a Boolean function as a binary decision diagram (BDD) and translating the resulting BDD structure into a reversible circuit (Wille et al., 2009). Since

variable ordering strongly affects the resulting BDD, and the BDD size provides an upper bound on the circuit size, minimizing the BDD size is a common objective for ordering methods. Previous studies on variable ordering methods, including metaheuristics (Bollig et al., 1995; Awad et al., 2022) and learning-based methods (Miao et al., 2026), have used the BDD size as an objective. However, BDD size is an imperfect proxy for QCC. A smaller BDD does not necessarily imply a lower QCC. This motivates the search for BDD variable ordering heuristics that directly reduce downstream QCC.

We formulate this as a heuristic design problem. The goal is to discover a reusable BDD variable ordering heuristic that maps each Boolean function to a variable ordering with low QCC. Recent LLM-based heuristic design methods use LLMs to explore a broader space of heuristic programs beyond a predefined set (Novikov et al., 2025; Ye et al., 2024; Liu et al., 2026; Huang et al., 2026). Building on this idea, we propose `QuantumEvo`[1], an evolutionary framework that uses an LLM as a heuristic generator. It initializes the search from multiple heuristic families and evaluates executable heuristics that manipulate variable ordering using standard BDD operations.

Our main contributions are as follows: (1) We formulate QCC-aware BDD variable ordering as a heuristic design problem, where candidates are selected by downstream QCC rather than BDD size; (2) `QuantumEvo` discovers a novel hybrid genetic algorithm, which replaces standard sifting inside a genetic algorithm with a targeted sifting procedure better aligned with QCC, achieving strict-best performance on 13.5% of functions and a 70.9% tie-or-win rate against the per-function best baseline across the benchmark set; (3) `QuantumEvo` provides an interpretable heuristic design process by producing heuristic programs whose evolution can be traced and analyzed, rather than directly returning predicted variable orders; and (4) our analysis provides empirical insights into framework design, showing that QCC-based fitness and diverse heuristic initialization lead to stronger discovered heuristics.

2 Background and Related Work

This section reviews the background and related work relevant to our approach.

2.1 Reversible Logic and Circuit Synthesis

A Boolean function $f : \mathbb{B}^n \to \mathbb{B}^m$ maps n binary input bits to m binary output bits, where $\mathbb{B} = \{0, 1\}$. A reversible function is a bijection with the same number of input and output bits. While general Boolean functions are not necessarily bijective, they can be embedded into reversible functions by introducing ancilla inputs and garbage outputs (Wille & Drechsler, 2010). Due to its information-preserving property, reversible logic is relevant to a range of applications, most notably quantum computing, low power design, optical computing, and nanotechnologies (Landauer, 1961; Saeedi & Markov, 2013; Cuykendall & Andersen, 1987; Merkle, 1993).

Reversible circuit synthesis realizes Boolean functions as circuits over a gate library. We focus on the widely used *NCT library*, which comprises NOT, CNOT, and Toffoli gates; see Section A.1. Each gate in the NCT library with $k \in \{0, 1, 2\}$ control bits is assigned a QCC of $\text{Cost}(k) = 2k^2 - 2k + 1$. based on its decomposition into elementary quantum gates, following Szyprowski & Kerntopf (2013). The total QCC of a circuit is the sum of its gate costs.

Before synthesis, a Boolean function must be represented in a form suitable for manipulation. Although a truth table provides a canonical representation of Boolean functions by enumerating all 2^n rows, they become impractical at scale due to their exponential size. BDDs provide compact intermediate

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[1]An anonymized implementation is available at `https://anonymous.4open.science/r/QuantumEvo-Circuit`.

(a) BDD under $\pi_1 = \{x_3, x_2, x_1, x_4\}$

(b) BDD under $\pi_2 = \{x_4, x_2, x_1, x_3\}$

(c) Reversible circuit for (a)

(d) Reversible circuit for (b)

Figure 1: Impact of variable ordering on QCC for the function `sf_232`. Solid/dashed edges denote 1/0-branches. BDD size is 8 for both orderings, whereas QCC is 43 for π_1 and 36 for π_2. Here, $y = x_2 \oplus x_3 \oplus x_4 \oplus x_1 \cdot (x_2 \oplus x_3 \oplus x_2 x_3 \oplus x_2 x_4)$

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representations for scalable reversible circuit synthesis (Wille et al., 2009; Soeken et al., 2016). Following Wille et al. (2009), the synthesis procedure considered in this work constructs reversible circuits by replacing each non-terminal BDD node with a cascade of reversible gates.

2.2 Binary Decision Diagrams and Variable Ordering

A BDD is a directed acyclic graph representation of a Boolean function (Bryant, 1986). Each non-terminal node is labeled by one variable and has two branches pointing to terminal or non-terminal child nodes. When the same subfunction appears in multiple places, it is represented once and reused as a shared subgraph. The variable ordering is important when constructing a BDD, as it strongly affects the size of the resulting graph. The BDD size is typically measured by the number of non-terminal nodes, and finding an optimal ordering that minimizes this size is $\mathcal{NP}$-complete (Bollig & Wegener, 1996). Although the BDD size is a natural proxy for the QCC because the size of the resulting circuit is bounded by the BDD size (Wille et al., 2009), it still remains an indirect proxy. Figure 1 shows that two orderings of the same function can produce BDDs with the same size but different QCCs.

Prior work on BDD variable ordering includes exact algorithms (Friedman & Supowit, 1987; Jeong et al., 1993), heuristic algorithms such as sifting (Rudell, 1993; Panda et al., 1994; Panda & Somenzi, 1995; Meinel et al., 1997), and metaheuristics including genetic algorithms (GA), simulated annealing (SA), and swarm-based search (Drechsler et al., 1996; Bollig et al., 1995; Awad et al., 2022). More recently, BDD2Seq (Miao et al., 2026) introduced a learning-based approach that trains a graph-to-sequence model using supervised learning to predict the orderings. However, all of these methods primarily select the orderings based on BDD size, while downstream QCC is observed only after synthesis. This motivates a QCC-aware objective that optimizes the orderings by the QCC rather than by the BDD size.

Figure 2: Overview of the `QuantumEvo` framework. Each individual heuristic reorders the initial variable order of BDD, and the resulting circuit is evaluated by QCC.

2.3 Related Work

We review the LLM-based approaches related to our work. The first line examines how LLMs have been used in quantum-computing workflows. Recent Quantum–LLM studies address broad problem settings within the quantum-computing workflow. Some works evaluate LLMs on generating quantum code from textual task prompts (Vishwakarma et al., 2024; Guo et al., 2025). Others fine-tune LLMs to generate parameterized quantum circuits for quantum optimization tasks (Jern et al., 2025). QCircuitBench (Yang et al., 2026) benchmarks LLMs on quantum algorithm design tasks, including oracle construction and the implementation of standard quantum algorithms. Oracle construction is related to our setting because it often requires implementing Boolean functions as reversible circuits. However, QCircuitBench focuses mainly on textbook-level or algorithm-specific oracles, whereas `QuantumEvo` discovers BDD variable ordering heuristics for synthesizing large Boolean functions under downstream QCC as the objective.

The second covers LLM-based algorithm design, which provides the methodological basis for our approach. A complementary line of work uses LLMs to generate heuristic programs rather than predict solutions directly. FunSearch (Romera-Paredes et al., 2024) introduced LLM-guided algorithm search for mathematical and heuristic design, and ReEvo (Ye et al., 2024) extended it with reflective evolution for combinatorial optimization. Recent frameworks have further advanced LLM-based algorithm design (Zheng et al., 2025; Novikov et al., 2025; Ye et al., 2025; Yao et al., 2025; Dat et al., 2025; Liu et al., 2026; Huang et al., 2026). This paradigm has also been applied to domain-specific heuristic design (Zhao et al., 2026; Shi & Zhen, 2026; Wang et al., 2026). `QuantumEvo` extends this line of work to BDD variable ordering in reversible circuit synthesis.

3 Methodology

This section describes our methodology. We first define the objective used to evaluate BDD variable orderings in terms of QCC. We then introduce `QuantumEvo`, an LLM-driven evolutionary framework for discovering BDD variable ordering heuristics. Figure 2 gives an overview of the framework.

3.1 Problem Formulation

Given a Boolean function $f : \mathbb{B}^n \to \mathbb{B}^m$, let $\mathcal{H}$ denote the search space of candidate heuristics for BDD variable ordering. An initial BDD $G_f^{\pi_0}$ is first constructed under an initial variable ordering π_0. A heuristic $h \in \mathcal{H}$ then reorders the BDD and produces a variable ordering π_h for f. The reordered BDD $G_f^{\pi_h}$ is synthesized into a reversible circuit by a fixed BDD-based synthesis procedure $\mathcal{S}$ following Wille et al. (2009), as implemented in RevKit (Soeken et al., 2012). The resulting circuit is denoted by $C_{h,f} = \mathcal{S}\left(G_f^{\pi_h}\right)$. Let $q(C)$ denote the QCC of a circuit C. For the given Boolean function f, the goal is to find a heuristic that minimizes the QCC of the synthesized circuit within a time budget:

$$\begin{aligned} h^* = \arg\min_{h \in \mathcal{H}} \quad & q(C_{h,f}) \\ \text{s.t.} \quad & t_{\mathrm{syn}}(h, f) \le \tau_U, \end{aligned} \tag{1}$$

where $t_{\mathrm{syn}}(h, f)$ denotes the total time required to construct the BDD for f using the ordering produced by heuristic h and synthesize the resulting circuit, and τ_U is the time budget.

3.2 QuantumEvo: LLM-Based Evolutionary Heuristic Design

`QuantumEvo` follows the general idea of LLM-based algorithm design, where LLMs generate heuristic programs that are improved through iterative evaluation and feedback (Ye et al., 2024; Novikov et al., 2025; Liu et al., 2026). Starting from seed heuristics, an LLM proposes new heuristics by revising existing ones, guided by natural-language feedback on previous fitness results. We instantiate this paradigm for BDD variable ordering through three design insights that shape the search process: how individual heuristics are represented, how the initial heuristics are constructed, and how the fitness function is defined.

Individual representation Each individual is represented as a `C` function linked with CUDD (Colorado University Decision Diagram), a widely used `C` library for constructing and manipulating decision diagrams. This allows generated heuristics to operate directly on the BDD manager, avoiding the need to implement low level BDD operations from scratch. The relevant CUDD API calls are listed in Section B.4.

Initial heuristics construction The initial population is seeded using three established BDD variable ordering methods: sifting (Rudell, 1993), GA (Drechsler et al., 1996), and SA (Bollig et al., 1995). Each method provides a different search strategy, and the LLM-driven evolution can recombine and adapt these strategies to generate new heuristics.

Fitness Function `QuantumEvo` defines the fitness function using the downstream QCC. Each valid individual is compiled, executed on the search set, and evaluated after full reversible circuit synthesis. Let $\mathcal{T}$ denote the search set used during evolution. For each instance $f \in \mathcal{T}$, let q_f^{base} denote the lowest QCC achieved on f among the classical baseline methods considered in this work. The fitness of a generated heuristic h is defined as the average relative QCC gap to the instance-wise best baseline:

$$\mathrm{fitness}(h) = \frac{1}{|\mathcal{T}|} \sum_{f \in \mathcal{T}} \frac{q(C_{h,f}) - q_f^{\mathrm{base}}}{q_f^{\mathrm{base}}}. \tag{2}$$

Lower values are better, and negative values indicate improvement over the best baseline for that instance. Because generated heuristics may contain stochastic components, each valid individual is evaluated multiple times per instance.

Figure 3: Fitness of the best individual over iterations, showing the family transition at iteration 8. Lower values indicate better fitness.

Table 1: Comparison of GA variants on the search set using best-of-3 seeds. The mean gaps are signed percentage differences relative to the best known value per instance. The win/tie/loss counts are relative to GA with sifting.

| | QCC | | BDD | | |
Method	Mean Gap (↓)	W/T/L	Mean Gap (↓)	W/T/L	Total Time (s)
GA with sifting	2.096%	-	−0.211%	-	23.24
GA with MiniSift	0.532%	34/53/13	−0.350%	4/94/2	21.95
GA with MiniSift + local refinement	0.527%	34/53/13	−0.354%	4/94/2	22.01

Note. The initial GA heuristic, corresponding to GA with sifting, uses a reduced population for runtime efficiency. The negative mean BDD gap indicates smaller BDDs than the best baseline value.

3.3 Analysis of the Discovered Algorithm

We analyze the best heuristic selected by `QuantumEvo`, tracing its evolution across iterations. Figure 3 shows the fitness trajectory of the overall best individual. In the best run, the best-so-far individual starts from an SA-family heuristic, transitions to a GA-family heuristic at iteration 8, and later incorporates a local refinement step. We refer to the best heuristic as HGA-QE, a hybrid GA discovered by `QuantumEvo` using a targeted sifting procedure. HGA-QE uses two generated subroutines, PartialSift and MiniSift. The algorithm is summarized in Algorithm 1, where the highlighted phase indicates the modification from the original GA. The subroutines are detailed in Section B.4. PartialSift reorders the current BDD according to a candidate variable order, identifies the affected levels, and calls MiniSift to locally sift only those levels. This replaces the standard sifting step used in the original GA, while keeping the same fitness function based on inverse BDD size.

Table 1 shows that this replacement improves QCC while reducing runtime. The main gain comes from MiniSift, while the additional local refinement step gives only marginal improvement. We infer that the improvement is related to greater diversity among candidate orderings and the resulting BDD structures. The standard sifting tends to pull different candidates toward similar BDD structures optimized for BDD size, whereas MiniSift produces more diverse orderings with comparable BDD size. The diagnostic analysis in Section C supports this interpretation and further examines cases where the BDD size is unchanged but MiniSift lowers QCC. In these cases, the resulting BDDs have synthesis-relevant structural differences, accompanied by fewer Toffoli gates and total controls. This suggests that MiniSift may expose a more diverse set of variable orderings, some of which are more favorable for synthesis and are associated with lower QCC, even though the GA itself still selects candidates by the BDD size.

Algorithm 1 HGA-QE: Hybrid GA discovered by `QuantumEvo`. Subroutines PARTIALSIFT and MINISIFT are defined in Section B.4.

Require: BDD manager T, variable range $[\ell, u]$
Ensure: Improved variable ordering in T

    Phase 1 — Baseline sifting
1: SIFT(T, ℓ, u) ▷ existing CUDD API; sift variables to reduce BDD size

    Phase 2 — Population initialization
2: $n \leftarrow u - \ell + 1$;   popsize $\leftarrow \max\{4, \min(2n, 40)\}$;   nCross $\leftarrow \min(3n, \text{popsize}, 60)$
3: $P[0] \leftarrow$ current ordering of T after sift; $P[0].\text{cost} \leftarrow |T.\text{nodes}|$
4: $P[1] \leftarrow$ reverse of $P[0]$
5: **for** $i = 2$ **to** popsize $- 1$ **do**
6:     $P[i] \leftarrow$ uniformly random permutation of variables in $[\ell, u]$
7: **end for**
8: **for** $i = 1$ **to** popsize $- 1$ **do**
9:     $(P[i], P[i].\text{cost}) \leftarrow$ PARTIALSIFT$(T, P[i], \ell, u)$ ▷ Algorithm 2
10: **end for**

    Phase 3 — Evolutionary loop
11: **for** $m = 1$ **to** nCross **do**
12:     $(p_1, p_2) \leftarrow$ ROULETTESELECT(P) such that $p_1 \neq p_2$ ▷ roulette selection weighted by $1/\text{cost}$
13:     $(c_1, c_2) \leftarrow$ PMX$(P[p_1], P[p_2])$ ▷ PMX crossover for valid permutations
14:     **for** each offspring $c \in \{c_1, c_2\}$ **do**
15:         $(c, c.\text{cost}) \leftarrow$ PARTIALSIFT(T, c, ℓ, u)
16:         $w \leftarrow$ worst replaceable individual in P ▷ do not discard the only tracked copy of a repeated order
17:         **if** $c.\text{cost} < P[w].\text{cost}$ **then**
18:             $P[w] \leftarrow c$ ▷ replace worst with offspring
19:         **end if**
20:     **end for**
21: **end for**
22: $b \leftarrow \arg\min_i P[i].\text{cost}$
23: $(P[b], P[b].\text{cost}) \leftarrow$ PARTIALSIFT$(T, P[b], \ell, u)$ ▷ restore T to the best ordering
24: $O^\star \leftarrow$ current ordering of T; $C^\star \leftarrow P[b].\text{cost}$

    Phase 4 — Local refinement
25: **if** $n \geq 2$ **then**
26:     Choose $x \sim \text{Unif}\{\ell, \ldots, u - 1\}$
27:     SWAPADJACENT$(T, x, x + 1)$ ▷ existing CUDD API; exchange two adjacent BDD levels
28:     MINISIFT$(T, \ell, u, \{0, \ldots, n - 1\})$
29:     newCost $\leftarrow |T.\text{nodes}|$
30:     **if** newCost $< C^\star$ **then**
31:         $O^\star \leftarrow$ current ordering of T
32:         $C^\star \leftarrow$ newCost
33:     **else**
34:         RESTOREORDER$(T, O^\star, \ell, u)$ ▷ revert to the saved best order
35:     **end if**
36: **end if**

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4 Experiment

We compare the best heuristic discovered by `QuantumEvo` against baselines, and conduct additional studies on the design of framework. All experiments are conducted using a single thread on an AMD Ryzen 9 5900X machine with 64 GB of RAM.

4.1 Experimental Setup

This subsection describes the datasets, baselines, and LLM configuration used in our experiments. The hyperparameters are summarized in Table 5.

Dataset We use benchmark functions from RevLib, LGSynth91, and ISCAS85/89. RevLib (Wille et al., 2008) provides benchmarks for reversible and quantum circuit design,[2] while LGSynth91 and ISCAS85/89 are taken from the `hdl-benchmarks` repository.[3] We augment RevLib functions using input negation, output complementation, and don't-care expansion. The resulting instances are split into a search set for evaluating generated heuristics during evolution and a validation set for selecting the best heuristic across independent runs. Final evaluation uses a benchmark set of 148 reversible functions not used during search or validation, consisting of 96 RevLib, 48 LGSynth91, and 4 ISCAS85/89 circuits. A detailed comparison between the search/validation set and the benchmark set is provided in Section B.3.

Baseline Algorithms We compare against five classical BDD variable ordering methods implemented in CUDD: sifting, symmetric sifting, group sifting, GA and SA (details in Section B.1). We also compare against a learning-based method, BDD2Seq (Miao et al., 2026), using the QCC reported in the original paper; our benchmark set is constructed to match the functions evaluated therein.

LLM configuration The best selected heuristic is obtained from an evolutionary run using the local LLM `gpt-oss-120b`,[4] an open-source 120B-parameter mixture-of-experts model released by OpenAI under the Apache 2.0 license. Section D compares independent runs with two model settings: the local model `gpt-oss-120b` and the commercial model `gpt-5.4-mini`. [5]

4.2 Experimental Results

We organize the experimental evidence around three findings. Finding 1 evaluates the discovered heuristic on the benchmark. Finding 2 and 3 draw from the additional studies on search design.

4.2.1 Finding 1: Evolved heuristics can match or improve strong BDD variable ordering baselines

We select the heuristic with the best validation fitness across independent runs, corresponding to the one analyzed in Section 3.3. This heuristic is fixed and evaluated once on the benchmark set. Table 2 reports per-suite results, where Best counts ties and Strict counts unique wins. For stochastic methods, QCC statistics are computed over 5 random seeds; for BDD2Seq, we take the lower of BDD2Seq(B*) and BDD2Seq(E*) from the original paper. The evolved heuristic achieves the highest Best count and lowest mean QCC across all three suites. On RevLib, it is best on 70 of 96 functions with mean QCC comparable to SA, though BDD2Seq leads in strict wins. On LGSynth91 and ISCAS85/89, the advantage over baselines is clearer, with our method best on all 4 ISCAS functions. The performance profile in Figure 4 further confirms this trend. Notably, LGSynth91 and ISCAS85/89 functions are structurally distinct from and often larger than the search set, yet the evolved heuristic maintains its advantage, demonstrating transfer beyond the distribution seen during evolution. Table 3 details the 20 functions on which our method strictly dominates all baselines in QCC. The average time reports the mean runtime per seed, and the total Time reports the cumulative runtime over all seeds. The runtime is a secondary consideration in this setting because the resulting low-QCC circuit can be saved and reused once obtained.

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[2]`https://www.revlib.org/index.php`
[3]`https://github.com/ispras/hdl-benchmarks`
[4]`https://huggingface.co/openai/gpt-oss-120b`
[5]`https://platform.openai.com/docs/models/gpt-5.4-mini`

Table 2: Per-suite benchmark performance. Best includes ties for the lowest Best QCC; Strict is counted only when the method is the unique best (5 random seeds for stochastic methods).

Suite	Method	Best	Strict	Mean Best QCC	Mean Avg QCC	Mean Worst QCC
RevLib (96)	HGA-QE	**70**	6	**1416.04**	**1434.31**	**1450.76**
	SA	68	5	1416.25	1437.27	1463.44
	BDD2Seq	65	**13**	1456.03	–	–
	GA	58	1	1445.28	1457.55	1468.35
	SIFT	42	0	1767.24	–	–
	SYMM SIFT	41	0	1757.90	–	–
	GROUP SIFT	41	1	1688.56	–	–
LGSynth91 (48)	HGA-QE	**31**	13	**2088.54**	**2218.05**	**2344.25**
	SA	22	3	2153.58	2267.06	2447.79
	BDD2Seq	22	5	2466.00	–	–
	GA	23	6	125739.94	202778.95	293303.17
	SIFT	16	1	4687.90	–	–
	SYMM SIFT	16	0	4682.71	–	–
	GROUP SIFT	15	0	5762.81	–	–
ISCAS85/89 (4)	HGA-QE	**4**	**1**	**12134.25**	**12211.23**	**12288.75**
	SA	3	0	12286.00	12403.25	12589.00
	BDD2Seq	1	0	13549.50	–	–
	GA	3	0	12296.00	12518.60	12681.25
	SIFT	0	0	19230.00	–	–
	SYMM SIFT	0	0	19230.00	–	–
	GROUP SIFT	1	0	23486.00	–	–

Figure 4: Performance profile over the whole benchmark set, using the ratio between each method's QCC and the best QCC achieved on the same function.

4.2.2 Finding 2: Optimizing QCC directly as the fitness function outperforms BDD size as a proxy

Figure 5 examines the relationship between BDD size and QCC on the 140 search/validation functions with BDD size at most 1000. The residual plot shows the difference between the actual and fitted QCC, with the horizontal line indicating zero residual. QCC is strongly correlated with BDD size, as indicated by Spearman $\rho = 0.989$, but residuals around the fitted trend are substantial. The mean absolute deviation is 79.3 up to 980. Moreover, 35% of these functions exhibit at least one ordering pair in which a smaller BDD yields a higher QCC, suggesting that BDD size is not a reliable proxy.

Table 3: Compact comparison for functions where `QuantumEvo` is strictly best.

Function	I	O	HGA-QE			GA			SA		
			Best	Avg	Avg Time	Best	Avg	Avg Time	Best	Avg	Avg Time
apex5_104[R]	117	88	**9184**	9812.40	0.97	9551	10041.40	1.24	9397	**9809.00**	1.97
apex6_orig[L]	135	99	**3736**	**3787.20**	0.65	4159	4368.40	0.64	4392	4551.40	1.08
apex7_orig[L]	49	37	**1431**	**1440.60**	0.28	1441	1443.40	0.58	1703	1759.20	0.44
b9_orig[L]	41	21	**713**	719.40	0.17	715	**719.00**	0.27	804	804.00	0.19
c432[I]	36	7	**10494**	**10797.50**	227.83	11141	11170.60	6.20	11101	11148.60	1.46
comp_orig[L]	32	3	**796**	**796.00**	2.41	824	824.00	2.28	897	936.00	2.01
cordic_138[R]	23	2	**304**	**308.00**	0.16	306	312.00	0.23	316	323.20	0.17
cordic_orig[L]	23	2	**304**	**308.00**	0.12	306	312.00	0.20	316	323.20	0.14
cu_orig[L]	14	11	**219**	**219.80**	0.08	224	224.00	0.09	220	220.00	0.09
dalu_orig[L]	75	16	**5230**	5301.60	71.73	5240	**5291.20**	71.72	5316	5663.20	69.72
frg1_160[R]	28	3	**559**	**597.40**	0.17	560	610.20	0.24	567	637.40	0.17
frg1_orig[L]	28	3	**559**	**597.40**	0.15	560	610.20	0.27	567	637.40	0.17
k2_orig[L]	45	45	**10156**	**10212.60**	0.97	10200	10265.40	1.78	10276	10410.00	1.49
misex1_178[R]	8	7	**279**	290.40	0.06	287	**287.00**	0.06	288	288.00	0.07
pair_orig[L]	173	137	**20236**	**22344.20**	6.24	47110	602976.40	8.28	20729	22438.00	20.16
rot_orig[L]	135	107	**23926**	26838.00	845.19	5928978	9069721.00	62.87	24124	**25464.60**	25.99
seq_201[R]	41	35	**9080**	9151.00	1.17	9113	**9142.00**	3.03	9165	9237.40	1.99
urf4_89[R]	11	11	**28406**	**28537.40**	0.41	28538	28591.80	0.54	28489	28590.80	0.97
x3_orig[L]	135	99	**3736**	**3787.20**	0.62	4159	4368.40	0.66	4392	4551.40	1.15
x4_orig[L]	94	71	**2437**	**2513.60**	0.37	2505	2809.40	0.65	2738	2863.20	0.69

Function	HGA-QE		BDD2Seq(B*)		BDD2Seq(E*)		SIFT		SYMM SIFT		GROUP SIFT	
	Best	Total Time	Best	Time	Best	Time	Best	Time	Best	Time	Best	Time
apex5_104[R]	**9184**	4.87	9852	18.27	10697	0.20	10358	0.06	10228	0.03	10227	0.06
apex6_orig[L]	**3736**	3.24	3895	23.86	4021	0.16	4822	0.05	4810	0.03	4269	0.03
apex7_orig[L]	**1431**	1.40	1455	3.41	1456	0.07	2653	0.04	2653	0.04	2377	0.04
b9_orig[L]	**713**	0.83	737	2.43	737	0.05	804	0.03	810	0.03	836	0.03
c432[I]	**10494**	499.72	12255	11.39	20391	6.79	11101	0.04	11101	0.02	11141	0.04
comp_orig[L]	**796**	12.04	824	45.14	824	28.75	961	1.77	1071	1.96	884	1.79
cordic_138[R]	**304**	0.82	344	0.87	446	0.05	325	0.02	323	0.02	318	0.04
cordic_orig[L]	**304**	0.58	320	0.86	320	0.06	325	0.01	323	0.01	318	0.03
cu_orig[L]	**219**	0.39	224	0.36	231	0.02	220	0.01	220	0.01	224	0.03
dalu_orig[L]	**5230**	358.67	5297	67.32	5652	28.17	31145	66.13	31145	71.95	47170	75.07
frg1_160[R]	**559**	0.85	598	1.34	629	0.03	747	0.03	747	0.03	827	0.03
frg1_orig[L]	**559**	0.77	757	1.35	762	0.03	747	0.03	747	0.02	827	0.02
k2_orig[L]	**10156**	4.85	10432	3.33	10686	0.19	11058	0.07	11058	0.13	11275	0.09
misex1_178[R]	**279**	0.30	281	0.16	289	0.02	288	0.01	288	0.03	289	0.03
pair_orig[L]	**20236**	31.21	20754	39.87	20799	1.06	46491	0.11	46391	0.17	49917	0.15
rot_orig[L]	**23926**	4225.96	38453	47.58	48344	15.69	78639	0.56	78374	0.62	110936	0.74
seq_201[R]	**9080**	5.85	9349	2.89	9908	0.33	19362	0.24	19350	0.28	15309	0.27
urf4_89[R]	**28406**	2.06	28488	0.77	28833	0.47	28523	0.06	28523	0.10	28523	0.11
x3_orig[L]	**3736**	3.09	3941	23.93	4645	0.19	4822	0.03	4810	0.05	4269	0.03
x4_orig[L]	**2437**	1.85	2662	11.84	2782	0.11	4470	0.02	4470	0.04	4334	0.04

Superscripts denote source dataset: [I] ISCAS85/89; [L] LGSynth91; [R] RevLib.

(a) Overall trend with Spearman $\rho = 0.989$ and linear fit

(b) Residuals from the linear fit, showing function-level deviations

Figure 5: BDD size vs. QCC for the 140 search and validation functions with BDD size at most 1000.

Table 4: Benchmark evaluation of the best-validation individual from each setting on the full benchmark set with five random seeds.

Initialization	Fitness	Best/Total (%)	Strictly Best/Total (%)	Mean Best QCC	Mean Avg QCC	Mean Worst QCC
Sifting, GA, SA	QCC	**70.9**	**13.5**	**1923.83**	**1979.76**	**2033.46**
GA	QCC	63.5	3.4	2061.84	2172.50	2264.26
SA	QCC	66.2	0.7	1949.11	2004.30	2084.68
Sifting	QCC	55.4	4.1	2203.34	2435.04	2670.19
Sifting, GA, SA	BDD size	61.5	5.4	1944.65	2003.03	2099.68

The following results confirm this difference. Figure 6 shows the validation set distribution of QCC fitness and win-or-tie rate across runs. From each configuration, we select the individual with the lowest validation QCC fitness and evaluate it on the full 148 functions (Table 4). The fitness based on the BDD size achieves comparable validation QCC but lower win-or-tie rate. The same trend holds on the full benchmark, where QCC-based fitness outperforms on every metric, reducing mean QCC by 1.1%. Although modest in magnitude, the improvement is consistent across all metrics, confirming that evaluating by actual synthesis output selects better heuristics than optimizing a proxy.

4.2.3 Finding 3: Mixed initialization improves transfer across structurally different benchmarks

The selected heuristic in Section 3.3 is discovered from a mixed initialization of sifting, GA, and SA heuristics. To isolate the contribution of seed diversity, we compare mixed initialization against initialization from each individual heuristic family, running ten independent searches for each setting. As shown in Figure 6, SA-only collapses to zero variance, with every run converging to the same individual despite LLM stochasticity. This indicates that a single SA heuristic can constrain the search to a narrow region. On validation, mixed initialization does not achieve the lowest QCC fitness, which is obtained by GA-only, but it shows a higher win-or-tie rate. The benefit of seed diversity becomes clear at benchmark evaluation. On the full benchmark, which includes LGSynth91 and ISCAS85/89 functions absent from the search set, GA-only no longer leads despite its strong validation fitness, while SA-only and sifting-only remain weak. Mixed initialization achieves the strongest benchmark performance across all metrics, including the highest Strictly Best rate, despite not having the best validation fitness. This suggests that seed diversity produces heuristics that transfer more robustly to structurally different functions.

(a) QCC fitness (lower is better, ↓)   (b) Win-or-tie rate (higher is better, ↑)

Figure 6: Validation-set distributions over ten independent runs per setting. The star marks the run selected by the lowest validation QCC fitness; the corresponding win-or-tie rate is reported for the same run.

5 Discussion

While the results demonstrate that LLM-driven evolutionary search can discover competitive BDD variable ordering heuristics, several limitations remain. First, the discovered heuristic is still influenced by the initial heuristic families used to seed the search. Although the best lineage transitions from SA-family to GA-family individuals, the best heuristic combines and refines ideas related to sifting, SA, and GA rather than introducing an entirely new class of ordering strategy. Whether alternative framework designs can discover substantially different and more effective ordering strategies remains an open question. Second, `QuantumEvo` optimizes an important component of BDD-based reversible synthesis, namely the variable ordering. While this choice affects the BDD structure and downstream QCC, other resource measures, such as ancilla count, also depend on the synthesis procedure itself. Future work could combine `QuantumEvo` with post-synthesis ancilla reduction techniques or alternative synthesis methods that produce lower-cost circuits.

6 Conclusion

We presented `QuantumEvo`, an LLM-driven evolutionary framework that discovers interpretable and reusable BDD variable ordering heuristics for reversible circuit synthesis. The best discovered heuristic achieves strong performance across benchmark circuits, improving over classical CUDD ordering methods and learning-based baselines in the QCC. As quantum hardware and compilation workflows continue to mature, the efficient synthesis of complex Boolean functions into low-cost reversible circuits will become increasingly important. `QuantumEvo` suggests that QCC-aware heuristic design can help make BDD-based synthesis more effective for this expanding design space.

Beyond this specific synthesis setting, `QuantumEvo` illustrates that LLM-driven heuristic design can be applied not only to standalone optimization problems, but also to optimization decisions embedded inside specialized synthesis pipelines. The same paradigm could in principle extend to other BDD-based workflows where variable ordering affects downstream performance, such as classical logic synthesis and technology mapping.

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A   Example of Reversible Circuit Synthesis

A.1   NCT library and Circuit

The NCT library consists of three gate types: NOT, CNOT, and Toffoli. The NOT gate flips a single bit ($x \mapsto \bar{x}$). The CNOT gate flips the target bit when the control bit is 1, $(x, t) \mapsto (x, t \oplus x)$, and the Toffoli gate flips the target bit when both control bits are 1, $(x_1, x_2, t) \mapsto (x_1, x_2, t \oplus (x_1 \wedge x_2))$. Figure 7 illustrates this with a 3-bit reversible function. The truth table lists all $2^3 = 8$ input combinations (x_1, x_2, x_3) and their outputs (y_1, y_2, y_3); each output row appears exactly once, confirming bijectivity. The right panel shows the NCT circuit synthesized from this function, with a total QCC of 8 under the cost model of Szyprowski & Kerntopf (2013).

x_1	x_2	x_3	y_1	y_2	y_3
0	0	0	0	0	1
0	0	1	0	0	0
0	1	0	0	1	1
0	1	1	0	1	0
1	0	0	1	0	1
1	0	1	1	1	1
1	1	0	1	0	0
1	1	1	1	1	0

(a) Truth table of a 3-bit reversible function   (b) Synthesized reversible circuit (QCC = 8)

Figure 7: A 3-bit reversible function and its synthesized NCT circuit.

A.2   BDD Variable Ordering and Circuit

Figure 8 provides another example illustrating the impact of variable ordering on BDD size and QCC. The same function under two orderings yields BDD sizes of 9 and 6, and QCC of 52 and 24, respectively. This shows that variable ordering can affect both BDD size and QCC.

B   Detailed Experimental Setup

This section provides supplementary details needed to reproduce and interpret the experiments.

(a) BDD under $\pi_1 = \{x_1, x_2, x_3, x_4\}$

(b) BDD under $\pi_2 = \{x_3, x_1, x_4, x_2\}$

(c) Reversible circuit for (a)

(d) Reversible circuit for (b)

Figure 8: Impact of variable ordering on QCC for the function `4mod5_8`. Solid/dashed edges denote 1/0-branches. The BDD sizes are 9 under π_1 and 6 under π_2, whereas the corresponding QCC are 52 and 24. Here, $y = \overline{(x_1 \oplus x_3)} \cdot \overline{(x_2 \oplus x_4)}$

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B.1 Baseline Ordering Methods in CUDD

CUDD (Somenzi, 2023) is a `C` library for constructing and manipulating BDDs. The classical baselines implemented in CUDD are:

- **Sifting** (Rudell, 1993): Moves each variable through all possible levels and retains the position minimizing node count.
- **Symmetric Sifting** (Panda et al., 1994): Extends sifting by detecting symmetric variable pairs during traversal and grouping them for joint sifting.
- **Group Sifting** (Panda & Somenzi, 1995): Generalizes symmetric sifting to arbitrary dependent variable groups, sifting each group as a whole.
- **Genetic Algorithm** (Drechsler et al., 1996): Maintains a population of variable orderings; generates new individuals via crossover and sifts each before fitness evaluation, replacing the worst individual if the offspring improves.
- **Simulated Annealing** (Bollig et al., 1995): Randomly perturbs the ordering and accepts or rejects moves according to an annealing schedule.

B.2 Experimental Setting

Hyperparameters Table 5 summarizes the hyperparameters used in all runs.

Individual Representation Each individual is a program linked directly to CUDD. Rather than implementing individuals at the Python level, we use functions linked into the BDD process, which avoids

Table 5: Hyperparameters used for all runs.

Hyperparameter	Value	Hyperparameter	Value
Max. function evaluations	200	Per-iteration timeout	40 s
Population size	10	Evaluation seeds per instance	3
Initial population size	30	Search instances	100
Mutation rate	0.5	Validation instances	50

per-step interface overhead and provides more direct access to CUDD ordering routines and relevant internal data structures.

Initial Heuristics   The initial heuristics are generated from three functions corresponding to sifting, GA, and SA. The sifting and simulated annealing seeds follow the corresponding CUDD implementations, while the GA seed is a lightweight variant of GA in CUDD that only reduces the population size to keep evaluation cost manageable.

B.3   Analysis of Dataset and Benchmarks

RevLib is aligned with reversible circuit synthesis and provides functions that can be repeatedly evaluated within the limited search budget. To increase the diversity of the search pool, we augment RevLib. Figure 9 compares the 100 search functions with the 148 benchmark functions by input count, output count, and gate count. The search set is drawn from augmented RevLib functions, whereas the benchmark set consists of RevLib, LGSynth91, and ISCAS85/89 functions with broader structural variety. Although the two sets partially overlap in coarse metadata such as input and output counts, they share no Boolean functions. The benchmark set extends to substantially larger and more irregular functions, including instances whose gate counts far exceed those seen during evolution. Strong benchmark performance, therefore, reflects transfer to unseen function structures rather than overfitting to the search set.

(a) Input-gate coverage   (b) Output-gate coverage   (c) Input-output coverage

Figure 9: Structural comparison of the search set (100 functions) and the benchmark set (148 functions)

B.4   Mapping to CUDD Implementations

Algorithm 2 and Algorithm 3 define the subroutines called by Algorithm 1. Table 6 maps each pseudocode function to its corresponding CUDD function and source file.

C   Diagnostic Analysis of MINISIFT Algorithm

This section provides additional diagnostics for the comparison between the standard sifting and MINISIFT. The goal is to understand why MINISIFT can improve downstream QCC.

Algorithm 2 PARTIALSIFT: Individual-order evaluator with MINISIFT (subroutine of Algorithm 1)

Require: BDD manager T, individual permutation σ, range $[\ell, u]$
Ensure: σ is replaced by the post-sift ordering; return the resulting BDD size
 1: $n \leftarrow u - \ell + 1$
 2: $\pi[i] \leftarrow$ variable currently at level $\ell + i$, for $i = 0, \ldots, n - 1$ ▷ snapshot before any move
 3: **for** $j = 0$ **to** $n - 1$ **do**
 4:      Move variable $\sigma[j]$ to target level $\ell + j$ via adjacent swaps
 5: **end for**
 6: $\Delta \leftarrow \{i \in [0, n) : \sigma[i] \neq \pi[i]\}$ ▷ levels changed by imposing the target order, before MINISIFT
 7: **if** $\Delta \neq \emptyset$ **then**
 8:      MINISIFT(T, ℓ, u, Δ)
 9: **end if**
10: $\sigma[i] \leftarrow$ variable currently at level $\ell + i$, for $i = 0, \ldots, n - 1$
11: **return** $|T.\text{nodes}|$

Algorithm 3 MINISIFT: targeted local sifting (subroutine of Algorithm 1 and Algorithm 2)

Require: BDD manager T, range $[\ell, u]$, changed-level set $\Delta \subseteq [0, n)$
Ensure: Marked levels are used as starting levels for local CUDD sifting within $[\ell, u]$
      Upward pass: low levels first
 1: **for** $i = \ell$ **to** u **do**
 2:      **if** $i - \ell \notin \Delta$ **then skip**
 3:      **end if**
 4:      Sift variable at level i upward within $[\ell, u]$; restore the best position found
 5: **end for**
      Downward pass: high levels first
 6: **for** $i = u$ **downto** ℓ **do**
 7:      **if** $i - \ell \notin \Delta$ **then skip**
 8:      **end if**
 9:      Sift variable at level i downward within $[\ell, u]$; restore the best position found
10: **end for**

Metrics The diversity statistics are computed over all 100 functions in the search set and three independent seeds. The unique fraction is the fraction of distinct candidate orderings among the final GA population. The Hamming fraction measures the average normalized Hamming distance between pairs of candidate orderings, where a larger value indicates that the candidate orderings differ at more variable positions. The remaining metrics are computed on the 31 circuits where the GA with the standard sifting and MINISIFT produce the same BDD size, but MINISIFT achieves lower QCC. Child-node line preservation cases are cases where the synthesized value of a child node must be kept for later reuse during the synthesis. Fewer such cases impose fewer constraints on the use of circuit lines, and are associated with fewer Toffoli gates and total controls. The toffoli gates, total controls, and QCC are computed from the synthesized reversible circuits.

Statistical tests All statistical tests are paired tests between GA with the standard sifting and MINISIFT on the same set of circuits or circuit-seed pairs. The Wilcoxon signed-rank test additionally uses the ranks of the absolute paired differences, and therefore accounts for both the direction and relative magnitude of the changes. Smaller p-values indicate stronger evidence that the observed changes are systematic rather than random paired fluctuations.

Table 7 shows that MINISIFT produces more diverse set of the candidate orderings than the standard sifting. On the same BDD size subset where MINISIFT achieves lower QCC, it also reduces the child-node line preservation cases, Toffoli gates, total controls, and QCC. These results suggest that MINISIFT can expose ordering candidates that have comparable BDD size but are more favorable for synthesis.

Table 6: Correspondence between pseudocode functions and CUDD source. $W =$ wrapped (unmodified API call), $A =$ adapted, $N =$ new.

Pseudocode	CUDD function	Source file	Type	Notes
SIFT	cuddSifting	cuddReorder.c	W	—
SWAPADJACENT	cuddSwapInPlace	cuddReorder.c	W	—
PMX	PMX	cuddGenetic.c	W	—
ROULETTESELECT	roulette	cuddGenetic.c	A	Spin range unified to popsize for both parent selections
PARTIALSIFT	build_dd	cuddGenetic.c	A	Replaces full cuddSifting with MINISIFT over marked starting levels
RESTOREORDER	restoreOrder	cuddAnneal.c	A	Reimplemented locally; based on annealing restore logic (cf. ddShuffle in cuddReorder.c)
MINISIFT	—	—	N	New function; internally calls ddSiftingUp, ddSiftingDown, ddSiftingBackward

Table 7: Diagnostic comparison between the standard sifting and MINISIFT. Diversity rows use all search-set functions and three seeds; other rows use the same-BDD-size cases where MINISIFT lowers QCC. The change is relative to the standard sifting.

Statistic	Sifting	MINISIFT	Change (%)	Wilcoxon p
Unique fraction	0.553	0.615	$+11.2\%$	4.35×10^{-11}
Hamming fraction	0.507	0.537	$+5.9\%$	1.04×10^{-4}
Child-node line preservation cases	999	983	-1.6%	0.0100
Toffoli gates	3847	3797	-1.3%	4.70×10^{-5}
Total controls	10591	10479	-1.1%	4.98×10^{-6}
QCC	22550	22274	-1.2%	1.12×10^{-6}

D   Study on the Choice of LLM

We compare two LLM choices under the same pipeline, prompt structure, and seed initialization: `gpt-oss-120b` (120B-parameter open-source MoE, Apache 2.0) and `gpt-5.4-mini` (OpenAI API). For each model, we run ten independent searches, select the best individual on the validation set, and evaluate it on the 148 benchmark functions.

Figure 10 shows the distribution of validation results across ten independent runs. Runs using `gpt-5.4-mini` show wider variance and a lower median validation score, and even the best run does not reach the best result obtained with `gpt-oss-120b`. Figure 11 traces the search trajectory of the best `gpt-5.4-mini` run, showing how its best heuristic evolved during the search.

(a) QCC fitness   (b) Win-or-tie rate

Figure 10: Validation comparison between the two LLM choices over ten independent runs. The star marks the run selected by the lowest validation QCC fitness.

Figure 11: Evolution of the Best Discovered Ordering Heuristic for the choice of `gpt-5.4-mini`

The search runs using `gpt-oss-120b` produce stronger validation results than those using `gpt-5.4-mini`, with a higher win-or-tie rate and lower mean QCC. The two settings also tend to converge to different heuristic families. Runs with `gpt-oss-120b` select SA-family individuals in 9 of 10 runs, whereas runs with `gpt-5.4-mini` more often converge to sifting-family individuals. Based on these validation results, we use the best individual generated with `gpt-oss-120b` for the main evaluation.

Table 8: Benchmark comparison of LLM choices. Family counts summarize the best individual selected in each of ten independent runs per model.

| LLM choice | Benchmark | | | | | Family Type | | |
	Best/Total (%)	Strictly Best/Total (%)	Mean Best QCC	Mean Avg QCC	Mean Worst QCC	GA	SA	Sifting
gpt-oss-120b	**70.9**	**13.5**	**1923.83**	**1979.76**	**2033.46**	1	9	0
gpt-5.4-mini	66.9	6.1	2080.71	2147.33	2211.11	2	3	5

Family Composition across Iterations We classify each valid individual by its code-level family. Figure 12 traces the family composition over iterations for the best run from each LLM. In the `gpt-oss-120b` run, the population shifts from diverse seed families toward SA-family and later GA-family individuals. In contrast, `gpt-5.4-mini` tends to converge toward simpler sifting-style variants.

(a) Best run from `gpt-oss-120b` run

(b) Best run from `gpt-5.4-mini` run

Figure 12: Code-level family composition of valid individuals across iterations

E Prompts

We show the problem-specific prompts used in `QuantumEvo`, which are function signature, function description, and external knowledge. They are tailored to BDD variable ordering heuristic design and to the `C` language individual representation. The remaining operator prompts, including mutation, crossover, and reflection, are available in the accompanying code.

```
int cuddFunc(DdManager *table, int lower, int upper)
```

Prompt 1: Function Signature

```
Implement 'cuddFunc()' for CUDD BDD reordering.
Primary goal:  reduce native BDD size and Quantum Cost.
If output hooks are present, you may also consult 'CURRENT_SIZE(table)', but native BDD
heap size is a valid primary objective.

Requirements:
- Return '1' on success and '0' on failure.
- On OOM, set 'table->errorCode = CUDD_MEMORY_OUT' and return '0'.
- On return, the live order must equal the best order found.
- Respect the active level range '[lower, upper]'.
- Free owned arrays with 'FREE' and owned move lists with 'cuddDeallocMove'.

Key CUDD contracts:
- 'cuddSwapInPlace(table, x, x + 1)' takes LEVELS, not variable indices.  Pass the lower
 adjacent level first; 'cuddSwapInPlace(table, curLevel, curLevel - 1)' is wrong.
- Keep 'Cudd_SetFuncOutputs(...)' and 'Cudd_ClearFuncOutputs(...)' exactly named.
- 'ddUndoMoves(moves)' returns an inverse history; free the original move list after calling
 it, and free the full list, not just the head node.
- For plain sifting, compose the rollback list as 'down + inv' before
'ddSiftingBackward(...)'.
- 'ddSiftingBackward(...)' does not free the move list for you.
- For annealing-style code, keep an explicit best-order snapshot and restore the best order
 before returning.
```

Prompt 2: Function Description

```
'cuddFunc()' should behave like a native CUDD reorderer.

Common mistakes:
- Passing a variable index where a level is required, especially in 'cuddSwapInPlace()'.
- Passing adjacent levels in reverse order.  Use 'cuddSwapInPlace(table, curLevel - 1,
 curLevel)', not 'cuddSwapInPlace(table, curLevel, curLevel - 1)'.
- Calling 'Cudd_ReduceHeap()' or another nested top-level reorderer inside 'cuddFunc()'.
- Returning without restoring the best order found.
- Mishandling move ownership:  freeing '(Move *)CUDD_OUT_OF_MEM', freeing only the head
 after 'ddUndoMoves(...)', or double-freeing move lists.
- Building the wrong rollback order for plain sifting; use 'down + inv', not 'inv + down'.

Do not:
- Swap outside '[lower, upper]'.
- Manually increment 'table->ddTotalNumberSwapping'.
- Redefine 'Move', 'IndexKey', 'ddMin', 'ddMax', 'cuddDynamicAllocNode', or
'cuddDeallocMove'.
- Use headers other than 'util.h', 'cuddInt.h', 'math.h', and 'string.h'.
```

Prompt 3: External Knowledge

F   Full Benchmark Results

Table 9–Table 14 report the full per-function results for all 148 benchmark functions, grouped by the suite. RevLib instances are listed in Table 9–Table 13. ISCAS85/89 and LGSynth91 instances are listed in Table 11 and Table 14 and denoted by superscripts [I] and [L], respectively. For ordering methods with stochastic components, we report best QCC, average QCC, and runtime; for BDD2Seq and deterministic methods, we report best QCC and runtime.

Table 9: Full benchmark results, RevLib functions, stochastic methods, part 1 of 2. Best and average QC compared among HGA-QE, GA, and SA; lowest values bold.

Function	I	O	HGA-QE			GA			SA		
			Best	Avg	Time	Best	Avg	Time	Best	Avg	Time
4gt10_22	4	1	**17**	**17.00**	0.04	**17**	**17.00**	0.06	**17**	**17.00**	0.05
4gt11_23	4	1	**5**	**5.00**	0.06	**5**	**5.00**	0.06	**5**	**5.00**	0.05
4gt12_24	4	1	**17**	**17.00**	0.06	**17**	**17.00**	0.06	**17**	**17.00**	0.06
4gt13_25	4	1	**10**	**10.00**	0.06	**10**	**10.00**	0.06	**10**	**10.00**	0.05
4gt4_20	4	1	**19**	**19.00**	0.06	**19**	**19.00**	0.06	**19**	**19.00**	0.06
4gt5_21	4	1	**12**	**12.00**	0.05	**12**	**12.00**	0.06	**12**	**12.00**	0.06
4mod5_8	4	1	**24**	**24.00**	0.06	**24**	**24.00**	0.06	**24**	**24.00**	0.05
4mod7_26	4	3	**86**	**86.00**	0.06	**86**	**86.00**	0.05	**86**	**86.00**	0.06
5xp1_90	7	10	**254**	**254.00**	0.06	280	280.00	0.06	**254**	**254.00**	0.07
9symml_91	9	1	**206**	**206.00**	0.06	**206**	**206.00**	0.06	**206**	**206.00**	0.07
add6_92	12	7	**118**	**118.00**	0.07	**118**	**118.00**	0.08	**118**	122.60	0.09
adr4_93	8	5	**74**	**74.00**	0.05	**74**	**74.00**	0.06	**74**	**74.00**	0.07
alu1_94	12	8	**139**	**139.00**	0.06	**139**	**139.00**	0.07	**139**	**139.00**	0.07
alu2_96	10	6	**1266**	**1276.80**	0.08	1269	1287.00	0.09	**1266**	1278.80	0.11
alu3_97	10	8	**430**	434.80	0.06	**430**	**430.00**	0.07	**430**	431.60	0.10
alu4_98	14	8	4385	4422.00	0.19	**4255**	4341.20	0.20	**4255**	4361.40	0.33
alu_9	5	1	**29**	**29.00**	0.05	**29**	**29.00**	0.06	**29**	**29.00**	0.06
apex2_101	39	3	2663	**2697.20**	0.62	2726	2929.00	1.82	**2660**	2776.00	0.89
apex4_103	9	19	8236	8243.60	0.13	**8231**	**8235.80**	0.12	**8231**	8240.40	0.26
apex5_104	117	88	**9184**	9812.40	0.97	9551	10041.40	1.24	9397	9809.00	1.97
apla_107	10	12	**731**	734.60	0.07	**731**	**732.20**	0.08	**731**	**732.20**	0.10
bw_116	5	28	**926**	930.40	0.06	**926**	934.80	0.06	**926**	930.40	0.08
C17_117	5	2	**37**	**37.00**	0.06	**37**	**37.00**	0.06	**37**	**37.00**	0.06
C7552_119	5	16	**202**	**202.00**	0.06	**202**	**202.00**	0.05	**202**	**202.00**	0.06
clip_124	9	5	515	521.40	0.06	**512**	**517.80**	0.07	519	522.20	0.09
cm150a_128	21	1	**186**	**186.00**	0.52	**186**	**186.00**	0.33	**186**	**186.00**	0.30
cm151a_129	19	9	**298**	**298.00**	0.10	**298**	**298.00**	0.12	**298**	**298.00**	0.09
cm152a_130	11	1	**62**	**62.00**	0.07	**62**	**62.00**	0.07	**62**	**62.00**	0.07
cm163a_133	16	13	**244**	254.40	0.08	**244**	253.20	0.09	**244**	261.20	0.07
cm42a_125	4	10	**117**	**117.00**	0.06	**117**	**117.00**	0.05	**117**	**117.00**	0.06
cm82a_126	5	3	**44**	**44.00**	0.05	**44**	**44.00**	0.06	**44**	**44.00**	0.06
cm85a_127	11	3	**203**	205.40	0.06	207	210.20	0.07	**203**	208.20	0.08
cmb_134	16	4	**153**	**153.00**	0.08	**153**	**153.00**	0.11	**153**	**153.00**	0.09
co14_135	14	1	**159**	**159.00**	0.08	**159**	**159.00**	0.10	**159**	**159.00**	0.08
con1_136	7	2	**89**	**90.20**	0.06	**89**	**90.20**	0.06	**89**	93.80	0.06
cordic_138	23	2	**304**	**308.00**	0.16	306	312.00	0.23	316	323.20	0.17
cu_141	14	11	219	219.80	0.08	224	224.00	0.09	**220**	**220.00**	0.09
dc1_142	4	7	**186**	**186.00**	0.06	**186**	**186.00**	0.06	**186**	**186.00**	0.06
dc2_143	8	7	**431**	**431.00**	0.06	**431**	**431.00**	0.06	**431**	**431.00**	0.07
decod24-enable_32	3	4	**38**	**38.00**	0.06	**38**	**38.00**	0.05	**38**	**38.00**	0.06
decod_137	5	16	**202**	**202.00**	0.05	**202**	**202.00**	0.05	**202**	**202.00**	0.06
dist_144	8	5	975	975.80	0.06	979	979.00	0.07	**975**	**975.00**	0.09
dk17_145	10	11	**426**	**426.00**	0.06	**426**	**426.00**	0.07	**426**	**426.00**	0.09
dk27_146	9	9	**140**	140.80	0.06	144	144.00	0.06	**140**	**140.00**	0.06
e64_149	65	65	**886**	**886.00**	0.39	**886**	**886.00**	1.14	**886**	890.20	1.07
ex1010_155	10	10	9681	9724.60	0.23	**9643**	**9703.20**	0.22	9717	9803.60	0.36
ex1_150	5	1	**8**	**8.00**	0.06	**8**	**8.00**	0.05	**8**	**8.00**	0.06
ex2_151	5	1	**60**	**60.00**	0.06	**60**	**60.00**	0.06	**60**	64.00	0.06

Table 10: Full benchmark results, RevLib functions, stochastic methods, part 2 of 2.

Function	I	O	HGA-QE			GA			SA		
			Best	Avg	Time	Best	Avg	Time	Best	Avg	Time
ex3_152	5	1	**38**	**40.00**	0.06	**38**	44.00	0.06	**38**	45.00	0.06
ex5p_154	8	63	**1843**	**1843.00**	0.11	**1843**	**1843.00**	0.11	**1843**	**1843.00**	0.14
example2_156	10	6	**1266**	**1276.80**	0.08	1269	1287.00	0.09	**1266**	1278.80	0.11
f2_158	4	4	**108**	111.00	0.05	**108**	**108.00**	0.05	113	113.00	0.05
f51m_159	14	8	**2422**	2429.20	0.14	2427	2429.40	0.20	**2422**	2430.60	0.25
frg1_160	28	3	**559**	**597.40**	0.17	560	610.20	0.24	567	637.40	0.17
frg2_161	143	139	7017	**7601.60**	1.47	9429	9429.00	1.63	**6972**	7689.80	3.22
in0_162	15	11	2299	2313.40	0.11	2299	2317.40	0.16	**2271**	**2295.80**	0.19
inc_170	7	9	**592**	**592.00**	0.06	**592**	**592.00**	0.06	**592**	**592.00**	0.08
life_175	9	1	**204**	**204.00**	0.06	210	210.00	0.07	**204**	**204.00**	0.07
majority_176	5	1	**41**	**41.00**	0.06	**41**	**41.00**	0.05	**41**	**41.00**	0.06
max46_177	9	1	556	562.80	0.07	546	558.00	0.07	**545**	**552.60**	0.09
misex1_178	8	7	**279**	290.40	0.06	287	**287.00**	0.06	288	288.00	0.07
misex3_180	14	14	**3789**	**3791.40**	0.16	**3789**	3792.60	0.22	**3789**	3833.40	0.35
misex3c_181	14	14	**3936**	**3950.40**	0.16	**3936**	**3950.40**	0.24	**3936**	3979.60	0.32
mlp4_184	8	8	**1158**	1158.40	0.07	1159	1159.00	0.07	**1158**	**1158.00**	0.10
mux_185	21	1	**170**	**170.00**	0.55	**170**	**170.00**	0.41	**170**	**170.00**	0.41
one-two-three_27	3	3	**44**	**44.00**	0.06	**44**	**44.00**	0.06	**44**	**44.00**	0.05
parity_188	16	1	**31**	**31.00**	0.50	**31**	**31.00**	0.49	**31**	**31.00**	0.49
pcler8_190	16	5	**124**	**124.00**	0.08	**124**	**124.00**	0.08	**124**	**124.00**	0.08
pdc_191	16	40	**6599**	**6599.00**	0.41	**6599**	**6599.00**	0.60	**6599**	**6599.00**	0.84
pm1_192	4	10	**117**	**117.00**	0.06	**117**	**117.00**	0.06	**117**	**117.00**	0.06
radd_193	8	5	**74**	**74.00**	0.06	**74**	**74.00**	0.06	**74**	**74.00**	0.06
rd53_68	5	3	**98**	**98.00**	0.05	**98**	**98.00**	0.06	**98**	**98.00**	0.05
rd73_69	7	3	**217**	**217.00**	0.06	**217**	**217.00**	0.06	**217**	**217.00**	0.06
rd84_70	8	4	**304**	**304.00**	0.06	**304**	**304.00**	0.06	**304**	**304.00**	0.07
root_197	8	5	**444**	**444.00**	0.06	446	446.00	0.06	**444**	**444.00**	0.08
ryy6_198	16	1	**107**	**109.40**	0.09	**107**	112.60	0.11	**107**	117.20	0.09
sao2_199	10	4	**653**	**653.00**	0.07	**653**	654.80	0.07	**653**	655.80	0.11
seq_201	41	35	**9080**	9151.00	1.17	9113	**9142.00**	3.03	9165	9237.40	1.99
sf_232	4	1	**21**	26.80	0.06	43	43.00	0.06	36	36.00	0.06
spla_202	16	46	**5858**	5860.00	0.33	**5858**	5860.00	0.35	**5858**	5861.00	0.56
sqn_203	7	3	**356**	358.00	0.06	**356**	358.00	0.06	**356**	**357.00**	0.07
sqr6_204	6	12	486	486.00	0.06	**482**	**482.00**	0.06	486	486.00	0.07
sqrt8_205	8	4	**221**	233.80	0.06	**221**	230.60	0.06	**221**	230.60	0.07
squar5_206	5	8	**232**	**232.00**	0.06	**232**	**232.00**	0.05	**232**	**232.00**	0.06
sym10_207	10	1	**253**	**253.00**	0.07	**253**	**253.00**	0.08	**253**	**253.00**	0.07
sym6_63	6	1	**93**	**93.00**	0.06	**93**	**93.00**	0.06	**93**	**93.00**	0.06
sym9_71	9	1	**206**	**206.00**	0.06	**206**	**206.00**	0.07	**206**	**206.00**	0.06
t481_208	16	1	**147**	**147.00**	0.10	**147**	**147.00**	0.12	**147**	**147.00**	0.09
table3_209	14	14	**5898**	**5899.40**	0.18	**5898**	5902.20	0.22	**5898**	5908.80	0.43
tial_214	14	8	4460	4487.60	0.13	4354	4401.00	0.21	**4231**	**4299.20**	0.38
urf4_89	11	11	**28406**	**28537.40**	0.41	28538	28591.80	0.54	28489	28590.80	0.97
wim_220	4	7	**107**	**107.00**	0.06	**107**	**107.00**	0.06	**107**	**107.00**	0.05
x2_223	10	7	**191**	**191.00**	0.06	**191**	**191.00**	0.07	**191**	**191.00**	0.08
xor5_195	5	1	**8**	**8.00**	0.05	**8**	**8.00**	0.06	**8**	**8.00**	0.06
z4_224	7	4	**66**	**66.00**	0.06	**66**	**66.00**	0.05	**66**	**66.00**	0.06
z4ml_225	7	4	**66**	**66.00**	0.05	**66**	**66.00**	0.06	**66**	**66.00**	0.06

Table 11: Full benchmark results, ISCAS85/89 and LGSynth91 functions, stochastic methods.

Function	I	O	HGA-QE			GA			SA		
			Best	Avg	Time	Best	Avg	Time	Best	Avg	Time
apex6_orig[L]	135	99	**3736**	**3787.20**	0.65	4159	4368.40	0.64	4392	4551.40	1.08
apex7_orig[L]	49	37	**1431**	**1440.60**	0.28	1441	1443.40	0.58	1703	1759.20	0.44
b1_orig[L]	3	4	**22**	23.00	0.06	**22**	**22.00**	0.05	**22**	**22.00**	0.06
b9_orig[L]	41	21	**713**	719.40	0.17	715	**719.00**	0.27	804	804.00	0.19
c17[I]	5	2	**37**	**37.00**	0.06	**37**	**37.00**	0.06	**37**	**37.00**	0.06
c432[I]	36	7	**10494**	**10797.50**	227.83	11141	11170.60	6.20	11101	11148.60	1.46
c880[I]	60	26	**34241**	**34241.00**	382.75	**34241**	35095.20	44.13	**34241**	34610.60	12.39
c8_orig[L]	28	18	**436**	**436.00**	0.14	**436**	446.40	0.18	**436**	450.80	0.15
cc_orig[L]	21	20	283	289.00	0.08	**267**	**280.00**	0.11	278	287.20	0.08
cht_orig[L]	47	36	**714**	**714.00**	0.12	**714**	**714.00**	0.21	**714**	**714.00**	0.14
cm138a_orig[L]	6	8	**104**	**104.00**	0.06	**104**	**104.00**	0.06	**104**	**104.00**	0.06
cm150a_orig[L]	21	1	**186**	**186.00**	0.52	**186**	**186.00**	0.45	**186**	**186.00**	0.36
cm151a_orig[L]	12	2	**92**	**92.00**	0.07	**92**	**92.00**	0.08	**92**	**92.00**	0.07
cm152a_orig[L]	11	1	**62**	**62.00**	0.07	**62**	**62.00**	0.07	**62**	**62.00**	0.07
cm162a_orig[L]	14	5	222	222.00	0.08	**219**	**220.20**	0.09	**219**	220.80	0.09
cm163a_orig[L]	16	5	**124**	**124.00**	0.08	**124**	**124.00**	0.08	**124**	**124.00**	0.07
cm42a_orig[L]	4	10	**117**	**117.00**	0.05	**117**	**117.00**	0.05	**117**	**117.00**	0.06
cm82a_orig[L]	5	3	**44**	**44.00**	0.06	**44**	**44.00**	0.06	**44**	**44.00**	0.06
cm85a_orig[L]	11	3	**203**	**205.40**	0.07	207	210.20	0.07	**203**	208.20	0.08
cmb_orig[L]	16	4	**153**	**153.00**	0.09	**153**	**153.00**	0.11	**153**	**153.00**	0.09
comp_orig[L]	32	3	**796**	**796.00**	2.41	824	824.00	2.28	897	936.00	2.01
cordic_orig[L]	23	2	**304**	**308.00**	0.12	306	312.00	0.20	316	323.20	0.14
count_orig[L]	35	16	**441**	**441.00**	0.17	**441**	**441.00**	0.29	**441**	**441.00**	0.21
cu_orig[L]	14	11	**219**	**219.80**	0.08	224	224.00	0.09	220	220.00	0.09
dalu_orig[L]	75	16	**5230**	5301.60	71.73	5240	**5291.20**	71.72	5316	5663.20	69.72
decod_orig[L]	5	16	**202**	**202.00**	0.06	**202**	**202.00**	0.06	**202**	**202.00**	0.06
example2_orig[L]	85	66	1790	1844.40	0.28	**1784**	**1835.80**	0.54	1894	1954.00	0.45
frg1_orig[L]	28	3	**559**	**597.40**	0.15	560	610.20	0.27	567	637.40	0.17
frg2_orig[L]	143	139	7017	**7601.60**	1.44	9429	9429.00	1.69	**6972**	7689.80	3.08
k2_orig[L]	45	45	**10156**	**10212.60**	0.97	10200	10265.40	1.78	10276	10410.00	1.49
lal_orig[L]	26	19	496	496.00	0.10	**478**	**490.80**	0.14	484	507.80	0.12
majority_orig[L]	5	1	**41**	**41.00**	0.06	**41**	**41.00**	0.06	**41**	**41.00**	0.05
mux_orig[L]	21	1	**170**	**170.00**	0.52	**170**	**170.00**	0.39	**170**	**170.00**	0.40
my_adder_orig[L]	33	17	**352**	**352.00**	1.24	**352**	**352.00**	0.66	**352**	**352.00**	0.37
pair_orig[L]	173	137	**20236**	**22344.20**	6.24	47110	602976.40	8.28	20729	22438.00	20.16
parity_orig[L]	16	1	**31**	**31.00**	0.09	**31**	**31.00**	0.11	**31**	**31.00**	0.06
pcle_orig[L]	19	9	**298**	**298.00**	0.09	**298**	**298.00**	0.11	**298**	**298.00**	0.10
pcler8_orig[L]	27	17	639	647.00	0.15	674	674.00	0.23	**638**	**638.00**	0.16
pm1_orig[L]	16	13	**244**	254.40	0.08	**244**	**253.20**	0.09	**244**	261.20	0.08
rot_orig[L]	135	107	**23926**	26838.00	845.19	5928978	9069721.00	62.87	24124	**25464.60**	25.99
s1196_orig[I]	14	14	**3765**	**3769.40**	0.14	**3765**	3771.60	0.18	**3765**	3816.80	0.29
sct_orig[L]	19	15	375	386.80	0.10	373	**375.80**	0.12	**369**	382.40	0.11
tcon_orig[L]	17	16	**88**	**88.00**	0.06	**88**	**88.00**	0.07	**88**	**88.00**	0.06
term1_orig[L]	34	10	497	500.80	0.18	**473**	**489.60**	0.33	563	639.80	0.25
too_large_orig[L]	38	3	2676	**2736.40**	0.71	**2675**	2830.80	1.83	2694	2816.60	0.90
ttt2_orig[L]	24	21	736	738.80	0.13	**711**	**727.20**	0.16	727	730.20	0.15
unreg_orig[L]	36	16	**494**	494.40	0.12	495	495.00	0.19	**494**	**494.00**	0.11
vda_orig[L]	17	39	4169	4181.00	0.16	4169	4177.00	0.26	**4107**	**4176.20**	0.28
x1_orig[L]	51	35	3062	3133.60	0.50	**3030**	**3089.80**	1.06	3144	3308.40	0.76
x2_orig[L]	10	7	**191**	**191.00**	0.06	**191**	**191.00**	0.07	**191**	**191.00**	0.08
x3_orig[L]	135	99	**3736**	**3787.20**	0.62	4159	4368.40	0.66	4392	4551.40	1.15
x4_orig[L]	94	71	**2437**	**2513.60**	0.37	2505	2809.40	0.65	2738	2863.20	0.69

Superscripts denote source dataset: [I] ISCAS85/89; [L] LGSynth91.

Table 12: Full benchmark results, RevLib functinos, prior baselines, part 1 of 2. Best QC compared among HGA-QE, BDD2Seq, and deterministic CUDD basselines; lowest values bold.

Function	HGA-QE		BDD2Seq(B*)		BDD2Seq(E*)		SIFT		SYMM SIFT		GROUP SIFT	
	Best	Time	Best	Time	Best	Time	Best	Time	Best	Time	Best	Time
4gt10_22	**17**	0.04	**17**	1.22	**17**	2.13	**17**	0.02	23	0.01	23	0.02
4gt11_23	**5**	0.06	**5**	0.07	**5**	0.01	**5**	0.01	**5**	0.01	**5**	0.03
4gt12_24	17	0.06	17	0.06	17	0.01	17	0.01	**16**	0.01	**16**	0.03
4gt13_25	**10**	0.06	**10**	0.07	**10**	0.01	**10**	0.01	**10**	0.01	**10**	0.03
4gt4_20	**19**	0.06	**19**	0.06	25	0.01	**19**	0.03	23	0.01	23	0.03
4gt5_21	**12**	0.05	**12**	0.07	**12**	0.01	**12**	0.03	18	0.01	18	0.03
4mod5_8	**24**	0.06	**24**	0.07	**24**	0.02	**24**	0.03	**24**	0.01	**24**	0.02
4mod7_26	86	0.06	86	0.06	86	0.01	86	0.03	86	0.01	**80**	0.03
5xp1_90	**254**	0.06	**254**	0.13	280	0.01	**254**	0.03	**254**	0.01	**254**	0.03
9symml_91	**206**	0.06	**206**	0.20	**206**	0.03	**206**	0.03	**206**	0.01	**206**	0.03
add6_92	**118**	0.07	**118**	0.29	566	0.01	499	0.03	499	0.02	474	0.03
adr4_93	**74**	0.05	**74**	0.16	**74**	0.02	**74**	0.03	**74**	0.01	**74**	0.03
alu1_94	**139**	0.06	**139**	0.29	**139**	0.02	**139**	0.03	**139**	0.01	**139**	0.03
alu2_96	1266	0.08	**1218**	0.23	1415	0.03	1436	0.03	1436	0.01	1366	0.03
alu3_97	**430**	0.06	**430**	0.22	464	0.01	644	0.03	644	0.01	497	0.03
alu4_98	**4385**	0.19	4455	0.44	4934	0.09	7222	0.04	7222	0.02	7623	0.04
alu_9	**29**	0.05	**29**	0.08	**29**	0.02	**29**	0.03	**29**	0.01	35	0.03
apex2_101	**2663**	0.62	3103	2.37	3178	0.16	5922	0.11	5393	0.08	6439	0.16
apex4_103	**8236**	0.13	8409	0.42	8813	0.19	8343	0.02	8343	0.02	8381	0.04
apex5_104	**9184**	0.97	9852	18.27	10697	0.20	10358	0.06	10228	0.03	10227	0.06
apla_107	**731**	0.07	732	0.22	955	0.03	1002	0.03	1002	0.03	979	0.03
bw_116	926	0.06	**924**	0.08	937	0.02	943	0.03	943	0.03	943	0.03
C17_117	**37**	0.06	**37**	0.08	**37**	0.01	49	0.03	49	0.01	**37**	0.03
C7552_119	**202**	0.06	**202**	0.08	**202**	0.01	**202**	0.03	**202**	0.01	**202**	0.03
clip_124	**515**	0.06	695	0.18	698	0.02	704	0.03	704	0.02	520	0.03
cm150a_128	186	0.52	**136**	0.84	**136**	0.24	186	0.21	186	0.16	186	0.20
cm151a_129	**298**	0.10	**298**	0.61	**298**	0.02	**298**	0.03	310	0.02	**298**	0.03
cm152a_130	**62**	0.07	**62**	0.25	**62**	0.02	**62**	0.03	**62**	0.02	**62**	0.03
cm163a_133	**244**	0.08	**244**	0.45	267	0.02	273	0.03	**244**	0.02	267	0.03
cm42a_125	**117**	0.06	**117**	0.07	**117**	0.01	**117**	0.03	**117**	0.02	**117**	0.03
cm82a_126	**44**	0.05	**44**	0.07	**44**	0.01	82	0.03	**44**	0.01	**44**	0.03
cm85a_127	**203**	0.06	227	0.25	283	0.03	275	0.03	221	0.01	221	0.03
cmb_134	**153**	0.08	**153**	0.45	**153**	0.02	158	0.03	**153**	0.01	**153**	0.03
co14_135	**159**	0.08	**159**	0.36	**159**	0.02	**159**	0.02	**159**	0.01	**159**	0.03
con1_136	89	0.06	**88**	0.13	103	0.03	96	0.02	96	0.02	96	0.01
cordic_138	**304**	0.16	344	0.87	446	0.05	325	0.02	323	0.02	318	0.04
cu_141	**219**	0.08	**219**	0.36	230	0.02	220	0.01	220	0.01	224	0.02
dc1_142	186	0.06	**160**	0.07	168	0.01	**160**	0.03	**160**	0.03	186	0.03
dc2_143	**431**	0.06	**431**	0.15	**431**	0.03	**431**	0.03	**431**	0.02	**431**	0.03
decod24-enable_32	**38**	0.06	**38**	0.05	**38**	0.01	**38**	0.03	**38**	0.03	**38**	0.03
decod_137	**202**	0.05	**202**	0.08	**202**	0.01	**202**	0.03	**202**	0.03	**202**	0.03
dist_144	**975**	0.06	**975**	0.17	979	0.04	**975**	0.03	**975**	0.03	979	0.03
dk17_145	**426**	0.06	429	0.21	429	0.03	**426**	0.03	**426**	0.03	574	0.03
dk27_146	140	0.06	**131**	0.18	**131**	0.02	140	0.03	140	0.03	141	0.03
e64_149	**886**	0.39	1019	5.90	1208	0.17	907	0.05	907	0.05	1124	0.04
ex1010_155	9681	0.23	**9633**	0.41	9810	0.14	9766	0.07	9766	0.04	9696	0.05
ex1_150	**8**	0.06	**8**	0.10	**8**	0.01	**8**	0.03	**8**	0.01	**8**	0.03
ex2_151	**60**	0.06	**60**	0.09	68	0.01	73	0.03	70	0.03	**60**	0.03

Table 13: Full benchmark results, RevLib functions, prior baselines, part 2 of 2.

Function	HGA-QE		BDD2Seq(B*)		BDD2Seq(E*)		SIFT		SYMM SIFT		GROUP SIFT	
	Best	Time	Best	Time	Best	Time	Best	Time	Best	Time	Best	Time
ex3_152	**38**	0.06	**38**	0.10	**38**	0.02	61	0.03	61	0.03	43	0.03
ex5p_154	1843	0.11	**1837**	0.22	1871	0.06	1843	0.04	1843	0.05	1843	0.05
example2_156	1266	0.08	**1218**	0.24	1415	0.03	1436	0.03	1436	0.03	1366	0.02
f2_158	**108**	0.05	**108**	0.07	113	0.01	113	0.03	113	0.03	113	0.02
f51m_159	**2422**	0.14	2427	0.60	2517	0.14	5392	0.05	5392	0.05	4603	0.05
frg1_160	**559**	0.17	598	1.34	629	0.03	747	0.03	747	0.03	827	0.03
frg2_161	**7017**	1.47	8584	26.96	10768	0.37	12468	0.07	12361	0.08	12224	0.09
in0_162	2299	0.11	2296	0.42	2417	0.04	**2283**	0.01	**2283**	0.03	2328	0.04
inc_170	592	0.06	**579**	0.13	**579**	0.01	**579**	0.01	**579**	0.03	592	0.03
life_175	**204**	0.06	**204**	0.19	210	0.01	**204**	0.01	210	0.01	**204**	0.03
majority_176	**41**	0.06	**41**	0.08	**41**	0.01	**41**	0.01	**41**	0.02	**41**	0.03
max46_177	**556**	0.07	562	0.19	596	0.02	598	0.01	598	0.03	598	0.03
misex1_178	279	0.06	281	0.16	289	0.02	288	0.01	288	0.03	289	0.03
misex3_180	**3789**	0.16	3830	0.44	4054	0.09	4661	0.03	4661	0.05	4619	0.05
misex3c_181	**3936**	0.16	3969	0.40	4457	0.05	4769	0.02	4769	0.04	4823	0.04
mlp4_184	**1158**	0.07	1191	0.17	1228	0.03	**1158**	0.02	**1158**	0.03	**1158**	0.03
mux_185	170	0.55	**135**	1.18	224	0.26	170	0.23	170	0.25	170	0.35
one-two-three_27	**44**	0.06	**44**	0.05	**44**	0.01	**44**	0.01	**44**	0.03	**44**	0.01
parity_188	**31**	0.50	**31**	0.99	**31**	0.44	**31**	0.18	**31**	0.21	**31**	0.17
pcler8_190	**124**	0.08	**124**	0.46	137	0.01	**124**	0.01	**124**	0.03	137	0.01
pdc_191	6599	0.41	6742	0.72	6742	0.23	**6500**	0.13	**6500**	0.19	**6500**	0.13
pm1_192	**117**	0.06	**117**	0.06	**117**	0.02	**117**	0.01	**117**	0.01	**117**	0.01
radd_193	**74**	0.06	**74**	0.16	140	0.01	217	0.01	192	0.03	212	0.01
rd53_68	**98**	0.05	**98**	0.07	**98**	0.01	**98**	0.01	**98**	0.03	**98**	0.01
rd73_69	**217**	0.06	**217**	0.12	**217**	0.02	**217**	0.01	**217**	0.03	**217**	0.01
rd84_70	**304**	0.06	**304**	0.17	**304**	0.02	**304**	0.01	**304**	0.03	**304**	0.01
root_197	**444**	0.06	**444**	0.16	446	0.01	**444**	0.01	**444**	0.03	**444**	0.01
ryy6_198	107	0.09	**103**	0.46	107	0.02	133	0.02	132	0.03	119	0.03
sao2_199	**653**	0.07	657	0.22	698	0.01	667	0.01	667	0.03	684	0.03
seq_201	**9080**	1.17	9349	2.89	9908	0.33	19362	0.24	19350	0.28	15309	0.27
sf_232	**21**	0.06	31	0.07	36	0.02	36	0.01	36	0.03	**21**	0.03
spla_202	**5858**	0.33	5947	0.66	6092	0.19	5925	0.08	5925	0.11	5925	0.10
sqn_203	**356**	0.06	374	0.13	426	0.02	426	0.01	426	0.03	426	0.03
sqr6_204	486	0.06	**470**	0.11	524	0.02	486	0.01	486	0.03	486	0.03
sqrt8_205	**221**	0.06	237	0.16	283	0.01	240	0.01	240	0.03	240	0.03
squar5_206	**232**	0.06	**232**	0.09	264	0.02	253	0.01	253	0.03	**232**	0.03
sym10_207	**253**	0.07	**253**	0.24	**253**	0.03	**253**	0.02	**253**	0.03	**253**	0.03
sym6_63	**93**	0.06	**93**	0.11	**93**	0.01	**93**	0.01	**93**	0.03	**93**	0.03
sym9_71	**206**	0.06	**206**	0.20	**206**	0.03	**206**	0.01	**206**	0.03	**206**	0.03
t481_208	147	0.10	**139**	0.47	140	0.03	147	0.02	152	0.03	152	0.03
table3_209	**5898**	0.18	5954	0.48	7005	0.08	6276	0.02	6276	0.04	5927	0.03
tial_214	**4460**	0.13	4475	0.43	5157	0.09	7609	0.02	7609	0.04	4852	0.04
urf4_89	**28406**	0.41	28488	0.77	28833	0.47	28523	0.06	28523	0.10	28523	0.11
wim_220	107	0.06	**103**	0.07	108	0.01	107	0.01	107	0.03	107	0.03
x2_223	**191**	0.06	**191**	0.23	**191**	0.03	273	0.01	273	0.03	283	0.03
xor5_195	**8**	0.05	**8**	0.07	**8**	0.01	**8**	0.01	**8**	0.03	**8**	0.03
z4_224	**66**	0.06	**66**	0.13	**66**	0.01	**66**	0.01	**66**	0.03	**66**	0.03
z4ml_225	**66**	0.05	**66**	0.13	**66**	0.02	**66**	0.01	**66**	0.03	**66**	0.03

Table 14: Full benchmark results, ISCAS85/89 and LGSynth91 functions, prior baselines.

Function	HGA-QE		BDD2Seq(B*)		BDD2Seq(E*)		SIFT		SYMM SIFT		GROUP SIFT	
	Best	Time	Best	Time	Best	Time	Best	Time	Best	Time	Best	Time
apex6_orig^L	**3736**	0.65	3895	23.86	4021	0.16	4822	0.05	4810	0.03	4269	0.03
apex7_orig^L	**1431**	0.28	1455	3.41	1456	0.07	2653	0.04	2653	0.04	2377	0.04
b1_orig^L	**22**	0.06	**22**	0.05	**22**	0.01	**22**	0.03	**22**	0.03	**22**	0.03
b9_orig^L	**713**	0.17	737	2.43	737	0.05	804	0.03	810	0.03	836	0.03
c17^I	**37**	0.06	**37**	0.08	49	0.01	49	0.03	49	0.03	**37**	0.03
c432^I	**10494**	227.83	12255	11.39	20391	6.79	11101	0.04	11101	0.02	11141	0.04
c880^I	**34241**	382.75	37802	26.22	39992	15.52	61079	0.53	61079	0.65	78113	0.59
c8_orig^L	436	0.14	**432**	1.21	436	0.04	445	0.03	445	0.02	486	0.03
cc_orig^L	283	0.08	**272**	0.72	278	0.03	385	0.03	385	0.01	379	0.03
cht_orig^L	**714**	0.12	**714**	3.15	**714**	0.06	**714**	0.03	**714**	0.02	**714**	0.03
cm138a_orig^L	**104**	0.06	**104**	0.11	**104**	0.01	**104**	0.02	**104**	0.02	**104**	0.03
cm150a_orig^L	186	0.52	**136**	0.93	**136**	0.34	186	0.38	186	0.25	186	0.32
cm151a_orig^L	92	0.07	**70**	0.29	**70**	0.02	92	0.03	92	0.02	92	0.03
cm152a_orig^L	**62**	0.07	**62**	0.25	81	0.02	**62**	0.03	**62**	0.02	**62**	0.03
cm162a_orig^L	222	0.08	222	0.36	222	0.03	**192**	0.03	258	0.02	244	0.03
cm163a_orig^L	**124**	0.08	136	0.45	136	0.03	**124**	0.03	**124**	0.02	137	0.03
cm42a_orig^L	**117**	0.05	**117**	0.05	**117**	0.02	**117**	0.03	**117**	0.01	**117**	0.03
cm82a_orig^L	**44**	0.06	**44**	0.07	**44**	0.01	82	0.03	**44**	0.01	**44**	0.03
cm85a_orig^L	**203**	0.07	225	0.25	225	0.02	275	0.03	221	0.01	221	0.03
cmb_orig^L	**153**	0.09	**153**	0.45	**153**	0.03	158	0.03	**153**	0.01	**153**	0.03
comp_orig^L	**796**	2.41	824	45.14	824	28.75	961	1.77	1071	1.96	884	1.79
cordic_orig^L	**304**	0.12	320	0.86	320	0.06	325	0.01	323	0.01	318	0.03
count_orig^L	**441**	0.17	**441**	1.80	**441**	0.04	**441**	0.01	**441**	0.01	**441**	0.02
cu_orig^L	**219**	0.08	224	0.36	231	0.02	220	0.01	220	0.01	224	0.03
dalu_orig^L	**5230**	71.73	5297	67.32	5652	28.17	31145	66.13	31145	71.95	47170	75.07
decod_orig^L	**202**	0.06	**202**	0.07	**202**	0.01	**202**	0.03	**202**	0.03	**202**	0.03
example2_orig^L	**1790**	0.28	1800	9.74	1806	0.08	2066	0.03	2066	0.02	2041	0.03
frg1_orig^L	**559**	0.15	757	1.35	762	0.03	747	0.03	747	0.02	827	0.02
frg2_orig^L	**7017**	1.44	7189	27.54	7189	0.35	12468	0.03	12361	0.06	12224	0.07
k2_orig^L	**10156**	0.97	10432	3.33	10686	0.19	11058	0.07	11058	0.13	11275	0.09
lal_orig^L	496	0.10	**478**	1.07	496	0.04	527	0.01	768	0.02	724	0.03
majority_orig^L	**41**	0.06	**41**	0.07	**41**	0.01	**41**	0.01	**41**	0.03	**41**	0.03
mux_orig^L	170	0.52	**135**	1.03	**135**	0.40	170	0.24	170	0.35	170	0.26
my_adder_orig^L	**352**	1.24	**352**	64.22	**352**	48.54	**352**	0.15	**352**	0.35	**352**	0.16
pair_orig^L	**20236**	6.24	20754	39.87	20799	1.06	46491	0.11	46391	0.17	49917	0.15
parity_orig^L	**31**	0.09	**31**	0.92	**31**	0.61	**31**	0.01	**31**	0.03	**31**	0.01
pcle_orig^L	**298**	0.09	**298**	0.61	**298**	0.02	**298**	0.01	310	0.03	**298**	0.01
pcler8_orig^L	639	0.15	765	1.13	765	0.02	**638**	0.01	**638**	0.04	639	0.01
pm1_orig^L	244	0.08	**234**	0.46	273	0.03	273	0.01	244	0.01	267	0.01
rot_orig^L	**23926**	845.19	38453	47.58	48344	15.69	78639	0.56	78374	0.62	110936	0.74
s1196_orig^I	**3765**	0.14	4104	0.41	4110	0.05	4691	0.02	4691	0.04	4653	0.04
sct_orig^L	**375**	0.10	405	0.61	513	0.02	545	0.01	545	0.03	548	0.03
tcon_orig^L	**88**	0.06	**88**	0.51	**88**	0.02	**88**	0.01	**88**	0.03	**88**	0.02
term1_orig^L	497	0.18	**480**	1.76	483	0.06	1193	0.02	1151	0.04	1070	0.04
too_large_orig^L	**2676**	0.71	3744	2.93	5379	0.46	5922	0.07	5393	0.12	6334	0.10
ttt2_orig^L	736	0.13	749	0.92	1006	0.04	**727**	0.02	**727**	0.04	1139	0.03
unreg_orig^L	**494**	0.12	**494**	1.93	**494**	0.04	**494**	0.01	**494**	0.03	**494**	0.03
vda_orig^L	**4169**	0.16	4224	0.56	4224	0.08	4477	0.03	4477	0.05	4460	0.05
x1_orig^L	**3062**	0.50	3567	3.76	3703	0.16	3678	0.02	4189	0.04	4172	0.05
x2_orig^L	**191**	0.06	**191**	0.20	**191**	0.02	273	0.01	273	0.03	283	0.03
x3_orig^L	**3736**	0.62	3941	23.93	4645	0.19	4822	0.03	4810	0.05	4269	0.03
x4_orig^L	**2437**	0.37	2662	11.84	2782	0.11	4470	0.02	4470	0.04	4334	0.04

Superscripts denote source dataset: ^{I} ISCAS85/89; ^{L} LGSynth91.