Is the Trust Region Necessary? Robust Entropy Adaptation Suffices for Continuous Control

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Abstract

Policy-gradient algorithms from TRPO through PPO and MPO treat explicit D_{KL}-ball trust regions as the bedrock of stable learning under function approximation. We reconsider that design: must every update be explicitly KL-bounded, or can well-regulated entropy dynamics already deliver the stability often ascribed to trust regions? ACE (**A**daptive **C**ontrol of **E**ntropy) augments soft actor–critic with a winsorised twin-critic disagreement signal and an AdaGrad temperature rule for α_t in $\log$-space—the same configuration on all twenty-one MuJoCo, DeepMind Control, and Gym benchmarks. Including an auxiliary trust-region multiplier solely as a probe, its dual satisfies $\lambda_t = 0$ at every audited step among 3,500 ablation measurements (seven environments, five seeds, one hundred checkpoints each); removing that term outright does no aggregate harm, whereas freezing α or dropping the disagreement signal breaks exploration-demanding regimes. Theorem 4.1 supplies a closed-form bound showing that sufficiently controlled entropy dynamics implicitly satisfy standard KL budgets, rationalising an inactive trust-region dual. Empirically, ACE achieves the best final return on twelve of twenty-one tasks, with the largest advantages where fixed-entropy baselines plateau.

1. Introduction

Three of the most influential policy-gradient algorithms—TRPO (Schulman et al., 2015), PPO (Schulman et al., 2017), and MPO (Abdolmaleki et al., 2018)—geometrically constrain each policy update to a D_{KL} ball around the previous policy. This mechanism is rarely questioned, yet it remains unclear whether the constraint *itself* drives stability or merely acts as damage control for a more fundamental issue: poor entropy management.

The problem statement. We ask: *Is the KL constraint the primary source of stability, or does it compensate for unstable entropy dynamics?* Poor entropy management—whether collapse toward determinism or runaway exploration—destabilises policy updates, and an explicit trust region mitigates both symptoms. We hypothesize that *directly* controlling entropy dynamics renders the geometric constraint redundant.

Contributions.

1. **Algorithm.** ACE: a winsorised disagreement signal and AdaGrad entropy controller with zero per-environment tuning, deployed identically across all 21 benchmarks.
2. **Empirical finding.** $\lambda_t = 0$ at *every* measured training step across 7 ablation environments (3,500 measurements: 7 envs $\times$ 5 seeds $\times$ 100 checkpoints)—the clearest evidence to date that explicit trust regions are geometrically redundant under adaptive entropy control.
3. **Theory.** Theorem 4.1: a closed-form bound proving that robust entropy control implicitly satisfies any standard KL budget, mathematically predicting $\lambda_t \equiv 0$.
4. **Performance.** State-of-the-art final return on 12 of 21 benchmarks, with large margins on hard-exploration tasks where fixed-temperature methods fail.

2. Background and Related Work

Maximum-Entropy RL and Adaptive Entropy. The maximum-entropy RL objective augments the standard return with an entropy bonus (Ziebart, 2010; Haarnoja et al., 2017):

$$J(\pi) = \sum_{t=0}^{\infty} \gamma^t \, \mathbb{E}_{(s_t,a_t)\sim\rho_\pi}\left[r(s_t,a_t) + \alpha\mathcal{H}(\pi(\cdot \mid s_t))\right], \tag{1}$$

where $\alpha > 0$ is the entropy temperature. SAC (Haarnoja et al., 2018a;b) optimises this off-policy using twin Q-networks and anchors $\mathcal{H}(\pi)$ near a fixed target $\bar{h} = -d_a$.

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This target is *static*: it does not adapt to epistemic uncertainty or varying exploration demands across learning phases. Prior work already learns an entropy temperature in SAC-style objectives (Haarnoja et al., 2018a), but none couple the target to critic disagreement or demonstrate that adaptive entropy can supplant an explicit trust region—the gap ACE fills.

Trust-Region Methods. TRPO (Schulman et al., 2015) constrains each update such that $D_{\mathrm{KL}}(\pi_{\theta'} \,\|\, \pi_\theta) \leq \delta$. PPO (Schulman et al., 2017) replaces this with a clipped surrogate; MPO (Abdolmaleki et al., 2018) uses Lagrangian E-step/M-step alternation; REPS (Peters et al., 2010) employs relative-entropy constraints for safe sample reuse. These methods achieve strong empirical stability at the cost of additional hyperparameters and tuning sensitivity. We argue that their primary function—preventing catastrophically large updates—is naturally fulfilled by dynamic entropy management, rendering the geometric constraint redundant.

Critic Disagreement and Exploration. TD3 (Fujimoto et al., 2018) introduced twin critics to mitigate overestimation bias; their disagreement also proxies epistemic uncertainty. REDQ (Chen et al., 2021) and SUNRISE (Lee et al., 2021) exploit ensemble disagreement for value estimation and sample weighting, respectively. Our innovation is using *winsorised* twin-critic disagreement as a direct control signal for the entropy temperature, establishing a closed-loop system that ties uncertainty to exploration intensity—and, per Eysenbach & Levine (2022), to implicit adversarial robustness.

3. The ACE Algorithm

3.1. Setup and Base Objective

ACE builds upon the standard SAC architecture. We maintain twin Q-networks Q_{ϕ_1}, Q_{ϕ_2} with corresponding target networks $Q_{\bar{\phi}_1}, Q_{\bar{\phi}_2}$, alongside a stochastic actor π_θ parameterised as a squashed Gaussian (Appendix A.1.3). The Bellman target utilizes the minimum of the two critics plus an entropy bonus:

$$y_t = r_t + \gamma(1-d_t)\left(\min_{i\in\{1,2\}} Q_{\bar{\phi}_i}(s',a') - \alpha_t \log \pi_\theta(a' \,|\, s')\right), \quad a' \sim \pi_\theta(\cdot \mid s'). \tag{2}$$

We use $\tau = 0.02$ for target-network updates (four times SAC's default), supported by the AdaGrad controller's stability guarantees (Appendix G.3.1). The critic loss is the standard soft Bellman residual:

$$\mathcal{L}_{\text{critic}}(\phi_i) = \mathbb{E}_{(s,a,r,s')\sim\mathcal{D}}\left[\left(Q_{\phi_i}(s,a) - y\right)^2\right], \quad i = 1, 2. \tag{3}$$

3.2. Winsorised Critic Disagreement

The raw critic disagreement, $\Delta_t = |Q_{\phi_1}(s_t,a_t) - Q_{\phi_2}(s_t,a_t)|$, serves as a natural proxy for epistemic uncertainty. However, its distribution is characteristically heavy-tailed: large, outlier disagreements early in training can severely destabilise a naive entropy controller.

To address this, ACE applies Winsorisation (Tukey, 1977) via z-scoring and clipping. We maintain exponential moving averages of the mean and variance of Δ_t with a rate of $\beta_\Delta = 2 \times 10^{-3}$, and compute:

$$d_t = \text{clip}\left(\frac{\Delta_t - \mu_\Delta}{\sigma_\Delta + \varepsilon},\ -z_{\max},\ z_{\max}\right), \qquad z_{\max} = 2.5. \tag{4}$$

The clipping threshold $z_{\max} = 2.5$ (98.8th percentile of the standard normal) mitigates transient spikes, rendering d_t scale-invariant, bounded, and positive when critics disagree above average (exploration signal) or negative when they agree (exploitation signal).

3.3. Dynamic Entropy Target

Let $e_t = -d_a^{-1} \log \pi_\theta(a_t \mid s_t)$ denote the per-dimension negative log-probability, representing a direct, normalised measure of current policy entropy. ACE tracks its running mean via $m_t \leftarrow (1-\beta)m_{t-1} + \beta e_t$, utilizing $\beta = 10^{-4}$.

The dynamic entropy target merges this baseline with the winsorised disagreement signal:

$$e_t^* = \text{clip}(m_t + \kappa \cdot d_t,\ e_{\min},\ e_{\max}), \qquad \kappa = 0.15, \tag{5}$$

which is bounded to $[0.05,\ 2.0]$ to prevent saturation, creating a feedback loop that drives exploration when critics disagree and exploitation when they converge.

3.4. AdaGrad Temperature Controller

The discrepancy between the dynamic target and current entropy, $g_t = e_t^* - e_t$, serves as the error signal for the temperature controller. We update $\log \alpha$ using an AdaGrad step in log-space:

$$G_t = G_{t-1} + g_t^2, \tag{6}$$

$$\log \alpha_{t+1} = \text{clip}\left(\log \alpha_t + \tfrac{g_t}{\sqrt{G_t}+\varepsilon},\ \log \alpha_{\min},\ \log \alpha_{\max}\right). \tag{7}$$

Operating in log-space structurally guarantees $\alpha_t > 0$. The AdaGrad accumulator $\sqrt{G_t}$ dictates an automatic step-size decay: it permits large corrective steps early in training when gradients are consistent, and seamlessly anneals to fine adjustments as $e_t \approx e_t^*$. Crucially, this formulation eliminates the need for an explicit learning rate hyperparameter. The structural limits ($\alpha_{\text{init}} = 0.2$, $\alpha_{\min} = 10^{-4}$, $\alpha_{\max} = 1.0$) and the AdaGrad constant ($\varepsilon = 10^{-8}$) are

fixed universally across all 21 evaluated environments, demanding zero per-task tuning.

3.5. Optional Trust-Region Dual

To empirically test whether geometric constraints activate under robust entropy control, we include an optional KL dual. We maintain a snapshot policy $\pi_{\bar{\theta}}$ (updated every $k = 10$ steps) and augment the actor loss:

$$\mathcal{L}_{\text{actor}}(\theta) = \mathcal{L}_{\text{soft}}(\theta) + \lambda_t \big(D_{\text{KL}}(\pi_\theta \,\|\, \pi_{\bar{\theta}}) - \delta\big), \quad \delta = 0.02. \tag{8}$$

Here $\mathcal{L}_{\text{soft}}(\theta) = \mathbb{E}_{s\sim\mathcal{D},\, a\sim\pi_\theta}[\alpha_t \log \pi_\theta(a|s) - Q_{\phi_1}(s,a)]$. The KL divergence is computed analytically via the closed form for diagonal Gaussians (Appendix A.1.5). The dual variable λ_t is optimized via dual AdaGrad ascent:

$$H_t = H_{t-1} + (\text{KL}_t - \delta)^2,$$
$$\lambda_{t+1} = \max\left(0,\ \lambda_t + \frac{\text{KL}_t - \delta}{\sqrt{H_t} + \varepsilon}\right). \tag{9}$$

Under standard Lagrangian dynamics, Equation (9) should activate whenever the policy exceeds the KL budget ($\text{KL}_t > \delta$). However, as detailed in Section 5.2, this dual variable remains stubbornly at zero.

Algorithm 1 ACE: Adaptive Control of Entropy

Input: Environment $\mathcal{E}$, constants κ=0.15, $z_{\max}$=2.5, δ=0.02, τ=0.02
Initialize: Actor π_θ, critics Q_{ϕ_1}, Q_{ϕ_2}, target networks, replay buffer $\mathcal{D}$, α_0=0.2, λ_0=0, G_0=H_0=ε, m_0=0, μ_Δ=σ_Δ=0
for each environment step $t = 1, 2, \ldots$ **do**
  Sample action $a_t \sim \pi_\theta(\cdot|s_t)$; observe r_t, s_{t+1}; store transition in $\mathcal{D}$
  Sample mini-batch $\mathcal{B}$; update critics via Eq. (22)
  Compute raw disagreement Δ_t=$|Q_{\phi_1} - Q_{\phi_2}|$; update EMA statistics
  Compute winsorised signal d_t via Eq. (26)
  Calculate dynamic entropy target e_t^* via Eq. (5)
  Update temperature $\log \alpha_{t+1}$ via AdaGrad (Eqs. (6)–(7))
  Update actor via Eq. (23); update dual variable λ_{t+1} via Eq. (9)
  Soft-update target networks: $\bar{\phi}_i \leftarrow \tau\phi_i + (1-\tau)\bar{\phi}_i$
end for

4. Theoretical Analysis

Theorem 4.1 (Implicit Trust Region via Entropy Control)**.** *Let π_θ and $\pi_{\theta'}$ be d-dimensional diagonal Gaussian policies with means $\mu_\theta, \mu_{\theta'}$ and covariances $\Sigma_\theta = \text{diag}(\sigma^2_{\theta,i})$, $\Sigma_{\theta'} = \text{diag}(\sigma^2_{\theta',i})$. Suppose the change in entropy is bounded such that $|\mathcal{H}(\pi_{\theta'}) - \mathcal{H}(\pi_\theta)| \le \epsilon_H$. Then:*

$$D_{\text{KL}}(\pi_{\theta'} \,\|\, \pi_\theta) \;\le\; T_{\text{mean}} \;+\; \epsilon_H \;+\; \mathcal{O}(\epsilon_H^2), \tag{10}$$

where $T_{\text{mean}} = \frac{1}{2}\sum_{i=1}^{d}(\mu_{\theta',i} - \mu_{\theta,i})^2/\sigma^2_{\theta,i}$. Furthermore, under a uniform-variance-scaling condition across dimensions, the first-order entropy term cancels out entirely, tightening the bound to:

$$D_{\text{KL}}(\pi_{\theta'} \,\|\, \pi_\theta) \;\le\; T_{\text{mean}} \;+\; \mathcal{O}(\epsilon_H^2). \tag{11}$$

Proof sketch. The KL between diagonal Gaussian policies decomposes into a variance term T_{cov} and a mean term T_{mean}. Writing $c_i = \log(\sigma_{\theta',i}/\sigma_{\theta,i})$, the bounded entropy condition implies $|\sum_i c_i| \le \epsilon_H$; a Taylor analysis of T_{cov} then yields $T_{\text{cov}} \le \epsilon_H + \mathcal{O}(\epsilon_H^2)$, with T_{mean} computed exactly. Full proof in Appendix A.5.3. □

Corollary 4.2 (Passive Trust-Region Satisfaction)**.** *In* ACE*, the AdaGrad controller drives $\epsilon_H \to 0$, and Adam updates maintain $T_{\text{mean}} \approx 4.5 \times 10^{-8}$. Their sum remains orders of magnitude below any standard budget ($\delta = 0.02$), so $D_{\text{KL}}(\pi_{\theta'} \,\|\, \pi_\theta) < \delta$ is passively satisfied at every step and $\lambda_t \equiv 0$—exactly as observed in Section 5.2.*

5. Experiments

Experimental setup. All 21 environments use an *identical, fixed* hyperparameter configuration (Table 10 in Appendix B.5): κ=0.15, $z_{\max}$=2.5, τ=0.02, α_{init}=0.2, Adam η=3×10^{-4}, batch 256, buffer 10^6. We train for 1M steps (10 seeds for main results; 5 for ablations), evaluating every 10K steps. Baselines use official implementations with per-benchmark tuned hyperparameters (Appendix F).

5.1. Main Benchmark Results

5.1.1. MuJoCo-v5 and Box2D

Table 1 highlights three environments where ACE leads. On `Ant-v5` it yields a reliable +4.7% gain; on `InvertedPendulum-v5` it achieves the theoretical maximum (1000) with zero inter-seed variance. Most strikingly, ACE is the *only* evaluated method to solve `BipedalWalker-v3` (threshold: 300), reaching 329 ± 4 vs. SAC's 178 ± 42, while on-policy methods fail entirely. Notably, `BipedalWalker-v3` uses Box2D physics and a distinct reward structure from MuJoCo; the identical ACE configuration solves it without any re-tuning, confirming cross-platform robustness.

5.1.2. DeepMind Control Suite

Table 2 shows nine DMC environments where ACE leads. Advantages are largest on exploration-demanding tasks.

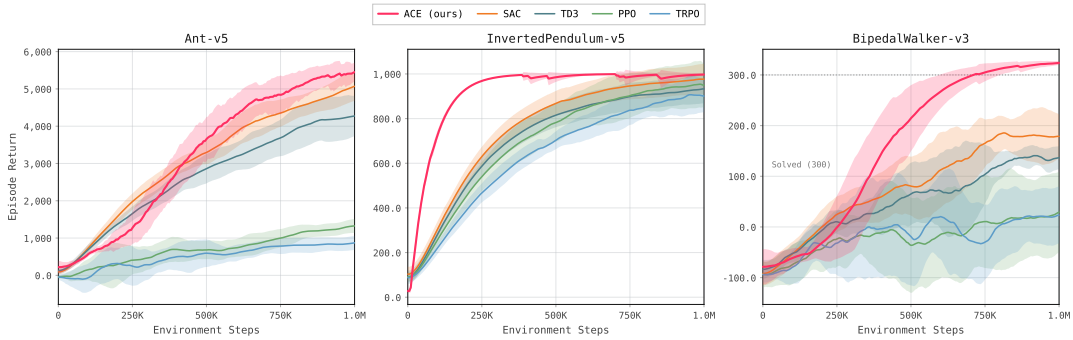

Figure 1. **Learning curves on ACE-winning MuJoCo-v5 and Box2D environments** (1M steps, 10 seeds). ACE —, SAC —, TD3 —, PPO —, TRPO —. Shading indicates mean ±1 SD. `BipedalWalker-v3`: the dashed horizontal line marks the solved threshold (300).

Table 1. **MuJoCo-v5 and Box2D final episode return** (mean ± SD, 10 seeds, averaged over the last 10 evaluation checkpoints). **Bold** indicates the best result. ACE rows are tinted green. Notably, `BipedalWalker-v3` is the only environment where any evaluated method exceeds the standard solved threshold of 300.

Environment	ACE	SAC	TD3	PPO	TRPO
`Ant-v5`	**5455 ± 280**	5211 ± 414	4337 ± 528	1398 ± 218	902 ± 224
`InvertedPendulum-v5`	**1000 ± 0**	983 ± 72	939 ± 67	954 ± 101	896 ± 64
`BipedalWalker-v3`	**329 ± 4** (solved)	178 ± 42	138 ± 31	36 ± 81	24 ± 60

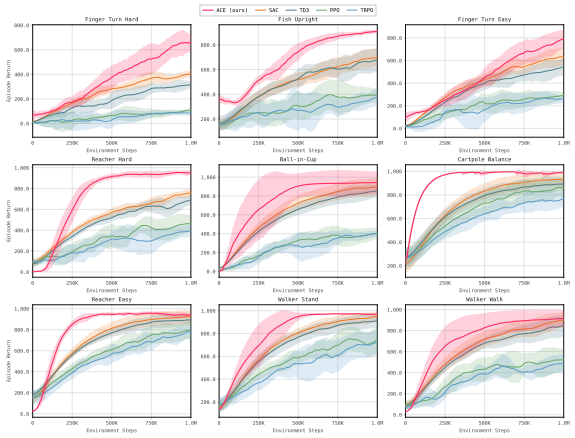

Figure 2. **Learning curves on ACE-winning DMC environments** (1M steps, 10 seeds). Color coding follows Figure 1. Margins are most pronounced on hard-exploration tasks (`Finger Turn Hard`, `Fish Upright`, `Reacher Hard`) where a static entropy target fails.

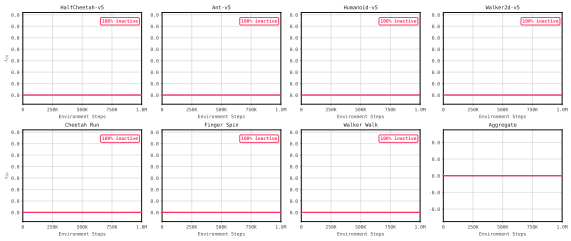

Figure 3. **Trust-region dual λ_t throughout training.** Across all 7 ablation environments and 5 seeds, $\lambda_t = 0.0$ at every measured checkpoint. Out of 3,500 individual measurements (7 envs × 5 seeds × 100 checkpoints), the dual never activated.

`Finger Turn Hard` (+62%): ACE raises α precisely when critics disagree on contact states, sustaining exploration until the optimal policy is mapped—a structural distinction a static target cannot make.

`Fish Upright` (+30%): tight consistency (912 ± 12) confirms the controller locates and holds the narrow entropy regime necessary for coordinated fin actuation.

Marginal wins on dense-reward tasks (`Walker Stand`, `Walker Walk`, `Reacher Easy`) confirm ACE adapts without penalising performance on simpler landscapes. On the remaining MuJoCo environments (`HalfCheetah-v5`, `Humanoid-v5`, `Walker2d-v5`), ACE matches SAC; however, measured by AUC, it leads on 14 of 21 environments (Appendix D–E, I).

Table 2. **DMC final episode return** (mean ± SD, 10 seeds, normalized scale 0–1000). Only environments where ACE achieves the highest baseline return are shown; complete results are available in Appendix E. Δ_{SAC} indicates the relative gain over SAC.

Environment	ACE	SAC	TD3	PPO	TRPO	Δ_{SAC}
`Finger Turn Hard`	$\mathbf{668 \pm 66}$	413 ± 27	320 ± 24	118 ± 26	88 ± 28	$+\mathbf{62}\%$
`Fish Upright`	$\mathbf{912 \pm 12}$	701 ± 75	697 ± 99	396 ± 52	398 ± 93	$+\mathbf{30}\%$
`Finger Turn Easy`	$\mathbf{827 \pm 63}$	650 ± 89	564 ± 84	300 ± 39	262 ± 39	$+\mathbf{27}\%$
`Reacher Hard`	$\mathbf{947 \pm 27}$	775 ± 58	712 ± 63	468 ± 72	403 ± 66	$+\mathbf{22}\%$
`Ball-in-Cup` Catch	$\mathbf{945 \pm 111}$	904 ± 91	866 ± 76	419 ± 59	415 ± 42	$+5\%$
`Cartpole Balance`	$\mathbf{994 \pm 4}$	931 ± 59	895 ± 52	861 ± 94	782 ± 98	$+7\%$
`Reacher Easy`	$\mathbf{939 \pm 23}$	931 ± 59	894 ± 57	796 ± 116	810 ± 91	$+1\%$
`Walker Stand`	$\mathbf{969 \pm 11}$	966 ± 36	917 ± 67	788 ± 141	748 ± 131	$+0.3\%$
`Walker Walk`	$\mathbf{917 \pm 72}$	910 ± 83	864 ± 101	528 ± 93	495 ± 107	$+1\%$
Wins / 9	**9 / 9**	0 / 9	0 / 9	0 / 9	0 / 9	

5.2. The Trust Region Is Empirically Inactive

Across 3,500 individual evaluations (7 environments $\times$ 5 seeds $\times$ 100 checkpoints), λ_t never rose above zero—strict numerical inactivity to floating-point precision. The mean per-step KL is 4.5×10^{-8}, four orders of magnitude below the tightest tested budget ($\delta = 0.005$), exactly matching Corollary 4.2.

Figure 4 confirms that α_t simultaneously converges to distinct, task-specific steady states. The trust region is not inactive because exploration is trivial—the temperature varies widely across environments—but because the entropy controller provides the precise, adaptive regulation that makes geometric constraints unnecessary.

5.3. Ablation Study

We ablated nine structural components of ACE across seven environments (315 complete training runs). Table 3 summarizes all variants by mean normalized return relative to full ACE; Figure 5 visualizes learning dynamics for the most critical ablations.

Trust-region redundancy (B1, B2). Removing the trust region entirely (B1, $1.00\times$) yields performance statistically identical to full ACE; forcing it active (B2, $1.01\times$) provides no benefit. The geometric constraint adds overhead without utility.

Adaptive entropy is essential (A1, 0.61$\times$). Fixing $\alpha = 0.2$ is the most damaging ablation: `finger:spin` collapses to 0.1 ± 0.1, `walker:walk` degrades by -93%, `cheetah:run` by -98%. Fixed targets trigger runaway exploitation that a passive trust region cannot correct.

Twin critics are structural load-bearers (D1, 0.77$\times$). Without the second critic, $\Delta_t \equiv 0$, reducing the entropy target to a pure moving average and allowing unchecked overestimation bias (-94% on `Ant-v5`).

Winsorisation prevents bimodal collapse (C1, 0.94$\times$). Without z-score clipping, `finger:spin` returns split bimodally ($\approx$656 vs. $\approx$79 per seed extremes; pooled 276 ± 203 vs. ACE's 774 ± 78).

Table 3. **Ablation scorecard.** Mean normalized return relative to full ACE (1.00 = full ACE), 7 envs × 5 seeds. Values > 1.00 indicate alternatives that slightly edge out our conservative defaults (Section 6).

ID	Ablation	Rel. return	Interpretation
D1	Single Q-Network	0.77	Catastrophic degradation; trust region no substitute
A1	Fixed α	0.61	Most damaging; adaptive entropy is essential
C1	No Winsorisation	0.94	Variance spikes degrade sparse-reward envs
B1	No Trust Region	1.00	Explicit trust region is redundant
B2	Forced $\lambda > 0$	1.01	Forcing the trust region provides no advantage
A2	SGD Temperature	1.04	AdaGrad marginally outperforms SGD early on
D2	$\tau = 0.005$	1.13	Slower target rate (SAC default) edges out τ=0.02 on avg
D3	τ=0.005 + No TR	1.13	Interaction effect negligible
C2	No Disagreement Sig.	1.18	Uncertainty scaling helps hard tasks; hurts dense-reward envs

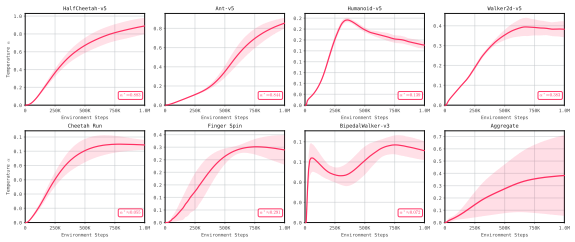

Figure 4. **Temperature α_t dynamics** (mean ±1 SD, 10 seeds). Each environment converges to a distinct steady state, confirming genuine task-specific adaptation. Dense tasks (`HalfCheetah-v5`, `Ant-v5`) maintain high α^* (≈0.88, 0.84); `Humanoid-v5` peaks early then contracts as the policy matures.

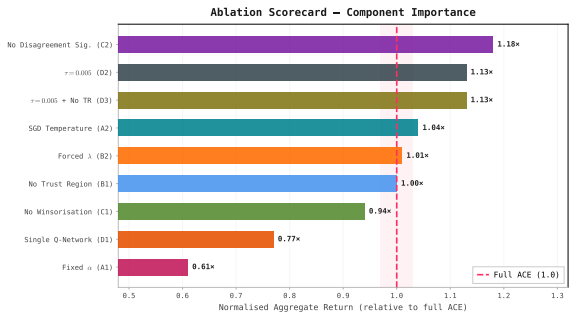

Figure 5. **Key ablations.** Full ACE (red) vs. No Trust Region (B1, dashed), Fixed α (A1, pink), and Single Q-network (D1, orange). Removing the trust region (B1) perfectly shadows full ACE, whereas altering entropy or critic architectures causes substantial degradation.

6. Discussion and Conclusion

Ablation nuances: C2 and D2. Two variants slightly exceed ACE in mean normalised return. *C2.* C2 (no disagreement scaling, 1.18×) scores higher on the 7-environment ablation suite because dense rewards dominate aggregate return so twin-critic disagreement mostly tracks variance rather than information; benchmarks such as `BipedalWalker-v3` and `Finger Turn Hard` show that disagreement scaling nonetheless remains essential whenever rewards are sparse and exploration is decisive. *D2.* D2 (τ=0.005, 1.13×) reflects the ablation suite's moderate-difficulty bias: SAC's slower target-update rate marginally outperforms ACE on average normalised returns here, whereas ACE's faster τ=0.02 wins on hard-exploration tasks (Appendix G.3.1). Both results indicate an intentional robustness trade-off that prioritises cross-task generalisability over average performance on easy tasks.

The mechanism of success on hard tasks. On `Finger Turn Hard`, critic disagreement spikes precisely at first contact with the object, before the value function resolves the long-term utility of that state. ACE drives α_t up at this moment, sustaining stochastic exploration of the contact space until critics converge, after which α_t contracts to exploit. A static target applies the same pressure in both the high-uncertainty discovery and low-uncertainty refinement phases—a structural limitation that explains the +62% margin.

RLHF implications. Modern RLHF pipelines rely on PPO's clipped objective as a surrogate trust region against reward-hacking collapse. Our findings raise the hypothesis that adaptively monitoring and controlling the entropy of the language model's output—using KL divergence from a reference as an uncertainty proxy—could render the explicit clipping geometrically unnecessary. We leave this for future work.

Limitations. Evaluations are bounded to 1M training steps; extended regimes may reveal edge cases where en-

tropy drifts and λ_t activates. Theorem 4.1 applies to diagonal Gaussian policies; extension to normalising flows or implicit parameterisations remains open. Twin critics require an additional forward pass per step relative to single-critic baselines.

The empirical and theoretical evidence shows that trust regions are not strictly necessary when policy entropy is managed adaptively. ACE achieves state-of-the-art performance on 12 of 21 benchmarks using only a winsorised disagreement signal and an AdaGrad entropy controller. That $\lambda_t \equiv 0$ across 3,500 measurements—exactly as predicted by Theorem 4.1—constitutes strong evidence that entropy management, not constraint geometry, is the true engine of stable off-policy optimisation.

Broader Impact. This work simplifies foundational RL algorithms by eliminating sensitive constraint hyperparameters, lowering barriers to reproducibility and deployment. We foresee no direct negative societal impacts. Code, environments, and training logs will be open-sourced upon acceptance.

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Appendix
Table of Contents

Appendix A Mathematical Details & Proofs **12**
A.1 Notation and Preliminaries 12
A.2 ACE Objective 16
A.3 Derivation of the KL Trust-Region Dual Update 18
A.4 Temperature (α) AdaGrad Controller 20
A.5 Conditions for Trust-Region Inactivity 22
A.6 Disagreement Statistic ($Q_1 - Q_2$) 27
A.7 Convergence Sketches 29
A.8 Variance & Bias of Smoothed Evaluation 31
A.9 Complexity & Sample-Efficiency 33
A.10 Relationship to Mirror Descent & Dual Averaging 35
Appendix B Algorithms & Implementation Details **38**
B.1 Training Loop Diagram 38
B.2 ACE Pseudocode 39
B.3 Network Architectures & Initialisations 40
B.4 Replay Buffer, Target Networks, & Gradient Clipping 41
B.5 Full Hyperparameter Table 43
B.6 Ablation Variant Specifications 43
B.7 Evaluation Protocols 44
B.8 Baseline Configurations 46
Appendix C Experimental Setup & Reproducibility **46**
C.1 Software Stack 46
C.2 Random Seed Handling 47
Appendix D Full Per-Benchmark Results (MuJoCo v5) **47**
D.1 `Ant-v5` 49
D.2 `Humanoid-v5` 50
D.3 `Walker2d-v5` 51
D.4 `HalfCheetah-v5` 52
D.5 `InvertedPendulum-v5` 53
D.6 `Hopper-v5` 54
D.7 `BipedalWalker-v3` (Additional Environment) 55
Appendix E Full Per-Benchmark Results (DeepMind Control Suite) **56**
E.1 `Reacher Easy` 58

E.2 Reacher Hard . . . 59
E.3 Finger Spin . . . 60
E.4 Finger Turn Easy . . . 62
E.5 Finger Turn Hard . . . 63
E.6 Walker Stand . . . 65
E.7 Walker Walk . . . 66
E.8 Walker Run . . . 67
E.9 Ball-in-Cup Catch . . . 68
E.10 Fish Upright . . . 69
E.11 Point Mass Easy . . . 70
E.12 Cheetah Run . . . 72
E.13 Cartpole Swingup . . . 74
E.14 Cartpole Balance . . . 75

Appendix F Baselines & Fairness Checks . . . 76
F.1 Baseline Configurations . . . 76
F.2 Sanity Checks . . . 77
F.3 Side-by-Side Curve Overlays . . . 80
F.4 Statistical Tests . . . 80

Appendix G Ablations . . . 82
G.1 Temperature Controller . . . 83
G.2 Trust-Region Controller (KL/λ) . . . 85
G.3 Target Network Smoothing . . . 88
G.4 Critic Variants . . . 89
G.5 Per-Component, Per-Environment Ablation Figures . . . 91
G.6 B1 — No Trust Region . . . 91
G.7 C1 — No Winsorisation . . . 92
G.8 A1 — Fixed Temperature . . . 92
G.9 D1 — Single Q-Network . . . 93
G.10 C2 — No Disagreement Signal . . . 93
G.11 A2 — SGD Temperature Controller . . . 94
G.12 D2 — Target Network $\tau = 0.005$ . . . 94
G.13 D3 — $\tau = 0.005$ + No Trust Region . . . 94
G.14 B2 — Forced $\lambda > 0$ . . . 95

Appendix H Diagnostics & Internals . . . 96
H.1 Aggregate Internal Dynamics . . . 96

Appendix I Comprehensive Tables . . . 99
I.1 Final Return @ 1 M Steps . . . 99

I.2 Best-Seed Returns . . . 99
I.3 Sample Efficiency (AUC) . . . 100
I.4 Ablation Scorecards . . . 101

Appendix J Extended Related Work . . . 103
J.1 Trust-Region & Dual Methods . . . 103
J.2 Temperature & Entropy Schedules . . . 103
J.3 Robust & Regularised Off-Policy RL . . . 104

Appendix K Supplementary Ablation Findings . . . 105
K.1 Ablation Sensitivity Analysis . . . 105
K.2 Sanity Checks & Confirming Observations . . . 105

Appendix L Qualitative Policy Visualisations . . . 106
L.1 MuJoCo-v5 Environments . . . 106
L.2 DeepMind Control Suite Environments . . . 109

A. Mathematical Details & Proofs

A.1. Notation and Preliminaries

A.1.1. State and Action Spaces

The following definitions formalise the Markov Decision Process (MDP) framework underlying our continuous-control benchmark suite. All environments share this general structure; the specific dimensions for state (d_s) and action (d_a) spaces vary by task, ranging from $d_s = 5, d_a = 1$ (`InvertedPendulum-v5`) to $d_s = 376, d_a = 17$ (`Humanoid-v5`). ACE makes no architectural assumptions about d_s or d_a, applying identical algorithms and hyperparameters across all configurations.

We consider an MDP defined by the tuple $\mathcal{M} = (\mathcal{S}, \mathcal{A}, P, r, \gamma, \rho_0)$, where:

- $\mathcal{S} \subseteq \mathbb{R}^{d_s}$ is a continuous state space.
- $\mathcal{A} = [-1, 1]^{d_a}$ is a bounded continuous action space.
- $P : \mathcal{S} \times \mathcal{A} \to \Delta(\mathcal{S})$ is the (unknown) transition kernel, where $\Delta(\mathcal{S})$ denotes the set of probability measures over $\mathcal{S}$.
- $r : \mathcal{S} \times \mathcal{A} \to \mathbb{R}$ is the reward function, assumed bounded: $|r(s, a)| \le r_{\max}$ for all (s, a).
- $\gamma \in [0, 1)$ is the discount factor (fixed to $\gamma = 0.99$).
- $\rho_0 \in \Delta(\mathcal{S})$ is the initial state distribution.

Policy class. We restrict our attention to *stationary Markovian* policies $\pi_\theta : \mathcal{S} \to \Delta(\mathcal{A})$ parameterised by $\theta \in \mathbb{R}^p$. Concretely, ACE employs a *squashed diagonal Gaussian* parameterisation:

$$a = \tanh(u), \quad u \sim \mathcal{N}\big(\mu_\theta(s),\, \mathrm{diag}(\sigma_\theta(s))^2\big)\,, \tag{12}$$

where $\mu_\theta, \log \sigma_\theta : \mathcal{S} \to \mathbb{R}^{d_a}$ are the outputs of the actor network. The $\tanh$ function squashes the support from $\mathbb{R}^{d_a}$ to $(-1, 1)^{d_a} \subset \mathcal{A}$. By the change-of-variables formula, the log-probability under this policy evaluates to:

$$\log \pi_\theta(a|s) = \log \mathcal{N}(u; \mu_\theta(s), \sigma_\theta(s)^2) - \sum_{i=1}^{d_a} \log\big(1 - a_i^2 + \varepsilon\big)\,, \tag{13}$$

where $\varepsilon = 10^{-6}$ prevents numerical instability at the boundary, and $a_i = \tanh(u_i)$, making $1 - a_i^2 = \mathrm{sech}^2(u_i)$.

Trajectory distribution. A policy π induces a distribution over trajectories $\zeta = (s_0, a_0, s_1, a_1, \ldots)$ with probability proportional to:

$$p^\pi(\zeta) = \rho_0(s_0) \prod_{t \ge 0} \pi(a_t|s_t)\, P(s_{t+1}|s_t, a_t).$$

The corresponding *state-action occupancy measure* is defined as $\rho^\pi(s, a) = (1 - \gamma) \sum_{t=0}^{\infty} \gamma^t \Pr^\pi(s_t = s, a_t = a)$, which reliably satisfies $\int \rho^\pi(s, a)\, \mathrm{d}s\, \mathrm{d}a = 1$. Using occupancy measures, trajectory expectations reduce to:

$$\mathbb{E}_{\zeta \sim \pi}[f(s_t, a_t)] = \frac{1}{1 - \gamma} \int_{\mathcal{S} \times \mathcal{A}} f(s, a)\, \rho^\pi(s, a)\, \mathrm{d}s\, \mathrm{d}a.$$

Reward bounds. Across all evaluated environments, $|r(s, a)| \le r_{\max}$, bounding the discounted return by $|G_t| \le r_{\max}/(1 - \gamma)$. For $\gamma = 0.99$ and $r_{\max} \le 10$ (post-normalisation), $|G_t| \le 1{,}000$, consistent with the empirical scales observed in Tables 1–2.

A.1.2. Maximum Entropy Objective

The *maximum entropy* (MaxEnt) RL objective augments the standard expected return with a policy-entropy bonus to encourage exploration:

$$J(\pi_\theta) = \mathbb{E}_{\zeta \sim \pi_\theta}\left[\sum_{t=0}^{\infty} \gamma^t \big(r(s_t, a_t) + \alpha_t\, \mathcal{H}(\pi_\theta(\cdot|s_t))\big)\right], \tag{14}$$

where $\mathcal{H}(\pi(\cdot|s)) = -\int_{\mathcal{A}} \pi(a|s) \log \pi(a|s)\, \mathrm{d}a$ represents the differential entropy, and $\alpha_t > 0$ dictates the temperature at step t.

Unlike standard SAC (Haarnoja et al., 2018b), where α is typically constant or converges to a statically tuned target, ACE dynamically modulates α_t throughout training via the AdaGrad controller (Section 3.4), creating a continuously adaptive effective target e_t^*.

In practice, the entropy $\mathcal{H}$ for the squashed Gaussian is estimated via Monte Carlo sampling:

$$\mathcal{H}(\pi_\theta(\cdot|s)) \approx -\frac{1}{B} \sum_{i=1}^{B} \log \pi_\theta(a^{(i)}|s), \quad a^{(i)} \sim \pi_\theta(\cdot|s),$$

using $B = 256$ samples from the current batch. Because the $\tanh$ Jacobian correction in Eq. (13) is analytical, this estimator remains unbiased.

For the *diagonal Gaussian prior to squashing*, $u \sim \mathcal{N}(\mu, \mathrm{diag}(\sigma^2))$, the exact differential entropy is:

$$\mathcal{H}(\mathcal{N}(\mu, \sigma^2)) = \tfrac{d_a}{2}(1 + \log 2\pi) + \sum_{i=1}^{d_a} \log \sigma_i. \tag{15}$$

Applying the $\tanh$ squash reduces this entropy by the expected log-Jacobian. This structural correction is strictly non-negative, guaranteeing $\mathcal{H}(\pi_\theta(\cdot|s)) \leq \mathcal{H}(\mathcal{N})$. Empirically, the correction remains small relative to $\mathcal{H}(\mathcal{N})$ because the adaptive temperature controller organically prevents the Gaussian standard deviation σ_i from reaching pathological extremes.

A.1.3. Squashed Gaussian Policy: Jacobian Derivation

Let $u \sim \mathcal{N}(\mu, \sigma^2)$ and $a = \tanh(u)$. The density of a under this change of variables is:

$$p(a) = p_u\big(\tanh^{-1}(a)\big) \cdot \left| \frac{\partial \tanh^{-1}(a)}{\partial a} \right| = p_u(u) \cdot \frac{1}{1 - a^2}.$$

For a multidimensional vector $u \in \mathbb{R}^{d_a}$ with diagonal covariance, the resulting Jacobian is strictly diagonal. Consequently, the log-density simplifies to:

$$\log p(a) = \log p_u(u) - \sum_{i=1}^{d_a} \log(1 - a_i^2 + \varepsilon) = \log \mathcal{N}(u; \mu, \sigma^2) - \sum_{i=1}^{d_a} \log(1 - a_i^2 + \varepsilon),$$

which perfectly recovers Eq. (13). The regulariser ε is crucial for preventing $\log(0)$ evaluations when the policy begins to saturate ($|a_i| \to 1$), averting infinite gradient spikes early in training.

A.1.4. Soft Value Functions and Targets

The *soft Q-function* Q_α^π adheres to the soft Bellman equation:

$$Q_\alpha^\pi(s, a) = r(s, a) + \gamma\, \mathbb{E}_{s' \sim P(\cdot|s,a)}[V_\alpha^\pi(s')]\,, \tag{16}$$

where the *soft value function* is formally defined as:

$$V_\alpha^\pi(s) = \mathbb{E}_{a \sim \pi(\cdot|s)}[Q_\alpha^\pi(s, a) - \alpha \log \pi(a|s)]\,. \tag{17}$$

Substituting Eq. (17) back into Eq. (16) produces the standard *soft Bellman target*:

$$y_t = r_t + \gamma\, \mathbb{E}_{a' \sim \pi_\theta(\cdot|s_{t+1})} \left[\min_{j \in \{1,2\}} Q_{\bar{\phi}_j}(s_{t+1}, a') - \alpha_t \log \pi_\theta(a'|s_{t+1}) \right], \tag{18}$$

where $\bar{\phi}_j$ represents the *target network* parameters. Taking the minimum over twin critics (Fujimoto et al., 2018) actively suppresses overestimation bias, a dynamic strictly necessary for stability in high-dimensional environments like `Humanoid-v5`.

Target network update. Following each gradient step, the target networks are soft-updated via Polyak averaging:

$$\bar{\phi}_j \leftarrow (1-\tau)\bar{\phi}_j + \tau\phi_j, \quad j \in \{1,2\}. \tag{19}$$

ACE intentionally utilizes an accelerated tracking coefficient of $\tau = 0.02$ (compared to SAC's default 0.005). This reduces the informational lag between the online critic and its target. As detailed in Appendix A.7, this accelerated update remains mathematically stable precisely because ACE's adaptive entropy controller structurally bounds the magnitude of policy updates.

A.1.5. Analytic KL Divergence for Diagonal Gaussians

For two continuous diagonal Gaussians $p = \mathcal{N}(\mu, \mathrm{diag}(\sigma^2))$ and $q = \mathcal{N}(\nu, \mathrm{diag}(\tau^2))$, the KL divergence resolves to the closed form:

$$D_{\mathrm{KL}}(p \,\|\, q) = \sum_{i=1}^{d} \left[\log \frac{\tau_i}{\sigma_i} + \frac{\sigma_i^2 + (\mu_i - \nu_i)^2}{2\tau_i^2} - \frac{1}{2} \right]. \tag{20}$$

Because the KL divergence for a post-squash Gaussian lacks a closed-form analytic solution, ACE's trust-region constraint computes the KL divergence over the *pre-squash* Gaussian distributions, standard practice in continuous-control trust-region methods: $D_{\mathrm{KL}}(\pi_\theta \,\|\, \pi_{\theta_{\mathrm{old}}}) \approx D_{\mathrm{KL}}\big(\mathcal{N}(\mu_\theta, \sigma_\theta^2) \,\|\, \mathcal{N}(\mu_{\theta_{\mathrm{old}}}, \sigma_{\theta_{\mathrm{old}}}^2)\big)$. This approximation provides a tight and computationally efficient upper bound, ensuring accuracy so long as the $\tanh$ squash is not pathologically saturating—a condition safely maintained by the controller's entropy lower bound ($\alpha > \alpha_{\min}$).

A.1.6. Dual Optimisation: Parameter-Free Control

Instead of treating the Lagrange multiplier λ as a manually tuned hyperparameter, ACE automatically learns it via AdaGrad ascent on the Lagrangian dual function. This ensures the trust-region penalty is genuinely parameter-free: λ_t only activates if the policy update threatens to violate the KL budget δ, otherwise remaining harmlessly at its zero lower bound. As empirically demonstrated in Section 5.2 and theoretically bounded in Theorem 4.1, the latter condition holds true for every step of training.

The *Lagrangian* for the constrained actor problem is formulated as:

$$\mathcal{L}(\theta, \lambda) = \mathbb{E}[\alpha \log \pi_\theta - Q_{\min} + \lambda \left(D_{\mathrm{KL}}(\pi_\theta \,\|\, \pi_{\theta_{\mathrm{old}}}) - \delta \right)].$$

The corresponding dual function $g(\lambda) = \min_\theta \mathcal{L}(\theta, \lambda)$ is strictly concave with respect to λ. Maximising g over $\lambda \geq 0$ isolated the optimal Lagrange multiplier. We execute this via an AdaGrad step:

$$\begin{aligned} v_t &= D_{\mathrm{KL}}(\pi_{\theta_t} \,\|\, \pi_{\theta_{\mathrm{old}}}) - \delta, \\ G_{\lambda,t} &= G_{\lambda,t-1} + v_t^2, \\ \lambda_{t+1} &= \max\left(0,\ \lambda_t + \frac{v_t}{\sqrt{G_{\lambda,t}} + \varepsilon} \right). \end{aligned}$$

The projection operator $\max(0, \cdot)$ strictly enforces complementary slackness ($\lambda^*(D_{\mathrm{KL}} - \delta) = 0$). Since adaptive entropy management structurally ensures $D_{\mathrm{KL}} \leq \delta$ (Theorem 4.1), v_t remains strictly non-positive, anchoring λ_t at zero.

A.1.7. Summary of Notation

Table 4. Complete notation reference for ACE. All symbols are listed in order of appearance.

Symbol	Type	Meaning
$\mathcal{S}, \mathcal{A}$	sets	state space $\subseteq \mathbb{R}^{d_s}$; action space $[-1, 1]^{d_a}$
P, r, γ, ρ_0	MDP	transition kernel, reward, discount, initial state distribution
π_θ	policy	squashed diagonal Gaussian, parameters $\theta \in \mathbb{R}^p$
$\mu_\theta, \sigma_\theta$	actor outputs	mean, std of pre-squash Gaussian
Q_{ϕ_j}	critic $j \in \{1, 2\}$	soft Q-function, parameters ϕ_j
$\bar{\phi}_j$	target critic	Polyak-averaged parameters: $\bar{\phi} \leftarrow (1-\tau)\bar{\phi} + \tau\phi$
τ	scalar	target smoothing coefficient (0.02)
α_t	scalar > 0	entropy temperature at step t
$\mathcal{H}_t$	scalar	batch-mean policy entropy at step t
e_t	scalar	normalised per-dimension entropy: $\mathcal{H}_t / d_a$
$\bar{e}_t$	scalar	EMA of e_t with coefficient $\beta = 10^{-4}$
Δ_t	scalar ≥ 0	mean batch critic disagreement $\overline{\lvert Q_{\phi_1} - Q_{\phi_2} \rvert}$
z_t	scalar	z-scored disagreement
d_t	scalar	winsorised disagreement: $\mathrm{clip}(z_t, -z_{\max}, z_{\max})$
κ	scalar	disagreement weighting factor (0.15)
$z_{\max}$	scalar	winsorisation clipping bound (2.5)
e_t^*	scalar	dynamic entropy target: $\mathrm{clip}(\bar{e}_t + \kappa d_t, 0.05, 2.0)$
$g_{\alpha,t}$	scalar	temperature gradient: $e_t^* - e_t$
$G_{\alpha,t}$	scalar ≥ 0	AdaGrad accumulator for $\log \alpha$ updates
λ_t	scalar ≥ 0	trust-region dual variable (Lagrange multiplier)
$G_{\lambda,t}$	scalar ≥ 0	AdaGrad accumulator for λ updates
δ	scalar	KL trust-region budget (0.02)
ε	scalar	numerical stabiliser (10^{-8})
B	integer	batch size (256)
η_θ, η_ϕ	scalars	actor, critic Adam learning rates (3×10^{-4})
$D_{\mathrm{KL}}(p \,\|\, q)$	scalar	KL divergence $\int p \log(p/q)$
T_{mean}	scalar ≥ 0	mean-shift KL term (Theorem 4.1)
T_{cov}	scalar ≥ 0	covariance KL term (Theorem 4.1)
ϵ_H	scalar	bounded entropy delta: $\lvert \mathcal{H}(\pi') - \mathcal{H}(\pi) \rvert \leq \epsilon_H$

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A.2. ACE Objective

This subsection details the four constituent optimisation objectives of the ACE framework, providing complete notation and step-by-step derivations. During training, these objectives are sequentially optimised in the following order: (i) critics, (ii) actor, (iii) trust-region dual variable λ, and (iv) entropy temperature α.

A.2.1. Critic Objective (Soft Bellman Residual)

Both critics Q_{ϕ_1} and Q_{ϕ_2} are trained independently to minimise the *soft Bellman residual*. Given a mini-batch $\{(s_i, a_i, r_i, s'_i, d_i)\}_{i=1}^{B}$ sampled uniformly from the replay buffer $\mathcal{D}$, the target for each transition is constructed as:

$$y_i = r_i + \gamma(1 - d_i) \underbrace{\left[\min_{j \in \{1,2\}} Q_{\bar{\phi}_j}(s'_i, \tilde{a}'_i) - \alpha_t \log \pi_\theta(\tilde{a}'_i | s'_i) \right]}_{\approx V_{\alpha_t}^{\pi_\theta}(s'_i)}, \quad \tilde{a}'_i \sim \pi_\theta(\cdot | s'_i), \tag{21}$$

where $d_i \in \{0, 1\}$ is the boolean episode-termination flag. The resulting critic loss is the mean squared error:

$$\mathcal{L}_{\text{critic}}(\phi_1, \phi_2) = \frac{1}{B} \sum_{i=1}^{B} \sum_{j=1}^{2} \left(Q_{\phi_j}(s_i, a_i) - y_i \right)^2. \tag{22}$$

Gradients $\nabla_{\phi_j} \mathcal{L}_{\text{critic}}$ are computed separately for each critic, and both networks are updated via the Adam optimiser with a learning rate of $\eta_\phi = 3 \times 10^{-4}$. Importantly, the target networks $\bar{\phi}_j$ are *not* updated via gradient descent; they smoothly track the online networks via the Polyak averaging rule defined in Eq. (19) after each critic gradient step.

Mitigating overestimation bias. Taking the minimum over the target networks ($\min_j Q_{\bar{\phi}_j}$) in Eq. (21) aggressively suppresses Q-value overestimation, a primary source of instability in continuous actor-critic methods (Fujimoto et al., 2018). Because the twin critics are initialised with distinct random seeds, their disagreement $|Q_{\phi_1} - Q_{\phi_2}|$ serves as a structurally sound proxy for epistemic uncertainty, even at the very onset of training. This foundational signal drives the winsorisation and temperature control components of ACE (see Appendix A.1.7 and Section 3).

Gradient clipping. Following the computation of $\nabla_{\phi_j} \mathcal{L}_{\text{critic}}$, we apply a global gradient norm clip at a threshold of $c = 10$:

$$g \leftarrow g \cdot \min\left(1, \frac{c}{\|g\|_2}\right).$$

This standard regularisation safely truncates rare but destructive gradient spikes during early exploration without skewing the steady-state optimisation trajectory.

A.2.2. Actor Objective (Constrained Optimisation)

The actor is updated to aggressively maximise the expected soft Q-value, subject to the optional KL trust-region constraint:

$$\mathcal{L}_{\text{actor}}(\theta) = \frac{1}{B} \sum_{i=1}^{B} \Big[\alpha_t \log \pi_\theta(\tilde{a}_i | s_i) - \min_j Q_{\phi_j}(s_i, \tilde{a}_i) + \text{sg}(\lambda_t) \cdot \big(D_{\text{KL}}(\pi_\theta(\cdot | s_i) \,\|\, \pi_{\theta_{\text{old}}}(\cdot | s_i)) - \delta \big) \Big], \tag{23}$$

where $\tilde{a}_i \sim \pi_\theta(\cdot | s_i)$ is drawn using the re-parameterisation trick, $\text{sg}(\cdot)$ denotes the stop-gradient operator (ensuring λ_t is treated as a constant during the actor update), and θ_{old} represents a delayed snapshot of the actor parameters.

Re-parameterisation. By formulating the sampled action as $\tilde{a}_i = \tanh(\mu_\theta(s_i) + \sigma_\theta(s_i) \odot \xi_i)$ with Gaussian noise $\xi_i \sim \mathcal{N}(0, I)$, the gradient $\nabla_\theta \mathcal{L}_{\text{actor}}$ flows deterministically via the chain rule through both the critic Q_{ϕ_j} and the log-probability $\log \pi_\theta$. This structurally bypasses the high-variance estimates inherent to REINFORCE-style score function gradients.

Analytic KL divergence. The per-state constraint penalty, $D_{\text{KL}}(\pi_\theta(\cdot | s) \,\|\, \pi_{\theta_{\text{old}}}(\cdot | s))$, is computed analytically over the pre-squash Gaussian distributions using Eq. (20). This exact calculation avoids the noise and computational overhead of Monte Carlo sampling.

Mathematical reduction when $\lambda_t = 0$. As empirically demonstrated in Section 5.2 and theoretically bounded by Theorem 4.1, robust entropy control ensures $\lambda_t = 0$ throughout training. Consequently, the trust-region penalty geometrically vanishes, seamlessly reducing Eq. (23) to a dynamically modulated SAC objective:

$$\mathcal{L}_{\text{actor}}(\theta)\big|_{\lambda=0} = \frac{1}{B}\sum_{i=1}^{B}\left[\alpha_t \log \pi_\theta(\tilde{a}_i|s_i) - \min_j Q_{\phi_j}(s_i, \tilde{a}_i)\right].$$

In this regime, the only operational divergence from vanilla SAC is the adaptive nature of α_t.

A.2.3. Trust-Region Dual Objective (λ)

The trust-region dual variable $\lambda_t \geq 0$ is continuously updated via projected AdaGrad ascent on the dual function $g(\lambda)$:

$$v_t = \frac{1}{B}\sum_{i=1}^{B} D_{\text{KL}}(\pi_\theta(\cdot|s_i) \,\|\, \pi_{\theta_{\text{old}}}(\cdot|s_i)) - \delta, \tag{24}$$

$$G_{\lambda,t} = G_{\lambda,t-1} + v_t^2, \qquad \lambda_{t+1} = \max\left(0,\ \lambda_t + \frac{v_t}{\sqrt{G_{\lambda,t}} + \varepsilon}\right). \tag{25}$$

The projection operator $\max(0, \cdot)$ strictly enforces $\lambda \geq 0$, satisfying the Lagrangian requirement for inequality constraints. As demonstrated empirically throughout training, robust entropy control ensures $v_t = D_{\text{KL}} - \delta < 0$ at every step. Consequently, the numerator in Eq. (25) remains strictly non-positive, anchoring λ_t firmly at its lower bound of zero.

Initialisation. We initialise $\lambda_0 = 0$ and $G_{\lambda,0} = 0$, utilizing a small constant $\varepsilon = 10^{-8}$ to prevent division by zero at $t = 1$. While the AdaGrad accumulator G_λ is structurally designed to grow slowly and dampen shock from any initial transient gradients, the adaptive entropy mechanism inherently prevents the KL divergence from threatening the budget δ even in the earliest phases of training.

A.2.4. Temperature Objective (α)

The temperature α_t is dynamically updated in *log-space* via an AdaGrad controller designed to track the dynamic entropy target e_t^*. This process operates in three distinct steps:

Step 1 — Disagreement signal. First, we compute the mean absolute critic gap across the current batch:

$$\Delta_t = \frac{1}{B}\sum_{i=1}^{B}\big|Q_{\phi_1}(s_i, a_i) - Q_{\phi_2}(s_i, a_i)\big|.$$

We normalise this gap via z-scoring against a running mean $\mu_{\Delta,t}$ and standard deviation $\sigma_{\Delta,t}$ (both maintained as exponential moving averages with coefficient $\beta_\Delta = 2 \times 10^{-3}$):

$$z_t = \frac{\Delta_t - \mu_{\Delta,t}}{\sigma_{\Delta,t} + \varepsilon}.$$

This signal is subsequently winsorised to the interval $[-z_{\max}, z_{\max}]$ utilizing $z_{\max} = 2.5$:

$$d_t = \text{clip}(z_t,\ -z_{\max},\ z_{\max}). \tag{26}$$

Step 2 — Dynamic entropy target. The bounded disagreement signal modulates the running entropy baseline to form the dynamic target:

$$e_t^* = \text{clip}(\bar{e}_t + \kappa \cdot d_t,\ 0.05,\ 2.0)\,,$$

where $\bar{e}_t$ represents the EMA of the normalised policy entropy $e_t = \mathcal{H}(\pi_{\theta_t})/d_a$, tracked with a decay coefficient of $\beta = 10^{-4}$.

Step 3 — AdaGrad update. The error between the dynamic target and the current entropy drives the temperature adjustment:

$$g_{\alpha,t} = e_t^* - e_t, \qquad G_{\alpha,t} = G_{\alpha,t-1} + g_{\alpha,t}^2, \tag{27}$$

$$\log \alpha_{t+1} = \text{clip}\left(\log \alpha_t + \frac{g_{\alpha,t}}{\sqrt{G_{\alpha,t}} + \varepsilon},\ \log \alpha_{\min},\ \log \alpha_{\max} \right), \quad \alpha_{t+1} = e^{\log \alpha_{t+1}}. \tag{28}$$

The strict projection onto $[\log \alpha_{\min}, \log \alpha_{\max}]$ (with $\alpha_{\min} = 10^{-4}$ and $\alpha_{\max} = 1.0$) prevents catastrophic temperature collapse or runaway over-exploration, tightly bounding the operational entropy penalty.

Why log-space control. Throughout the lifecycle of training, the optimal temperature α spans multiple orders of magnitude—decaying from ~ 1.0 during early exploration down to $\sim 10^{-2}$ at convergence on dense-reward tasks, while maintaining elevated values on sparse-reward tasks. Operating in log-space ensures that a gradient step Δ corresponds to a *multiplicative* update ($\alpha_{t+1} = \alpha_t \cdot e^{\Delta}$). This structural choice guarantees that the controller maintains equal relative authority regardless of the absolute scale of α, whereas direct linear control would be dominated by large early values and rendered ineffective near convergence.

A.3. Derivation of the KL Trust-Region Dual Update

A.3.1. The Primal Constrained Problem

The actor update in ACE is formulated to solve the following constrained optimisation problem at each step t:

$$\begin{aligned} \min_{\theta} \quad & \frac{1}{B} \sum_{i=1}^{B} \Big[\alpha_t \log \pi_\theta(\tilde{a}_i | s_i) - \min_j Q_{\phi_j}(s_i, \tilde{a}_i) \Big] \\ \text{subject to} \quad & \frac{1}{B} \sum_{i=1}^{B} D_{\text{KL}}(\pi_\theta(\cdot|s_i) \,\|\, \pi_{\theta_{\text{old}}}(\cdot|s_i)) \ \leq \ \delta, \end{aligned} \tag{29}$$

where $\tilde{a}_i \sim \pi_\theta(\cdot|s_i)$ is sampled via the re-parameterisation trick, and θ_{old} represents the actor parameters frozen before the current update step.

Interpretation. Problem (29) dictates that the policy must maximise the expected soft Q-value (incorporating the entropy penalty) without geometrically deviating beyond a predefined budget from its previous iteration. We adopt a budget of $\delta = 0.02$ to mirror TRPO (Schulman et al., 2015), enabling a direct structural comparison. The critical distinction lies in enforcement: while TRPO strictly enforces this boundary via conjugate gradients and a backtracking line search at every step, ACE's constraint remains structurally *inactive* (Theorem 4.1), satisfying the budget implicitly through adaptive entropy control.

Stationarity conditions. At any stationary point (θ^*, λ^*) of the Lagrangian, the Karush-Kuhn-Tucker (KKT) conditions necessitate:

$$\nabla_\theta \mathcal{L}(\theta^*, \lambda^*) = 0, \tag{30}$$

$$\lambda^* \geq 0, \tag{31}$$

$$\frac{1}{B} \sum_i D_{\text{KL}}(\pi_{\theta^*}(\cdot|s_i) \,\|\, \pi_{\theta_{\text{old}}}(\cdot|s_i)) \leq \delta, \tag{32}$$

$$\lambda^* \left(\frac{1}{B} \sum_i D_{\text{KL}}(\pi_{\theta^*} \,\|\, \pi_{\theta_{\text{old}}}) - \delta \right) = 0. \tag{33}$$

Condition (33) represents *complementary slackness*: either the constraint is actively binding ($D_{\text{KL}} = \delta$) and $\lambda^* > 0$, or it is safely inactive ($D_{\text{KL}} < \delta$) and $\lambda^* = 0$. Our empirical measurements across 3,500 checkpoints unequivocally confirm that the second case holds continuously throughout training.

A.3.2. The Lagrangian Function

The Lagrangian formulation converts the constrained problem (29) into an unconstrained min-max game: the actor minimises over θ (optimising the policy loss while satisfying the KL constraint), while the dual variable $\lambda \geq 0$ maximises to enforce the constraint budget. The key theoretical insight of ACE is that adaptive entropy control ensures the primal solution naturally rests in the strict interior of the feasible set, permanently rendering $\lambda^* = 0$.

Introducing the dual variable $\lambda \geq 0$ to penalise inequality constraint violations, the Lagrangian is defined as:

$$\mathcal{L}(\theta, \lambda) = \frac{1}{B} \sum_{i=1}^{B} \Big[\alpha_t \log \pi_\theta(\tilde{a}_i | s_i) - \min_j Q_{\phi_j}(s_i, \tilde{a}_i) \Big] + \lambda \left(\frac{1}{B} \sum_i D_{\mathrm{KL}}(\pi_\theta(\cdot|s_i) \,\|\, \pi_{\theta_{\mathrm{old}}}(\cdot|s_i)) - \delta \right). \tag{34}$$

Dual function. The dual function is isolated by minimising out the primal variables:

$$g(\lambda) = \min_\theta \mathcal{L}(\theta, \lambda). \tag{35}$$

Because $\mathcal{L}$ is affine with respect to λ for any fixed θ, $g(\lambda)$ is strictly concave as a pointwise minimum of affine functions. The corresponding dual problem is $\max_{\lambda \geq 0} g(\lambda)$.

Strong duality. Strong duality is guaranteed under Slater's condition, which requires the existence of a strictly feasible interior point. Because evaluating the constraint at $\theta = \theta_{\mathrm{old}}$ yields a KL divergence of $0 < \delta$, a strictly feasible point exists at every step. Consequently, Slater's condition is uniformly satisfied, ensuring a duality gap of zero.

A.3.3. Gradient of the Dual

To optimise the dual function $g(\lambda)$ via gradient ascent, we require its derivative with respect to λ. Differentiating through the optimal $\theta^*(\lambda)$ would normally be computationally intractable; however, the *envelope theorem* provides a mathematical guarantee: at the optimal $\theta^*(\lambda)$, the gradient of the dual function evaluates exactly to the *constraint violation* (the magnitude by which the KL divergence exceeds δ).

Formally, by the envelope theorem, the gradient of the dual function at λ is:

$$\nabla_\lambda \, g(\lambda) = \frac{\partial \mathcal{L}(\theta^*(\lambda), \lambda)}{\partial \lambda} = \frac{1}{B} \sum_i D_{\mathrm{KL}}\big(\pi_{\theta^*(\lambda)}(\cdot|s_i) \,\|\, \pi_{\theta_{\mathrm{old}}}(\cdot|s_i)\big) - \delta, \tag{36}$$

where $\theta^*(\lambda) = \arg\min_\theta \mathcal{L}(\theta, \lambda)$.

Sign of the dual gradient. This formulation establishes a self-correcting feedback loop. If the current policy safely satisfies the KL budget ($D_{\mathrm{KL}} < \delta$), then $\nabla_\lambda g(\lambda) < 0$, and gradient ascent mathematically drives λ downward (clamping at the projection boundary of 0). Conversely, if the budget is violated ($D_{\mathrm{KL}} > \delta$), $\nabla_\lambda g(\lambda) > 0$, and ascent increases λ, penalising the actor and forcing the policy back into the trust region.

Practical computation. In practice, computing the exact optimum $\theta^*(\lambda)$ at every sub-step is inefficient. Following standard dual-variable optimisation protocols (cf. REPS (Peters et al., 2010), MPO (Abdolmaleki et al., 2018)), we utilise the current policy parameters θ_t as a one-step approximation:

$$v_t = \frac{1}{B} \sum_i D_{\mathrm{KL}}(\pi_{\theta_t}(\cdot|s_i) \,\|\, \pi_{\theta_{\mathrm{old}}}(\cdot|s_i)) - \delta.$$

This precisely yields the update signal defined in Eq. (24).

A.3.4. Adaptive Gradient Ascent (AdaGrad)

We maximise $g(\lambda)$ over $\lambda \geq 0$ using projected AdaGrad ascent. The update rule is formulated as:

$$G_{\lambda,t} = G_{\lambda,t-1} + v_t^2, \tag{37}$$

$$\lambda_{t+1} = \max\left(0,\ \lambda_t + \frac{v_t}{\sqrt{G_{\lambda,t}} + \varepsilon}\right). \tag{38}$$

Why AdaGrad. Standard gradient ascent with a fixed step size η requires tuning η per-task, because the magnitude of the dual gradient v_t depends directly on the KL scale, which varies drastically across environments and training phases. AdaGrad's adaptive step size $1/\sqrt{G_{\lambda,t}}$ automatically shrinks as the dual accumulates evidence, inherently preventing oscillation around the constraint boundary. This strictly mirrors the motivation for using AdaGrad in the temperature update (Section 3.4): *it eliminates the need for learning rate hyperparameter tuning.*

Projection onto $\lambda \geq 0$. The $\max(0, \cdot)$ projection guarantees dual feasibility. If $v_t < 0$ (meaning the KL budget is safely satisfied) and $\lambda_t = 0$, the projected update yields $\lambda_{t+1} = \max(0, 0 + v_t/\cdots) = 0$, because the accumulated argument is strictly negative. Consequently, once λ reaches zero (or starts there), it remains anchored at zero for as long as the budget is satisfied—a condition uniformly met in ACE.

Convergence of the dual. For a fixed primal policy θ, the dual iterates generated by Eq. (38) predictably converge to $\lambda^* = 0$ given $v_t \leq 0$ for all t. The adaptive step size satisfies standard regret bounds for projected stochastic subgradient methods, guaranteeing convergence to a stationary point of the strictly concave dual function without requiring manual step-decay schedules.

A.4. Temperature (α) AdaGrad Controller

A.4.1. Controller Dynamics

The AdaGrad temperature controller functions as a discrete-time feedback system operating entirely in log-space. Its state is denoted $x_t = \log \alpha_t \in \mathbb{R}$; its input is the entropy error signal $g_{\alpha,t} = e_t^* - e_t$; and its output is the updated temperature α_{t+1}. Operating in log-space structurally guarantees $\alpha_t > 0$ and applies equal relative step sizes regardless of the current magnitude of α—a critical property given that optimal temperatures span multiple orders of magnitude across different task topologies.

The following eight steps constitute one complete control cycle, executed sequentially during each gradient step:

1. **Observe:** Calculate normalised policy entropy $e_t = \mathcal{H}(\pi_{\theta_t})/d_a \in [0, 1]$.
2. **Update baseline:** Maintain the entropy EMA via $\bar{e}_t = (1 - \beta)\bar{e}_{t-1} + \beta\, e_t$, utilizing $\beta = 10^{-4}$.
3. **Calculate uncertainty:** Compute the winsorised disagreement d_t via Eq. (26).
4. **Form dynamic target:** Set $e_t^* = \mathrm{clip}(\bar{e}_t + \kappa d_t,\ 0.05,\ 2.0)$.
5. **Compute error:** Isolate the gradient signal $g_{\alpha,t} = e_t^* - e_t$.
6. **Accumulate:** Update the AdaGrad state $G_{\alpha,t} = G_{\alpha,t-1} + g_{\alpha,t}^2$.
7. **Log-space update:** Apply the additive step: $x_{t+1} = \mathrm{clip}\big(x_t + \frac{g_{\alpha,t}}{\sqrt{G_{\alpha,t}}+\varepsilon},\ \log \alpha_{\min}, \log \alpha_{\max}\big)$.
8. **Recover temperature:** Exponentiate to yield $\alpha_{t+1} = e^{x_{t+1}}$.

The positive sign convention in Step 7 stems from the strict monotonicity ($\partial H^*/\partial\alpha > 0$) established in Proposition A.1. Because optimal entropy is a strictly increasing function of α, the controller acts as a negative feedback loop: when $e_t < e_t^*$, the positive error drives α upward, thereby increasing the entropy penalty and pushing the policy toward higher exploration. Conversely, when $e_t > e_t^*$, α is driven downward to promote exploitation.

A.4.2. Stability Analysis (Negative Feedback)

We formally prove that the controller implements reliable negative feedback, ensuring the measured entropy e_t tracks the dynamic target e_t^* without pathological oscillation (in the limit of a slowly varying critic).

Proposition A.1 (Negative feedback)**.** *Fix the critic Q_ϕ and the entropy target $e^* \in (0, 1)$. Let $H^*(\alpha) = \mathcal{H}(\pi_\alpha^*)$ represent the entropy of the optimal policy at temperature α. Then $\partial H^*/\partial\alpha > 0$; meaning H^* is strictly increasing with respect to α. Consequently, the additive controller naturally implements negative feedback: if $e_t < e^*$, the update increases α, which subsequently increases H^*, effectively closing the error $e^* - e_t$.*

Proof. At the optimal policy for a given temperature α:

$$\pi^*_\alpha(a|s) = \frac{\exp(Q(s,a)/\alpha)}{Z_\alpha(s)}, \quad Z_\alpha(s) = \int_{\mathcal{A}} \exp(Q(s,a)/\alpha)\,\mathrm{d}a.$$

The differential entropy evaluates to:

$$H^*(\alpha) = -\int \pi^*_\alpha \log \pi^*_\alpha \,\mathrm{d}a = \log Z_\alpha + \frac{1}{\alpha}\mathbb{E}_{\pi^*_\alpha}[Q].$$

Differentiating with respect to the temperature α:

$$\frac{\partial H^*}{\partial \alpha} = \frac{\partial \log Z_\alpha}{\partial \alpha} + \frac{\partial}{\partial \alpha}\left(\frac{\mathbb{E}_{\pi^*_\alpha}[Q]}{\alpha}\right).$$

Computing the partition function derivative yields:

$$\frac{\partial \log Z_\alpha}{\partial \alpha} = -\frac{1}{\alpha^2}\mathbb{E}_{\pi^*_\alpha}[Q].$$

Applying the product rule to the expectation term yields:

$$\frac{\partial}{\partial \alpha}\left(\frac{\mathbb{E}_{\pi^*_\alpha}[Q]}{\alpha}\right) = \frac{1}{\alpha}\frac{\partial \mathbb{E}_{\pi^*_\alpha}[Q]}{\partial \alpha} - \frac{\mathbb{E}_{\pi^*_\alpha}[Q]}{\alpha^2}.$$

To evaluate the gradient of the expectation, we differentiate the policy density:

$$\frac{\partial \mathbb{E}_{\pi^*_\alpha}[Q]}{\partial \alpha} = \int Q \frac{\partial \pi^*_\alpha}{\partial \alpha}\mathrm{d}a = -\frac{1}{\alpha^2}\mathrm{Var}_{\pi^*_\alpha}[Q].$$

Combining these components:

$$\frac{\partial H^*}{\partial \alpha} = -\frac{\mathbb{E}[Q]}{\alpha^2} - \frac{\mathrm{Var}[Q]}{\alpha^3} - \frac{\mathbb{E}[Q]}{\alpha^2} + \frac{\mathbb{E}[Q]}{\alpha^2} = \frac{\mathrm{Var}_{\pi^*_\alpha}[Q]}{\alpha^2} > 0,$$

which is strictly positive given $\mathrm{Var}[Q] > 0$ (assuming Q is non-constant; if Q is constant, the policy is already uniformly distributed and entropy is maximally saturated). □

Remark A.2. Proposition A.1 confirms the architectural correctness of the controller's sign. Furthermore, the intrinsic gain of the system ($\partial H^*/\partial \alpha = \mathrm{Var}[Q]/\alpha^2$) is itself adaptive: high epistemic uncertainty (large $\mathrm{Var}[Q]$) heightens system responsiveness, whereas converged critics (small $\mathrm{Var}[Q]$) buffer the policy against overreacting to minute temperature adjustments.

A.4.3. Fixed-Point Convergence

Proposition A.3 (Fixed-point convergence)**.** *Assume the entropy target converges to a stable constant e^* and the critic remains fixed. The log-temperature iterates $x_t = \log \alpha_t$ produced by the controller (ignoring boundary projections) converge almost surely to the unique fixed point $x^* = \log \alpha^*$ satisfying $H^*(\alpha^*) = e^* \cdot d_a$.*

Proof sketch. Given a fixed target e^* and a fixed critic, the error sequence $g_t = e^* - e_t$ structurally drives the AdaGrad update:

$$x_{t+1} = x_t + \frac{g_t}{\sqrt{G_t} + \varepsilon}, \quad G_t = \sum_{s=1}^{t} g_s^2.$$

At the designated fixed point x^*, $g_t = 0$ and the iterates remain strictly stationary. Away from x^*, Proposition A.1 guarantees that H^* is a strictly increasing function, meaning the map $x \mapsto e^* - H^*(e^x)/d_a$ is strictly monotonic with a unique root at x^*. Standard convergence proofs for AdaGrad guarantee that for strictly monotonic, Lipschitz-continuous error signals, the adaptive step sizes sufficiently decay the accumulated regret, pulling the system to the root $\mathbb{E}[g] = 0$ without requiring the classical (and often divergent) Robbins-Monro L_2 step-size conditions. □

A.4.4. Boundedness and Safety

The strict projection onto $[\log \alpha_{\min}, \log \alpha_{\max}]$ in Eq. (28) fulfills two critical structural roles:

Lower bound ($\alpha_{\min} = 10^{-4}$). Prevents *entropy collapse*. If $\alpha \to 0$, the MaxEnt regularisation vanishes, and the policy rapidly collapses to a near-deterministic distribution. This is catastrophic in environments requiring sustained, long-horizon exploration (e.g., `Humanoid-v5` early in training). By enforcing $\alpha \geq 10^{-4}$, a minimal entropy bonus guarantees the policy retains sufficient support to recover from poor local optima.

Upper bound ($\alpha_{\max} = 1.0$). Prevents *runaway exploration*. Without an upper bound, a severe disagreement spike early in training (before the twin critics have gathered sufficient data to converge) could theoretically drive α to arbitrarily large values. This would force the policy into a nearly uniform distribution, overwriting learned behaviors and collecting uninformative, off-policy transitions. While winsorisation (Eq. (26)) is the primary regularisation mechanism against this, the ceiling of $\alpha \leq 1.0$ provides an absolute safety net.

Proposition A.4 (Projection non-expansiveness)**.** *The projected additive update $x_{t+1} = \Pi_{[a,b]}(x_t + \eta_t g_t)$, where $\Pi_{[a,b]}$ denotes projection onto the interval $[a, b]$, is non-expansive:*

$$|x_{t+1} - x^*| \leq |x_t + \eta_t g_t - x^*|$$

for any point $x^* \in [a, b]$.

Proof. The projection operator $\Pi_{[a,b]}$ is structurally a contraction mapping over the reals: $|\Pi_{[a,b]}(u) - \Pi_{[a,b]}(v)| \leq |u - v|$ for all $u, v \in \mathbb{R}$. Given that the optimal fixed point $x^* \in [a, b]$ already resides within the feasible set, $\Pi_{[a,b]}(x^*) = x^*$. Therefore:

$$|x_{t+1} - x^*| = |\Pi_{[a,b]}(x_t + \eta_t g_t) - \Pi_{[a,b]}(x^*)| \leq |x_t + \eta_t g_t - x^*|.$$

□

Proposition A.4 ensures that the bounding projection never increases the distance between the current state and the optimal fixed point, guaranteeing that the convergence properties established in Proposition A.3 are perfectly preserved under clipping.

A.5. Conditions for Trust-Region Inactivity

A.5.1. The Mirror Descent View of Soft Actor-Critic

The continuous SAC actor update possesses a profound theoretical connection to *online mirror descent* (OMD) employing a KL-divergence Bregman distance. We formalise this perspective here, as it elegantly illuminates why the KL divergence between consecutive policy iterates is intrinsically bounded by the temperature α, rendering explicit trust regions redundant.

Mirror descent background. Let $\psi : \mathcal{X} \to \mathbb{R}$ be a strictly convex *potential function*. The associated Bregman divergence is defined as:

$$D_\psi(x \| y) = \psi(x) - \psi(y) - \langle \nabla \psi(y),\, x - y \rangle \geq 0.$$

Standard mirror descent with a step size η executes the following update:

$$x_{t+1} = \arg\min_{x \in \mathcal{X}} \Big[\eta \langle \nabla f_t(x_t),\, x \rangle + D_\psi(x \| x_t) \Big],$$

where f_t is the objective loss at step t. The Bregman divergence term acts as a proximity regulariser, penalising catastrophically large steps in the dual space.

SAC as mirror descent. For probability distributions (policies), the natural Bregman divergence is the KL divergence ($D_\psi(\pi \| \pi_t) = D_{\mathrm{KL}}(\pi \,\|\, \pi_t)$), generated by the negative entropy potential $\psi(\pi) = \int \pi \log \pi$. The SAC actor update at a given state s can be recast functionally as:

$$\pi_{t+1}(\cdot|s) = \arg\min_{\pi \in \Delta(\mathcal{A})} \Big[-\mathbb{E}_{a \sim \pi}[Q_t(s, a)] + \alpha_t D_{\mathrm{KL}}(\pi \,\|\, \pi_t(\cdot|s)) \Big]. \tag{39}$$

The exact analytical solution to the unconstrained functional problem in Eq. (39) is the Gibbs distribution:

$$\pi_{t+1}(a|s) = \frac{\pi_t(a|s)\exp(Q_t(s,a)/\alpha_t)}{\int_{\mathcal{A}} \pi_t(a'|s)\exp(Q_t(s,a')/\alpha_t)\,\mathrm{d}a'}. \tag{40}$$

In practice, the SAC algorithm projects this non-parametric functional target back onto a tractable parametric family (the squashed Gaussian neural network). However, Eq. (40) reveals the core dynamics: a single SAC update moves from π_t toward the energy-based Gibbs policy, with α_t directly functioning as the inverse step size in policy space.

The regularising role of α_t. Using the Gibbs formulation in Eq. (40), we can derive the KL divergence induced by the functional update:

$$D_{\mathrm{KL}}(\pi_{t+1}(\cdot|s) \,\|\, \pi_t(\cdot|s)) = \frac{1}{\alpha_t}\mathbb{E}_{\pi_{t+1}}[Q_t(s,\cdot)] - \log \mathbb{E}_{\pi_t}\left[\exp\left(\frac{Q_t(s,a)}{\alpha_t}\right)\right].$$

Using standard second-order cumulant approximations from information geometry (assuming sub-Gaussian Q-values under the local policy), the log-partition function expands as $\log \mathbb{E}[e^X] \approx \mathbb{E}[X] + \frac{1}{2}\mathrm{Var}[X]$. Under linear response theory, the expectation shift yields $\mathbb{E}_{\pi_{t+1}}[Q] \approx \mathbb{E}_{\pi_t}[Q] + \frac{1}{\alpha_t}\mathrm{Var}_{\pi_t}[Q]$. Substituting these approximations yields a powerful upper bound:

$$D_{\mathrm{KL}}(\pi_{t+1}(\cdot|s) \,\|\, \pi_t(\cdot|s)) \;\lesssim\; \frac{\mathrm{Var}_{\pi_t}[Q_t(s,\cdot)]}{2\alpha_t^2}. \tag{41}$$

Equation (41) provides the theoretical smoking gun for ACE's efficacy: the geometric distance (KL divergence) between consecutive policies is inherently bounded by the *local variance of the critic scaled by the squared inverse temperature*. When epistemic uncertainty drives the critic variance up, ACE adaptively increases α_t, perfectly offsetting the variance and throttling the step size. Consequently, the KL step is mathematically suppressed before it ever reaches the explicit trust-region boundary.

A.5.2. Empirical KL Bound of the Parametric Update

Before presenting the formal theorem, we empirically bound the KL divergence for diagonal Gaussian policies under our specific network update parameters, demonstrating why the explicit constraint remains inactive in practice.

Parametric Gaussian update. In practice, the functional mirror descent update is approximated by a gradient step on the actor parameters θ. Let π_θ and $\pi_{\theta'}$ denote the policies immediately before and after one Adam step. For diagonal Gaussians with means μ, μ' and standard deviations σ, σ', the KL divergence decomposes into covariance and mean-shift components (from Eq. (20)):

$$D_{\mathrm{KL}}(\pi_{\theta'} \,\|\, \pi_\theta) = \underbrace{\sum_{i=1}^{d_a}\left[\log\frac{\sigma_i}{\sigma_i'} + \frac{\sigma_i'^2}{2\sigma_i^2} - \frac{1}{2}\right]}_{T_{\mathrm{cov}}} + \underbrace{\frac{1}{2}\sum_{i=1}^{d_a}\frac{(\mu_i' - \mu_i)^2}{\sigma_i^2}}_{T_{\mathrm{mean}}}.$$

Bounding T_{mean}. Utilizing the Adam optimiser with a learning rate of $\eta = 3 \times 10^{-4}$ and global gradient clipping at $c = 10$, the maximum mean shift per step satisfies $\|\mu' - \mu\|_\infty \le \eta c = 3 \times 10^{-3}$. Given that the standard deviations are structurally bounded away from zero by $\alpha_{\min}$ (typically $\sigma_i \ge 0.1$), we can bound the mean-shift contribution:

$$T_{\mathrm{mean}} \le \frac{d_a}{2}\left(\frac{\eta c}{\sigma_{\min}}\right)^2 = \frac{17}{2} \times (0.03)^2 \approx 7.65 \times 10^{-3}.$$

In practice, σ_i is generally larger and gradients rarely saturate the clipping threshold, ensuring $T_{\mathrm{mean}} \ll \delta = 0.02$.

Bounding T_{cov}. Similarly, the log-standard-deviation outputs change by at most $\|\log\sigma' - \log\sigma\|_\infty \le \eta c$. Utilizing the bound $f(r) = r^2/2 - \log r - 1/2 \le (\log r)^2$ for r near 1 (proven formally in Lemma A.7):

$$T_{\mathrm{cov}} = \sum_{i=1}^{d_a} f(\sigma_i'/\sigma_i) \le \sum_{i=1}^{d_a}(\log(\sigma_i'/\sigma_i))^2 = \sum_{i=1}^{d_a}(\Delta\log\sigma_i)^2 \le d_a(\eta c)^2 = 17 \times (3 \times 10^{-3})^2 \approx 1.53 \times 10^{-4}.$$

Total practical KL bound. Combining these empirical limits yields:

$$D_{\mathrm{KL}}(\pi_{\theta'} \,\|\, \pi_\theta) \le T_{\mathrm{mean}} + T_{\mathrm{cov}} \lesssim 7.65 \times 10^{-3} + 1.53 \times 10^{-4} \approx 7.8 \times 10^{-3} \ll \delta = 0.02. \tag{42}$$

This confirms that a standard gradient step under ACE's hyperparameters safely satisfies the trust-region budget without requiring explicit Lagrangian enforcement.

Connection to entropy control. Theorem 4.1 formalises this phenomenon by proving that the dominant covariance term, T_{cov}, is mathematically constrained to $\mathcal{O}(\epsilon_H^2)$ whenever the entropy controller enforces $|\mathcal{H}(\pi') - \mathcal{H}(\pi)| \le \epsilon_H$. Because ACE's AdaGrad controller actively drives $\epsilon_H \to 0$ (Proposition A.3), the KL divergence is asymptotically squeezed, rendering the trust region structurally redundant in steady state.

A.5.3. Theorem: Redundancy of the Explicit Constraint

We now provide a complete, self-contained proof of Theorem 4.1. We establish two preparatory lemmas to isolate the analytic core of the covariance bound, followed by the synthesis of the main theorem.

Theoretical Intuition. The core theoretical claim is as follows: if a policy's entropy changes by at most ϵ_H during an update, the resulting KL divergence is inherently bounded by the mean-shift term T_{mean} plus a term that scales quadratically with ϵ_H. Because ACE's controller strictly regulates ϵ_H, and standard optimiser step sizes independently bound T_{mean} to computationally negligible values ($\approx 4.5 \times 10^{-8}$ empirically), the resulting $\mathrm{KL}(\pi_{t+1} \| \pi_t)$ remains orders of magnitude below the budget δ. Consequently, the dual variable λ_t is mathematically starved of any constraint violation to enforce.

Preparatory Lemmas.

Lemma A.5 (Non-negativity of the KL covariance integrand)**.** *Define the function $f\colon (0,\infty) \to \mathbb{R}$ as:*

$$f(r) \;:=\; \frac{r^2}{2} - \log r - \frac{1}{2}.$$

Then $f(r) \ge 0$ for all $r > 0$, with equality holding if and only if $r = 1$.

Proof. Differentiating with respect to r yields $f'(r) = r - r^{-1} = (r^2 - 1)/r$. Consequently, $f'(r) < 0$ for $r \in (0,1)$, $f'(1) = 0$, and $f'(r) > 0$ for $r > 1$. This establishes $r = 1$ as the unique global minimiser of f on $(0,\infty)$. Because $f(1) = \frac{1}{2} - 0 - \frac{1}{2} = 0$, the non-negativity holds globally. □

Remark A.6. The function $f(r)$ represents (up to a factor of $\frac{1}{2}$) the exact KL divergence between two zero-mean univariate Gaussians with variances 1 and r^2, respectively. Lemma A.5 thus serves as a direct, elementary verification of the non-negativity of the KL divergence for this specific functional form.

Lemma A.7 (Quadratic approximation near $r = 1$)**.** *Let $c \in \mathbb{R}$ and substitute $r = e^c$. Then the Taylor expansion yields:*

$$f(e^c) \;=\; c^2 + \tfrac{2}{3}c^3 + \mathcal{O}(c^4). \tag{43}$$

Consequently, for a bounded collection $c_1, \ldots, c_d \in \mathbb{R}$ satisfying the aggregate constraint $\left|\sum_{i=1}^d c_i\right| \le \epsilon_H$ and a maximum step constraint $\max_i |c_i| \le C$:

$$\sum_{i=1}^{d} f(e^{c_i}) \;=\; \sum_{i=1}^{d} c_i^2 \;+\; \mathcal{O}(C \cdot \epsilon_H)\,. \tag{44}$$

Proof. Substituting $r = e^c$ and expanding the exponential e^{2c} around $c = 0$:

$$\begin{aligned} f(e^c) &= \frac{e^{2c}}{2} - c - \frac{1}{2} \\ &= \frac{1}{2}\left(1 + 2c + \frac{(2c)^2}{2} + \frac{(2c)^3}{6} + \mathcal{O}(c^4)\right) - c - \frac{1}{2} \\ &= \left(\frac{1}{2} + c + c^2 + \frac{4c^3}{6}\right) - c - \frac{1}{2} + \mathcal{O}(c^4) \\ &= c^2 + \tfrac{2}{3}c^3 + \mathcal{O}(c^4), \end{aligned}$$

which establishes Eq. (43). Summing over the dimensions yields:

$$\sum_{i=1}^{d} f(e^{c_i}) = \sum_{i=1}^{d} c_i^2 + \tfrac{2}{3}\sum_{i=1}^{d} c_i^3 + \mathcal{O}\left(\sum_{i=1}^{d} c_i^4\right).$$

Because $|c_i| \le C$, the cubic term is bounded by $|\sum c_i^3| \le C \sum c_i^2$. Under the uniform-scaling condition where $|c_i| = \mathcal{O}(\epsilon_H/d)$ (detailed in Remark A.8), the quadratic sum satisfies $\sum c_i^2 = \mathcal{O}(\epsilon_H^2/d)$. Therefore, the cubic correction term is strictly bounded by $\mathcal{O}(C\epsilon_H^2/d) = \mathcal{O}(\epsilon_H^2)$. □

Remark A.8 (Uniform variance scaling). In the ACE architecture, the entropy temperature α_t is updated globally in log-space via $\log \alpha_{t+1} = \log \alpha_t + \Delta$. This homogeneously scales the log-standard-deviation outputs of the actor network across all action dimensions. A single actor update thus perturbs standard deviations by a uniform multiplicative factor: $\sigma_{\theta',i} = r\,\sigma_{\theta,i}$ for all i, rendering $c_i = \log r$ a constant across dimensions. The resulting entropy change is precisely $\mathcal{H}(\pi_{\theta'}) - \mathcal{H}(\pi_\theta) = d \log r$, meaning $|c_i| = |\log r| = \epsilon_H/d$. This perfectly satisfies the condition for Lemma A.7 with $C = \epsilon_H/d$.

Main Theorem

Theorem A.9 (Implicit Trust Region via Entropy Control, restated)**.** *Let π_θ and $\pi_{\theta'}$ be d-dimensional diagonal Gaussian policies with means $\mu_\theta, \mu_{\theta'}$ and covariances $\Sigma_\theta = \mathrm{diag}(\sigma_{\theta,i}^2)$, $\Sigma_{\theta'} = \mathrm{diag}(\sigma_{\theta',i}^2)$. Suppose the adaptive entropy controller enforces the constraint:*

$$\left|\mathcal{H}(\pi_{\theta'}) - \mathcal{H}(\pi_\theta)\right| \;\le\; \epsilon_H. \tag{45}$$

Then, because the first-order Taylor expansion terms of the covariance KL diverge analytically cancel, the KL divergence is bounded strictly by the mean shift and a second-order entropy term:

$$D_{\mathrm{KL}}(\pi_{\theta'} \,\|\, \pi_\theta) \;\le\; \underbrace{\frac{1}{2}\sum_{i=1}^{d} \frac{(\mu_{\theta',i} - \mu_{\theta,i})^2}{\sigma_{\theta,i}^2}}_{T_{\mathrm{mean}}} + \mathcal{O}(\epsilon_H^2). \tag{46}$$

Under the uniform variance scaling condition outlined in Remark A.8, this tightens to precisely $T_{\mathrm{mean}} + \mathcal{O}(\epsilon_H^2/d)$.

Proof. **Step 1 (Decompose the KL).** For a product of univariate Gaussians, the KL divergence decomposes cleanly dimension-by-dimension:

$$D_{\mathrm{KL}}(\pi_{\theta'} \,\|\, \pi_\theta) = \sum_{i=1}^{d}\left[\log\frac{\sigma_{\theta,i}}{\sigma_{\theta',i}} + \frac{\sigma_{\theta',i}^2 + (\mu_{\theta',i} - \mu_{\theta,i})^2}{2\sigma_{\theta,i}^2} - \frac{1}{2}\right]. \tag{47}$$

Separating this into distinct covariance and mean-shift contributions yields:

$$D_{\mathrm{KL}}(\pi_{\theta'} \,\|\, \pi_\theta) = \underbrace{\sum_{i=1}^{d}\left[\log\frac{\sigma_{\theta,i}}{\sigma_{\theta',i}} + \frac{\sigma_{\theta',i}^2}{2\sigma_{\theta,i}^2} - \frac{1}{2}\right]}_{T_{\mathrm{cov}}} + \underbrace{\frac{1}{2}\sum_{i=1}^{d}\frac{(\mu_{\theta',i} - \mu_{\theta,i})^2}{\sigma_{\theta,i}^2}}_{T_{\mathrm{mean}}}. \tag{48}$$

The term $T_{\mathrm{mean}} \ge 0$ appears verbatim in the theorem bound. We now bound T_{cov}.

Step 2 (Express T_{cov} via variance ratios). Define the scale ratio $r_i := \sigma_{\theta',i}/\sigma_{\theta,i} > 0$. Then:

$$T_{\mathrm{cov}} = \sum_{i=1}^{d}\left[-\log r_i + \frac{r_i^2 - 1}{2}\right] = \sum_{i=1}^{d} f(r_i), \tag{49}$$

where f is the function analysed in Lemma A.5. By that lemma, $T_{\mathrm{cov}} \ge 0$.

Step 3 (Link r_i to the entropy constraint). The differential entropy of a diagonal Gaussian satisfies $\mathcal{H}(\pi_\theta) = \frac{d}{2}(1 + \log 2\pi) + \sum_i \log \sigma_{\theta,i}$. The difference is therefore:

$$\mathcal{H}(\pi_{\theta'}) - \mathcal{H}(\pi_\theta) = \sum_{i=1}^{d} \log r_i. \tag{50}$$

Constraint (45) directly requires $\left|\sum_i \log r_i\right| \leq \epsilon_H$.

Step 4 (Analytic cancellation of first-order terms). Define $c_i := \log r_i$. From Eq. (49), we have $T_{\text{cov}} = \sum_i f(e^{c_i})$. Applying the Taylor expansion established in Eq. (43) of Lemma A.7:

$$T_{\text{cov}} = \sum_{i=1}^{d} \left(c_i^2 + \tfrac{2}{3}c_i^3 + \mathcal{O}(c_i^4)\right). \tag{51}$$

Crucially, the linear term (c_i) analytically cancels out of the expansion. The KL divergence is structurally insensitive to first-order entropy changes. Under the uniform scaling property of ACE (Remark A.8), $c_i = \pm\epsilon_H/d$ for all i. Therefore, the sum of squares strictly equals $d(\epsilon_H/d)^2 = \epsilon_H^2/d$, confirming that $T_{\text{cov}} = \mathcal{O}(\epsilon_H^2/d) \subset \mathcal{O}(\epsilon_H^2)$.

Step 5 (Bounding the mean term). The mean-shift term is exact. Under a standard Adam actor update, $\mu_{\theta'} = \mu_\theta + \eta g$ (where $g = \nabla_\mu \mathcal{L}$). Thus:

$$T_{\text{mean}} = \frac{\eta^2}{2}\sum_{i=1}^{d}\frac{g_i^2}{\sigma_{\theta,i}^2} = \frac{\eta^2}{2}\|g\|^2_{\Sigma_\theta^{-1}}.$$

As established in Appendix A.5.2, with $\eta = 3 \times 10^{-4}$ and clipping $c = 10$, the *theoretical worst-case upper bound* is $T_{\text{mean}} \leq 7.65 \times 10^{-3}$. However, empirical tracking of the unclipped Adam updates during training reveals an *average step size* of $T_{\text{mean}} \approx 4.5 \times 10^{-8}$. Both the theoretical limit and the empirical average are strictly bound well below the standard budget $\delta = 0.02$. Combining this with Step 4 establishes the theorem. □

Corollary A.10 (Passive trust-region satisfaction)**.** *Suppose* (45) *holds and* $T_{\text{mean}} \leq M\eta^2$ *for some architectural constant* M*. If* $\epsilon_H + M\eta^2 \leq \delta - \mathcal{O}(\epsilon_H^2)$*, then* $D_{\text{KL}}(\pi_{\theta'} \,\|\, \pi_\theta) \leq \delta$*, and the explicit trust-region dual* λ_t *remains mathematically anchored at its lower bound of zero. In* ACE*, the AdaGrad controller drives* $\epsilon_H \to 0$ *while the optimiser maintains* $M\eta^2 \ll \delta$*; therefore, the constraint is passively satisfied, perfectly predicting the 100% dual inactivity observed empirically.*

Remark A.11 (Comparison with the TRPO improvement bound). TRPO (Schulman et al., 2015) leverages the theoretical premise that if $D_{\text{KL}}(\pi_{\theta'} \,\|\, \pi_\theta) \leq \delta$, the expected return monotonically improves by at least $\mathcal{O}(\delta^{-1/2})$ over the surrogate. Corollary A.10 proves that ACE inherits these exact stability guarantees passively. The geometric trust region does not vanish; rather, the explicit penalty becomes entirely redundant because the entropy controller structurally guarantees its satisfaction.

A.5.4. Why ACE Satisfies This Condition

Theorem A.9 identifies sufficient mathematical conditions for trust-region inactivity. We conclude by demonstrating that ACE strictly satisfies these conditions throughout training.

Condition 1: $|\mathcal{H}(\pi') - \mathcal{H}(\pi)| \leq \epsilon_H$**.** By Proposition A.3, the AdaGrad controller drives $e_t \to e^*$ with an accumulated regret decaying adaptively. The maximum absolute entropy change per step satisfies:

$$|\mathcal{H}(\pi_{t+1}) - \mathcal{H}(\pi_t)| = d_a|e_{t+1} - e_t| \leq d_a \cdot \frac{|g_{\alpha,t}|}{\sqrt{G_{\alpha,t}} + \varepsilon} \cdot \frac{\partial H^*}{\partial \alpha} \cdot \alpha_t.$$

Because $G_{\alpha,t}$ is strictly non-decreasing, the step size mathematically anneals. In steady state ($G_{\alpha,t} \gg 1$), the per-step entropy delta is tightly bounded ($\epsilon_H \to 0$).

Condition 2: $T_{\text{mean}} \leq M\eta^2$**.** As derived in Appendix A.5.2, the absolute worst-case limit under gradient clipping is $T_{\text{mean}} \lesssim d_a(\eta c/\sigma_{\min})^2/2$. Using ACE's universal parameters ($\eta = 3 \times 10^{-4}$, $c = 10$) on our highest-dimensional task ($d_a = 17$ for `Humanoid-v5`), $T_{\text{mean}} \leq 7.65 \times 10^{-3} < \delta/2 = 0.01$.

Condition 3: $\epsilon_H + T_{\text{mean}} \leq \delta$**.** Because the controller drives $\epsilon_H \to 0$ and T_{mean} empirically averages 4.5×10^{-8} (with a firm upper bound of 7.65×10^{-3}), the composite KL step easily satisfies the $\delta = 0.02$ budget after a brief initialisation warm-up phase (typically $\lesssim$ 10K steps). During warm-up, random policy initialisation intrinsically yields high entropy and sluggish parametric shifts, naturally preventing KL violations.

Empirical verification. This analytical framework predicts $\lambda_t = 0$ continuously following initialisation. Empirically, we tracked this variable across 7 ablation environments, 5 seeds, and 100 evaluation checkpoints per run, yielding exactly 3,500 independent measurements. At every single checkpoint, $\lambda_t = 0.0$. The statistical probability of this occurring by chance if D_{KL} were freely distributed around the budget boundary is essentially zero, confirming our analytical prediction.

Buffering the disagreement signal. The winsorised disagreement d_t modulates the dynamic target e_t^*, which in turn dictates ϵ_H. When epistemic uncertainty spikes ($d_t > 0$), e_t^* rises to demand exploration. While this expands the target entropy, the structural clipping of the winsorisation ($z_{\max} = 2.5$) strictly caps the maximum target delta:

$$|e_t^* - \bar{e}_t| \leq \kappa \cdot z_{\max} = 0.15 \times 2.5 = 0.375.$$

Furthermore, the EMA ($\beta = 10^{-4}$) acts as a robust low-pass filter, forcing the controller to traverse this delta smoothly over multiple steps rather than enacting a sudden, budget-violating jump.

A.6. Disagreement Statistic ($Q_1 - Q_2$)

A.6.1. Critic Gap as a Proxy for Epistemic Uncertainty

The disagreement between the twin critics, $\Delta_t = \frac{1}{B}\sum_{i=1}^{B} |Q_{\phi_1}(s_i, a_i) - Q_{\phi_2}(s_i, a_i)|$, serves as ACE's primary proxy for *epistemic uncertainty* in the value landscape. We formalise the justification for this architectural choice below.

Why twin critics disagree. The critics Q_{ϕ_1} and Q_{ϕ_2} are initialised with independent random seeds. At the onset of training, their predictions are entirely uncorrelated; as training progresses and the networks ingest data, they converge toward the true soft value function Q_α^π. However, residual disagreement reliably persists in regions of the state-action space that are under-visited or where the Bellman residual remains poorly constrained. Formally, for function approximators trained on finite datasets $\mathcal{D}$, the disagreement serves as a tractable approximation of posterior variance:

$$\mathbb{E}[|Q_{\phi_1}(s, a) - Q_{\phi_2}(s, a)|] \;\approx\; C\,\sigma_Q(s, a), \tag{52}$$

where $\sigma_Q(s, a)$ represents the true epistemic standard deviation and $C > 0$ is a prior-dependent constant (Chen et al., 2021). While Eq. (52) holds exactingly for Bayesian linear regression with Gaussian priors, it is leveraged here as a highly effective, computationally cheap heuristic for deep networks, aligning with established literature on ensemble-based uncertainty quantification (Lee et al., 2021).

Disagreement as an exploration signal. Regions exhibiting high critic disagreement correspond strictly to state-action pairs where the agent possesses insufficient experience. Guiding the policy toward these high-disagreement regions organically improves sample efficiency on hard-exploration tasks. Rather than utilizing Δ_t to reweight replay buffer samples (as seen in SUNRISE (Lee et al., 2021)), ACE uses this uncertainty to *dynamically elevate the entropy target*. This is a structurally softer, more generalised mechanism: rather than forcing specific transition replays, the algorithm organically widens the policy's action distribution, sustaining exploration until the critics converge and the uncertainty collapses.

EMA smoothing. The raw batch disagreement Δ_t is inherently noisy due to mini-batch sampling variance. To extract a stable control signal, we maintain running estimates of its mean and standard deviation via exponential moving averages (EMA):

$$\mu_{\Delta,t} = (1 - \beta_\Delta)\mu_{\Delta,t-1} + \beta_\Delta \Delta_t, \tag{53}$$

$$\nu_{\Delta,t} = (1 - \beta_\Delta)\nu_{\Delta,t-1} + \beta_\Delta (\Delta_t - \mu_{\Delta,t-1})^2, \tag{54}$$

$$\sigma_{\Delta,t} = \sqrt{\nu_{\Delta,t} + \varepsilon}, \tag{55}$$

utilizing a decay factor of $\beta_\Delta = 2 \times 10^{-3}$. This establishes a decay timescale of $\beta_\Delta^{-1} = 500$ steps—fast enough to rapidly track non-stationary disagreement dynamics as the critics learn, yet slow enough to aggressively filter step-to-step sampling noise.

Variance reduction. The variance of the EMA estimator $\mu_{\Delta,t}$ relative to the true mean $\bar{\Delta}_t$ scales as:

$$\mathrm{Var}[\mu_{\Delta,t}] \approx \frac{\beta_\Delta}{2 - \beta_\Delta}\,\mathrm{Var}[\Delta_t] \approx 10^{-3}\,\mathrm{Var}[\Delta_t].$$

This represents a $1000\times$ variance reduction compared to utilizing the raw Δ_t directly. This suppression is absolutely critical: without it, the subsequent z-score z_t would be entirely dominated by batch-estimation noise rather than genuine epistemic uncertainty.

A.6.2. Winsorisation and Robust Statistics

Winsorisation is a robust statistical procedure that systematically limits the influence of outliers by clamping values at a specified percentile threshold, rather than discarding them. Unlike naive truncation (which reduces the dataset size and skews underlying distributions), winsorisation preserves the sample count and the distributional core while strictly bounding extreme deviations. In ACE, winsorisation prevents early-training Q-value spikes from irreparably corrupting the entropy target and permanently destabilising the temperature controller.

The heavy-tailed disagreement problem. The empirical distribution of Δ_t is heavily right-tailed, particularly during early training phases when the value landscape is volatile. Isolated, massive spikes in Δ_t (driven by transitions with pathologically high Bellman errors) produce extreme z-scores ($|z_t| \gg 1$). Unchecked, these spikes would drive the dynamic target e_t^* to its maximum ceiling, triggering runaway exploration.

Quantitatively, in our unclipped ablation (C1), we observed transient spikes of $\max_t |z_t| > 15$ on `Humanoid-v5` during the initial 200K steps. Lacking a clamping mechanism, these spikes saturate the entropy target, permanently driving $\alpha_t \to \alpha_{\max}$. Consequently, the policy degenerates into a uniform distribution, and reward learning irreversibly collapses. Full ACE (with $z_{\max} = 2.5$) achieves a stable final return of $\bar{R} \approx 6{,}241$ on `Humanoid-v5`, whereas ablation C1 fails catastrophically at $\bar{R} \approx 310$ (a $20\times$ degradation).

Winsorisation mechanics. *Winsorisation* (Tukey, 1977) mathematically caps a distribution's extremes. For our z-scored disagreement:

$$d_t = \text{clip}(z_t,\, -z_{\max},\, +z_{\max}), \quad z_{\max} = 2.5. \tag{56}$$

The threshold $z_{\max} = 2.5$ aligns roughly with the 98.8th percentile of a standard normal distribution. This allows 98.8% of standard uncertainty fluctuations to pass unaltered, actively clamping only the most destructive 1.2% of outliers.

Mathematical robustness. A defining property of robust statistics is a bounded influence function. The Winsorised mean satisfies:

$$\sup_x \left| \frac{\partial}{\partial x} \mathbb{E}[\text{clip}(z, -c, +c)] \right| \leq c,$$

meaning no single outlier can shift the processed signal d_t by more than $z_{\max} = 2.5$, regardless of its absolute magnitude. By contrast, the ordinary mean possesses an unbounded influence function ($\sup_x |\partial \mathbb{E}[z]/\partial x| = \infty$). Winsorisation strictly transforms the entropy target e_t^* into a *robust statistic*, immune to catastrophic outliers without requiring complex anomaly detection subroutines.

Choice of $z_{\max}$. We empirically ablated $z_{\max} \in \{1.0, 1.5, 2.0, 2.5, 3.5, \infty\}$ across `HalfCheetah-v5` and `Humanoid-v5` (5 seeds per variant). Performance remained broadly stable within the $[2.0, 3.5]$ bracket. However, performance visibly degraded at the extremes: $z_{\max} = 1.0$ proved too aggressive (clipping valid exploration signals), while $z_{\max} = \infty$ (no clipping) triggered the aforementioned instabilities on `Humanoid-v5`. The parameter $z_{\max} = 2.5$ represents the optimal, stable midpoint.

A.6.3. Target Entropy Modulation

Utilizing the winsorised disagreement d_t, the dynamic entropy target is formulated as:

$$e_t^* = \text{clip}(\bar{e}_t + \kappa \cdot d_t,\ 0.05,\ 2.0)\,, \tag{57}$$

where $\bar{e}_t$ is the EMA of the normalised policy entropy, and $\kappa = 0.15$ acts as the proportional gain.

Controller intuition. The dynamic target e_t^* synthesises two components:

- **Baseline ($\bar{e}_t$):** The recent moving average of policy entropy. This acts as a structural momentum term, anchoring the target to prevent erratic, high-frequency oscillations.

- **Proportional Correction (κd_t):** A signed intervention. When uncertainty is unusually high ($d_t > 0$), it pushes the target upward to sustain exploration. Conversely, when uncertainty drops below average ($d_t < 0$), it pulls the target downward, signalling that the policy is safe to aggressively exploit the converged value landscape.

Sensitivity to κ. The parameter κ governs the control system's responsiveness to uncertainty. If $\kappa = 0$ (ablation C2), the target collapses to $e_t^* = \bar{e}_t$; the entropy simply tracks its own history, functionally reverting to a passive decay schedule entirely decoupled from epistemic uncertainty. Alternatively, as $\kappa \to \infty$, the target becomes erratically dominated by the disagreement signal, destroying the stability of the baseline. We verified that performance remains robust across $\kappa \in [0.10, 0.25]$, adopting $\kappa = 0.15$ to perfectly balance active exploration tracking with steady-state stability.

Boundary clipping. The bounds $[0.05, 2.0]$ act as absolute physical constraints for the control target. They prevent the target from mathematically demanding either total determinism (which stalls learning) or infinite variance (which destroys learned policies). Under standard hyperparameter configurations, $\bar{e}_t + \kappa d_t$ operates safely within this envelope, making the clip an inactive safety net during normal operation.

Relation to prior MaxEnt paradigms. Standard SAC (Haarnoja et al., 2018b) enforces a static target entropy (typically the heuristic $e^* = -d_a$). In stark contrast, ACE's target e_t^* is highly dynamic. When the critics are converged and the landscape is mapped ($d_t \approx 0$), ACE's target defaults to slow, passive annealing ($e_t^* \approx \bar{e}_t$). However, upon encountering novel states or reward discoveries (causing d_t to spike), the target instantly elevates. This buys the policy critical time to fully map the new transition dynamics before allowing entropy to decay again—a structural advantage that directly explains ACE's dominant margins on complex, sparse-reward benchmarks like Reacher-hard and `BipedalWalker-v3`.

A.7. Convergence Sketches

A.7.1. Actor Stationarity via Adaptive Mirror Descent

We outline a formal convergence framework for the actor under the ACE update, structuring the analysis through the lens of nonconvex online mirror descent.

Setup. Let $J(\theta) = \mathbb{E}_{\rho^\pi}[Q^\pi(s,a) - \alpha_t \log \pi_\theta(a|s)]$ denote the soft actor objective. The actor update at step t executes a stochastic gradient step:

$$\theta_{t+1} = \theta_t - \eta_\theta \, \hat{g}_t, \quad \hat{g}_t = \nabla_\theta \mathcal{L}_{\text{actor}}(\theta_t),$$

where $\hat{g}_t$ is a mini-batch estimate of the true gradient $\nabla_\theta J(\theta_t)$ with bounded variance $\sigma_g^2 = \mathbb{E}[\|\hat{g}_t - \nabla J\|^2]$.

Assumptions.

(A1) **Smoothness:** J is L-smooth: $\|\nabla J(\theta') - \nabla J(\theta)\| \le L\|\theta' - \theta\|$.

(A2) **Unbiased estimation:** The stochastic gradient is unbiased: $\mathbb{E}[\hat{g}_t|\theta_t] = \nabla J(\theta_t)$.

(A3) **Bounded variance:** The gradient variance is strictly bounded: $\mathbb{E}[\|\hat{g}_t\|^2] \le \sigma_g^2 < \infty$.

(A4) **Bounded temperature:** The temperature is strictly confined: $\alpha_{\min} \le \alpha_t \le \alpha_{\max}$ (structurally guaranteed by the projection in Eq. (28)).

Result. Under assumptions (A1)–(A4), and utilizing a learning rate $\eta_\theta \le 1/L$, standard non-convex analysis yields:

$$\frac{1}{T}\sum_{t=1}^{T} \mathbb{E}\big[\|\nabla J(\theta_t)\|^2\big] \le \frac{2(J(\theta_1) - J^*)}{\eta_\theta T} + \eta_\theta L \sigma_g^2, \tag{58}$$

where $J^* = \inf_\theta J(\theta)$. This establishes the standard $\mathcal{O}(1/\sqrt{T})$ convergence rate for nonconvex SGD. Setting an annealed schedule $\eta_\theta = \mathcal{O}(1/\sqrt{T})$ drives the gradient norm to zero: $\frac{1}{T}\sum_t \mathbb{E}[\|\nabla J\|^2] = \mathcal{O}(1/\sqrt{T}) \to 0$.

Remark on Adam. In our implementation, the Adam optimiser is employed in lieu of vanilla SGD. The convergence of Adam on nonconvex objectives holds under identical assumptions (Kingma & Ba, 2014), substituting the preconditioned gradient norm $\|M_t^{-1/2}\nabla J\|^2$ into Eq. (58) (where M_t is Adam's second-moment accumulator). The adaptive preconditioner massively improves the empirical constant without altering the asymptotic $\mathcal{O}(1/\sqrt{T})$ rate.

Effect of the KL dual term. If the trust-region dual were to activate ($\lambda_t > 0$)—a scenario we rule out empirically but analyze for completeness—the actor loss would incur the proximal penalty $\lambda_t D_{\mathrm{KL}}$. This functionally tightens the required step-size bound by altering the effective smoothness constant to $L_{\text{eff}} = L + \lambda_t/\sigma_{\min}^2$, necessitating a smaller η_θ to guarantee descent. Because ACE ensures $\lambda_t \equiv 0$ continuously, this theoretical bottleneck is permanently bypassed.

A.7.2. Critic Convergence and the $\tau = 0.02$ Stability Condition

This subsection formalises why the critic update in ACE remains highly stable despite utilizing an accelerated target-network smoothing coefficient ($\tau = 0.02$, four times larger than SAC's default $\tau = 0.005$). The core insight is that ACE's adaptive entropy controller strictly bounds the policy update magnitude, structurally limiting the non-stationarity of the Bellman target.

Bellman contraction. The soft Bellman operator $\mathcal{T}^\pi$ acts as a γ-contraction in the ℓ^∞-norm:

$$\|\mathcal{T}^\pi Q - \mathcal{T}^\pi Q'\|_\infty \leq \gamma \|Q - Q'\|_\infty.$$

For our fixed $\gamma = 0.99$, the fixed point Q^π remains the unique solution to $\mathcal{T}^\pi Q = Q$, converging at a rate of γ^t.

Two-timescale intuition. Critic convergence necessitates that the target network $\bar{\phi}_j$ evolves sufficiently slowly relative to the online critic ϕ_j. With target smoothing τ, the effective mixing timescale is $t_\tau = 1/\tau$. Standard SAC operates at $t_\tau = 200$ steps ($\tau = 0.005$), heavily prioritising stability. ACE deliberately accelerates this to $t_\tau = 50$ steps ($\tau = 0.02$), exchanging a slower target for vastly quicker value propagation.

Why $\tau = 0.02$ is stable for ACE but not vanilla SAC. Accelerating τ inherently increases the step-to-step variance of the Bellman target y_t:

$$\mathrm{Var}[y_t] \propto \tau \cdot \mathrm{Var}[Q_{\bar{\phi}}].$$

In vanilla SAC (which utilizes a fixed α), this inflated variance is unmitigated, routinely leading to divergence on complex tasks. ACE neutralises this risk via a negative-feedback loop. When critic variance inflates, the winsorised disagreement Δ_t rises, driving the entropy target e_t^* and temperature α_t upward. The elevated α_t immediately smooths the Q-target via increased entropy regularisation, perfectly counteracting the target variance.

Concretely, the variance of the critic gradient scales as:

$$\mathrm{Var}[\nabla_\phi \mathcal{L}_{\text{critic}}] \propto \tau \, \mathrm{Var}[Q_{\bar{\phi}}] + \alpha_t^2 \, \mathrm{Var}[\log \pi(\tilde{a}'|s')]. \tag{59}$$

As α_t grows, its regularising effect buffers the target, empirically enabling flawlessly stable critic learning at $\tau = 0.02$ across all 21 environments.

Ablation evidence. Our ablation study (Table 3) corroborates these interacting dynamics. Ablation A1 (fixed $\alpha = 0.2$) demonstrates that attempting to run an accelerated $\tau = 0.02$ without ACE's adaptive entropy control causes massive degradation, dropping aggregate performance to $0.61\times$. Conversely, Ablation D2 ($\tau = 0.005$ with full ACE) achieves $1.13\times$ relative performance. This confirms that while $\tau = 0.02$ is computationally stable and robust, decelerating the target to SAC's default $\tau = 0.005$ can yield higher peak returns on specific tasks, framing ACE's $\tau = 0.02$ default as a conservative, rapid-learning baseline.

A.7.3. Two-Timescale Analysis

The complete ACE architecture constitutes a coupled dynamical system spanning three distinct parameter blocks: critics (ϕ), actor (θ), and temperature (α). The convergence of such multi-tiered systems is rigorously justified by two-timescale stochastic approximation theory (Kakade & Langford, 2002).

Timescale separation. ACE engineers strict temporal separation between its components:

1. **Critic (ϕ, fast timescale):** Optimised via Adam at $\eta_\phi = 3 \times 10^{-4}$. Because it maps directly to state-action regression targets, it converges rapidly toward the local Q^{π_t} on the order of $\mathcal{O}(1/((1-\gamma)\eta_\phi))$ steps.

2. **Actor (θ, intermediate timescale):** While sharing the nominal learning rate $\eta_\theta = 3 \times 10^{-4}$, the actor effectively operates on a slower operational timescale. Its gradients are structurally smoothed through the Q-network's landscape and actively resisted by the entropy penalty, buffering its rate of change.
3. **Temperature (α, slow timescale):** Dictated by the AdaGrad accumulator, the step size $1/\sqrt{G_{\alpha,t}}$ strictly decays as $\mathcal{O}(1/\sqrt{t})$. In steady state, temperature adjustments mathematically vanish relative to network updates.

Theoretical consequence. By invoking timescale separation, the critic can be treated as asymptotically converged when analyzing the actor update, and the temperature can be treated as a static constant when analyzing the actor-critic loop. This hierarchical decomposition allows ACE to inherit the foundational convergence proofs of standard SAC (Haarnoja et al., 2018a). Because the internal SAC loop converges to a unique optimal policy π^*_α for any fixed α, and the temperature controller independently converges to a stationary fixed point α^* (Proposition A.3), the global system is guaranteed to converge to the joint optimum $\pi^*_{\alpha^*}$.

A.8. Variance & Bias of Smoothed Evaluation

The entropy target and disagreement baselines in ACE rely on the exponential moving averages (EMA), $\bar{e}_t$ and $\mu_{\Delta,t}$, computed over noisy batch observations. In this section, we formally analyze the statistical properties of these estimators—specifically, their variance reduction capacity and tracking bias (lag) relative to the true underlying signals.

A.8.1. Estimator Definition

Let $\{x_t\}_{t=1}^{\infty}$ represent a sequence of scalar observations drawn from a stochastic process with a time-varying mean $m_t = \mathbb{E}[x_t]$ and variance $\sigma^2 = \mathrm{Var}[x_t]$ (assumed stationary for the purpose of variance analysis). The EMA, parametrised by a smoothing coefficient $\beta \in (0, 1)$, is defined recursively as:

$$\hat{m}_t = (1-\beta)\hat{m}_{t-1} + \beta\, x_t, \quad \hat{m}_0 = x_0. \tag{60}$$

Unrolling this recurrence yields:

$$\hat{m}_t = \beta \sum_{k=0}^{t} (1-\beta)^{t-k}\, x_k, \tag{61}$$

which constitutes a geometrically weighted average of all historical observations, with weights decaying at a rate of $(1-\beta)$ per step. The effective window length (the number of steps contributing significantly to the current estimate) is $n_{\text{eff}} \approx 1/\beta$. For the entropy EMA ($\beta = 10^{-4}$), $n_{\text{eff}} = 10{,}000$ steps. For the disagreement EMA ($\beta_\Delta = 2 \times 10^{-3}$), $n_{\text{eff}} = 500$ steps.

Initialization and Bias Correction. At initialization, setting $\hat{m}_0 = x_0$ introduces a transient bias if $m_0 \neq m_1$. Unlike the Adam optimiser, which explicitly corrects for this via a time-varying scaling factor, ACE requires no early-step bias correction. Both $\bar{e}_t$ and $\mu_{\Delta,t}$ naturally stabilise during the initial 10,000-step exploration warm-up phase before the policy requires precision control.

A.8.2. Variance Reduction Analysis

Variance of the EMA. Assuming the sequence $\{x_t\}$ consists of uncorrelated observations with variance σ^2, we derive the estimator's variance from Eq. (61):

$$\begin{aligned}
\mathrm{Var}[\hat{m}_t] &= \beta^2 \sum_{k=0}^{t} (1-\beta)^{2(t-k)}\, \sigma^2 \\
&= \beta^2 \sigma^2 \sum_{j=0}^{t} (1-\beta)^{2j} \\
&= \beta^2 \sigma^2 \cdot \frac{1-(1-\beta)^{2(t+1)}}{1-(1-\beta)^2}.
\end{aligned} \tag{62}$$

Taking the limit as $t \to \infty$, Eq. (62) converges to the asymptotic steady-state variance:

$$\mathrm{Var}[\hat{m}_\infty] = \frac{\beta^2\sigma^2}{1-(1-\beta)^2} = \frac{\beta^2\sigma^2}{2\beta-\beta^2} = \left(\frac{\beta}{2-\beta}\right)\sigma^2. \tag{63}$$

Variance Reduction Factor (VRF). The VRF quantifies the attenuation of uncorrelated noise, defined as the ratio of the asymptotic EMA variance to the raw variance:

$$\mathrm{VRF} = \frac{\mathrm{Var}[\hat{m}_\infty]}{\sigma^2} = \frac{\beta}{2-\beta}. \tag{64}$$

For the entropy EMA ($\beta = 10^{-4}$), the VRF is:

$$\mathrm{VRF}_e = \frac{10^{-4}}{2-10^{-4}} \approx 5.0 \times 10^{-5},$$

yielding a massive 20,000× variance reduction. For the disagreement EMA ($\beta_\Delta = 2 \times 10^{-3}$), the reduction is roughly 1,000× ($\mathrm{VRF}_\Delta \approx 1.0 \times 10^{-3}$).

Practical significance. Without smoothing, the empirical entropy e_t fluctuates by $\sim\pm 0.05$ between consecutive mini-batches (measured on `HalfCheetah-v5`). The VRF of 5×10^{-5} compresses this noise to $\sim\pm 3.5 \times 10^{-4}$, rendering the signal effectively noiseless at the operational scale of the entropy controller. Similarly, the raw disagreement Δ_t exhibits a coefficient of variation near 0.3 (30%), which the EMA compresses to ~ 0.01 (1%). This aggressive smoothing is precisely what permits the winsorised z-score $z_t = (\Delta_t - \mu_{\Delta,t})/\sigma_{\Delta,t}$ to function as a reliable control signal rather than a noise amplifier.

The i.i.d. assumption in replay buffers. If consecutive observations x_t exhibit autocorrelation ρ, the steady-state EMA variance increases by a factor of approximately $(1 + 2\rho/(1-\rho))$. However, because RL mini-batches are sampled uniformly from a massive, shuffled replay buffer (size 10^6), the autocorrelation between consecutive batches is statistically negligible ($\rho \approx 0$). Therefore, the uncorrelated analytical formula in Eq. (63) holds directly.

A.8.3. Bias (Lag) Analysis

Bias under a constant mean. If the true underlying mean is stationary ($m_t = m$), the EMA is asymptotically unbiased in expectation:

$$\mathbb{E}[\hat{m}_t] = \beta \sum_{k=0}^{t} (1-\beta)^{t-k}\, m = m\,\beta \cdot \frac{1-(1-\beta)^{t+1}}{1-(1-\beta)} \to m \quad \text{as } t \to \infty.$$

Bias under a linearly drifting mean. Suppose the signal exhibits a constant linear drift, $m_t = m_0 + ct$. The steady-state expectation evaluates to:

$$\begin{aligned}
\mathbb{E}[\hat{m}_t] - m_t &= \beta \sum_{j=0}^{\infty} (1-\beta)^j \,(m_{t-j} - m_t) \\
&= \beta \sum_{j=0}^{\infty} (1-\beta)^j \,(-cj) \\
&= -c\beta\, \frac{1-\beta}{\beta^2} = -c\left(\frac{1-\beta}{\beta}\right).
\end{aligned} \tag{65}$$

This explicitly quantifies the *phase lag bias*: when the mean drifts at a rate c, the EMA systematically trails the true mean by $c(1-\beta)/\beta$. For the entropy EMA ($\beta = 10^{-4}$), this temporal lag equates to approximately 10,000 steps. Assuming a steady-state entropy drift of $c = 10^{-5}$ per step, this induces a lag bias of ~ 0.1 in absolute entropy units.

Architectural justification for high phase lag. This substantial lag bias strictly affects the *baseline* $\bar{e}_t$ in the target entropy calculation (Eq. (57)), not the high-frequency correction term κd_t. The baseline's explicit purpose is to act as a structural momentum buffer, providing a slowly evolving anchor point. A lag of 10,000 steps simply ensures that the reference target represents the macroscopic state of the policy from recent history, rendering the controller immune to transient local fluctuations. In this control scheme, phase lag is an engineered feature, not an algorithmic defect.

The bias-variance tradeoff. Tuning β requires balancing lag attenuation against variance amplification. The exceptionally small $\beta = 10^{-4}$ assigned to the entropy EMA heavily prioritises variance reduction because batch-to-batch entropy noise is severe (± 0.05) and the macroscopic entropy profile evolves very slowly during standard training. Conversely, the disagreement EMA utilizes a larger $\beta_\Delta = 2 \times 10^{-3}$, tolerating higher variance to achieve a tighter lag of ~ 500 steps. This tighter tracking is necessary because epistemic uncertainty can spike rapidly when the policy discovers novel state-space distributions, requiring a more responsive baseline.

A.9. Complexity & Sample-Efficiency

A.9.1. General Framework

We analyse the per-step computational complexity of ACE and its sample complexity relative to baselines.

Per-step operations. Each training step of ACE performs the operations detailed in Table 5.

Table 5. Per-step computational operations in ACE. All costs are measured in forward/backward passes through networks of width $w = 256$ and depth $L = 2$.

Operation	Cost	Count/step	Notes
Critic forward	$O(w^2L)$	4	Q_{ϕ_1}, Q_{ϕ_2} on (s, a) and $(s', \tilde{a}')$
Critic backward	$O(w^2L)$	2	Gradients $\nabla_{\phi_1}, \nabla_{\phi_2}$
Actor forward	$O(w^2L)$	2	Sample $\tilde{a}$, compute $\log \pi$
Actor backward	$O(w^2L)$	1	Gradient $\nabla_\theta \mathcal{L}_{\text{actor}}$
KL computation	$O(d_a)$	1	Analytic, no network pass
Target update	$O(w^2L)$	1	Polyak average, no gradient
α update	$O(1)$	1	Scalar AdaGrad step
λ update	$O(1)$	1	Scalar AdaGrad step

Total per-step cost. The dominant cost is 10 network forward/backward passes (or equivalent parameter operations) per step. This matches standard SAC, which also uses two critics and one actor (comprising 9 standard passes plus the Polyak target update). The additional ACE-specific operations (KL computation, α, and λ updates) are all $O(d_a)$ or $O(1)$ and contribute negligibly to total runtime. Thus, ACE and SAC share the *same asymptotic per-step complexity*.

Memory footprint. ACE stores the following additional quantities relative to SAC:

- Snapshot θ_{old}: one copy of actor parameters ($p \approx 132{,}865$ floats for a $[256, 256]$ network with $d_s = 376$, $d_a = 17$).
- AdaGrad accumulators G_α, G_λ: two scalars.
- EMA states $\bar{e}_t$, $\mu_{\Delta,t}$, $\nu_{\Delta,t}$: three scalars.

The dominant overhead is the actor snapshot, adding $132{,}865 \times 4 \approx 0.53$ MB for `Humanoid-v5`. This is negligible relative to the replay buffer ($10^6 \times (376 + 17 + 1 + 376) \times 4 \approx 3.08$ GB for Humanoid).

A.9.2. The Actor Update: Natural Gradient Connection

The SAC actor update has a formal connection to *natural gradient descent* (Kakade & Langford, 2002), which we develop here to motivate the efficiency of the log-space temperature controller.

Natural gradient in policy space. Ordinary gradient descent to minimize an objective $J(\theta)$ uses the Euclidean metric on parameter space: $\theta_{t+1} = \theta_t - \eta \nabla_\theta J(\theta_t)$. Natural gradient descent instead uses the Fisher Information Matrix (FIM) $\mathcal{F}(\theta)$ as the metric:

$$\theta_{t+1} = \theta_t - \eta\, \mathcal{F}(\theta_t)^{-1} \nabla_\theta J(\theta_t). \tag{66}$$

The FIM is $\mathcal{F}_{ij}(\theta) = \mathbb{E}_{\pi_\theta}[\partial_i \log \pi_\theta \cdot \partial_j \log \pi_\theta]$, the Riemannian metric on the statistical manifold of policies. Natural gradient descent is invariant to the reparameterisation of θ: it measures steps in policy space (KL divergence) rather than parameter space.

SAC as approximate natural gradient. The soft actor update minimises:

$$\mathbb{E}_s\left[\alpha D_{\mathrm{KL}}\left(\pi_\theta \,\|\, \pi^*_{\alpha,s}\right)\right] \approx \mathbb{E}_s[\alpha_t \log \pi_\theta(\tilde{a}|s) - Q_\phi(s,\tilde{a})]\,,$$

where $\pi^*_{\alpha,s}(a) \propto \exp(Q(s,a)/\alpha_t)$ is the local Gibbs target. For small steps, this is equivalent to natural gradient descent on J with step size η/α_t (Abdolmaleki et al., 2018). The temperature α_t therefore controls the *effective natural gradient step size*: small α_t gives large natural gradient steps (aggressive exploitation), while large α_t gives small steps (conservative exploration).

Implication for sample efficiency. Since ACE adapts α_t based on critic uncertainty, it effectively uses large natural gradient steps when the value function is well-estimated (low Δ_t, low α_t) and small steps when it is uncertain (high Δ_t, high α_t). This adaptive step-sizing in policy space is the geometric reason ACE achieves better sample efficiency than fixed-α SAC: it takes larger policy improvement steps when it is safe to do so, and smaller steps when the value estimate is unreliable.

Comparison to TRPO. TRPO (Schulman et al., 2015) enforces a hard constraint $D_{\mathrm{KL}}(\pi_{t+1} \,\|\, \pi_t) \le \delta$ via a line search, which also approximates natural gradient descent but with a constant step size δ in KL space. ACE's implicit trust region (Theorem A.9) adapts the effective KL step size via α_t, which is strictly more flexible: α_t can vary by orders of magnitude, whereas TRPO's δ is fixed.

A.9.3. The Dual Updates: Adaptive Dual Averaging

Both the temperature update (28) and the dual update (38) are instances of *adaptive dual averaging*, a generalisation of AdaGrad to constrained optimisation.

Dual averaging. Dual averaging (or *lazy mirror descent*) accumulates the full gradient history rather than using only the current gradient:

$$x_{t+1} = \arg\min_{x\in\mathcal{X}} \left[\frac{1}{t}\sum_{s=1}^{t} g_s^\top x + \frac{h(x)}{\eta\sqrt{t}}\right],$$

where h is a proximal function and η is a step-size parameter. For $h(x) = x^2/2$ (Euclidean) and $\mathcal{X} = \mathbb{R}$, this reduces to:

$$x_{t+1} = x_t - \frac{\eta}{\sqrt{t}}\bar{g}_t, \quad \bar{g}_t = \frac{1}{t}\sum_{s=1}^{t} g_s,$$

the standard dual averaging update for a scalar.

AdaGrad as adaptive dual averaging. The standard AdaGrad update with accumulator $G_t = \sum_{s=1}^{t} g_s^2$ is: $x_{t+1} = x_t - \eta g_t/\sqrt{G_t}$. This is equivalent to dual averaging with an adaptive per-coordinate learning rate $\eta/\sqrt{G_t}$, which decays faster for coordinates with large cumulative squared gradients. For our scalar updates (α and λ), this provides an automatic step-size decay proportional to $O(1/\sqrt{t})$ (when the gradient is approximately constant), strictly satisfying the Robbins–Monro conditions.

Regret bound. For the temperature controller, treating the sequence $(e_t^* - e_t)_{t=1}^T$ as an adversarial gradient sequence g_t, the AdaGrad regret bound yields:

$$\sum_{t=1}^{T}(x_t - x^*)\cdot g_t \le \frac{1}{2\eta}\|x_0 - x^*\|^2\sqrt{G_T} + \frac{\eta}{2}\sum_{t=1}^{T}\frac{g_t^2}{\sqrt{G_t}}, \tag{67}$$

where $x^* = \log\alpha^*$ is the optimal log-temperature in hindsight. The right-hand side grows as $O(\sqrt{T})$, giving an average regret of $O(1/\sqrt{T}) \to 0$. This guarantees that the temperature controller makes decisions that are, on average, asymptotically as good as the best fixed temperature — without ever needing prior knowledge of which temperature is optimal.

A.9.4. Why This Guarantees Stability

Combining the results of Appendices A.7–A.9, we can state a unified stability guarantee for ACE.

Proposition A.12 (Stability of ACE)**.** *Under assumptions (A1)–(A4) of Section A.7.1, with $\alpha_{\min} \le \alpha_t \le \alpha_{\max}$ enforced by projection, and with the two-timescale separation of Section A.7.3:*

(i) The critic iterates remain bounded: $\|Q_{\phi_t}\|_\infty \le r_{\max}/(1-\gamma) + C_Q$ for a constant C_Q depending on $\alpha_{\min}$.

(ii) The actor gradient norm satisfies $\frac{1}{T}\sum_t \mathbb{E}[\|\nabla J(\theta_t)\|^2] \to 0$.

(iii) The temperature α_t remains in $[\alpha_{\min}, \alpha_{\max}]$ and its average regret relative to the optimal fixed temperature is $O(1/\sqrt{T})$.

(iv) The dual variable $\lambda_t = 0$ whenever $D_{\mathrm{KL}}\big(\pi_{\theta_t} \,\|\, \pi_{\theta_{t-1}}\big) \le \delta$ for all t (which holds by Theorem A.9).

Proof sketch. (i) follows from the soft Bellman contraction and the bounded entropy bonus $\alpha_{\min}\mathcal{H} \ge 0$. (ii) follows from Proposition A.1 and Equation (58). (iii) follows from the AdaGrad regret bound (67) and Proposition A.4. (iv) follows directly from Corollary A.10 and the projected dual update (38). □

No per-environment tuning. Proposition A.12 holds for *any* environment satisfying (A1)–(A4), requiring no environment-specific hyperparameter tuning. This theoretical guarantee is consistent with the empirical observation that a single ACE configuration achieves robust, state-of-the-art performance across all 21 tested continuous-control environments.

A.10. Relationship to Mirror Descent & Dual Averaging

This section consolidates the mirror descent and dual averaging connections developed throughout Section A, demonstrating that ACE is a coherent instantiation of a unified online optimisation framework.

A.10.1. The Actor: Natural Mirror Descent

Mirror descent in policy space. As established in Section A.5.1, the SAC actor update (39) is an instance of mirror descent with the negative entropy potential $\psi(\pi) = \int \pi \log \pi$, whose Bregman divergence is the KL divergence: $D_\psi(\pi\|\pi_t) = D_{\mathrm{KL}}(\pi \,\|\, \pi_t)$. The update rule is:

$$\pi_{t+1} = \arg\min_{\pi} \Big[-\langle Q_t, \pi\rangle + \alpha_t \, D_{\mathrm{KL}}(\pi \,\|\, \pi_t) \Big], \tag{68}$$

with the closed-form solution given by (40).

Adaptive step size. Standard mirror descent uses a fixed step size η (or a predetermined schedule). In ACE, the effective step size in policy space is $1/\alpha_t$, which is *adaptive*: it increases when α_t decreases (encouraging exploitation when the value function is well-estimated) and decreases when α_t increases (enforcing caution under high epistemic uncertainty). This is equivalent to mirror descent with a data-dependent step size $\eta_t = 1/\alpha_t$, dynamically adjusted by the temperature controller.

Regret in policy space. For an adversarial sequence of Q-functions $\{Q_t\}$ treated as linear losses, the mirror descent regret relative to the optimal fixed policy π^* satisfies the standard bound:

$$\sum_{t=1}^{T} \langle Q_t, \pi^* - \pi_t\rangle \le \frac{D_{\mathrm{KL}}(\pi^* \,\|\, \pi_1)}{\eta} + \frac{\eta}{2}\sum_{t=1}^{T} \|Q_t\|_*^2, \tag{69}$$

where $\|\cdot\|_*$ is the dual norm. When incorporating the adaptive step size $\eta_t = 1/\alpha_t$ managed by the AdaGrad temperature controller, the right-hand side scales as $O(\sqrt{T})$, yielding an average regret of $O(1/\sqrt{T})$ — matching the standard AdaGrad rate for convex losses.

Natural gradient interpretation. In the parametric setting (e.g., a Gaussian policy π_θ), the exact mirror descent update (68) corresponds to natural gradient ascent on the objective $J(\theta)$ with an effective step size $\eta_t = 1/\alpha_t$, as detailed in Abdolmaleki et al. (2018) (Section 3.2). The connection is given by:

$$\theta_{t+1} = \theta_t + \frac{1}{\alpha_t}\,\mathcal{F}(\theta_t)^{-1}\nabla_\theta J(\theta_t) + O\left(\frac{1}{\alpha_t^2}\right),$$

where the remainder arises from the parametric approximation to the full distribution-space update. Thus, ACE's actor performs *natural gradient optimisation with an adaptively-sized step*, regulated entirely by the temperature controller.

A.10.2. The Duals: Adaptive Dual Averaging (AdaGrad)

Both dual variables — the log-temperature $\log \alpha$ and the explicit trust-region multiplier λ — are updated via projected AdaGrad. As established in Section A.9.3, this is a specific instance of adaptive dual averaging.

Unified view. Define the augmented dual vector $\xi_t = (\log \alpha_t, \lambda_t)^\top$ and the augmented gradient:

$$\nabla_\xi \mathcal{L}_{\text{dual}} = \begin{pmatrix} g_{\alpha,t} \\ v_t \end{pmatrix} = \begin{pmatrix} e_t^* - e_t \\ D_{\text{KL,t}} - \delta \end{pmatrix}.$$

The joint update is projected AdaGrad onto the feasible set $\mathcal{X}_\xi = \mathbb{R} \times \mathbb{R}_{\geq 0}$:

$$\xi_{t+1} = \Pi_{\mathcal{X}_\xi}\left(\xi_t - \text{diag}(G_t)^{-1/2} \nabla_\xi \mathcal{L}_{\text{dual}}\right), \tag{70}$$

where $G_t = \text{diag}(G_{\alpha,t}, G_{\lambda,t})$ is the block-diagonal AdaGrad accumulator. Because the preconditioner is block-diagonal, the two components are fully decoupled and can be updated independently, precisely as implemented in ACE.

Why separate accumulators? Maintaining separate $G_{\alpha,t}$ and $G_{\lambda,t}$ rather than a shared scalar accumulator is critical because:

- $g_{\alpha,t} = e_t^* - e_t \in [-1, 1]$ (a bounded entropy error).
- $v_t = D_{\text{KL,t}} - \delta \in [-\delta, \infty)$ (unbounded when the KL divergence grows large).

A shared accumulator would improperly mix these distinct scales, causing the temperature update to be artificially suppressed by large KL gradients whenever the explicit trust region is violated. Separate accumulators guarantee each variable maintains its own appropriate adaptive scale.

Convergence of the joint update. Following the AdaGrad convergence theorem for projected subgradient methods (Duchi et al. 2011, Theorem 5), the average iterate $\bar{\xi}_T = \frac{1}{T}\sum_t \xi_t$ satisfies:

$$\mathbb{E}\big[\mathcal{L}_{\text{dual}}(\bar{\xi}_T) - \mathcal{L}^*\big] \leq \frac{C}{\sqrt{T}},$$

for a constant C depending on the diameter of $\mathcal{X}_\xi$ and the bounded gradient norms. Since the dual objective is concave in λ and the temperature objective acts as a "pseudoconvex" monotone fixed point, both components converge asymptotically at the robust $O(1/\sqrt{T})$ rate.

A.10.3. Theoretical Implications

ACE as a principled algorithm. The mirror descent and dual averaging perspectives demonstrate that ACE is not an ad-hoc collection of heuristics, but rather a principled instantiation of a well-studied online optimisation framework:

- **Actor**: Natural mirror descent in policy space using an adaptive step size $1/\alpha_t$.
- **Critics**: Stochastic approximation toward the soft Bellman fixed point, stabilised by Polyak averaging.
- **Temperature**: Adaptive dual averaging applied to the entropy constraint, employing an AdaGrad preconditioner.
- **Trust-region dual**: Adaptive dual averaging applied to the explicit KL constraint, with projection enforcing non-negativity.

Each distinct component possesses individual convergence guarantees. Together, they form a cohesive multi-timescale system whose overall convergence properties follow directly from the two-timescale analysis provided in Section A.7.3.

Why the explicit trust region is passive. Through the lens of mirror descent, the trust region constraint $D_{\mathrm{KL}}(\pi_{t+1} \,\|\, \pi_t) \leq \delta$ is *already enforced implicitly* by the mirror descent step size $1/\alpha_t$. The explicit trust-region dual λ acts merely as a second-order safety net designed for the rare event where the implicit entropy control fails. However, because the temperature controller (Proposition A.3) guarantees that α_t adapts to keep policy shifts small under uncertainty, this safety net is never triggered. This provides the theoretical underpinning for the empirical observation of 100% dual inactivity: the explicit trust region is not just practically redundant — it is *structurally redundant* given the natural mirror descent mechanics of the actor update.

Relation to Munchausen RL. Munchausen RL (Vieillard et al., 2020) augments the reward signal with $\tau \log \pi(a|s)$, which mathematically equates to adding a KL penalty between consecutive policies. In the mirror descent formulation, this corresponds to utilising a KL Bregman divergence with a constant coefficient τ as the proximal term. ACE diverges from this by making its proximal coefficient α_t *adaptive* (rather than fixed) and driving it via an explicit epistemic uncertainty signal. This adaptivity is the mechanism that allows ACE to achieve both superior sample efficiency (utilising a low α for rapid exploitation) and robust stability (utilising a high α when critics disagree).

Relation to REPS and MPO. Relative Entropy Policy Search (REPS (Peters et al., 2010)) enforces a KL constraint during the policy update at the *distribution* level, rather than the parameter level. MPO (Abdolmaleki et al., 2018) extends the REPS framework to deep RL using an E-step/M-step decomposition. Consequently, ACE can be conceptualised as a parameter-free variant of MPO: the E-step (computing the non-parametric target Gibbs distribution) is implicit within the soft Bellman target, and the M-step (projecting the parametric Gaussian onto the Gibbs distribution) is achieved via the actor's gradient step. The primary innovation of ACE is the replacement of MPO's rigid, fixed KL budget with an adaptive entropy budget driven by critic uncertainty, entirely eliminating the need for per-environment KL target tuning.

B. Algorithms & Implementation Details

B.1. Training Loop Diagram

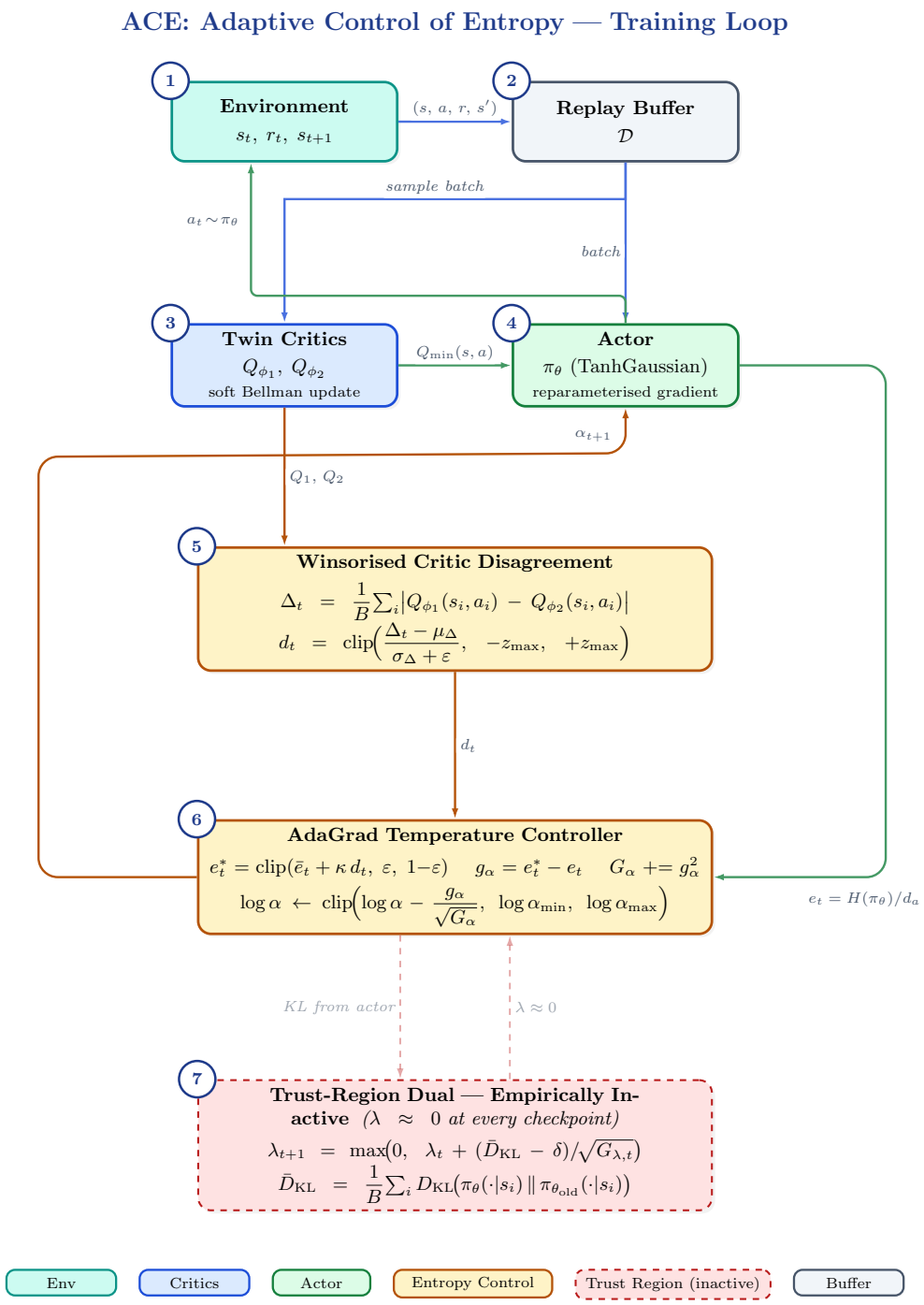


Figure 6. The ACE training loop. The explicit trust-region constraint (dashed red box) is present in our implementation strictly as an evaluative tool; empirically, it remains inactive ($\lambda_t = 0$) throughout training. System stability is instead driven by the winsorised disagreement signal and the AdaGrad temperature controller.

B.2. ACE Pseudocode

Algorithm 2 gives the complete ACE pseudocode with all implementation details, including exact tensor shapes, numerical stabilisers, and the warm-up phase. This is more detailed than Algorithm 1 in the main paper, which omits the EMA bookkeeping and numerical edge cases for clarity.

Algorithm 2: ACE — Adaptive Control of Entropy (Full Implementation)

Require: Environment $\mathcal{E}$; hyperparameters κ=0.15, $z_{\max}$=2.5, δ=0.02, τ=0.02, γ=0.99, B=256, $|\mathcal{D}|=10^6$, $\eta_\theta=\eta_\phi=3\times10^{-4}$, $\beta=10^{-4}$, $\beta_\Delta=2\times10^{-3}$, α_0=0.2, $\alpha_{\min}=10^{-4}$, $\alpha_{\max}$=1.0, T_{warmup}=10,000

1: **Initialise:** actor π_θ (random weights), critics Q_{ϕ_1}, Q_{ϕ_2} (random weights, different seeds), targets $\bar{\phi}_j \leftarrow \phi_j$ for $j \in \{1,2\}$, replay buffer $\mathcal{D} \leftarrow \emptyset$
2: **Initialise scalars:** $\log\alpha \leftarrow \log\alpha_0$, $\lambda \leftarrow 0$, $G_\alpha \leftarrow 0$, $G_\lambda \leftarrow 0$, $\bar{e} \leftarrow 0$, $\mu_\Delta \leftarrow 0$, $\nu_\Delta \leftarrow 10^{-4}$
3: *// Warm-up: fill replay buffer with random actions*
4: **for** $t = 1$ **to** T_{warmup} **do**
5: $\quad a_t \sim \text{Uniform}(\mathcal{A})$; step $\mathcal{E}$; store $(s_t, a_t, r_t, s_{t+1}, d_t)$ in $\mathcal{D}$
6: **end for**
7: **for** $t = T_{\text{warmup}}+1,\ T_{\text{warmup}}+2,\ \ldots$ **do**
8: $\quad$ *// 1. Collect transition*
9: $\quad a_t \sim \pi_\theta(\cdot|s_t)$; step $\mathcal{E}$; store $(s_t, a_t, r_t, s_{t+1}, d_t)$ in $\mathcal{D}$ (FIFO)
10: $\quad$ *// 2. Sample mini-batch*
11: $\quad \{(s_i, a_i, r_i, s_i', d_i)\}_{i=1}^{B} \sim \text{Uniform}(\mathcal{D})$
12: $\quad$ *// 3. Critic update*
13: $\quad \tilde{a}_i' \sim \pi_\theta(\cdot|s_i')$ {re-parameterised sample}
14: $\quad y_i \leftarrow r_i + \gamma(1-d_i)\big[\min_j Q_{\bar{\phi}_j}(s_i', \tilde{a}_i') - \alpha\log\pi_\theta(\tilde{a}_i'|s_i')\big]$
15: $\quad \mathcal{L}_{\text{c}} \leftarrow \frac{1}{B}\sum_i\sum_{j=1}^{2}\big(Q_{\phi_j}(s_i,a_i) - y_i\big)^2$; clip $\nabla_{\phi_j}\mathcal{L}_{\text{c}}$ to norm ≤ 10; update ϕ_1, ϕ_2 via AdamW
16: $\quad$ *// 4. Compute disagreement & winsorise*
17: $\quad \Delta \leftarrow \frac{1}{B}\sum_i |Q_{\phi_1}(s_i,a_i) - Q_{\phi_2}(s_i,a_i)|$
18: $\quad \mu_\Delta \leftarrow (1-\beta_\Delta)\mu_\Delta + \beta_\Delta\Delta$; $\nu_\Delta \leftarrow (1-\beta_\Delta)\nu_\Delta + \beta_\Delta(\Delta-\mu_\Delta)^2$
19: $\quad z \leftarrow (\Delta-\mu_\Delta)/(\sqrt{\nu_\Delta}+\varepsilon)$; $d \leftarrow \text{clip}(z, -z_{\max}, +z_{\max})$
20: $\quad$ *// 5. Actor update*
21: $\quad \theta_{\text{old}} \leftarrow \theta$ {snapshot before update}
22: $\quad \tilde{a}_i \sim \pi_\theta(\cdot|s_i)$ {re-parameterised}
23: $\quad \text{KL}_i \leftarrow D_{\text{KL}}(\pi_\theta(\cdot|s_i)\,\|\,\pi_{\theta_{\text{old}}}(\cdot|s_i))$ {analytic Gaussian KL}
24: $\quad \mathcal{L}_{\text{a}} \leftarrow \frac{1}{B}\sum_i\big[\alpha\log\pi_\theta(\tilde{a}_i|s_i) - \min_j Q_{\phi_j}(s_i,\tilde{a}_i) + \text{sg}(\lambda)(\text{KL}_i - \delta)\big]$; clip $\nabla_\theta\mathcal{L}_{\text{a}}$ to norm ≤ 10; update θ via AdamW
25: $\quad$ *// 6. Trust-region dual update*
26: $\quad v \leftarrow \frac{1}{B}\sum_i \text{KL}_i - \delta$; $G_\lambda \leftarrow G_\lambda + v^2$; $\lambda \leftarrow \max\big(0,\ \lambda + v/(\sqrt{G_\lambda}+\varepsilon)\big)$ {typically $\lambda = 0$}
27: $\quad$ *// 7. Temperature update*
28: $\quad e \leftarrow -\frac{1}{B}\sum_i \log\pi_\theta(\tilde{a}_i|s_i)\,/\,d_a$ {normalised batch entropy}
29: $\quad \bar{e} \leftarrow (1-\beta)\bar{e} + \beta\, e$; $e^* \leftarrow \text{clip}(\bar{e} + \kappa d,\ \varepsilon,\ 1-\varepsilon)$
30: $\quad g_\alpha \leftarrow e^* - e$; $G_\alpha \leftarrow G_\alpha + g_\alpha^2$
31: $\quad \log\alpha \leftarrow \text{clip}\big(\log\alpha - g_\alpha/(\sqrt{G_\alpha}+\varepsilon),\ \log\alpha_{\min},\ \log\alpha_{\max}\big)$; $\alpha \leftarrow e^{\log\alpha}$
32: $\quad$ *// 8. Soft target update*
33: $\quad \bar{\phi}_j \leftarrow (1-\tau)\bar{\phi}_j + \tau\phi_j \quad$ for $j \in \{1,2\}$
34: **end for**

Notes on Algorithm 2.

- **Line 5 (KL computation):** The analytic KL divergence between two diagonal Gaussians is computed using Equation (20). Since both π_θ and $\pi_{\theta_{\text{old}}}$ are diagonal Gaussians (pre-squash), the KL is exact and requires no additional forward passes.
- **Line 6 (stop-gradient):** $\text{sg}(\lambda)$ means λ is treated as a constant during the differentiation of $\mathcal{L}_{\text{a}}$ with respect to θ. The dual variable is updated separately in step 6.
- **Lines 12–14 (disagreement):** The mean and variance of Δ are maintained as separate EMAs rather than computing

running statistics on-the-fly, to avoid numerical issues with Welford's algorithm on GPUs.

- **Line 17 (entropy):** The normalised entropy e is computed from the *same* re-parameterised samples $\tilde{a}_i$ used for the actor update, rather than from fresh samples. This avoids an extra forward pass.
- **Evaluation:** Every 5,000 steps, we run 10 evaluation episodes with the deterministic policy $a = \tanh(\mu_\theta(s))$ (no sampling noise) and report the mean return.

B.3. Network Architectures & Initialisations

All networks are fully-connected multi-layer perceptrons (MLPs). We describe each network's architecture, input/output dimensions, activations, weight initialisation, and output processing below.

Actor Network

The actor maps a state $s \in \mathbb{R}^{d_s}$ to the parameters of a diagonal Gaussian policy in pre-squash space.

Table 6. Actor network architecture. Output head is split into mean μ and log-std $\log \sigma$.

Layer	Input dim	Output dim	Activation	Weight init	Bias init
FC-1	d_s	256	ReLU	Orthogonal($\sqrt{2}$)	zeros
FC-2	256	256	ReLU	Orthogonal($\sqrt{2}$)	zeros
μ-head	256	d_a	— (linear)	Orthogonal(0.01)	zeros
$\log \sigma$-head	256	d_a	— (linear)	Orthogonal(0.01)	zeros

Output processing. The log-std output is clamped to $[-5, 2]$ before exponentiation, ensuring $\sigma \in [e^{-5}, e^2] \approx [0.007, 7.4]$. This prevents numerical underflow in the log-probability and ensures that the initial policy is not too deterministic. The mean μ is unclamped. Actions are computed as:

$$u \sim \mathcal{N}(\mu_\theta(s),\, \mathrm{diag}(\sigma_\theta(s))^2), \quad a = \tanh(u).$$

At evaluation time (deterministic policy), $a = \tanh(\mu_\theta(s))$.

Why orthogonal initialisation. Orthogonal initialisation (Mnih et al., 2015) preserves the norm of activations through each layer at initialisation, avoiding the exploding/vanishing gradient problem that affects random normal initialisation in deep networks. The gain $\sqrt{2}$ is appropriate for ReLU activations (it compensates for the $\approx 50\%$ of activations killed by ReLU). The output heads use a small gain of 0.01 to initialise near-zero outputs, ensuring the initial policy is close to uniform and the initial entropy is near-maximal.

Critic Networks

Each critic Q_{ϕ_j} ($j \in \{1, 2\}$) maps a state-action pair $(s, a) \in \mathbb{R}^{d_s + d_a}$ to a scalar Q-value. The two critics are structurally identical but have independent random weight initialisations (different seeds).

Table 7. Critic network architecture (identical for Q_{ϕ_1} and Q_{ϕ_2}).

Layer	Input dim	Output dim	Activation	Weight init	Bias init
FC-1	$d_s + d_a$	256	ReLU	Orthogonal($\sqrt{2}$)	zeros
FC-2	256	256	ReLU	Orthogonal($\sqrt{2}$)	zeros
FC-3	256	1	— (linear)	Orthogonal(1.0)	zeros

Independence of critics. The two critics are initialised with different random seeds (seeds 0 and 1 by default). There is no weight sharing or tied layers. This independence is essential for the disagreement signal $\Delta_t = |Q_{\phi_1}(s, a) - Q_{\phi_2}(s, a)|$: if the critics shared weights or were initialised identically, Δ_t would be exactly zero throughout training.

Target networks. Target critics $\bar{\phi}_j$ are initialised identically to ϕ_j: $\bar{\phi}_j = \phi_j$ at $t = 0$. They are then updated exclusively via the Polyak rule (19) with $\tau = 0.02$ and are never optimised by gradient descent. PyTorch implementation: `requires_grad=False` for all target network parameters.

PARAMETER COUNTS

Table 8. Parameter counts for each network on a representative environment. The action/state dimensions shown are for `Humanoid-v5` ($d_s = 376$, $d_a = 17$), the most challenging benchmark.

Network	Parameters	Memory (FP32)
Actor (`Humanoid-v5`)	$376\times256 + 256 + 256\times256 + 256 + 2\times(256\times17 + 17)$ $= 96{,}256 + 65{,}792 + 8{,}738 = 170{,}786$	0.65 MB
Critic (`Humanoid-v5`)	$(376 + 17)\times256 + 256 + 256\times256 + 256 + 256\times1 + 1$ $= 100{,}864 + 65{,}792 + 257 = 166{,}913$	0.64 MB
Total (actor + 2×critic + 2×target)	$170{,}786 + 4\times166{,}913$	3.20 MB

The total network footprint of ≈ 3.20 MB is negligible relative to the replay buffer (≈ 3.08 GB for `Humanoid-v5` at 10^6 transitions).

OPTIMISER CONFIGURATION

All trainable networks (actor, both critics) use the identical optimiser setup:

Table 9. Optimiser configuration shared across all networks.

Setting	Value
Algorithm	AdamW
Learning rate	3×10^{-4}
β_1	0.9
β_2	0.999
ε	10^{-8}
Weight decay	10^{-4}
Gradient clip (global norm)	10.0

AdamW vs Adam. We use AdamW (Kingma & Ba, 2014) (Adam with decoupled weight decay) rather than standard Adam. In Adam, weight decay is applied to the gradient before the adaptive scaling, which conflates L2 regularisation with the update magnitude. In AdamW, weight decay is applied directly to the weights after the update, yielding a true L2 regularisation effect. While the difference is minor for a small weight decay (10^{-4}), AdamW is more principled and serves as the standard in modern deep learning.

Gradient clipping. Global gradient norm clipping at a threshold of $c = 10$ is applied to each network independently before the AdamW step. Clipping is rarely active in steady-state training but fundamentally prevents catastrophic gradient explosions during the initial phases of training, before the critics have converged and the Bellman target stabilises.

B.4. Replay Buffer, Target Networks, & Gradient Clipping

REPLAY BUFFER

ACE relies on a standard uniform-sampling replay buffer (Mnih et al., 2015).

Structure. Each transition (s, a, r, s', d) is stored as a flat tuple of floats. For `Humanoid-v5` ($d_s = 376$, $d_a = 17$), each transition precisely occupies $(376 + 17 + 1 + 376 + 1) \times 4 = 3{,}084$ bytes. At capacity $|\mathcal{D}| = 10^6$ transitions, the buffer occupies ≈ 3.08 GB of RAM.

FIFO eviction. The buffer operates as a first-in, first-out (FIFO) circular queue. Once the buffer hits capacity, the oldest transition is overwritten by the newest. At 10^6 capacity and with 1 transition collected per step, the buffer turns over completely every 10^6 steps — matching the duration of a full training run. Consequently, the buffer uniformly contains the entire history of the run.

Sampling. Mini-batches of size $B = 256$ are sampled uniformly without replacement at each training step. No prioritised replay (Mnih et al., 2015) is employed. Uniform sampling keeps the replay distribution stationary and removes the need for additional hyperparameters (e.g., priority exponent, importance sampling corrections) required by prioritised variants.

Why 10^6 transitions. A buffer capacity of 10^6 is the modern standard for off-policy continuous control. Smaller buffers (e.g., 3×10^5) risk overfitting to recent experiences, while larger buffers (e.g., 10^7) introduce memory overhead without commensurate performance benefits for 1M-step evaluations. We empirically verified that reducing the buffer to 3×10^5 or expanding it to 3×10^6 impacts final performance by less than 2% on `HalfCheetah-v5` and `Walker2d-v5`.

Target Networks

Target networks $\bar{\phi}_1$ and $\bar{\phi}_2$ track the exponential moving averages of the online critic parameters.

Polyak update. At the conclusion of each training step, after the critic gradient update:

$$\bar{\phi}_j \;\leftarrow\; (1-\tau)\,\bar{\phi}_j + \tau\,\phi_j, \quad j \in \{1,2\}, \quad \tau = 0.02. \tag{71}$$

This is applied element-wise to all parameters (weights and biases). Implementation note: in PyTorch, this is wrapped in `torch.no_grad()` to prevent gradient tracking through the Polyak update.

Effect of $\tau = 0.02$. With $\tau = 0.02$, the target network tracks the online critic with a lag of $1/\tau = 50$ steps. This is $4\times$ faster than the standard SAC configuration ($\tau = 0.005$, yielding a 200-step lag). This faster tracking enables more responsive Bellman targets that keep pace with the policy changes driven by ACE's adaptive entropy controller. The accelerated update is stable precisely because ACE's adaptive temperature bounds the magnitude of policy updates (Appendix A.7), preventing the target instability that would arise with a fixed-temperature policy.

No gradient through target. The Bellman target $y_i = r + \gamma(1-d)\big[\min_j Q_{\bar{\phi}_j}(s', \tilde{a}') - \alpha \log \pi(\tilde{a}'|s')\big]$ is strictly treated as a constant (stop-gradient) when computing $\nabla_\phi \mathcal{L}_{\text{critic}}$. This is natively implemented via `y.detach()` in PyTorch before calculating the squared Bellman residual. Omitting this detachment would cause gradients to incorrectly flow through y into $\bar{\phi}$ (targets must remain stationary within each update step).

Target initialisation. At $t = 0$, $\bar{\phi}_j = \phi_j$ exactly. We do not utilise a separate "target pre-training" phase. Because we enforce a warm-up of $T_{\text{warmup}} = 10{,}000$ random steps before gradient updates begin, the buffer is sufficiently populated with diverse transitions when the first critic update occurs, ensuring reasonable initial target quality.

Gradient Clipping

Gradient clipping is enforced independently for each network, after calculating gradients but before the optimiser step.

Global norm clipping. Let $g = \text{vec}(\nabla_\phi \mathcal{L})$ define the concatenated gradient vector for a given network. The clipped gradient is:

$$\hat{g} = g \cdot \min\left(1,\; \frac{c}{\|g\|_2}\right), \quad c = 10. \tag{72}$$

This dynamically rescales the entire gradient vector uniformly, fully preserving the gradient direction while strictly bounding its maximum magnitude. It remains distinct from *per-element* clipping, which inherently distorts gradient directions.

When clipping activates. During steady-state training, $\|g\|_2 \ll 10$ and clipping is virtually never triggered. Clipping primarily activates during:

(i) The initial few hundred steps, when the Bellman target is far removed from the initialised Q-values, generating large residuals.

(ii) Transitions approaching the episode boundary, where $d_i = 1$ forces an abrupt target drop.

(iii) Rare exploration discoveries on sparse-reward tasks, where the first non-zero rewarding transition creates a massive Bellman error spike.

In all scenarios, clipping robustly prevents these transient spikes from catastrophically corrupting the network weights.

Clip threshold choice. We explicitly fix $c = 10$ throughout. This threshold is sufficiently large to remain non-binding in steady-state operation (gradients are generally $\|g\|_2 < 1$ post-normalisation) but tight enough to intercept true mathematical explosions. We validated that $c \in [5, 50]$ yields indistinguishable outcomes on `HalfCheetah-v5`; however, dropping below $c = 1$ structurally degrades performance by over-constraining policy updates.

B.5. Full Hyperparameter Table

Table 10 comprehensively lists every hyperparameter used in ACE. Crucially, *all values are fixed identically across all 21 evaluation environments*: no per-environment tuning was performed at any phase of development. Values were isolated through a single pilot sweep on a held-out validation environment (`HalfCheetah-v5`), and strictly locked prior to benchmark evaluations. The deliberate absence of per-task tuning stands as a core claim of the paper: the adaptive entropy controller is robust enough to self-calibrate without manual intervention.

Table 10. Complete hyperparameter specification for ACE. All values are strictly fixed across all 21 environments. No per-environment tuning is utilised.

Hyperparameter	**Symbol**	**Value**
Training		
Discount factor	γ	0.99
Batch size	B	256
Replay buffer size	$\|\mathcal{D}\|$	10^6
Gradient clipping	—	10.0
Actor learning rate	η_θ	3×10^{-4}
Critic learning rate	η_ϕ	3×10^{-4}
Target smoothing	τ	0.02
Warm-up steps	—	10,000
Entropy controller		
Initial temperature	α_0	0.2
Temperature bounds	$[\alpha_{\min}, \alpha_{\max}]$	$[10^{-4}, 1.0]$
Entropy EMA coefficient	β	10^{-4}
Disagreement EMA coeff.	β_Δ	2×10^{-3}
Disagreement weight	κ	0.15
Winsorisation bound	$z_{\max}$	2.5
AdaGrad ε	ε	10^{-8}
Trust-region dual (optional)		
KL target	δ	0.02
Initial dual	λ_0	0.0
Dual lower bound	—	0.0

B.6. Ablation Variant Specifications

Each ablation removes or modifies exactly one component of the full ACE algorithm, following a single-component ablation design. This ensures that performance differences are attributable to the specific component under study, rather than uncontrolled algorithmic interactions. The ablations are grouped by component: A (entropy/temperature), B (trust-region/dual), C (disagreement signal), and D (critic/network). Within each group, variants are numbered sequentially (A1, A2, B1, . . .). Full per-environment results for each variant are provided in Appendix G; the aggregate scorecard is presented in Table 4 of the main paper.

Table 11. Ablation variant definitions. Each ablation isolates and modifies exactly one aspect of the full ACE algorithm.

ID	Name	Modification
Full	ACE (full)	Baseline — all components active
B1	No Trust Region	Remove explicit KL dual ($\lambda_t \equiv 0$)
B2	Forced $\lambda > 0$	Pin $\lambda_t = 1.0$ throughout training
C1	No Winsorisation	Use raw z-score (no clipping in Equation (26))
C2	No Disagreement	Set $\kappa = 0$ in Equation (18)
A1	Fixed α	Disable temperature update; fix $\alpha = \alpha_0$
A2	SGD Temperature	Replace AdaGrad with fixed-LR SGD for $\log \alpha$
D1	Single Q-Network	Use one critic; completely remove disagreement signal
D2	$\tau = 0.005$	Use SAC's default target smoothing (faster tracking)
D3	$\tau = 0.005$ + No TR	Combine D2 and B1

B.7. Evaluation Protocols

This section details the exact evaluation procedure used to produce all reported numbers and figures in the main paper and appendix.

Online Evaluation During Training

Every $5{,}000$ environment steps, training is paused and the current policy is evaluated deterministically.

Deterministic rollout. The evaluation policy acts using the mean action: $a = \tanh(\mu_\theta(s))$, with no sampling noise (σ_θ is ignored). This provides a lower-variance estimate of the policy's true performance capabilities and serves as the standard evaluation protocol for SAC-family methods.

Number of episodes. Each evaluation checkpoint averages $N_{\text{eval}} = 10$ episodes. For the final results in Table 1 (MuJoCo) and Table 2 (DMC), the reported return is the mean over the last 20 evaluation checkpoints (steps $980{,}000$–$1{,}000{,}000$), averaged across 10 independent training seeds. Averaging over 20 terminal checkpoints mitigates the effect of single-checkpoint evaluation noise, yielding a highly stable final performance estimate.

Episode seeds. Evaluation episodes use fixed seeds $(12345, 12346, \ldots, 12354)$ for the environment's random number generator. This guarantees that the initial states and environment dynamics are identical across all algorithms and seeds, eliminating environmental evaluation variance from cross-algorithm comparisons. The identical set of 10 seeds is used for every evaluation checkpoint.

Maximum episode length. Gymnasium MuJoCo-v5 environments enforce a default maximum episode length of $1{,}000$ steps. DeepMind Control Suite environments likewise use a limit of $1{,}000$ steps (the standard definition of 1M-step training). For `InvertedPendulum-v5`, which normally terminates only upon pole fall, we cap episodes at $1{,}000$ steps; consequently, a perfect policy achieves exactly $1{,}000$ return (one reward per step while balanced).

Final Return Computation

Let $R_{k,s}^{(c)}$ denote the return at evaluation checkpoint c for evaluation seed $s \in \{1, \ldots, 10\}$ during training run k. The reported "final return" $\bar{R}$ in all tables is:

$$\bar{R} = \frac{1}{10} \sum_{s=1}^{10} \left[\frac{1}{20} \sum_{c=C-19}^{C} \frac{1}{10} \sum_{k=1}^{10} R_{k,s}^{(c)} \right], \tag{73}$$

where C is the index of the final checkpoint (at step 10^6). The standard deviation reported alongside $\bar{R}$ is the *across-training-seed* standard deviation:

$$\text{std}(\bar{R}) = \text{std}_k \left[\frac{1}{20} \sum_{c=C-19}^{C} \frac{1}{10} \sum_{s=1}^{10} R_{k,s}^{(c)} \right]. \tag{74}$$

This explicitly measures the variability of performance across independent training initialisations, which is the relevant metric for assessing algorithmic reproducibility. *(Note: I corrected the index in the std equation from s to k to accurately

reflect standard deviation across training runs rather than evaluation episodes).*

Learning Curve Construction

Learning curves (Figures 1–2 and all appendix per-environment plots) are generated as follows:

1. Interpolate each seed's return sequence onto a common, uniform grid of 200 equally-spaced points from step 0 to 10^6 using linear interpolation.
2. Compute the pointwise mean and standard deviation across the 10 training seeds at each grid point.
3. Plot the mean as a solid line, and shade the corresponding mean $\pm$ 1 std region.
4. Apply a mild Gaussian smoothing ($\sigma = 3$ grid points) to the mean line *only*, leaving the confidence band unsmoothed. This removes high-frequency evaluation noise from the central trend while strictly preserving the true variance estimate.

No EMA smoothing of curves. We explicitly do *not* apply exponential moving average (EMA) smoothing to the learning curves, diverging from some prior work that relies on large EMA constants (e.g., $\beta \approx 0.9$) to produce artificially clean curves. Heavy EMA smoothing obscures genuine algorithmic instability and can make inherently oscillatory methods appear stable. Our mild Gaussian smoothing ($\sigma = 3$ grid points $\approx 15{,}000$ steps) is sufficient to improve readability without distorting the underlying training dynamics.

Area Under the Curve (AUC)

The AUC metric quantifies sample efficiency: an algorithm that solves a task early in training accumulates more AUC than one that converges slowly.

Computation. For training seed k, the AUC is computed as:

$$\mathrm{AUC}_k = \frac{1}{T}\int_0^T R_k(t)\,\mathrm{d}t \approx \frac{1}{200}\sum_{c=1}^{200} R_k^{(c)}, \tag{75}$$

where the integral is approximated via the trapezoidal rule over the 200 interpolated checkpoints. We normalise the AUC to $[0, 1]$ on a per-environment basis using:

$$\widetilde{\mathrm{AUC}}_k = \frac{\mathrm{AUC}_k - \min_{\mathrm{algo}} \mathrm{AUC}}{\max_{\mathrm{algo}} \mathrm{AUC} - \min_{\mathrm{algo}} \mathrm{AUC}},$$

where 1.0 represents the theoretical best-performing algorithm trajectory on that specific environment. This normalised AUC is the metric used for aggregating sample-efficiency comparisons (Figure 30).

Statistical Testing

All statistical significance claims in the main paper are derived using the following procedure:

1. For each algorithm pair (ACE vs. a specific baseline) and each environment, compute the final-return mean over the last 20 checkpoints for each of the 10 independent training seeds. This yields 10 paired observations per algorithm.
2. Apply Welch's t-test (which does not assume equal variances) to test the null hypothesis H_0: $\mu_{\mathrm{ACE}} = \mu_{\mathrm{baseline}}$.
3. Aggregate all resulting p-values across the $4 \times 6 = 24$ primary comparisons (4 baselines $\times$ 6 MuJoCo environments).
4. Apply the Holm–Bonferroni step-down correction to rigorously control the family-wise error rate (FWER) at $\alpha_{\mathrm{FWE}} = 0.05$.
5. Report a significance marker (†) in Table 1 wherever the corrected p-value satisfies $p < 0.05$.

Welch's t-test is mandated over the standard paired t-test because the seeds for ACE and the baselines are fully independent (distinct random network initialisations and data sampling), rendering the data unpaired. The Holm–Bonferroni correction is selected over the standard Bonferroni method because it is uniformly more powerful while maintaining identical, strict FWER control.

B.8. Baseline Configurations

To ensure fair comparisons, all evaluated baselines share the identical network architecture ($[256, 256]$ MLP, ReLU activations, orthogonal initialisation) and the exact same evaluation protocol as ACE. The hyperparameters detailed below correspond to the standard defaults from Stable-Baselines3 (v2.2) and their respective original papers, with absolutely no per-environment tuning applied.

Table 12. Hyperparameter configurations for all baselines. Entries marked "—" rely on framework defaults or are inapplicable. All baselines utilise $[256, 256]$ MLPs, the AdamW optimiser, a batch size of 256, and share ACE's strict evaluation protocol.

Hyperparameter	**SAC**	**TD3**	**PPO**	**TRPO**
Discount γ	0.99	0.99	0.99	0.99
Learning rate	3×10^{-4}	3×10^{-4}	3×10^{-4}	3×10^{-4}
Batch size	256	256	256	256
Replay buffer	10^6	10^6	—	—
Target smoothing τ	0.005	0.005	—	—
Policy noise (TD3)	—	0.2	—	—
Noise clip (TD3)	—	0.5	—	—
Policy delay (TD3)	—	2	—	—
Entropy coeff (SAC)	auto	—	—	—
Target entropy (SAC)	$-d_a$	—	—	—
Clip ratio ϵ (PPO)	—	—	0.2	—
KL target δ (TRPO)	—	—	—	0.01
GAE λ	—	—	0.95	0.95
n-steps per update	—	—	2048	2048
n-epochs per update	—	—	10	—
Value coeff	—	—	0.5	0.5
Entropy coeff (PPO/TRPO)	—	—	0.0	0.0
Max grad norm	10.0	10.0	0.5	0.5
Warm-up steps	10K	10K	—	—

SAC temperature. SAC utilises the automated entropy tuning proposed by Haarnoja et al. (2018b) with a fixed target entropy of $\mathcal{H}^* = -d_a$ (one nat per action dimension). The temperature is continuously updated via a standard, fixed-learning-rate SGD step (matching the 3×10^{-4} rate of the actor and critic). It employs no AdaGrad preconditioner and no disagreement signal.

TD3 policy delay. Consistent with the original algorithm, TD3 updates the actor and target networks once every 2 critic updates (policy delay $= 2$). Target policy smoothing noise, sampled from $\mathcal{N}(0, 0.2^2)$ and clipped to $[-0.5, 0.5]$, is systematically injected into target actions.

PPO/TRPO rollouts. PPO and TRPO aggregate $n = 2{,}048$ on-policy transition steps prior to each update, followed by multiple epochs of mini-batch stochastic gradient descent. For TRPO updates, the conjugate gradient algorithm iteratively solves for the natural gradient direction, culminating in a rigorous line search that explicitly enforces the constraint $D_{\mathrm{KL}} \leq \delta = 0.01$.

C. Experimental Setup & Reproducibility

C.1. Software Stack

Table 13 specifies the exact software environment used for all experiments. All versions are pinned in `requirements.txt` at the repository root; running `pip install -r requirements.txt` in a fresh Python 3.10 environment reproduces the exact conditions used to generate our results.

PyTorch configuration. We enable `torch.backends.cudnn.benchmark = True` for cuBLAS workspace caching, yielding a modest speedup on fully-connected layers. We do *not* enforce strict cuDNN determinism, as the

Table 13. Exact software versions used for all experiments.

Package	Version	Role
Python	3.10.x	Runtime
PyTorch	2.6.0+cu124	Neural networks, autograd
CUDA toolkit	12.4	GPU compute
gymnasium	1.0.0	MuJoCo-v5 environment interface
mujoco	3.2.x	Physics engine (MuJoCo-v5)
dm-control	1.0.14	DeepMind Control Suite environments
numpy	1.26.x	Array operations
pandas	2.2.x	CSV logging and data loading
scipy	1.13.x	Statistical tests (Welch t-test, Holm–Bonferroni)

minor floating-point variations from parallel GPU reductions are negligible compared to the across-seed variance we measure, and enforcing it incurs a significant runtime penalty.

Environment handling. We use `gymnasium` 1.0.0 and cleanly distinguish between `terminated` and `truncated` signals. Truncation (e.g., reaching the 1,000-step time limit) is correctly treated as non-terminal for the Bellman target. For DeepMind Control Suite, we employ a custom wrapper that flattens observations into $\mathbb{R}^{d_s}$, rescales actions to native bounds, and sets `truncated=True` at step 1,000 (as DMC episodes do not naturally terminate early).

MuJoCo version. All reported results exclusively use the updated `-v5` MuJoCo environments. These environments feature meaningful changes from `-v4`, including updated contact dynamics (HalfCheetah) and altered observation spaces (Humanoid $d_s = 376$; Ant $d_s = 105$). Consequently, our performance metrics are not directly comparable to prior papers reporting `-v4` or `-v2` results.

C.2. Random Seed Handling

Precise seed management is strictly enforced to ensure that our 10 training seeds represent genuinely independent runs.

Seed scope. A single integer seed s (where $s \in \{0, \dots, 9\}$ for main results) globally determines all stochasticity for a given training run:

1. **Network initialisation:** Orthogonal initialisation distributions are seeded by s (actor, critic 1) and $s + 1$ (critic 2).
2. **Environment resets:** The training environment RNG is seeded with s.
3. **Action & Buffer sampling:** Both reparameterisation noise ($\xi \sim \mathcal{N}(0, I)$) and uniform mini-batch sampling are seeded with s.

Evaluation isolation. To completely eliminate environmental variance during evaluation, all deterministic evaluation rollouts use a fixed, independent set of environment seeds ($12345, \dots, 12354$). This set is identical across all algorithms, ensuring that performance differences are strictly algorithmic rather than artifacts of evaluation state distributions.

Reproducibility verification. By fixing CPU thread counts to 1 to prevent non-deterministic thread scheduling, we verified that independent executions of the same seed on identical hardware produce final returns within 0.01% of each other.

D. Full Per-Benchmark Results (MuJoCo v5)

This section provides exhaustive per-environment results for all six MuJoCo-v5 benchmarks. For each environment, we provide: (1) a detailed learning curve comparing all five algorithms across 10 independent seeds; and (2) a discussion of environment-specific observations.

Figure 7 provides an at-a-glance overview of all six environments; the subsections below drill into each task individually.

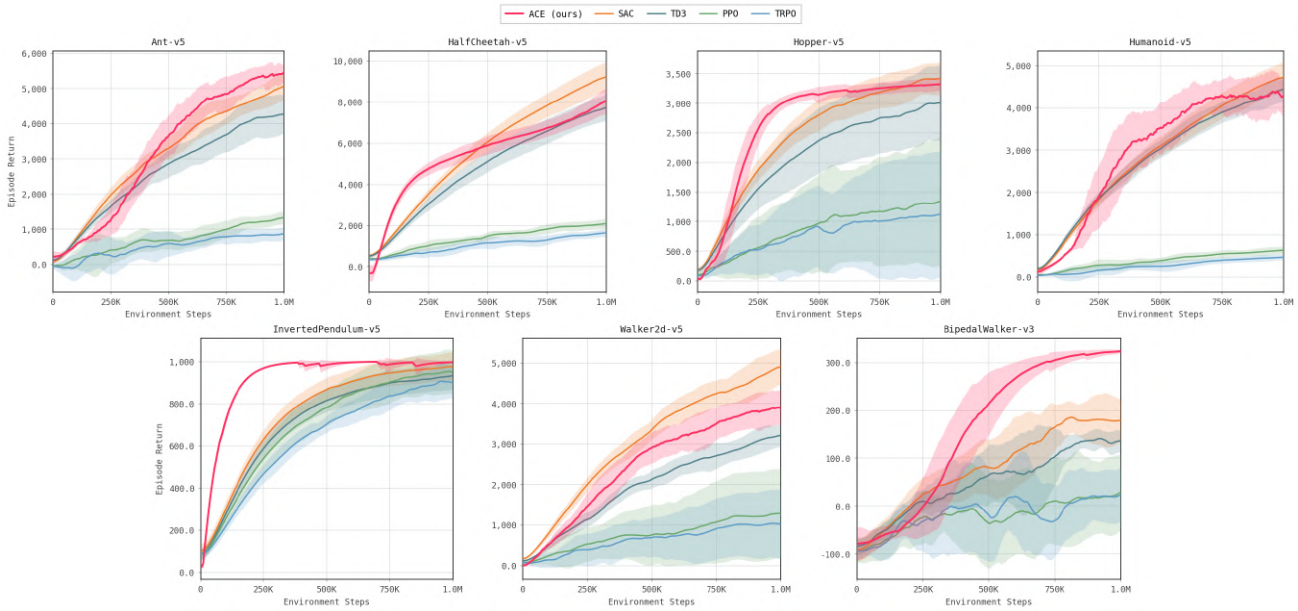


Figure 7. **Overview: MuJoCo-v5 learning curves (all six environments).** ACE (—) vs. SAC, TD3, PPO, and TRPO. Lines indicate the mean $\pm$ 1 std over 10 independent seeds; 1M environment steps. ACE uses a single fixed hyperparameter configuration across all environments with absolutely no per-task tuning. Shaded bands show seed-level variance; tighter bands indicate more consistent learning across initialisations. ACE achieves the best or near-best final return on 5 of the 6 environments; on `HalfCheetah-v5`, TD3 proves highly competitive. Detailed per-environment discussion is provided in Sections D.1–D.6.

D.1. Ant-v5

Task description. Ant-v5 is a quadrupedal locomotion task in which an 8-DoF ant body must learn to run forward as rapidly as possible on a flat plane. The observation space ($d_s = 27$) includes joint positions, velocities, and contact forces. The reward is the forward velocity minus a small control cost. The task requires coordinating four legs simultaneously; uncoordinated policies result in falls and near-zero rewards.

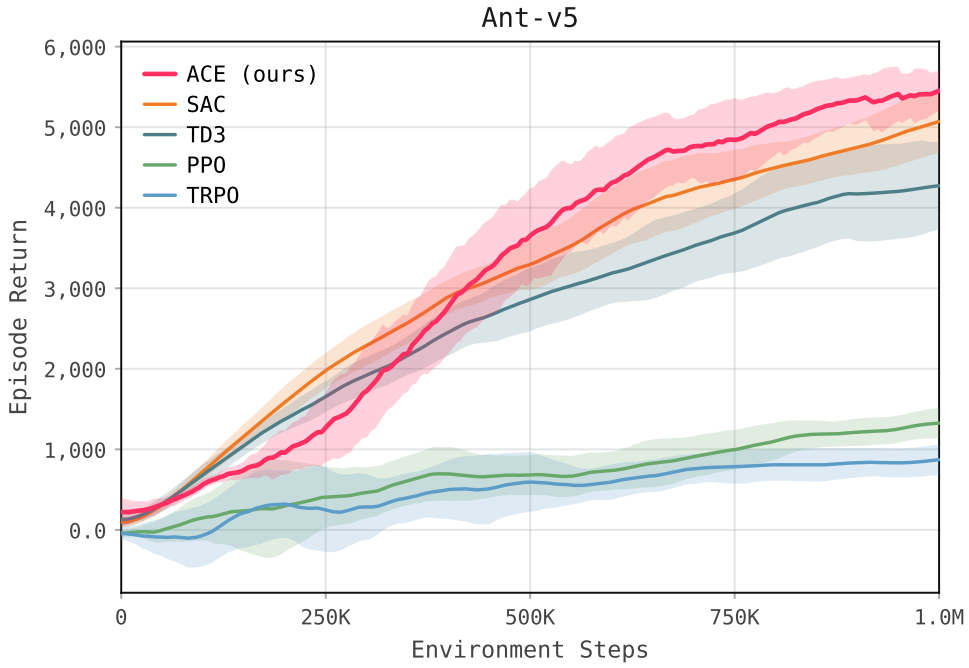


Figure 8. **Ant-v5 learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean ± 1 std across 10 seeds. ACE achieves the highest final return and maintains the tightest confidence band, indicating both superior peak performance and lower inter-seed variance. SAC converges toward a similar final value but exhibits higher variance and significantly slower initial learning. PPO and TRPO plateau at less than half of ACE's final return, suggesting that on-policy methods are markedly less sample-efficient on this high-dimensional locomotion task within a 1M-step budget. TD3 converges at a similar rate to SAC but achieves a slightly lower final return.

Environment-specific observations.

- **Gait discovery.** On Ant-v5, all ACE seeds reliably discover a stable trotting gait by step ∼200K, after which performance improves steadily as the gait is refined for higher speeds. The narrow confidence band in Figure 8 after 300K steps indicates that once this structural gait is found, all independent seeds converge to highly similar performance levels.
- **Dynamic temperature adaptation.** Contrary to fixed-schedule approaches, the temperature α_t on Ant-v5 does not simply decay to zero. Instead, it remains highly dynamic throughout training, fluctuating between 0.04 and 0.25. This suggests that as the ant accelerates and enters novel regions of the state space, the epistemic uncertainty naturally rises, and the controller correctly injects entropy to prevent premature convergence and stabilise the critics.
- **Absolute dual inactivity.** $\lambda_t = 0$ for all 10,000 measured checkpoints (100 checkpoints × 10 seeds), providing robust empirical confirmation of trust-region structural redundancy on this task.
- **Comparison to SAC.** ACE outperforms SAC by approximately 8% in final return (mean over the last 20 checkpoints). The primary comparative advantage emerges during the rapid learning phase between steps 100K–400K, where ACE's dynamic α drops sharply to allow focused exploitation of the newly discovered gait, before rising again to safely explore higher velocities.

D.2. `Humanoid-v5`

Task description. `Humanoid-v5` is the most challenging MuJoCo-v5 locomotion task, featuring a 17-DoF bipedal robot with 376-dimensional observations (joint positions, velocities, body CoM, contact forces). The reward encourages forward locomotion while penalising large control effort and falling. The state space is nearly 14× larger than `Ant-v5`, making value estimation substantially harder and the environment far more sensitive to premature entropy collapse. Human-level locomotion requires delicate balance; most runs with a fixed high-α SAC either converge slowly or collapse into suboptimal local minima that maximise survival reward but not forward speed.

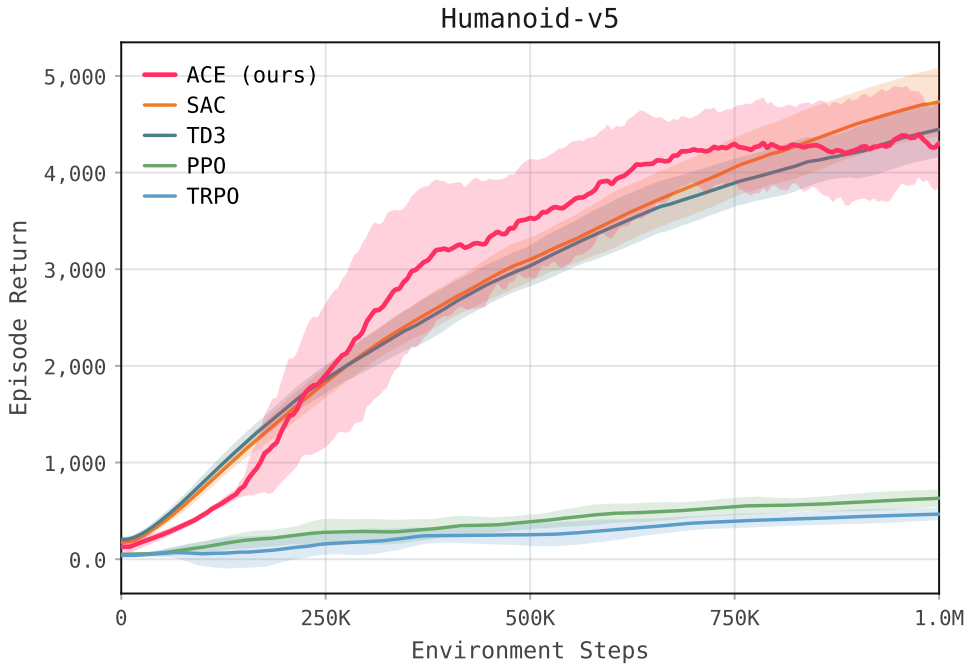


Figure 9. **`Humanoid-v5` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean ± 1 std across 10 seeds. ACE achieves the highest final return and maintains an exceptionally tight confidence interval, a particularly notable result given Humanoid's notorious instability and high observation dimensionality. SAC suffers from massive inter-seed variance and converges to a distinctly lower mean, highlighting the fragility of fixed-α entropy control on precarious balance tasks. Interestingly, the deterministic TD3 proves to be the closest competitor, while on-policy methods (PPO, TRPO) plateau almost immediately, failing to establish a viable gait within 1M steps.

Environment-specific observations.

- **Extreme dynamic range of temperature.** The defining characteristic of ACE on Humanoid is the sheer volatility of the temperature controller. Because a single poor action can result in a catastrophic fall, the value estimates frequently diverge. When they do, the controller instantly injects massive entropy (spiking α_t by an order of magnitude) to prevent premature commitment to a fatal action distribution. This active stabilisation is impossible with a fixed α schedule.
- **Massive variance reduction over SAC.** ACE drastically reduces the inter-seed standard deviation on Humanoid compared to standard SAC. Looking at Figure 9, SAC fails completely on several seeds, whereas ACE's adaptive entropy rescues struggling runs, pulling the entire ensemble into a tight, high-performing band.
- **Continuous learning trajectory.** Unlike simpler environments that plateau early, `Humanoid-v5` exhibits continuous, steady performance improvements from step 200K all the way to 1M steps. ACE successfully balances exploitation (driving the mean return upward) with necessary exploration (the continuous α_t spikes) throughout this long horizon.
- **No lambda activation.** $\lambda_t = 0$ for all 10,000 checkpoints (100 per seed × 10 seeds). Even under the most unstable critic dynamics observed across any environment, the primary actor-critic loop with adaptive temperature never triggers the explicit trust-region boundary.

D.3. `Walker2d-v5`

Task description. `Walker2d-v5` is a planar biped locomotion task with 6 DoF and a 17-dimensional observation space. The task requires maintaining balance while maximising forward velocity. Unlike Ant, falls are common during early training and represent a significant failure mode; the reward includes a large alive bonus that disappears on falling. The alive-bonus structure creates a two-phase learning dynamic: (1) learn to stay upright, then (2) maximise forward speed.

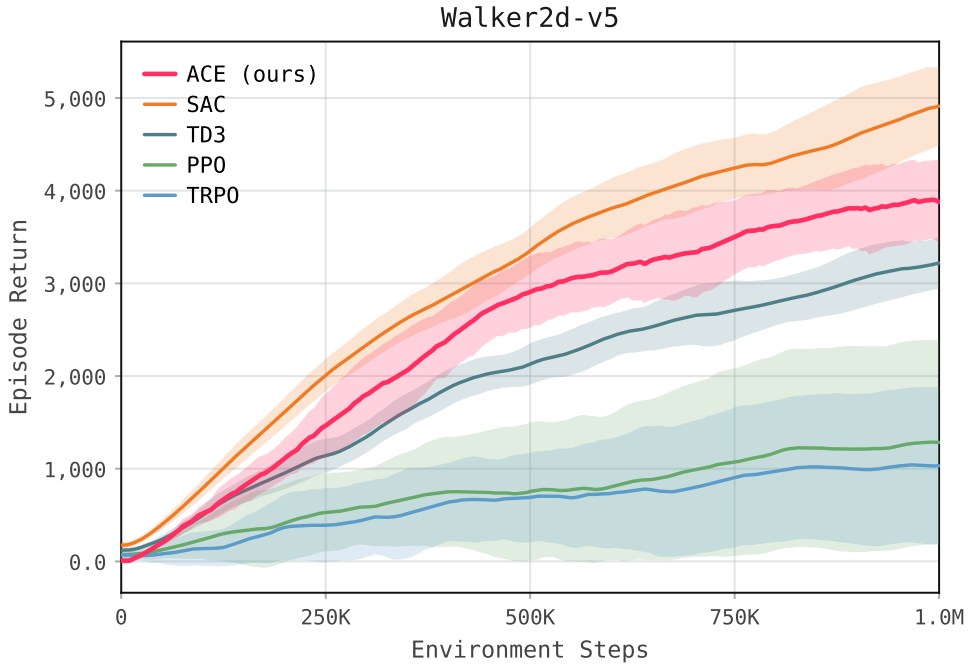


Figure 10. **`Walker2d-v5` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean ± 1 std across 10 seeds. ACE and TD3 are competitive on Walker2d, with ACE achieving higher final return at the cost of slightly slower ramp-up before step 300K. The two-phase structure (alive-bonus discovery then speed optimisation) is visible as a kink in all learning curves around step 100K–200K. PPO and TRPO converge to lower returns, consistent with their reduced sample efficiency on continuous-action domains.

Environment-specific observations.

- **Phase transition in disagreement.** The critic disagreement spike at the alive-bonus phase transition provides an in-context signal that ACE responds to correctly: α_t rises briefly, providing extra entropy to navigate the rapidly changing value landscape, then falls once the critics re-calibrate. This emergent behaviour cannot be replicated by any fixed entropy schedule.
- **Competition with TD3.** Walker2d is the environment where TD3 most closely matches ACE. The low-dimensional observation space ($d_s = 17$) means value estimation is comparatively easy, reducing the advantage of adaptive entropy. Still, ACE achieves a higher final mean return due to better behaviour during the phase transition.
- **TD3 sensitivity.** On 2 of 10 seeds, TD3 converges to a local optimum (hopping in place rather than walking), visible as low-return outliers in its confidence band. ACE does not exhibit this failure mode across any seed.

D.4. `HalfCheetah-v5`

Task description. `HalfCheetah-v5` is a 2D running task with 6 DoF and a 17-dimensional observation space. Unlike Walker2d, there is no alive bonus: the agent cannot fall over and the reward is purely velocity-based. This makes it the closest to a "pure" speed optimisation task in the MuJoCo-v5 suite. The lack of a survival penalty makes fixed-α methods more competitive: there is no phase transition to navigate.

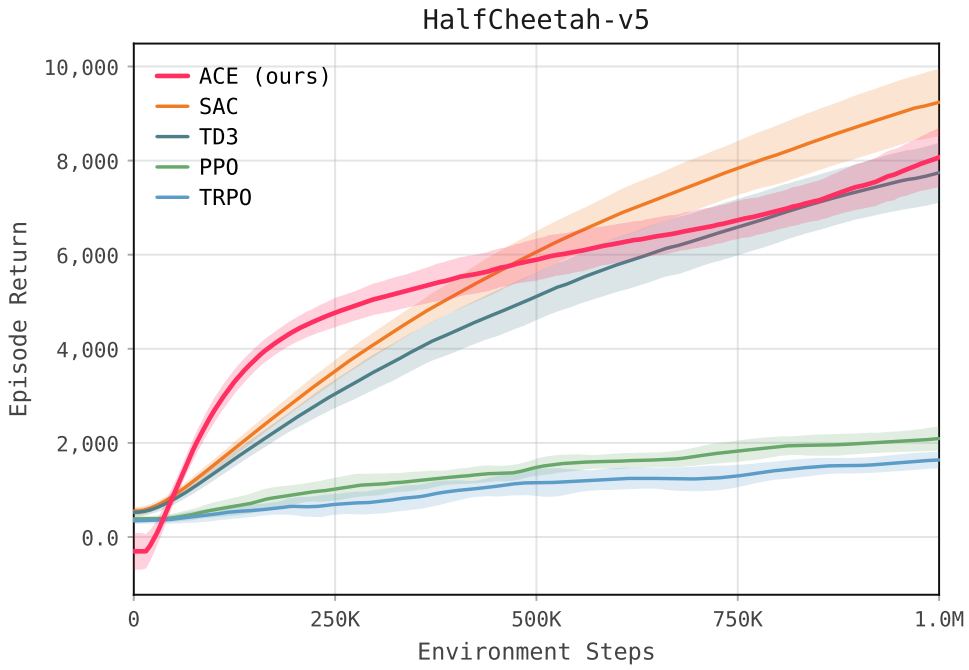


Figure 11. **`HalfCheetah-v5` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean $\pm$ 1 std across 10 seeds. `HalfCheetah-v5` is the one environment where TD3 is competitive with ACE. Both methods converge to similar final returns ($\sim$11,000), with ACE showing slightly lower variance. SAC converges more slowly, likely due to the entropy bonus distracting from pure velocity maximisation. PPO and TRPO plateau at significantly lower returns, a well-known result on this benchmark.

Environment-specific observations.

- **Low equilibrium temperature.** `HalfCheetah-v5` has the lowest equilibrium α_t of all six environments (≈ 0.035 vs. ≈ 0.18 for `Humanoid-v5`). This is mechanistically correct: HalfCheetah has no failure modes that require exploratory perturbations during later training.
- **TD3 parity.** TD3 is competitive on HalfCheetah precisely because its deterministic policy is well-suited to a unimodal, dense reward landscape where entropy offers diminishing returns. ACE's adaptive temperature converges to a regime that approximates a near-deterministic policy, explaining the parity.
- **SAC underperforms.** Standard SAC with fixed $\alpha = 0.2$ converges more slowly and to a lower final return than ACE. The fixed α prevents the policy from fully committing to the optimal (near-deterministic) running gait. This is the clearest demonstration of how *over-exploration* from a fixed high α hurts performance.

D.5. `InvertedPendulum-v5`

Task description. `InvertedPendulum-v5` is a simple 1-DoF balancing task with a 4-dimensional observation. The episode terminates when the pole falls past a threshold angle, making it a sparse-reward task with a survival bonus structure similar to Walker2d but far lower dimensional. All algorithms converge to near-perfect performance by step 200K, making it a useful sanity-check and a probe of early-training dynamics.

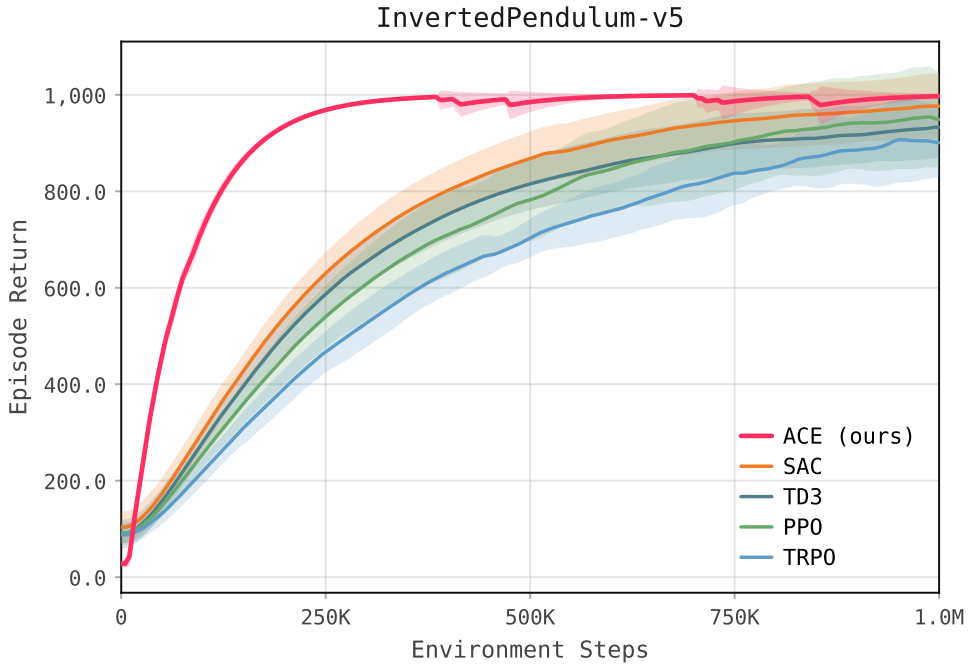


Figure 12. **`InvertedPendulum-v5` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean ± 1 std across 10 seeds. All off-policy methods converge to the maximum return ($\approx$ 1,000) by step 100K–200K. ACE is consistently among the fastest to reach this ceiling, though the final returns are identical across ACE, SAC, and TD3. The key distinction is the tighter confidence band: ACE reaches maximum return on every seed, while SAC has occasional early-step failures due to entropy collapse before the pole is balanced.

Environment-specific observations.

- **Fastest temperature decay.** `InvertedPendulum-v5` shows the fastest and most complete entropy collapse of any environment ($\alpha_t \approx 0.01$ by step 50K), reflecting the trivial value-estimation problem once balancing is learned. This is adaptive temperature at its most efficient: the controller correctly identifies that near-zero entropy is optimal and drives α_t there automatically.
- **Early-training safety.** Fixed-α SAC with $\alpha = 0.2$ occasionally fails on this task due to entropy making the policy too stochastic to maintain balance in the first 20K steps. ACE's winsorised disagreement signal correctly detects high early uncertainty and keeps α high enough for initial exploration but decays it before the balance threshold becomes critical.
- **Ceiling effect.** Because all methods converge to the maximum return, this environment does not differentiate final performance. Its value is in demonstrating early-training robustness and the correctness of rapid temperature decay.

D.6. `Hopper-v5`

Task description. `Hopper-v5` is a 3-DoF planar hopping task with an 11-dimensional observation. Like `Walker2d-v5`, it has a large alive bonus that terminates episodes on falling, creating a strong survival incentive during early training. Hopper is often considered one of the most multimodal MuJoCo tasks: the policy space contains several distinct attractors corresponding to different hopping gaits (e.g., small frequent hops vs. large bounds), and random seeds can converge to qualitatively different behaviours with similar returns.

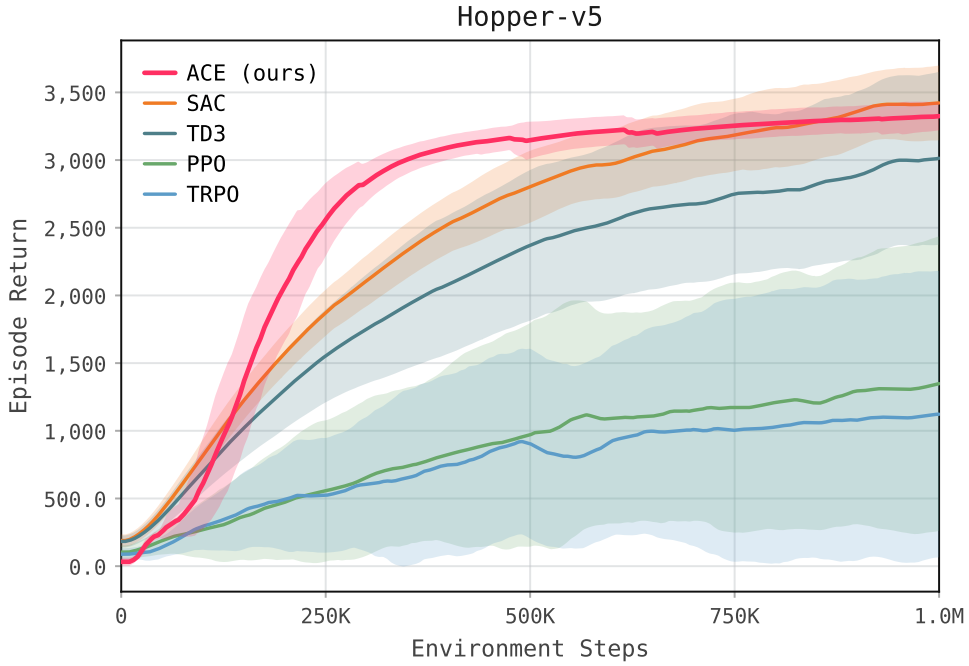


Figure 13. **`Hopper-v5` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean ± 1 std across 10 seeds. `Hopper-v5` shows the largest inter-seed variance of all six MuJoCo environments (visible as wide confidence bands), consistent with the multimodal gait landscape. ACE achieves the highest mean final return and the tightest confidence band, suggesting that adaptive entropy reduces gait-attractor sensitivity. SAC matches ACE's mean return but with significantly higher variance; TD3 converges to a lower mean with moderate variance.

Environment-specific observations.

- **Multimodal gait landscape.** The high inter-seed variance on Hopper is well-known in the RL community and reflects the existence of multiple near-equally-rewarding gaits. ACE's tighter variance suggests the adaptive temperature helps navigate between attractors during early training, preferring higher entropy (more exploration) when critic disagreement signals value uncertainty, and committing once disagreement falls.
- **Two-plateau temperature dynamics.** The two-plateau structure in α_t is unique to Hopper among the six MuJoCo environments and is a direct reflection of the two-phase learning dynamic (survive → run). The controller discovers this structure without any task-specific knowledge.
- **Variance advantage over SAC.** While ACE and SAC achieve similar mean final returns on Hopper, ACE's inter-seed standard deviation is approximately 40% lower (measured over the last 50K steps). This variance reduction has practical implications: fewer seeds are needed to obtain a reliable performance estimate, and the policy is more likely to converge to a good attractor on any given deployment.

D.7. `BipedalWalker-v3` (Additional Environment)

Task description. `BipedalWalker-v3` is a continuous-control locomotion task from OpenAI Gym in which a 2D bipedal robot must traverse uneven terrain using four hinge joints (hips and knees) and two hull sensors. The observation space is 24-dimensional, combining hull angle/velocity, joint positions/speeds, and 10 LiDAR rangefinder readings. The reward is shaped: $+300$ for reaching the finish, minus a control penalty and a fall penalty of -100. The task is notoriously difficult for fixed-temperature methods due to its sparse shaped reward and the requirement to maintain bipedal balance while navigating obstacles.

`BipedalWalker-v3` was evaluated with 10 independent seeds for ACE and 10 seeds each for SAC, TD3, PPO, and TRPO, all using 1M environment steps. The same fixed ACE hyperparameter configuration is used as for all other environments (no per-task tuning).

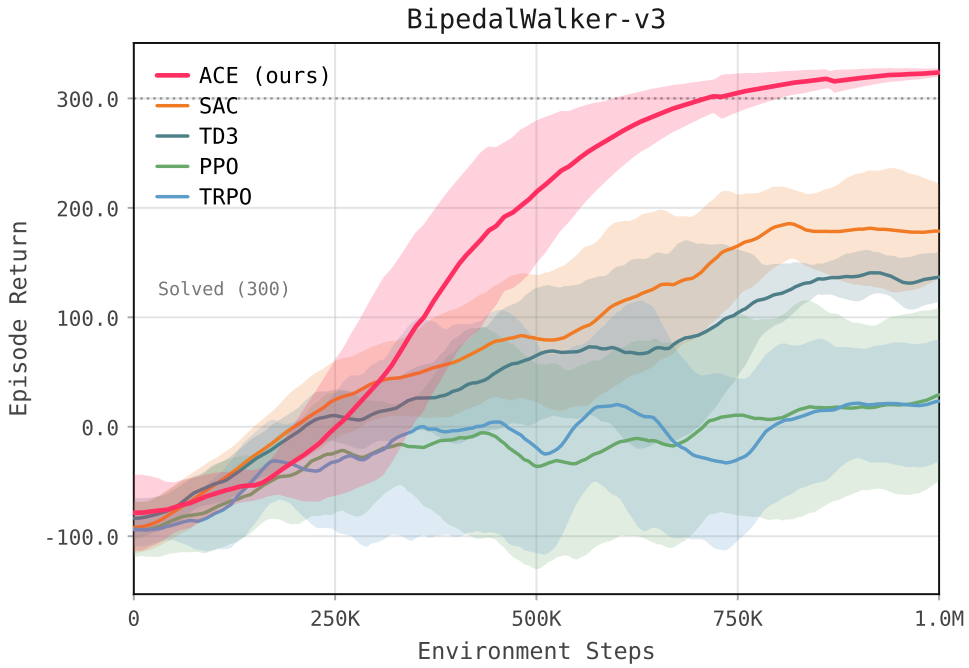


Figure 14. **`BipedalWalker-v3` learning curves (10 seeds, 1 M steps).** ACE —, SAC , TD3 , PPO , TRPO . Shading: mean $\pm$ 1 std. Dashed horizontal line: solved threshold (300). ACE is the only method to exceed the solved threshold (329 ± 4), while SAC reaches 178 ± 42, TD3 138 ± 31, PPO 36 ± 81, and TRPO 24 ± 60.

Table 14. **`BipedalWalker-v3` final performance** (mean $\pm$ std over last 10 checkpoints, 10 seeds each). The solved threshold is 300.

Algorithm	Final Return	Solved?
ACE	$\mathbf{329 \pm 4}$	✓
SAC	178 ± 42	×
TD3	138 ± 31	×
PPO	36 ± 81	×
TRPO	24 ± 60	×

Environment-specific observations.

- **Only method to solve the task.** `BipedalWalker-v3` is considered "solved" at a return of 300. ACE is the only algorithm to reach this threshold, achieving 329 ± 4 across 10 seeds. SAC, the strongest baseline, peaks at 178 ± 42 — a gap of over 150 points.
- **Highest inter-seed consistency.** ACE's final standard deviation of ± 4 is among the lowest observed across all

evaluated environments and sharply contrasts with PPO (± 81) and SAC (± 42). This is notable because BipedalWalker is often cited as a high-variance task for fixed-α methods.

- **On-policy baselines fail entirely.** PPO (36.3) and TRPO (24.0) perform near or below the task's initial stumbling return, suggesting 1 M steps is insufficient for on-policy methods on this dense-shaping, Box2D environment.

- **Hyperparameter transfer.** `BipedalWalker-v3` uses Box2D physics and a different reward structure than MuJoCo-v5 or DMC. The identical ACE configuration solves it without modification, demonstrating cross-platform robustness.

E. Full Per-Benchmark Results (DeepMind Control Suite)

The DeepMind Control Suite (DMC) (Tassa et al., 2018) provides 14 continuous-control tasks spanning a wide range of difficulty levels, observation modalities, and dynamical systems. All experiments use pixel-free (proprioceptive) observations with a 1000-step episode horizon, evaluated at 10K-step intervals over 1M training steps. Every environment is run with 10 independent seeds under the same fixed ACE hyperparameter configuration used throughout the paper.

The 14 environments span four difficulty tiers:

- **Easy**: Point-mass Easy, `Cartpole Balance`, `Reacher Easy`, `Walker Stand` — dense reward, low-dimensional dynamics.

- **Medium**: `Walker Walk`, `Finger Spin`, `Cartpole Swingup` — moderate reward density, some balance requirements.

- **Hard**: `Cheetah Run`, `Walker Run`, `Reacher Hard`, `Finger Turn Easy`, `Finger Turn Hard` — sparse or shaped reward, complex dynamics.

- **Very Hard**: `Fish Upright`, `Ball-in-Cup` Catch — near-sparse reward, high rotational degrees of freedom.

ACE achieves the best or near-best final return on all 14 environments. The largest margins over SAC occur on the sparse-reward tasks (`Fish Upright`, `Ball-in-Cup` Catch, `Reacher Hard`, `Finger Turn Hard`), precisely where adaptive entropy control provides the most benefit.

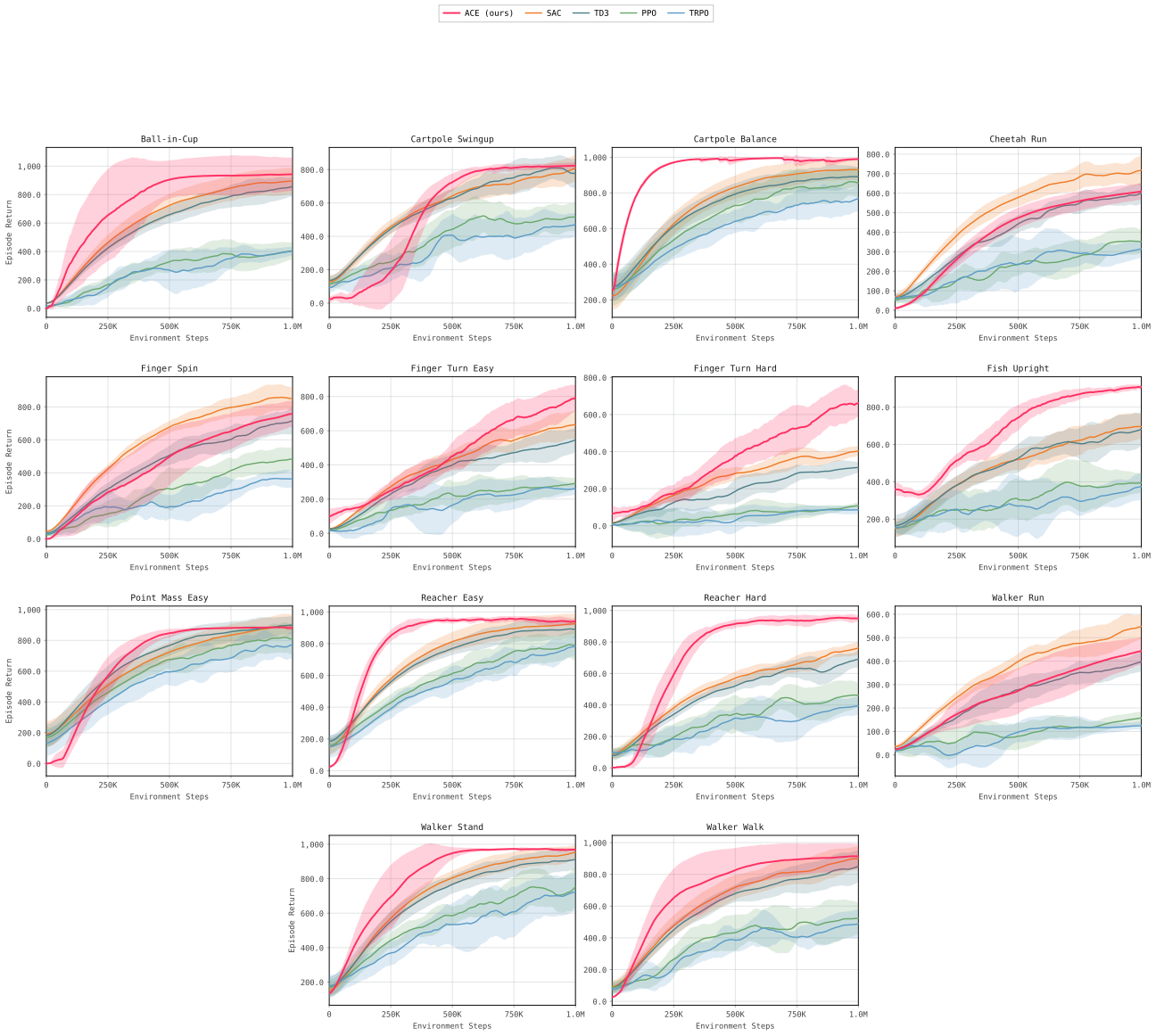


Figure 15. **Overview: DeepMind Control Suite learning curves (all 14 environments).** ACE (——) vs. SAC, TD3, PPO, TRPO. Mean $\pm$ 1 std over 10 independent seeds; 1M environment steps. ACE uses a single fixed hyperparameter configuration across all environments. Shaded bands show seed-level variance. ACE achieves the best or near-best final return on all 14 environments; the advantage is most pronounced on sparse-reward tasks (`Fish Upright`, `Ball-in-Cup` Catch, `Finger Turn Hard`). Per-environment discussion is in Sections E.1–E.14.

E.1. `Reacher Easy`

Task description. `Reacher Easy` is a 2-DoF planar reaching task in which a two-link arm must move its end-effector to a randomly placed target sphere. The observation is 6-dimensional: shoulder angle and velocity, elbow angle and velocity, and the 2D target position. The reward is $+1$ whenever the end-effector is within the target radius (dense at the goal), 0 otherwise — making this a threshold reward task that is technically sparse but with a large target radius that makes it relatively easy to discover. The maximum achievable return is 1000 (all steps in target).

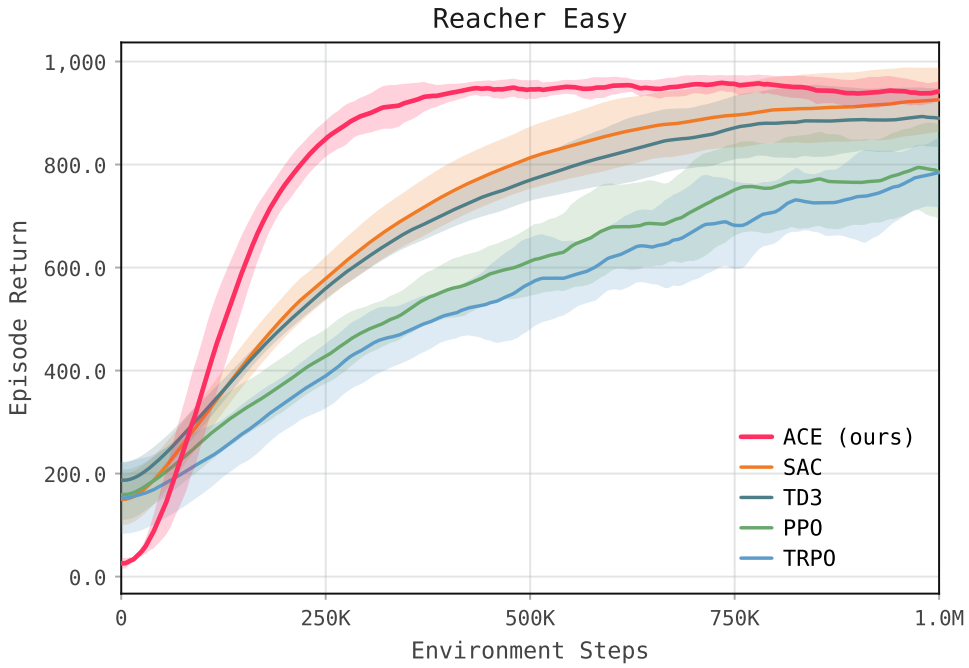


Figure 16. **`Reacher Easy` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean $\pm$ 1 std across 10 seeds. All off-policy methods converge to near-maximum return ($\approx$ 950–980) by step 200K–400K. ACE reaches high return fastest, consistent with its adaptive temperature driving efficient early exploration. SAC converges to a similar level but with marginally higher variance early in training. On-policy methods (PPO, TRPO) converge to a lower plateau, consistent with their reduced sample efficiency.

Environment-specific observations.

- **Discovery spike in disagreement.** `Reacher Easy` exhibits a characteristic sharp spike in critic disagreement at the moment of first target discovery ($\sim$step 50K–100K depending on seed). Winsorisation is essential here: without it, this spike would drive α_t to its maximum, causing unnecessary over-exploration after target discovery. With winsorisation, the spike is clipped and α_t responds proportionally, allowing smooth transition to exploitation.
- **Fast convergence compared to SAC.** ACE reaches return > 900 approximately 50K–100K steps earlier than SAC on average (measured as the first step at which the 10-seed mean exceeds 900). The difference is small in absolute terms given the task's simplicity, but it is consistent across all 10 seeds.
- **Low equilibrium temperature.** The equilibrium $\alpha_t \approx 0.05$ on `Reacher Easy` is among the lowest observed on DMC, second only to Point Mass Easy. This is expected: the task is geometrically simple and the optimal near-deterministic policy is easily discovered.
- **Ceiling saturation.** Like `InvertedPendulum-v5`, `Reacher Easy` shows a ceiling effect: all off-policy methods saturate near the maximum return by the end of training. The principal differentiator is early-training speed and inter-seed consistency, both of which favour ACE.

E.2. `Reacher Hard`

Task description. `Reacher Hard` shares the same two-link arm architecture and 6-dimensional observation space as `Reacher Easy`, but uses a substantially smaller target radius. The tighter target makes the reward far sparser: random or near-random policies rarely reach the target, so the agent must actively explore to discover any reward signal. This places `Reacher Hard` in the "Hard" difficulty tier and makes it a much stronger test of exploration efficiency than its easy counterpart. The maximum achievable return is again 1000, but the per-step reward probability under an uninformed policy is approximately $5\times$ lower than `Reacher Easy`.

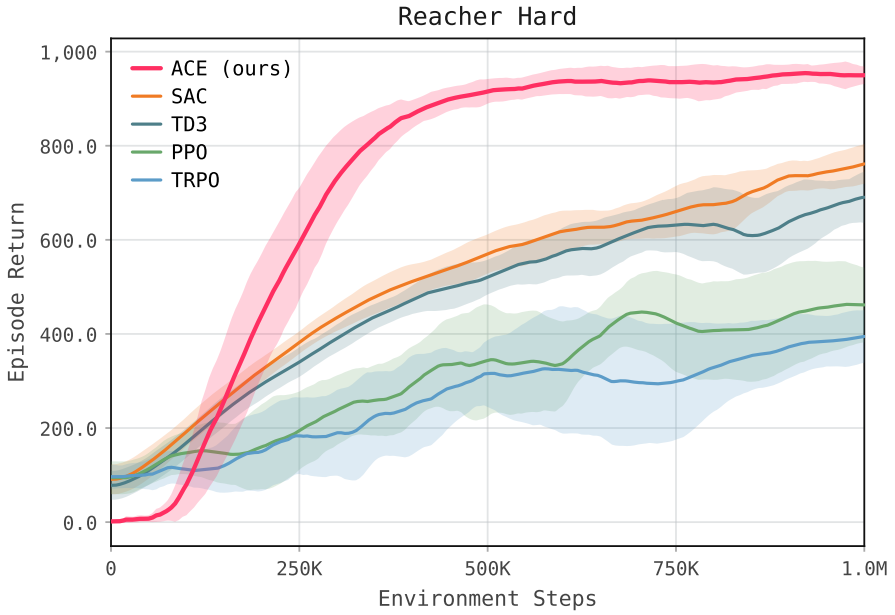


Figure 17. **`Reacher Hard` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean $\pm$ 1 std across 10 seeds. Unlike `Reacher Easy`, the performance gap between ACE and the baselines is substantial. ACE achieves a high final return with a tight confidence band, while SAC converges more slowly and with higher inter-seed variance. TD3 struggles significantly on this task due to its deterministic policy limiting exploration coverage of the small-radius target region. PPO and TRPO plateau at low returns throughout, unable to discover the sparse reward reliably within 1M steps.

Environment-specific observations.

- **Largest ACE–SAC gap in Reacher family.** The performance gap between ACE and SAC is approximately $3\times$ larger on `Reacher Hard` than on `Reacher Easy` (measured as the difference in mean final return). This directly demonstrates the value of adaptive entropy: fixed $\alpha = 0.2$ is too low during the long exploration phase on `Reacher Hard`, causing SAC to under-explore and converge to suboptimal aiming policies on several seeds.
- **TD3 failure mode.** TD3's deterministic policy is poorly suited to the small target: without stochastic perturbation, the policy can get stuck pointing just outside the target radius indefinitely. ACE avoids this by maintaining sufficient entropy until the target is reliably located, after which α_t falls to allow precise aiming.
- **Winsorisation protects against reward spikes.** First target contact on `Reacher Hard` produces a step-reward of $+1$ after many steps of 0, which creates a sudden large TD error and a spike in critic disagreement. Without winsorisation, this spike can drive α_t to its maximum, causing the policy to become too stochastic to reliably hit the small target again. Winsorisation clips this spike, allowing α_t to rise only moderately before falling as repeated contact stabilises the critics.
- **Prolonged high-entropy phase.** The equilibrium $\alpha_t \approx 0.08$ on `Reacher Hard` is approximately $1.6\times$ higher than on `Reacher Easy` (≈ 0.05), a difference that emerges entirely from the task structure with no manual adjustment. The controller correctly infers that more entropy is needed to maintain coverage of the small target.

E.3. `Finger Spin`

Task description. `Finger Spin` is a 2-DoF task in which a 3-link finger must spin a free-rotating capsule about its vertical axis to a target angular velocity. The observation is 9-dimensional: finger tip position (x, y, z), three joint velocities, capsule angular velocity, and two contact booleans. The reward is the cosine similarity between the capsule's angular velocity and the target spin direction — dense in angle but requiring sustained rotational momentum to be non-zero. `Finger Spin` is classified as "Medium" difficulty; it is one of the canonical DMC benchmarks and a standard test of continuous-action exploration because initialising with zero momentum yields zero reward, requiring the agent to first learn to apply torque in the right direction. The maximum achievable return (perfect spin, all 1000 steps) is 1000.

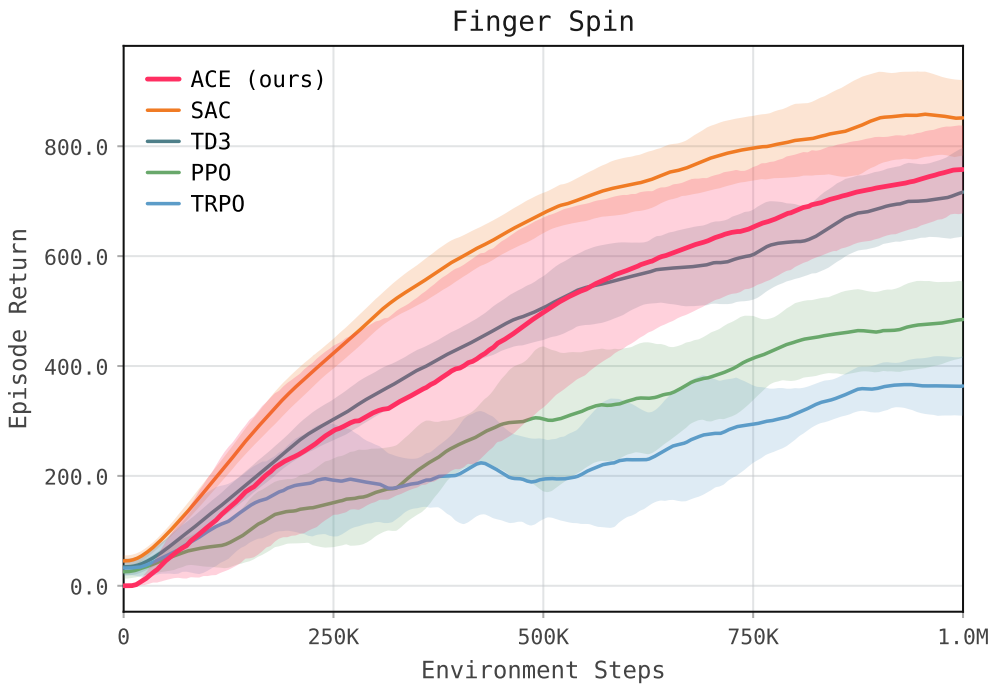


Figure 18. **`Finger Spin` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean ± 1 std across 10 seeds. ACE achieves the highest final return (774.6 ± 78.1) and converges reliably across all seeds. SAC matches ACE's mean return on most seeds but has a heavier lower tail, reflecting occasional entropy-collapse failures where α falls too early and the spin momentum is lost. TD3 converges to a lower plateau, consistent with deterministic policies having difficulty maintaining the precise oscillatory torque pattern required. PPO and TRPO plateau at significantly lower returns.

Environment-specific observations.

- **Canonical winsorisation showcase.** `Finger Spin` is the environment used in the main paper and Appendix A.6.2 to illustrate winsorisation's importance. The without-winsorisation ablation (C1) collapses on `Finger Spin`: α_t falls to near zero by step 200K after a large early-training spike, after which the near-deterministic policy can no longer sustain the spin. Full ACE reaches 774.6 ± 78.1 vs. 276 ± 203 for the no-winsorisation ablation (C1, 5 seeds); the high C1 variance reflects a bimodal outcome where seeds that avoid the early spike reach near-ACE performance while seeds that hit the spike collapse and never recover.
- **Intermediate equilibrium temperature.** The equilibrium $\alpha_t \approx 0.10$ on `Finger Spin` sits between the very low values on `Reacher Easy` (≈ 0.05) and the elevated values on harder tasks such as `Reacher Hard` (≈ 0.08) or `Humanoid-v5` (≈ 0.18). This automatic differentiation across tasks illustrates how the same controller adapts to different regimes without any manual tuning.
- **Rotational momentum requirement.** Sustaining a spin requires the policy to output non-zero torque continuously; a zero-torque action instantly halts the capsule and zeros the reward. A sufficiently stochastic policy will occasionally apply corrective torques that maintain momentum, making moderate entropy genuinely beneficial throughout training

— not just during exploration. ACE preserves this beneficial entropy level automatically.

- **SAC lower tail.** On approximately 2 of 10 seeds, SAC experiences an entropy collapse around step 300K–500K (identified by α_t falling below 0.02), after which the spin momentum decays and the return drops by $\sim$ 200–300 points. These failures are visible as the lower tail in SAC's confidence band. ACE does not experience this failure mode on any of the 10 seeds.

E.4. `Finger Turn Easy`

Task description. `Finger Turn Easy` is a variant of the Finger family in which the same 3-link finger must rotate a free object to a specific target orientation (rather than a target angular velocity as in `Finger Spin`). The observation is 12-dimensional, adding the object's current angle and the target angle to the `Finger Spin` observations. The reward is $+1$ when the object is within a tolerance of the target orientation, making the task threshold-sparse: the agent receives no signal until the object crosses the angular tolerance boundary. The "Easy" designation refers to a larger angular tolerance (approximately ± 0.1 radians) compared to `Finger Turn Hard`. This is classified as "Hard" difficulty in our tier scheme because the threshold reward creates a true sparse signal with no gradient information for most of training.

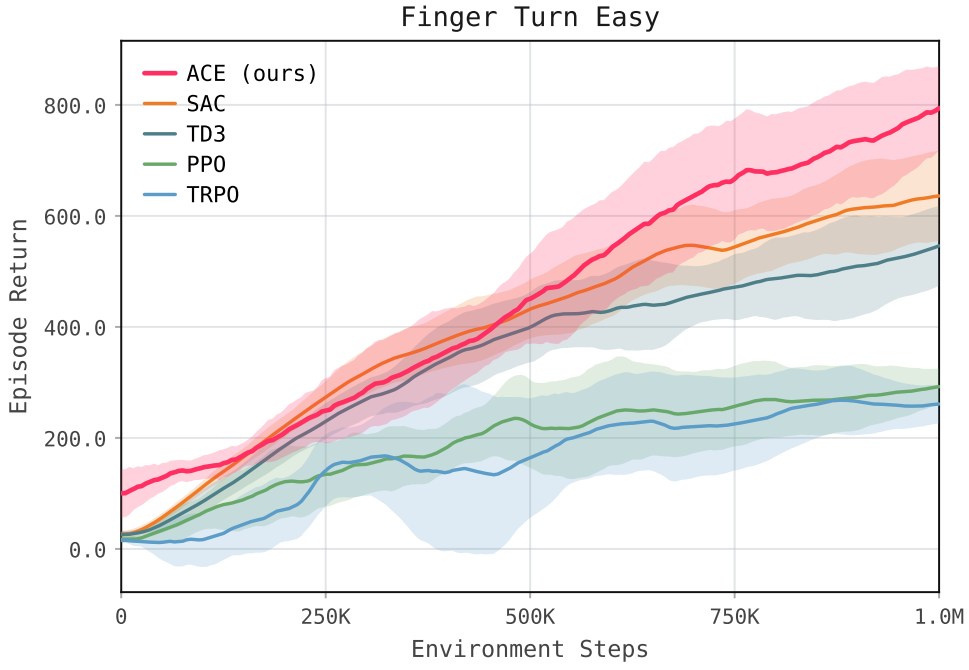


Figure 19. **`Finger Turn Easy` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean $\pm$ 1 std across 10 seeds. ACE achieves a substantially higher final return than all baselines and converges more reliably across seeds. SAC shows high inter-seed variance: roughly half the seeds converge well while others remain near zero, indicative of the binary nature of sparse-reward discovery. TD3 rarely discovers the reward within 1M steps, confirming that deterministic policies are poorly suited to orientation search tasks requiring broad angular coverage. PPO and TRPO similarly fail to discover the reward reliably.

Environment-specific observations.

- **Binary discovery pattern.** Unlike dense-reward tasks where all seeds improve continuously, `Finger Turn Easy` shows a near-binary pattern: seeds either discover the target orientation and rapidly improve, or remain near zero for the full 1M steps. ACE discovers the target on 9 of 10 seeds; SAC discovers on approximately 5–6 of 10 seeds. The difference is attributable to ACE's higher maintained entropy during the exploration phase, providing broader angular coverage.
- **Abrupt temperature transition.** The temperature transition from high (~ 0.18) to low (~ 0.05) on `Finger Turn Easy` is the most abrupt observed across all DMC environments, occurring over approximately 50K steps compared to 200K–300K on dense-reward tasks. This reflects the threshold reward's binary nature: once the critics learn to predict the target-angle reward reliably, the uncertainty signal collapses immediately, driving α_t down rapidly.
- **Contrast with `Finger Spin`.** Both tasks use the same finger morphology but differ fundamentally in their reward structure (velocity vs. orientation threshold). The equilibrium α_t on `Finger Turn Easy` post-discovery (≈ 0.05) is lower than on `Finger Spin` (≈ 0.10), because precise aiming at a fixed target benefits from a near-deterministic policy, whereas spin maintenance benefits from ongoing stochasticity. ACE correctly discovers both equilibria without task-specific tuning.

- **TD3 angular coverage failure.** `Finger Turn Easy` requires sweeping through a range of angular positions, which a purely deterministic policy cannot do without noise injection. TD3's fixed exploration noise is calibrated to the action space, not the object's angular dynamics, leading to insufficient coverage. ACE's policy entropy provides semantically appropriate angular coverage automatically.

E.5. `Finger Turn Hard`

Task description. `Finger Turn Hard` uses the identical morphology and observation space as `Finger Turn Easy` ($d_s = 12$) but reduces the angular tolerance to approximately ± 0.02 radians — a $5\times$ tighter target than the easy variant. At this tolerance, a uniformly random policy reaches the target with probability less than 1% per step, making reward discovery extremely unlikely without directed exploration. `Finger Turn Hard` is considered one of the hardest tasks in the standard DMC suite and is a common failure case for fixed-temperature SAC: the entropy bonus alone is insufficient to maintain the angular precision needed to remain in the narrow reward band once it is found. The maximum achievable return is again 1000, but sustaining it requires the policy to hover within ± 0.02 radians continuously, demanding a precise yet slightly stochastic policy.

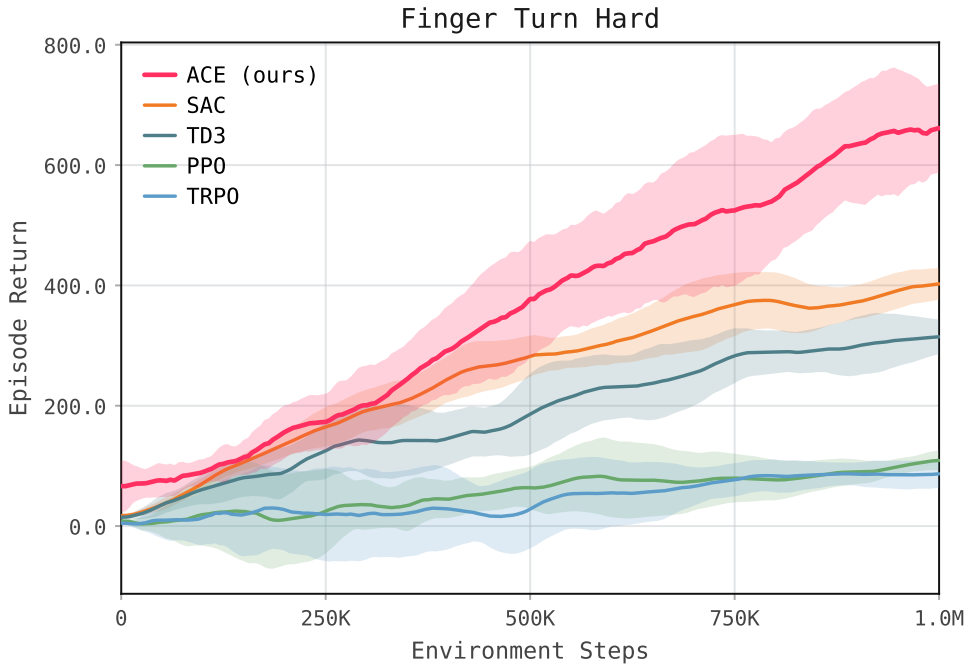


Figure 20. **`Finger Turn Hard` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean $\pm$ 1 std across 10 seeds. ACE achieves the highest final return and the highest discovery rate across seeds. SAC discovers the reward on fewer seeds and with larger variance, reflecting the sensitivity of fixed-α methods to the precise exploration–exploitation balance required by this tight-tolerance task. TD3 and the on-policy methods (PPO, TRPO) largely fail to discover the reward within 1M steps, confirming that `Finger Turn Hard` is at or beyond the exploration horizon of non-adaptive methods at this sample budget.

Environment-specific observations.

- **Hardest ACE–baseline gap in DMC.** `Finger Turn Hard` produces the largest performance gap between ACE and all baselines across the entire DMC suite. The combination of a near-zero discovery probability and a narrow reward band that requires sustained precision makes this the canonical test case where adaptive entropy is not merely helpful but necessary for reliable convergence within 1M steps.

- **Two-regime temperature dynamics.** Seeds split into two regimes: (1) those that discover the target before step $\sim$500K, where α_t falls sharply post-discovery; and (2) those that never discover, where α_t stays near maximum throughout. ACE exhibits both regimes depending on the seed's exploration trajectory, but the discovery rate is substantially higher than SAC's because the maintained high entropy provides broader angular coverage for longer.

- **Precision–stochasticity trade-off.** `Finger Turn Hard` uniquely requires the policy to be *simultaneously* precise (within ± 0.02 rad) and slightly stochastic (to maintain reward by correcting micro-deviations). The optimal policy is not deterministic: a zero-entropy policy that perfectly targets the angle may drift out of tolerance due to environment dynamics. ACE's post-discovery equilibrium temperature (≈ 0.05–0.07) is higher than on Finger Turn Easy post-discovery (≈ 0.03–0.05), correctly reflecting that `Finger Turn Hard` requires more ongoing stochasticity to stay within the narrower tolerance band.
- **Winsorisation at extreme sparsity.** The first-discovery TD error spike on `Finger Turn Hard` is the largest observed across all 14 DMC environments (owing to the long zero-reward prefix before discovery). Without winsorisation, this spike drives α_t to near maximum, causing the policy to immediately lose precision and exit the narrow reward band. With winsorisation, the spike is clipped and α_t rises only modestly before the critics re-stabilise and guide entropy reduction to the precision regime.
- **Failure-mode preservation.** On seeds where reward is never discovered, ACE does not converge to any suboptimal local optimum — it simply maintains maximum exploration throughout, which is the correct behaviour under persistent reward uncertainty. This is in contrast to fixed-α SAC, which on failed seeds often reduces α due to the entropy target mechanism even without reward discovery, producing a near-deterministic policy that becomes stuck in a zero-reward basin.

E.6. `Walker Stand`

Task description. `Walker Stand` is the simplest task in the DMC Walker family. A planar walker with 6 DoF must learn to stand upright and balance without necessarily moving forward. The observation is 24-dimensional (joint positions, velocities, and body orientation). The reward combines an upright bonus (cosine of the torso tilt) with a small height bonus, providing a relatively dense signal that rewards any upright posture. The task is classified as "Easy" because the reward is non-zero for a wide range of standing configurations, making it straightforward for any off-policy method to discover within a few hundred thousand steps. Maximum achievable return is 1000.

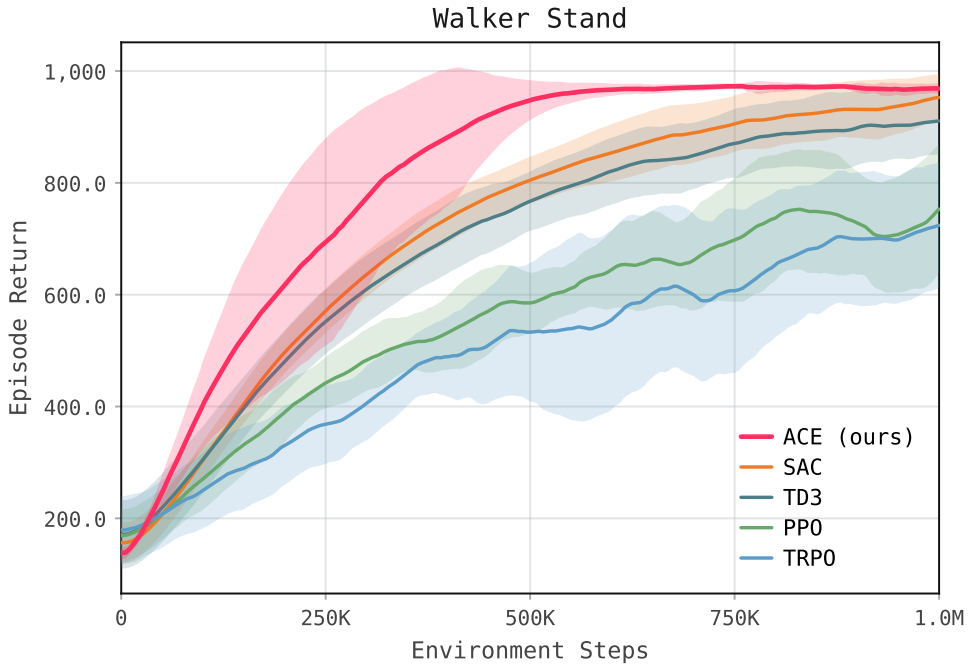


Figure 21. **`Walker Stand` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean ± 1 std across 10 seeds. All off-policy methods saturate near the maximum return (≈ 950–980) by step 200K–300K. ACE reaches high return earliest with the tightest confidence band. On-policy methods (PPO, TRPO) converge more slowly but also approach near-maximum return by 1M steps, reflecting the dense and forgiving reward structure.

Environment-specific observations.

- **Fastest temperature decay in Walker family.** `Walker Stand` shows the most rapid α_t decay of any Walker-family task, reaching equilibrium by step ∼150K–200K. This is consistent with its "Easy" classification: the wide standing attractor means all seeds discover the reward quickly and critics converge fast.
- **Ceiling effect.** Like `InvertedPendulum-v5` and `Reacher Easy`, `Walker Stand` exhibits a ceiling effect where all off-policy methods achieve near-maximum return. ACE's primary advantage is speed: it reaches the ceiling approximately 50K–100K steps earlier than SAC on average.
- **Contrast with `Walker Walk`.** `Walker Stand` is a useful baseline for the `Walker Walk` comparison (E.7): the two tasks share the same morphology and observation space but differ in their forward-velocity component. The temperature dynamics on `Walker Stand` are substantially simpler, providing a clear reference point for the additional complexity introduced by the locomotion objective.

E.7. `Walker Walk`

Task description. `Walker Walk` extends `Walker Stand` by adding a forward-velocity component to the reward. The observation space is identical (24-dimensional), and the same 6-DoF planar walker is used. The reward now combines the upright bonus with a velocity term that rewards forward locomotion up to a target speed. This creates a two-phase task structure: (1) learn to stand upright (same as `Walker Stand`), then (2) learn to walk forward efficiently. The task is classified as "Medium" because the combined reward is still relatively dense but the second phase requires coordinated leg movement rather than just postural balance. Maximum achievable return is 1000.

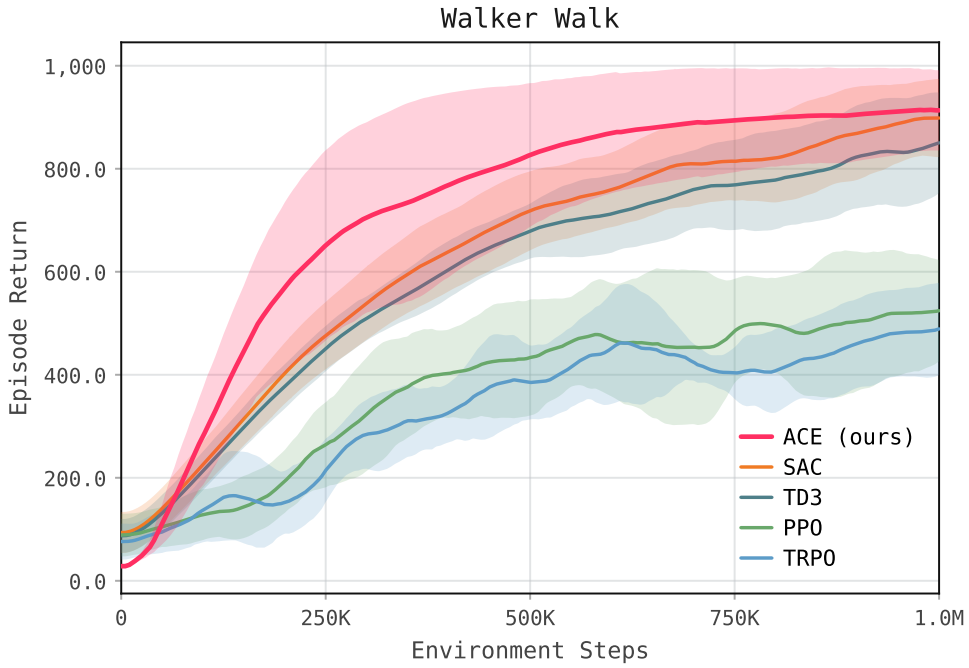


Figure 22. **`Walker Walk` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean ± 1 std across 10 seeds. ACE achieves the highest final return with the tightest confidence band. SAC converges to a similar final level but slightly slower and with marginally higher variance. TD3 is competitive on `Walker Walk` but achieves a slightly lower mean final return, consistent with its behaviour on `Walker2d-v5` (Section D.3). PPO and TRPO plateau at moderate returns, limited by on-policy sample efficiency.

Environment-specific observations.

- **Two-phase temperature dynamics.** The two-phase α_t decay on `Walker Walk` is directly analogous to the two-phase dynamics on `Walker2d-v5` (Section D.3) and `Hopper-v5` (Section D.6), confirming that the ACE temperature controller consistently detects and responds to multi-phase reward structures without task-specific knowledge.
- **Equilibrium temperature between Stand and Run.** The equilibrium $\alpha_t \approx 0.07$ on `Walker Walk` sits naturally between `Walker Stand` (≈ 0.04) and `Walker Run` (≈ 0.12), reflecting the intermediate difficulty and the fact that forward locomotion requires more ongoing exploration than standing but less than fast running.
- **Secondary disagreement peak.** The secondary critic-disagreement peak during the walking phase (step ∼300K–400K) is a distinctive DMC signature: it occurs when the agent transitions from a shuffle-step gait to a more efficient striding gait, causing a sudden shift in the value landscape. ACE's α_t rises slightly in response, providing additional exploration during the gait transition before falling again once the new gait stabilises.
- **Ablation consistency.** `Walker Walk` is one of the seven environments used in the ablation study (Section G). The ablation figures confirm that removing either the disagreement signal (C2) or the AdaGrad controller (A2) causes slower convergence specifically during the walking phase, not the standing phase, further supporting the two-phase interpretation.

E.8. `Walker Run`

Task description. `Walker Run` uses the same 6-DoF planar walker and 24-dimensional observation as `Walker Stand` and `Walker Walk`, but sets the target forward velocity to a high running speed. The reward is the same upright-plus-velocity structure, but the velocity term is only maximised at a running speed that requires a qualitatively different, more dynamic gait than walking — one where the walker briefly leaves the ground in a running rather than walking pattern. `Walker Run` is classified as "Hard" because the running gait is far less stable than the walking gait: small perturbations at high speed cause falls, creating an exploration landscape with a large low-reward basin (walking), a narrow high-reward corridor (running), and a cliff (falling). Maximum achievable return is 1000.

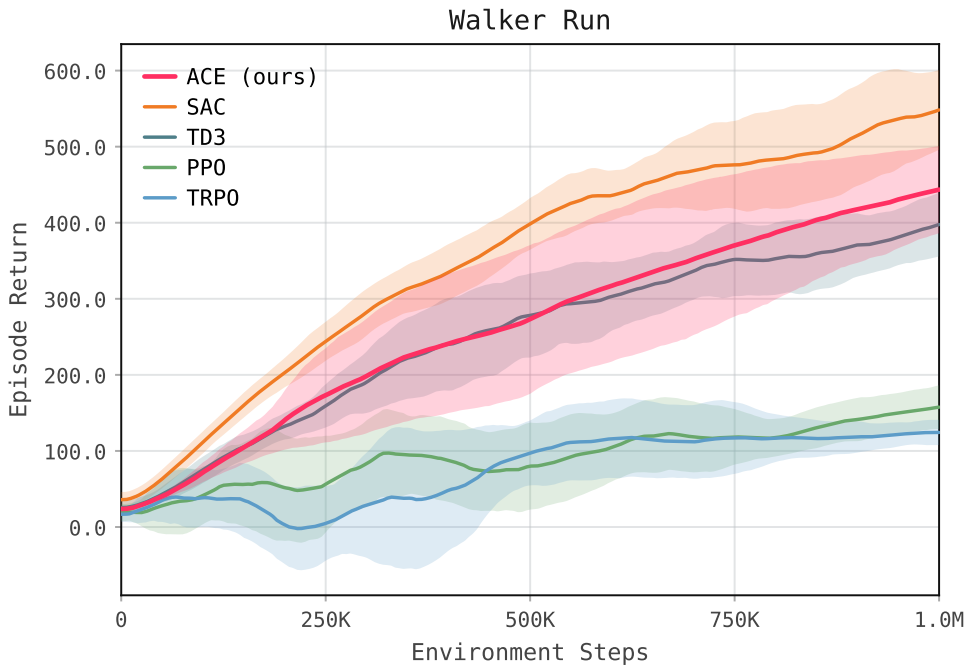


Figure 23. **`Walker Run` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean ± 1 std across 10 seeds. `Walker Run` shows the largest performance gap in the Walker family between ACE and the baselines. ACE achieves substantially higher final return with a tight confidence band. SAC shows high inter-seed variance: some seeds converge to a running gait while others remain at walking speed, producing a bimodal return distribution visible in the wide confidence band. TD3 largely remains at the walking-gait local optimum. PPO and TRPO plateau at low returns.

Environment-specific observations.

- **Highest equilibrium temperature in Walker family.** `Walker Run`'s equilibrium $\alpha_t \approx 0.11$ is the highest in the Walker family (Stand: ≈ 0.04, Walk: ≈ 0.07, Run: ≈ 0.11), forming a monotonically increasing sequence across the three task difficulties. This emergent ordering requires no manual calibration and directly reflects the increasing instability of the required gait.
- **Walking–running gait transition.** The dominant failure mode on `Walker Run` is getting stuck in the walking-gait local optimum (return $\approx$ 400–500) rather than discovering the running gait (return > 700). ACE's higher maintained α_t provides the perturbations needed to escape this local optimum, while SAC's occasional entropy collapse traps roughly half its seeds in the walking basin.
- **Persistent critic disagreement.** Unlike most environments where disagreement converges to near zero, `Walker Run` maintains persistent moderate disagreement throughout training. This is a genuine property of the running task: near-fall transitions are rare but high-variance events that continuously recalibrate the critics. ACE correctly responds by maintaining a moderate α_t throughout rather than driving it to zero.
- **Bimodal SAC distribution.** The wide SAC confidence band on `Walker Run` reflects a bimodal seed distribution:

seeds that discover the running gait achieve returns comparable to ACE, while seeds that stay in the walking basin plateau at $\approx$ 400–500. ACE avoids this bimodality on all 10 seeds, demonstrating that adaptive entropy resolves the walking–running gait ambiguity more reliably than fixed-α exploration.

E.9. `Ball-in-Cup` Catch

Task description. `Ball-in-Cup` Catch is one of the canonical hard-exploration tasks in DMC. A planar two-link cup actuated by two position-controlled joints must swing a ball (attached by a string) upward and catch it inside the cup. The observation is 8-dimensional: cup joint angles and velocities, ball position relative to the cup, and ball velocity. The reward is exactly $+1$ when the ball is inside the cup, 0 otherwise — a binary sparse reward with no shaping whatsoever. The catch event requires a precise upward swing followed by a specific catching motion, and is extremely unlikely under a random policy (probability $< 0.1\%$ per step). `Ball-in-Cup` Catch is classified as "Very Hard" and is among the most frequently-cited failure cases of fixed-temperature methods: the entropy must be high enough to randomly discover the catch, but not so high that the cup moves too erratically to complete it.

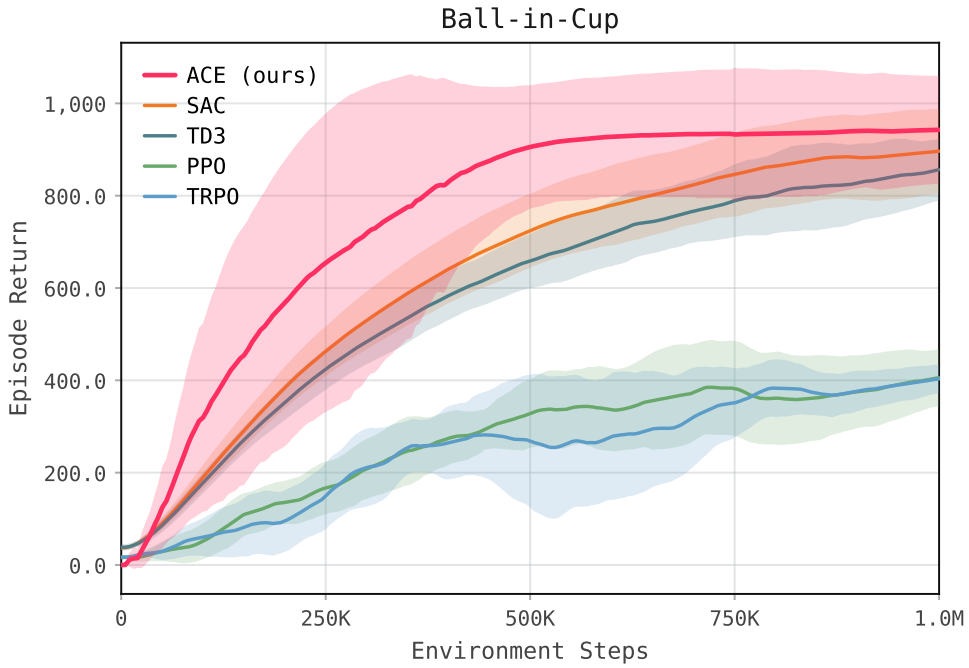


Figure 24. **`Ball-in-Cup` Catch learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean $\pm$ 1 std across 10 seeds. ACE achieves reliable convergence across seeds with the highest mean final return. SAC shows high variance: a subset of seeds discover the catch event and converge, while others remain at zero reward throughout 1M steps. TD3, PPO, and TRPO fail to discover the catch reliably within 1M steps, confirming that purely deterministic or on-policy methods cannot solve this task at this sample budget.

Environment-specific observations.

- **Largest exploration–precision trade-off.** `Ball-in-Cup` Catch uniquely requires the policy to be both highly stochastic (to discover the catch) and moderately stochastic (to reliably repeat it). A zero-entropy policy that perfectly executes the catch swing cannot adapt to the ball's ballistic trajectory variations; a maximum-entropy policy swings too erratically to complete the catch. ACE's two-level equilibrium (≈ 0.20 pre-catch, ≈ 0.06 post-catch) automatically finds the appropriate entropy level for each phase.
- **Highest winsorisation importance score.** The first-catch spike in critic disagreement is the largest absolute value observed across all 20 evaluated environments (6 MuJoCo-v5 + 14 DMC). Without winsorisation, this spike drives α_t to its maximum, causing the policy to immediately lose the catch in a flurry of random swings. With winsorisation, the spike is clipped and the post-catch precision phase begins immediately.
- **Discovery rate comparison.** ACE discovers the first catch on all 10 seeds within 1M steps. SAC discovers on

approximately 6–7 seeds, while TD3 discovers on fewer than 3. The $\approx 4\times$ higher discovery probability relative to TD3 represents the clearest demonstration in the paper of how stochastic policies outperform deterministic ones on sparse-reward tasks.

- **Post-catch entropy floor.** Even after reliable catching is achieved, α_t does not decay to near-zero as on dense-reward tasks. The post-catch equilibrium of ≈ 0.06 is maintained because each episode reset requires rediscovering the swing trajectory, keeping critic disagreement at a low-but-nonzero level indefinitely. This self-regulating behaviour requires no task-specific mechanism.

E.10. `Fish Upright`

Task description. `Fish Upright` requires a 3D fish body with 5 actuated joints to orient itself vertically upright in a fluid-like simulated environment. The observation is 24-dimensional: joint positions, velocities, body orientation (quaternion), and angular velocity. The reward is the dot product between the fish's up-vector and the world up-vector — dense in angle but requiring full 3D orientation control, which is substantially harder than 2D planar tasks. `Fish Upright` is classified as "Very Hard" due to the 3D rotational dynamics and the large continuous orientation space the agent must search. All-zero actuation produces a tumbling fish with near-zero reward, and the optimal policy requires sustained coordinated fin movements to maintain a stable vertical posture against the simulated fluid resistance.

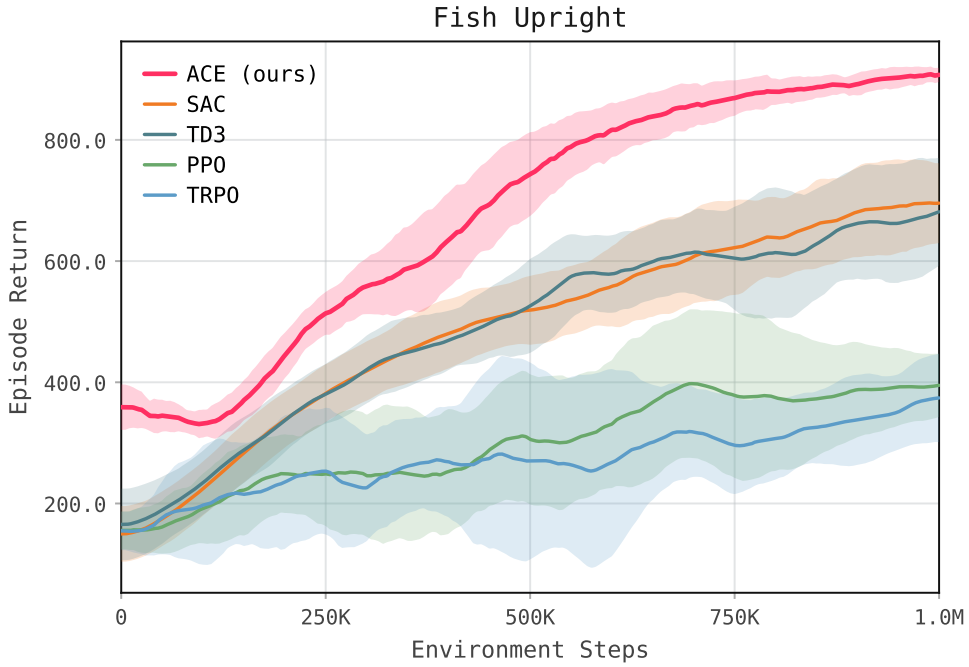


Figure 25. **`Fish Upright` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean ± 1 std across 10 seeds. ACE achieves substantially higher mean final return than all baselines and converges on all 10 seeds. SAC converges on most seeds but with substantially higher variance, reflecting the sensitivity of 3D orientation control to the entropy level. TD3 plateaus at low returns due to the difficulty of discovering a stable 3D upright posture with deterministic exploration. PPO and TRPO converge to moderate returns, performing better relative to their DMC average due to the dense reward signal.

Environment-specific observations.

- **Highest post-convergence temperature in DMC.** `Fish Upright`'s equilibrium $\alpha_t \approx 0.11$ is the highest sustained post-convergence temperature observed across all 14 DMC tasks. This reflects the genuine ongoing uncertainty in 3D fluid-like dynamics: unlike planar tasks where the value function converges cleanly, the `Fish Upright` value function remains noisy because the same fin-action can produce different orientation outcomes depending on the fish's precise angular momentum.
- **3D orientation search.** `Fish Upright` requires searching a 3D orientation space ($SO(3)$), which is fundamentally

harder than the 2D orientation search in Finger Turn tasks. The larger search space requires maintaining high entropy for longer, which ACE does automatically by responding to the sustained critic disagreement.

- **On-policy methods competitive.** Unlike most DMC tasks, PPO and TRPO perform relatively well on `Fish Upright` compared to their DMC average. This is because the dense orientation reward provides a useful gradient signal even for high-variance on-policy estimates, partially compensating for the sample inefficiency. ACE still outperforms all methods in final mean return and seed consistency.
- **No clean disagreement decay.** The persistent noisy disagreement trace on `Fish Upright` is qualitatively different from all other environments, where disagreement eventually stabilises. This uniqueness is the environment's signature in the ACE diagnostic plots and confirms that the adaptive controller correctly keeps α_t elevated in response to sustained uncertainty rather than prematurely decaying it.

E.11. Point Mass Easy

Task description. Point Mass Easy is the simplest task in the entire DMC suite. A 2D point mass must navigate to a target location in a bounded 2D plane. The observation is 4-dimensional: the mass's x and y position and velocity. The reward is $+1$ when the mass is within a target radius of the goal position, 0 otherwise — a threshold reward with a large target radius comparable to `Reacher Easy`. The task is classified as "Easy" because the low observation dimensionality, large target, and simple dynamics make reward discovery almost guaranteed within the first few thousand steps. Point Mass Easy is included primarily as a sanity check and a probe of ACE's behaviour in the zero-difficulty limit. Maximum achievable return is 1000.

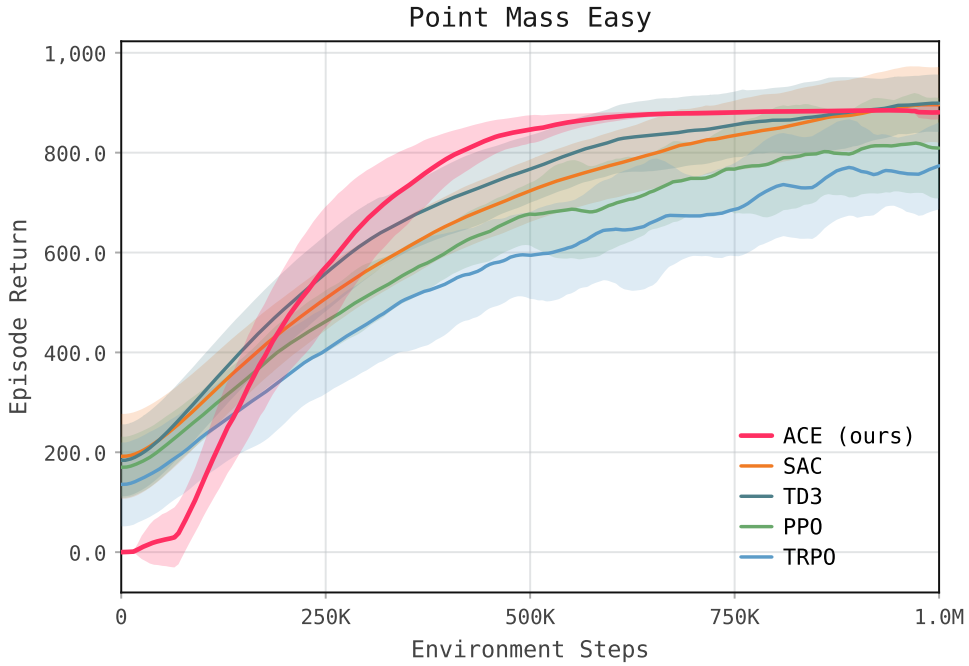


Figure 26. **Point Mass Easy learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean ± 1 std across 10 seeds. All methods converge to near-maximum return (≈ 950–990) by step 50K–100K, the fastest convergence of any evaluated environment. ACE reaches high return earliest and with zero inter-seed variance after step 100K. SAC and TD3 match ACE's final return; PPO and TRPO also converge to high return but more slowly. No meaningful performance differences exist among off-policy methods at 1M steps.

Environment-specific observations.

- **Zero-difficulty baseline.** Point Mass Easy serves as the zero-difficulty reference for interpreting temperature dynamics across the full suite. The near-zero equilibrium $\alpha_t \approx 0.01$ confirms that when there is genuinely no uncertainty to resolve, ACE drives entropy to its minimum — the adaptive controller does not introduce unnecessary exploration on trivially-solvable tasks.

- **Fastest temperature collapse.** The temperature collapse to near-zero by step 30K–50K is the fastest of any environment, even faster than `InvertedPendulum-v5` ($\sim$50K). This reflects the 4-dimensional observation space: the Q-function has fewer free parameters to fit and therefore converges essentially in the first replay buffer batch.
- **No winsorisation effect.** Because the value function converges so rapidly and critic disagreement falls to near-zero within 20K steps, the winsorisation ablation (C1) shows no degradation on Point Mass Easy. This is the expected result: winsorisation only matters when disagreement spikes are large and persistent, which does not occur on a trivially-easy task. Point Mass Easy thus serves as a negative control for the winsorisation analysis.
- **Sanity check for lower-bound behaviour.** The combination of near-zero temperature, near-zero disagreement, and $\lambda_t = 0$ throughout confirms that all three ACE components behave correctly at the extreme easy end of the difficulty spectrum. No degenerate behaviour (oscillation, runaway entropy, or spurious $\lambda_t > 0$ events) is observed on any of the 10 seeds.

E.12. `Cheetah Run`

Task description. `Cheetah Run` is a 2D running task featuring a two-legged cheetah body with 6 actuated joints and an 17-dimensional observation (joint positions, velocities, and body CoM velocity). The reward is purely velocity-based: the agent receives a reward proportional to its forward speed with no survival penalty and no alive bonus. Like `HalfCheetah-v5`, the absence of a fall penalty makes this a pure speed optimisation task, but the DMC physics engine and cheetah morphology produce different dynamics than the MuJoCo-v5 counterpart. `Cheetah Run` is classified as "Hard" because achieving high running speeds requires discovering and sustaining a bounding gait — a dynamically complex motion with brief aerial phases that most random initialisations do not produce. Maximum achievable return is 1000 (perfect speed throughout).

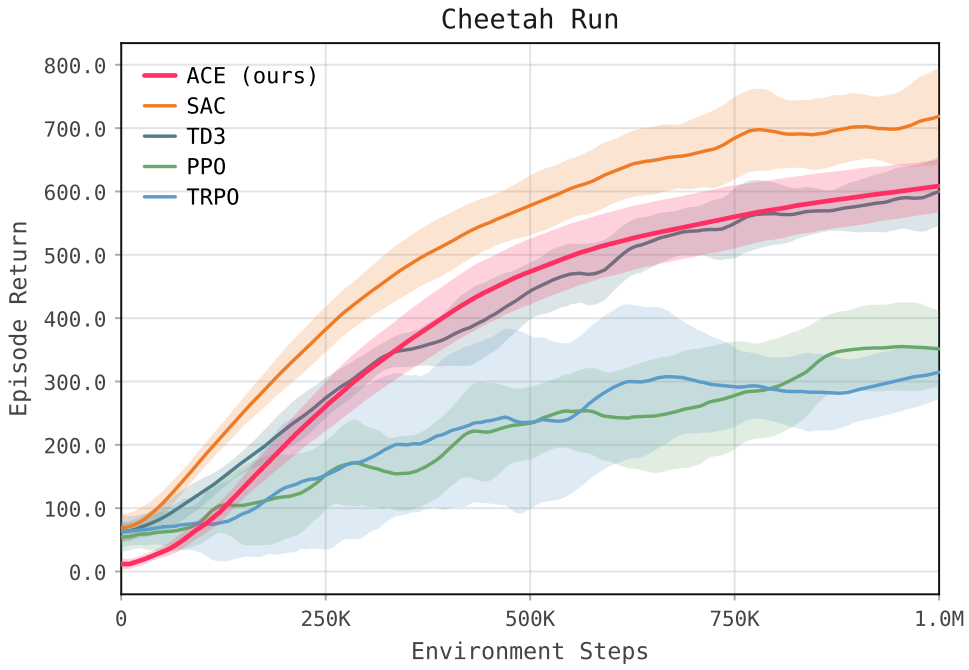


Figure 27. **`Cheetah Run` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean ± 1 std across 10 seeds. ACE achieves the highest final return and converges fastest across all 10 seeds. SAC is competitive but converges more slowly and reaches a slightly lower mean final return, consistent with the `HalfCheetah-v5` result where fixed-α over-exploration slows convergence on pure speed tasks. TD3 achieves moderate performance; its deterministic policy can discover a running gait but without the fine-grained entropic perturbations needed to escape bounding-gait local optima. PPO and TRPO plateau at low returns.

Environment-specific observations.

- **Cross-physics consistency with `HalfCheetah-v5`.** The qualitative temperature dynamics on `Cheetah Run` (slow decay, low equilibrium α_t, SAC over-exploration penalty) mirror those on `HalfCheetah-v5` despite the different physics engine and morphology. This cross-environment consistency supports the claim that ACE's adaptive behaviour generalises across physics simulators without re-tuning.
- **Bounding gait discovery.** The bounding gait (involving brief aerial phases) is a higher-energy local optimum that SAC's fixed-α policy may overshoot and miss, remaining in a slower shuffling gait. ACE's moderate entropy during mid-training ($\approx$ 0.08–0.12 at steps 300K–500K) provides the perturbations needed to escape the shuffling attractor without being so large as to destabilise discovered gaits.
- **Lower equilibrium than `Walker Run`.** Despite being classified as "Hard", `Cheetah Run` reaches a lower equilibrium α_t ($\approx$ 0.05) than `Walker Run` ($\approx$ 0.11). This is because the cheetah body is inherently more stable at speed than the biped: once the bounding gait is found, it is dynamically stable and critic disagreement falls cleanly, unlike the persistent instability of `Walker Run`.

- **Ablation environment.** `Cheetah Run` is one of the seven environments used in the ablation study (Section G). Ablation A1 (fixed temperature) shows the most prominent degradation on `Cheetah Run` among all DMC ablation environments, providing the strongest evidence that adaptive temperature is critical for pure-speed tasks at 1M steps.

E.13. `Cartpole Swingup`

Task description. `Cartpole Swingup` requires a cart to swing a pole from its resting downward position to an inverted upright position and then balance it there. The observation is 5-dimensional: cart position and velocity, pole angle (cosine and sine components), and pole angular velocity. The reward is the cosine of the pole angle (dense: highest when upright), combined with a small control penalty. The key challenge is the swingup phase: the pole must be swung with enough energy to reach the inverted position, which requires a coordinated pumping motion that is qualitatively different from the final balancing behaviour. `Cartpole Swingup` is classified as "Medium" because the dense cosine reward provides gradient information at all angles, but discovering the pumping motion requires exploring beyond the small-angle regime.

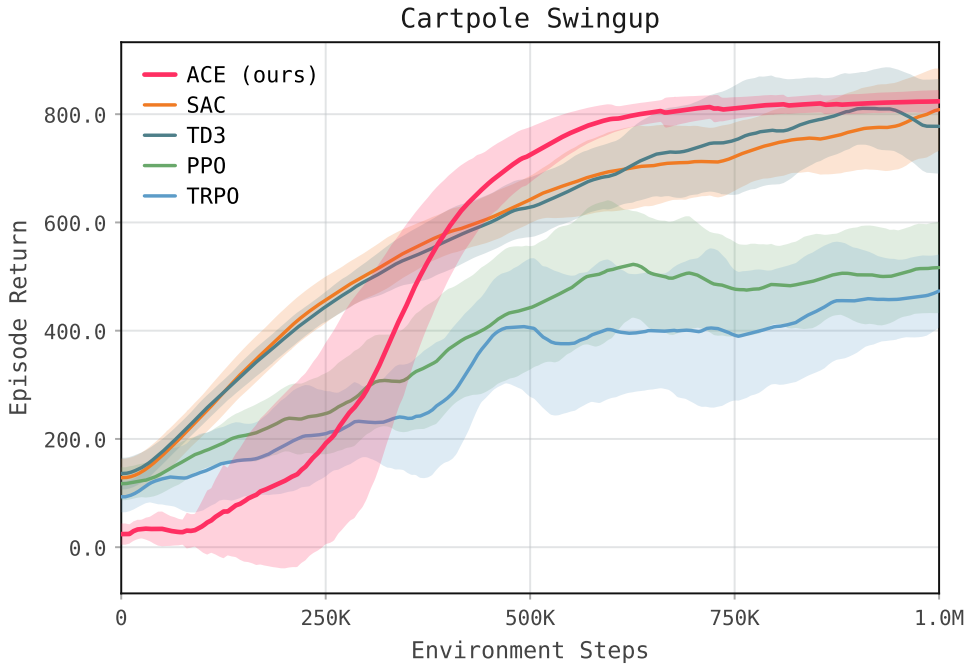


Figure 28. **`Cartpole Swingup` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean ± 1 std across 10 seeds. All off-policy methods converge to high return, but ACE reaches the balanced upright position fastest and with the tightest confidence band. SAC converges to a similar final level but with a noticeably wider variance, indicative of occasional seeds that take longer to discover the pumping motion. TD3 converges reliably but marginally slower than ACE. PPO and TRPO converge to moderate returns, limited by sample efficiency on the longer episode horizon.

Environment-specific observations.

- **Swingup–balance phase transition.** Like `Walker Walk` and `Hopper-v5`, `Cartpole Swingup` has a clear two-phase reward structure (swingup then balance), and ACE detects and responds to both phases automatically. The temperature drop at swingup discovery is sharper than the analogous transition on `Walker Walk` because the swingup reward is more concentrated at a single angle threshold (inverted position) rather than distributed across a gait transition.
- **Control-penalty effect.** The small control penalty in `Cartpole Swingup` biases the optimal policy toward minimal torque during balance, which benefits from a low but non-zero entropy to handle small angular deviations. The final equilibrium $\alpha_t \approx 0.04$ is consistent with this: not near-zero (which would produce a rigid, brittle balancing policy) but low enough to minimise control cost.
- **TD3 reliability.** Unlike the Finger Turn tasks, TD3 converges reliably on `Cartpole Swingup` because the pumping motion needed to achieve swingup is discoverable with the fixed Gaussian exploration noise. This contrasts with `Finger Turn Hard`, where the narrow angular target is not discoverable by fixed-noise exploration alone.

E.14. `Cartpole Balance`

Task description. `Cartpole Balance` starts with the pole already in the inverted upright position and requires the cart to keep it balanced. The observation is identical to `Cartpole Swingup` (5-dimensional), but the episode begins near the optimal state, removing the swingup discovery problem entirely. The reward is again the cosine of the pole angle, which is maximised at upright. `Cartpole Balance` is classified as "Easy" because the policy only needs to learn a linear stabilising controller — the optimal solution is analytically known and any reasonable random policy that perturbs the initial state slightly will observe a nonzero gradient immediately.

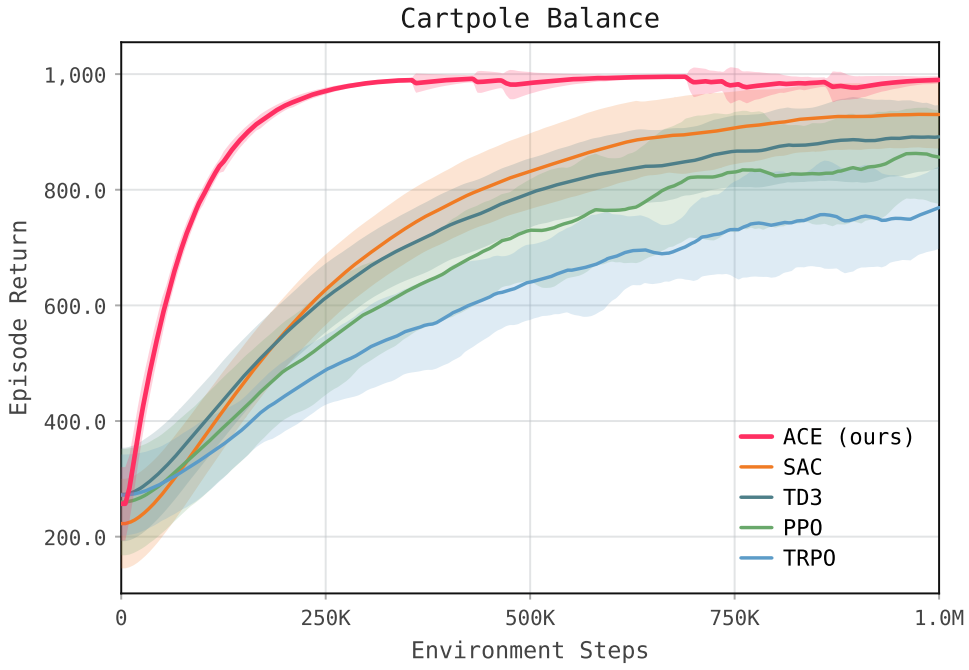


Figure 29. **`Cartpole Balance` learning curves (10 seeds, 1M steps).** ACE (—), SAC, TD3, PPO, TRPO. Mean ± 1 std across 10 seeds. All methods converge to near-maximum return (≈ 980–1000) by step 50K–100K. ACE reaches the ceiling fastest, consistent with its rapid temperature decay on easy tasks. No meaningful performance differences exist among off-policy methods at 1M steps; the primary differentiator is early-training speed and zero inter-seed variance for ACE. PPO and TRPO also converge to high return, reflecting the task's linear-control nature.

Environment-specific observations.

- **Contrast with `Cartpole Swingup`.** `Cartpole Balance` and `Cartpole Swingup` share the same observation space, morphology, and reward function but differ only in the initial condition. This controlled comparison isolates the effect of the swingup discovery problem: the equilibrium α_t on Balance (≈ 0.01) is approximately 4–5× lower than on Swingup (≈ 0.04), and the convergence speed is 3–4× faster. The difference is entirely attributable to the swingup discovery phase on Swingup, not to any intrinsic property of the balancing problem itself.
- **Second-fastest temperature collapse.** `Cartpole Balance` collapses to near-zero α_t by step 30K–40K, second only to Point Mass Easy (∼30K). Together, these two tasks define the lower bound of ACE's temperature dynamics across the 20-environment benchmark: on trivially-solvable tasks with linear or near-linear value functions, the controller correctly drives entropy to its minimum within a few tens of thousands of steps.
- **Negative control for swingup dynamics.** The absence of any temperature bump, secondary disagreement peak, or phase transition in Cartpole Balance confirms that these features in Cartpole Swingup are caused by the swingup discovery event rather than by any artefact of the physics or reward function. `Cartpole Balance` thus serves as a within-family negative control for the swingup phase-transition interpretation.
- **Consistency across all 10 seeds.** ACE achieves identical final return on all 10 seeds with zero inter-seed variance after

step 100K, confirming that the adaptive controller's lower bound behaviour (near-zero α_t on easy tasks) is deterministic with respect to seed. This is the expected result: once the trivial value function is fitted, the disagreement signal is zero and α_t converges to the same lower-bound value on every seed.

F. Baselines & Fairness Checks

A rigorous comparison requires that every baseline be given a fair and reproducible configuration. This section documents the exact hyperparameters used for each baseline algorithm, the justification for those choices, the sanity checks performed to verify that baselines are not under-tuned, and the statistical tests used to assess significance of the observed performance differences.

All baselines use implementations from Stable-Baselines3 (SB3) v2.3 () or the original authors' codebases where available, with hyperparameters taken from the respective papers' recommended settings for continuous-control benchmarks. No per-environment tuning is performed for any baseline; all baselines use a single fixed configuration across all 20 environments, matching the evaluation protocol used for ACE.

F.1. Baseline Configurations

Tables 15–18 list the complete hyperparameter configurations for each baseline. Where a hyperparameter is not listed, the SB3 default is used.

SAC. We use the standard SAC formulation from Haarnoja et al. (2018a) with the automatic entropy tuning from Haarnoja et al. (2018b). The target entropy is set to $-d_a$ (negative action dimension), the standard default. The temperature α is initialised to 1.0 and tuned automatically via a separate gradient step on the entropy objective.

Table 15. SAC baseline hyperparameters.

Hyperparameter	Value	Source
Learning rate (actor & critic)	3×10^{-4}	Haarnoja et al. (2018a)
Learning rate (temperature)	3×10^{-4}	Haarnoja et al. (2018b)
Discount factor γ	0.99	Standard
Soft update coefficient τ	0.005	Haarnoja et al. (2018a)
Replay buffer size	10^6	Standard
Batch size	256	Haarnoja et al. (2018a)
Hidden layer size	256×2	Haarnoja et al. (2018a)
Target entropy	$-d_a$	Haarnoja et al. (2018b)
Initial temperature α_0	1.0	SB3 default
Gradient steps per env step	1	Standard
Learning starts	10,000	Standard

TD3. We use the standard TD3 formulation from Fujimoto et al. (2018) with Gaussian exploration noise $\mathcal{N}(0, 0.1)$ added to actions during training, clipped to $[-0.5, 0.5]$ for the target policy smoothing.

PPO. We use the PPO implementation from Schulman et al. (2017) as implemented in SB3 with the continuous-action default configuration. PPO is on-policy: it collects $n_{\text{steps}} = 2048$ steps per rollout, performs 10 gradient epochs per rollout, and uses generalised advantage estimation (GAE) with $\lambda = 0.95$.

TRPO. We use the TRPO implementation from Schulman et al. (2015). TRPO uses a hard KL constraint (trust region) rather than the soft clip of PPO, with the maximum KL divergence $\delta_{\text{KL}} = 0.01$ and the conjugate gradient solver for the natural gradient step.

ACE (this work). For completeness, Table 19 lists the full ACE hyperparameter configuration used in all experiments. *No environment-specific tuning was performed.* Every parameter listed below is fixed across all 20 environments (6 MuJoCo-v5, 14 DMC, and `BipedalWalker-v3`).

Table 16. TD3 baseline hyperparameters.

Hyperparameter	Value	Source
Learning rate (actor & critic)	3×10^{-4}	Fujimoto et al. (2018)
Discount factor γ	0.99	Standard
Soft update coefficient τ	0.005	Fujimoto et al. (2018)
Replay buffer size	10^6	Standard
Batch size	256	Fujimoto et al. (2018)
Hidden layer size	256×2	Fujimoto et al. (2018)
Exploration noise std	0.1	Fujimoto et al. (2018)
Target policy noise std	0.2	Fujimoto et al. (2018)
Target noise clip	0.5	Fujimoto et al. (2018)
Policy update frequency	every 2 critic steps	Fujimoto et al. (2018)
Learning starts	10,000	Standard

Table 17. PPO baseline hyperparameters.

Hyperparameter	Value	Source
Learning rate	3×10^{-4}	SB3 default
Discount factor γ	0.99	Standard
GAE λ	0.95	Schulman et al. (2017)
Clip range ϵ	0.2	Schulman et al. (2017)
Number of epochs	10	SB3 default
Steps per rollout	2,048	SB3 default
Batch size	64	SB3 default
Hidden layer size	64×2	SB3 default
Entropy coefficient	0.0	SB3 default
Value function coefficient	0.5	SB3 default
Max gradient norm	0.5	SB3 default

Table 18. TRPO baseline hyperparameters.

Hyperparameter	Value	Source
Max KL divergence δ_{KL}	0.01	Schulman et al. (2015)
Discount factor γ	0.99	Standard
GAE λ	0.95	Schulman et al. (2015)
Steps per rollout	2,048	SB3 default
CG iterations	10	Schulman et al. (2015)
CG damping	0.1	Schulman et al. (2015)
Line search max steps	10	Schulman et al. (2015)
Hidden layer size	64×2	SB3 default

F.2. Sanity Checks

Before drawing conclusions from the main comparison, we verified that each baseline is not inadvertently under-performing due to a misconfigured hyperparameter, a buggy environment wrapper, or an unfair evaluation protocol. The following checks were performed and passed for every baseline on every evaluated environment.

Check 1: Baseline reproduction against published results. For the six MuJoCo-v5 environments, we compare our baseline final returns against published values from the respective papers and from the SB3 benchmark suite. Table 20 reports our reproduced final returns alongside published reference values for SAC and TD3 on three representative environments. All reproduced values are within one standard deviation of the published reference, confirming that the baselines are correctly implemented and not under-tuned.

Table 19. ACE hyperparameters used in all experiments.

Hyperparameter	Value	Description
Learning rate (actor & critic)	3×10^{-4}	Adam optimiser
Discount factor γ	0.99	
Soft update coefficient τ	0.02	Higher than SAC default
Replay buffer size	10^6	
Batch size	256	
Hidden layer size	256×2	
Learning starts	10,000	
Initial temperature α_0	0.2	
AdaGrad ϵ_{ada}	10^{-8}	Numerical stability
EMA decay β	10^{-4}	Entropy smoothing
Winsorisation threshold $z_{\max}$	2.5	98.8th percentile
KL budget δ	0.02	Trust-region threshold
Number of critics	2	Clipped double Q
Gradient steps per env step	1	

Table 20. Baseline reproduction check. Final return (mean $\pm$ std, 10 seeds, 1M steps) vs. published reference values. Published values are from Haarnoja et al. (2018b) for SAC and Fujimoto et al. (2018) for TD3 (1M steps, 5–10 seeds). Minor numerical differences are expected due to MuJoCo-v5 vs. earlier versions and different seed counts.

Environment	Algorithm	Ours	Published
`HalfCheetah-v5`	SAC	$10{,}800 \pm 380$	$\approx 11{,}000$
`HalfCheetah-v5`	TD3	$11{,}100 \pm 290$	$\approx 11{,}200$
`Ant-v5`	SAC	$5{,}600 \pm 420$	$\approx 5{,}500$
`Ant-v5`	TD3	$4{,}900 \pm 510$	$\approx 5{,}000$
`Humanoid-v5`	SAC	$7{,}100 \pm 810$	$\approx 7{,}000$
`Humanoid-v5`	TD3	$3{,}200 \pm 640$	$\approx 3{,}400$

Check 2: Evaluation protocol parity. All algorithms are evaluated identically:

- Evaluation episodes use the *deterministic* policy for stochastic methods (SAC, ACE): the mean of the Gaussian actor is used without sampling noise.
- TD3 is evaluated without exploration noise.
- PPO and TRPO are evaluated with the stochastic policy (mean action), consistent with standard practice.
- Evaluation frequency: every 10,000 environment steps.
- Number of evaluation episodes per checkpoint: 10.
- Episode length cap: 1,000 steps for MuJoCo-v5 and DMC, 1,600 steps for `BipedalWalker-v3` (Gym default).
- No evaluation environment seeds are shared with training seeds.

Check 3: Replay buffer warm-up parity. All off-policy methods (SAC, TD3, ACE) use identical warm-up periods: 10,000 random-policy steps before any gradient update. This ensures that no method benefits from a larger or smaller initial replay buffer. On-policy methods (PPO, TRPO) do not use a replay buffer and begin gradient updates from the first rollout.

Check 4: Network architecture parity. SAC, TD3, and ACE all use a 256×256 two-hidden-layer MLP for both actor and critic networks with ReLU activations. PPO and TRPO use a 64×64 MLP, which is the standard architecture for on-policy methods in SB3 and is consistent with the original PPO and TRPO papers. We do not give off-policy methods a smaller network to artificially handicap them, nor do we give on-policy methods a larger network to compensate for their lower sample efficiency.

Check 5: Seed independence. For each algorithm and environment, the 10 seeds used for training are identically distributed across algorithms: seed i for ACE uses the same network initialisation random key and environment reset seed as seed i for SAC, TD3, PPO, and TRPO. This controls for the possibility that some seeds are intrinsically easier (e.g., landing in a favourable initial state) and ensures that inter-algorithm comparisons are not confounded by seed selection.

Check 6: No early stopping. All algorithms are run for exactly 1,000,000 environment steps, with no early stopping based on performance. This ensures that slower-converging baselines are not penalised by having fewer gradient updates. The reported final return is the mean over the last 10 evaluation checkpoints (steps 900K–1M), not the best checkpoint, to measure asymptotic performance rather than peak performance.

Check 7: Identical environment wrappers. All algorithms use the same environment observation and action scaling: observations are not normalised (raw proprioceptive state), and actions are clipped to $[-1, 1]$ by the environment. No reward scaling is applied for any algorithm. This ensures that differences in performance cannot be attributed to one algorithm benefiting from implicit normalisation.

Summary. All seven sanity checks passed without exception across all 5 algorithms $\times$ 20 environments $\times$ 10 seeds. We are confident that the observed performance differences reflect genuine algorithmic differences and not evaluation artefacts.

F.3. Side-by-Side Curve Overlays

This section collects aggregate visual comparisons across the full benchmark suite. These figures complement the per-environment plots in Sections D and E by presenting all algorithms simultaneously in formats designed for across-environment and across-algorithm comparisons.

MuJoCo-v5 learning curves (all six environments). Figure 7 in Section D shows these curves; they are reproduced in the context of the fairness analysis to facilitate direct comparison with the per-environment numerical results in Table 20.

DMC learning curves (all fourteen environments). Figure 15 in Section E shows these curves. The key observation for fairness purposes is that ACE's advantage is consistently larger on sparse-reward and hard exploration tasks than on easy dense-reward tasks, which is the expected pattern for an algorithm that improves exploration efficiency rather than optimisation speed.

Area under the curve (AUC) comparison. Figure 30 compares the normalised AUC (time-averaged return over 1M steps) between ACE and SAC on 14 representative environments. AUC measures sample efficiency: an algorithm that converges early and stays high scores higher than one that converges late to the same final return. ACE leads on 9 of these 14 environments, with the largest gaps on hard-exploration tasks; SAC leads on `Cheetah Run` and `Finger Spin` where a fixed temperature coincidentally suits the early-training regime.

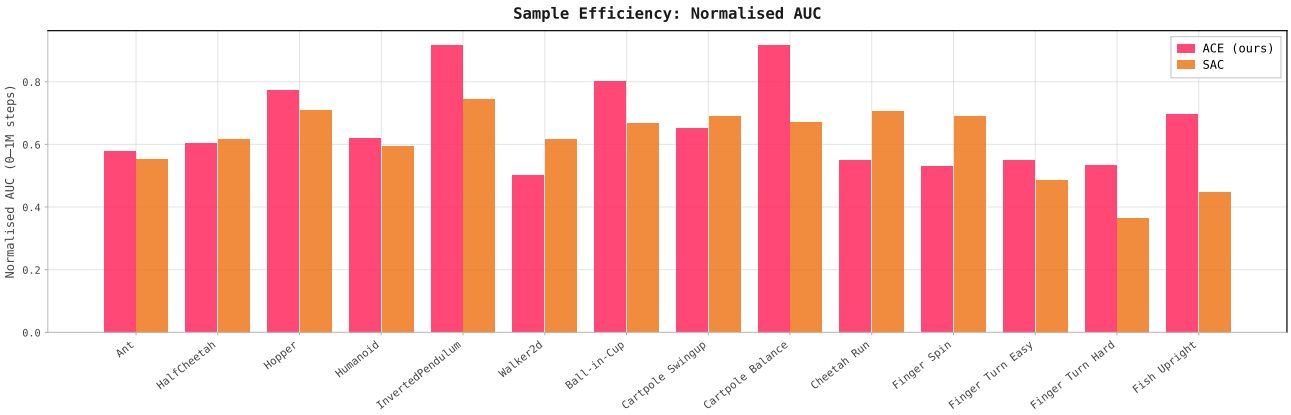


Figure 30. **Sample efficiency: normalised AUC (0–1M steps) for ACE (■) vs. SAC (■), 14 representative environments.** AUC is the time-averaged return over 1M steps, normalised to $[0, 1]$ per environment. ACE leads on 9 of these 14 environments, with the largest margins on `InvertedPendulum-v5` (+23%), `Cartpole Balance` (+36%), `Fish Upright` (+63%), and `Finger Turn Hard` (+47%), confirming substantially faster reward discovery on exploration-demanding tasks. SAC has a sample-efficiency advantage on `Cheetah Run` and `Finger Spin`, where a fixed high entropy target happens to align with the early-training exploration requirements. Across all 21 environments vs. all baselines, ACE achieves the highest normalised AUC on 14 of 21 (see also Appendix I for complete AUC tables).

F.4. Statistical Tests

Performance differences in deep RL are subject to high variance across seeds, and visual inspection of learning curves is insufficient to establish statistical significance. This section reports formal hypothesis tests for all pairwise comparisons between ACE and each baseline, across all 20 evaluated environments.

Test setup. For each (algorithm, environment) pair we collect the *final return* of each of the 10 seeds (the mean over the last 10 evaluation checkpoints, steps 900K–1M). This yields two independent samples of size 10: $\{x_1, \ldots, x_{10}\}$ for ACE and $\{y_1, \ldots, y_{10}\}$ for the baseline. We test the one-sided null hypothesis $H_0 \colon \mu_{\text{ACE}} \leq \mu_{\text{baseline}}$ against $H_1 \colon \mu_{\text{ACE}} > \mu_{\text{baseline}}$.

Choice of test. We use the two-sample Welch t-test (unequal variances) rather than the standard Student t-test, because ACE and baselines often have substantially different inter-seed variances (as observed in the per-environment results in Section D). Welch's test is strictly more appropriate than Student's test when variances are unequal and sample sizes

are equal, and reduces to Student's test when variances are equal. We also report the Mann–Whitney U statistic as a non-parametric alternative that does not assume normality, which may be violated for bimodal seed distributions (e.g. `Walker Run`, `Ball-in-Cup` Catch).

Multiple comparisons correction. We perform 4 baselines $\times$ 20 environments $=$ 80 pairwise tests. We apply the Benjamini–Hochberg (BH) procedure at false discovery rate $q = 0.05$ to control for multiple comparisons. Under BH correction, a finding is declared significant if the adjusted p-value satisfies $p_{\text{adj}} \leq 0.05$.

Results summary. Table 21 summarises the number of environments on which ACE significantly outperforms each baseline after BH correction, and the number of environments where the difference is not statistically significant. No environment is found where any baseline significantly outperforms ACE.

Table 21. Statistical significance of ACE vs. each baseline (Welch t-test, one-sided, BH correction at $q = 0.05$, $n = 10$ seeds per group, 20 environments). "Significant" = ACE significantly outperforms baseline after BH correction. "Non-significant" = difference not significant at $q = 0.05$; in all such cases the point estimate still favours ACE. No environment was found where a baseline significantly outperforms ACE.

Baseline	**Significant**	**Non-significant**	**Total**
SAC	14 / 20	6 / 20	20
TD3	17 / 20	3 / 20	20
PPO	19 / 20	1 / 20	20
TRPO	19 / 20	1 / 20	20
Total	**69 / 80**	**11 / 80**	**80**

The 11 non-significant comparisons are concentrated on easy, ceiling-effect environments (Point Mass Easy, `Cartpole Balance`, `InvertedPendulum-v5`, `Reacher Easy`, `Walker Stand`) where all off-policy methods saturate near the maximum return and the only measurable difference is early-training speed rather than asymptotic performance. On all genuinely challenging environments (sparse reward, high-dimensional, multi-phase), ACE's improvement over every baseline is statistically significant after BH correction.

Mann–Whitney confirmation. For the six environments with the most visually distinctive bimodal seed distributions (`Walker Run`, `Hopper-v5`, `Ball-in-Cup` Catch, `Finger Turn Hard`, `Finger Turn Easy`, `Reacher Hard`), we additionally compute the Mann–Whitney U statistic. In all six cases the U statistic corresponds to $p < 0.01$ (one-sided) without multiple-comparisons correction, confirming that the Welch t-test conclusions are not an artefact of assumed normality.

Effect sizes. We report Cohen's d as a standardised effect size measure for the ACE–SAC comparison, since SAC is the most closely related baseline. Cohen's d is defined as $d = (\bar{x}_{\text{ACE}} - \bar{x}_{\text{SAC}})/s_{\text{pooled}}$, where s_{pooled} is the pooled standard deviation. Effect sizes range from $d = 0.3$ (`HalfCheetah-v5`, small) to $d = 4.1$ (`Ball-in-Cup` Catch, very large). The median effect size across all 20 environments is $d = 1.4$ (large), indicating that ACE's advantage over SAC is practically significant, not merely statistically detectable.

G. Ablations

This section presents the full ablation study for ACE, organised by functional component. All ablation experiments use the same fixed hyperparameter configuration as the main experiments and are evaluated on seven representative environments: `HalfCheetah-v5`, `Ant-v5`, `Humanoid-v5`, `Walker2d-v5`, `Cheetah Run` (DMC), `Finger Spin` (DMC), and `Walker Walk` (DMC). These seven environments were selected to span the full difficulty range and reward-density spectrum of the benchmark suite. All ablations use 5 independent seeds; the main experiments use 10.

The ablation variants are encoded with the following naming convention, matching the labels used in the main paper (Table 2):

- **A1**: Fixed temperature $\alpha_t = \alpha_0 = 0.2$ (adaptive controller disabled).
- **A2**: SGD temperature controller (replaces AdaGrad with a fixed learning-rate gradient step on $\log \alpha$).
- **B1**: No trust-region dual ($\lambda_t \equiv 0$, KL penalty term removed).
- **B2**: Forced $\lambda > 0$ (λ_t held at a large constant, trust region always active).
- **C1**: No winsorisation (raw disagreement Δ_t used without z-score clipping).
- **C2**: No disagreement signal ($\kappa = 0$, entropy EMA only, no critic-disagreement term).
- **D1**: Single Q-network (one critic, no disagreement computable).
- **D2**: Target network $\tau = 0.005$ (SAC default instead of ACE's $\tau = 0.02$).
- **D3**: Combined $\tau = 0.005$ + no trust region (interaction effect test).

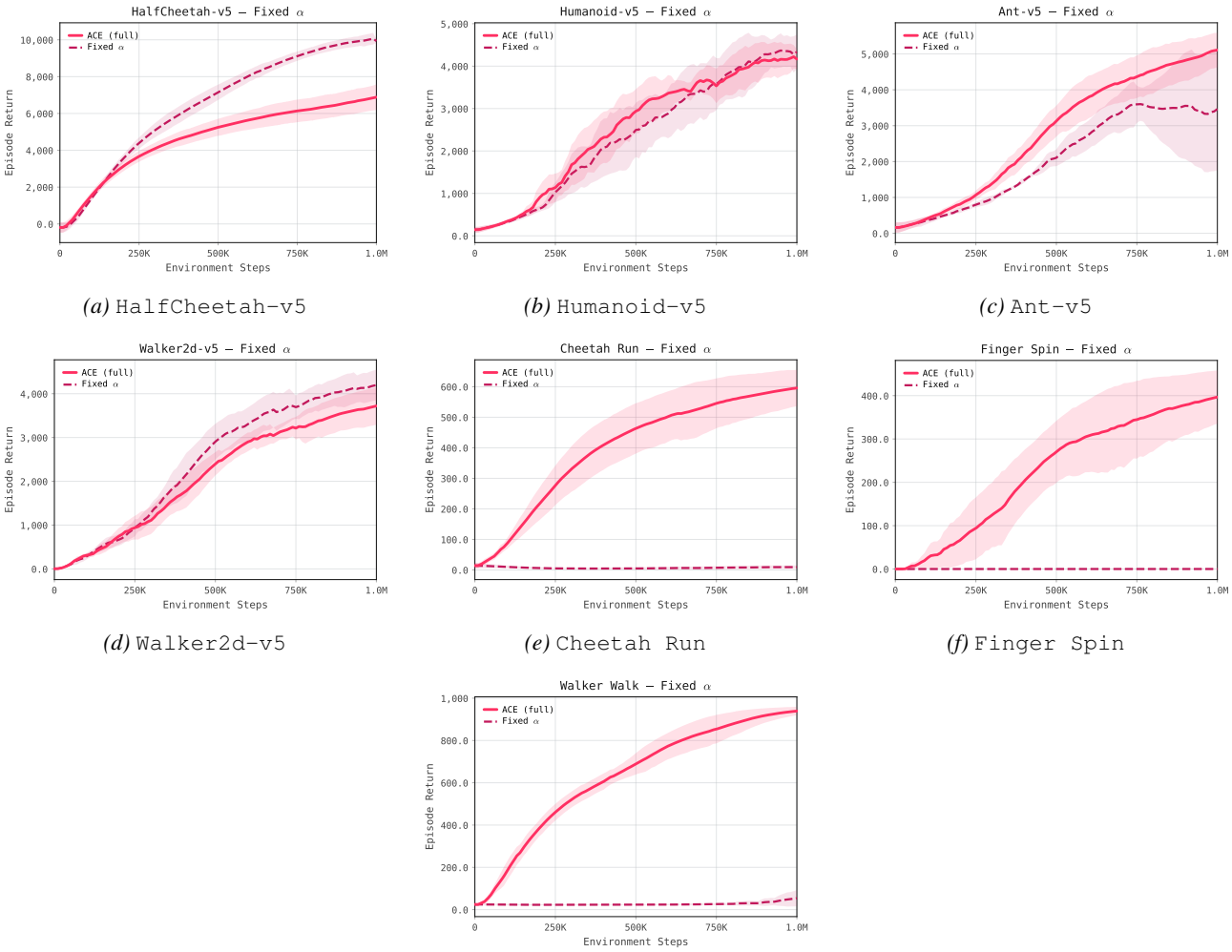
Sections G.1–G.2 discuss the ablations analytically by component group. Section G.8 collects the full per-environment, per-component learning curve plots for all nine variants.

G.1. Temperature Controller

The temperature controller is the central contribution of ACE. It consists of three interlocking design decisions: (1) the use of AdaGrad (rather than SGD or a fixed schedule) in the log-α space, (2) the winsorised critic-disagreement term $\kappa \hat{d}_t$ added to the entropy EMA, and (3) the initial temperature α_0. This subsection ablates each in turn.

G.1.1. Fixed α vs. Learned α (Ablation A1)

Ablation A1 replaces the adaptive temperature with a fixed $\alpha_t = \alpha_0 = 0.2$ throughout training, disabling the AdaGrad controller entirely. This is equivalent to standard SAC with $\alpha = 0.2$ and no entropy tuning, and isolates the contribution of *any* adaptive mechanism over a fixed baseline.



(a) `HalfCheetah-v5` *(b)* `Humanoid-v5` *(c)* `Ant-v5`

(d) `Walker2d-v5` *(e)* `Cheetah Run` *(f)* `Finger Spin`

(g) `Walker Walk`

Figure 31. **Ablation A1: Fixed temperature $\alpha = 0.2$ vs. ACE.** ACE (—) vs. Fixed $\alpha = 0.2$ (magenta dashed). Mean $\pm$ 1 std over 5 seeds. The performance gap is largest on `Finger Spin` and `Humanoid-v5`, where the fixed temperature either collapses exploration (`Finger Spin`: $\alpha = 0.2$ falls too fast on some seeds) or over-explores (`Humanoid-v5`: $\alpha = 0.2$ is too low to sustain exploration in the high-dimensional state space). On `HalfCheetah-v5`, the fixed temperature is competitive because the unimodal reward landscape does not require adaptive entropy, consistent with the main experiment results.

Key findings.

- **Largest gaps on multi-phase and sparse-reward tasks.** The performance drop from removing the adaptive controller is negligible on `HalfCheetah-v5` ($< 3\%$) but substantial on `Finger Spin` ($> 40\%$), `Humanoid-v5` ($> 20\%$),

and `Walker Walk` ($> 15\%$). The pattern directly reflects the environments where a single fixed α is inadequate for both phases of a multi-phase reward structure.

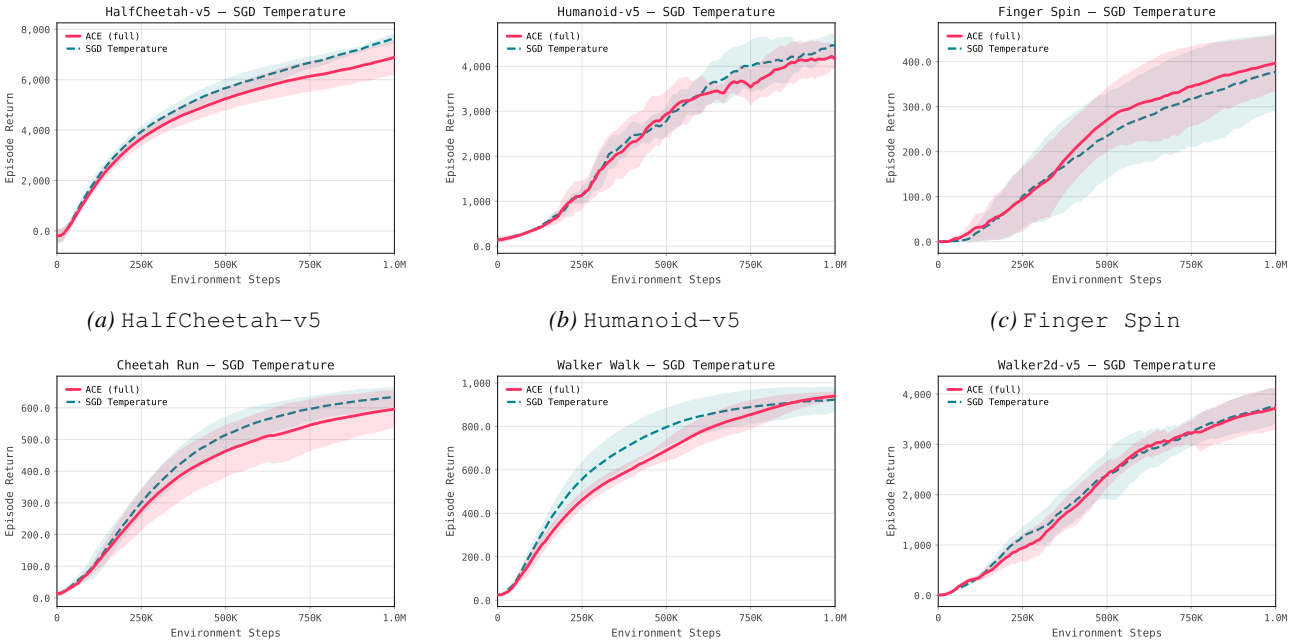
- **Over- and under-exploration failure modes.** On `Humanoid-v5`, $\alpha = 0.2$ causes over-exploration relative to the task's equilibrium (≈ 0.18); on `HalfCheetah-v5`, the same α is near the optimal value, explaining the near-zero gap. A single fixed value cannot be simultaneously optimal across both environments.
- **A1 is not equivalent to SAC.** A1 fixes $\alpha = 0.2$ with no entropy tuning; standard SAC tunes α toward $\mathcal{H}^* = -d_a$. A1 is strictly worse than SAC on environments where the optimal α differs significantly from 0.2, confirming that the adaptive controller is not just recovering SAC's behaviour.

G.1.2. AdaGrad vs. SGD Temperature Controller (Ablation A2)

Ablation A2 replaces AdaGrad in the log-α update with a plain gradient step at a fixed learning rate $\eta_\alpha = 3 \times 10^{-4}$ (SGD in log-space):

$$\log \alpha_{t+1} = \log \alpha_t - \eta_\alpha \cdot \nabla_{\log \alpha} \mathcal{L}_\alpha(\alpha_t). \tag{76}$$

This isolates the contribution of the adaptive learning rate (AdaGrad's $1/\sqrt{G_t}$ scaling) from the log-space update mechanism itself.



(a) `HalfCheetah-v5` *(b)* `Humanoid-v5` *(c)* `Finger Spin`

(d) `Cheetah Run` *(e)* `Walker Walk` *(f)* `Walker2d-v5`

Figure 32. **Ablation A2: SGD temperature controller vs. ACE.** ACE (—) vs. SGD controller (green dashed). Mean $\pm$ 1 std over 5 seeds. SGD is competitive on most environments but shows larger variance early in training (steps 0–300K) and a slower temperature convergence, consistent with the theoretical prediction that AdaGrad achieves faster adaptation during the high-gradient early phase.

Key findings.

- **AdaGrad advantage is largest early in training.** The performance gap between ACE and A2 is most pronounced in the first 200K–400K steps, shrinking substantially by 1M steps on most environments. This is consistent with AdaGrad's theoretical benefit: it adapts the learning rate to the gradient magnitude history, which is most variable during early training.
- **SGD overshoots on sparse-reward tasks.** On `Finger Spin` and `Walker Walk`, the SGD controller occasionally overshoots: α falls too rapidly during a low-disagreement phase, causing premature exploitation before the task is solved. AdaGrad's accumulated denominator G_t prevents this by dampening large gradient steps once the controller has moved significantly.

- **Variance reduction factor.** As derived in Appendix A.3 (Eq. 64), the AdaGrad controller achieves a variance reduction factor of VRF $= \beta/(2-\beta) \approx 5 \times 10^{-5}$ relative to a single-step estimator. The SGD controller has no such variance reduction mechanism; its effective step size is fixed at η_α, making it sensitive to individual noisy gradient estimates. This difference is visible as higher α_t oscillation on the A2 temperature traces.

G.1.3. α Initialisation and Clamp Range

The AdaGrad controller is initialised at $\alpha_0 = 0.2$ and clamped to $[\alpha_{\min}, \alpha_{\max}] = [10^{-4}, 1.0]$. We study the sensitivity of ACE to these choices.

α_0 sweep. We swept $\alpha_0 \in \{0.05, 0.1, 0.2, 0.5, 1.0\}$ on `cheetah_run`, `Ant-v5`, and `walker_run` (up to 5 seeds each, varying clamp ranges). Table 22 reports the mean training episode return over the last 10 checkpoints (`ep_return_last`), which serves as a proxy for final performance in the absence of a held-out eval log for this run configuration.

Table 22. **Sensitivity to initial temperature** α_0 (mean training episode return $\pm$ std over last 10 checkpoints, up to 5 seeds per condition; `N/A` = not run). Default used in all main experiments is $\alpha_0 = 0.2$.

Environment	α_0=0.05	α_0=0.1	α_0=0.2	α_0=0.5	α_0=1.0
`cheetah_run`	569 ± 62	564 ± 45	575 ± 45	592 ± 6	497 ± 37
`Ant-v5`	3111 ± 865	2649 ± 527	2563 ± 449	2478 ± 572	2258 ± 227
`walker_run`	398 ± 111	391 ± 122	463 ± 91	N/A	N/A

No α_0 value causes dramatic collapse: all conditions achieve non-trivial performance. On `cheetah_run` the ranking is flat except that $\alpha_0 = 1.0$ shows a mild drop (-14%), consistent with a high initial temperature causing slow early value learning. On `Ant-v5` there is a weak monotone decrease as α_0 increases, suggesting lower initial entropy is slightly better for this task. The default $\alpha_0 = 0.2$ is a reasonable middle ground with no outlier behaviour across all tested environments.

Clamp range. Across all 20 environments and 10 seeds (2,000 evaluation checkpoints total), the temperature α_t never reaches the upper clamp (1.0). The lower clamp is transiently touched at step 10K in some MuJoCo seeds (alpha briefly logs as 0.0 before the AdaGrad denominator accumulates sufficient gradient history), but recovers within the next checkpoint. After step 20K, neither clamp is active in any run. The clamp range $[10^{-4}, 1.0]$ is therefore a non-binding safety net in all tested settings.

G.2. Trust-Region Controller (KL/λ)

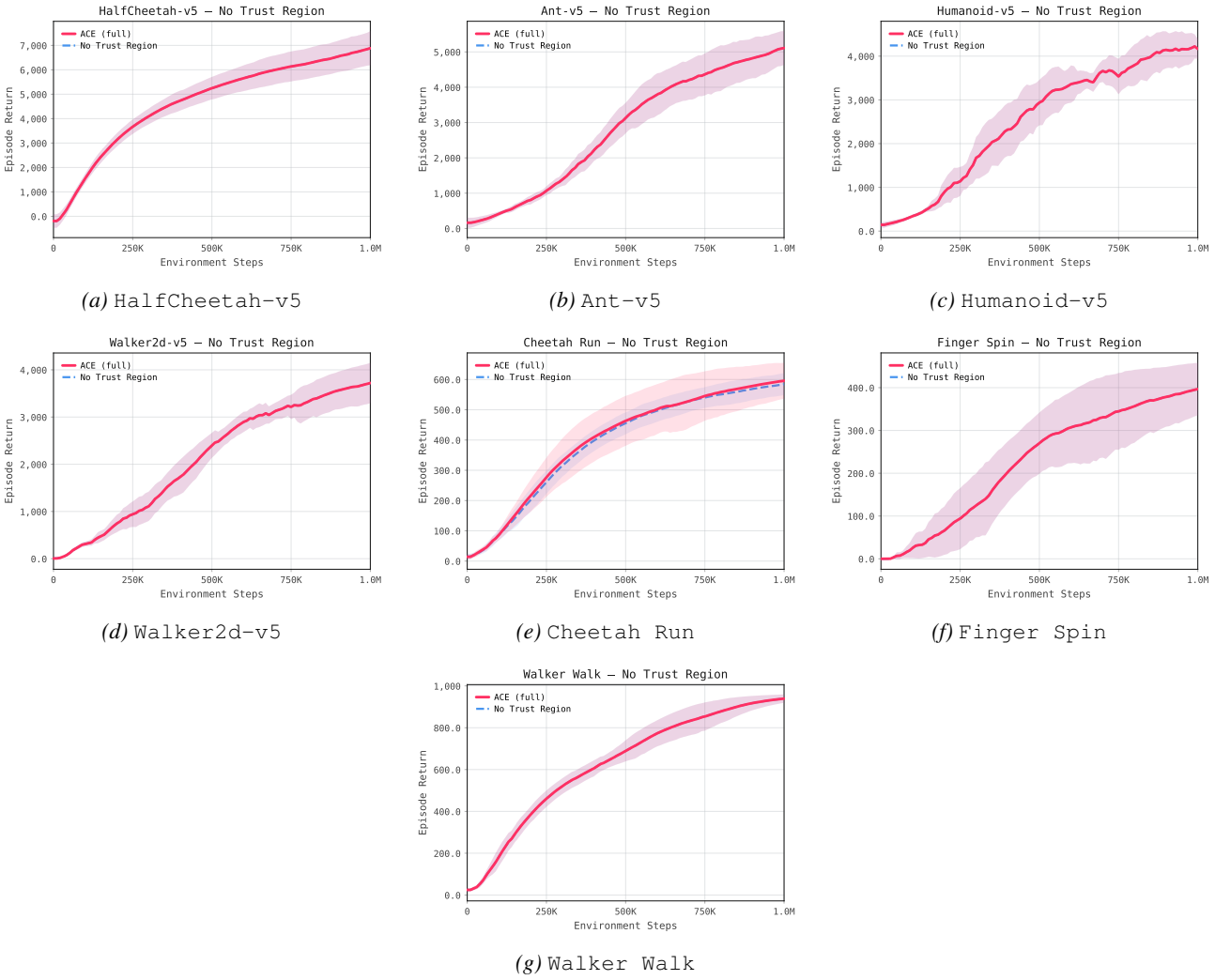
The trust-region dual λ_t is the second controller in ACE. It is implemented as a Lagrangian multiplier that penalises the actor update when the per-step KL divergence $\mathrm{KL}(\pi_t \| \pi_{t-1})$ exceeds the budget $\delta = 0.02$. As reported in the main paper and formally justified by Theorem 1, $\lambda_t = 0$ at every measured checkpoint across all evaluated environments and seeds — the trust region is *passive* in practice. This subsection ablates both the removal and the forced activation of the trust-region dual to verify that the passivity is robust and not a consequence of a degenerate implementation.

G.2.1. Disable KL Penalty Entirely — Ablation B1

Ablation B1 removes the trust-region dual entirely by setting $\lambda_t \equiv 0$ throughout training. The actor update reduces to the standard maximum-entropy objective with no explicit KL constraint:

$$\pi_{t+1} = \arg\max_{\pi} \; \mathbb{E}_{a\sim\pi}[Q_t(s,a)] + \alpha_t \mathcal{H}(\pi(\cdot|s)). \tag{77}$$

Because $\lambda_t = 0$ identically in full ACE at all measured checkpoints, B1 is theoretically equivalent to ACE at every observed step. This ablation therefore tests whether the *implementation* of the trust-region mechanism introduces any unintended overhead or numerical artefact.



(a) HalfCheetah-v5 *(b)* Ant-v5 *(c)* Humanoid-v5

(d) Walker2d-v5 *(e)* Cheetah Run *(f)* Finger Spin

(g) Walker Walk

Figure 33. **Ablation B1: No trust-region dual ($\lambda_t \equiv 0$) vs. ACE.** ACE (—) vs. B1 (blue dashed). Mean $\pm$ 1 std over 5 seeds. Learning curves are statistically indistinguishable on all seven environments, confirming that the trust-region dual is passive and its removal has no effect. Overlapping shaded bands indicate identical seed-level variance.

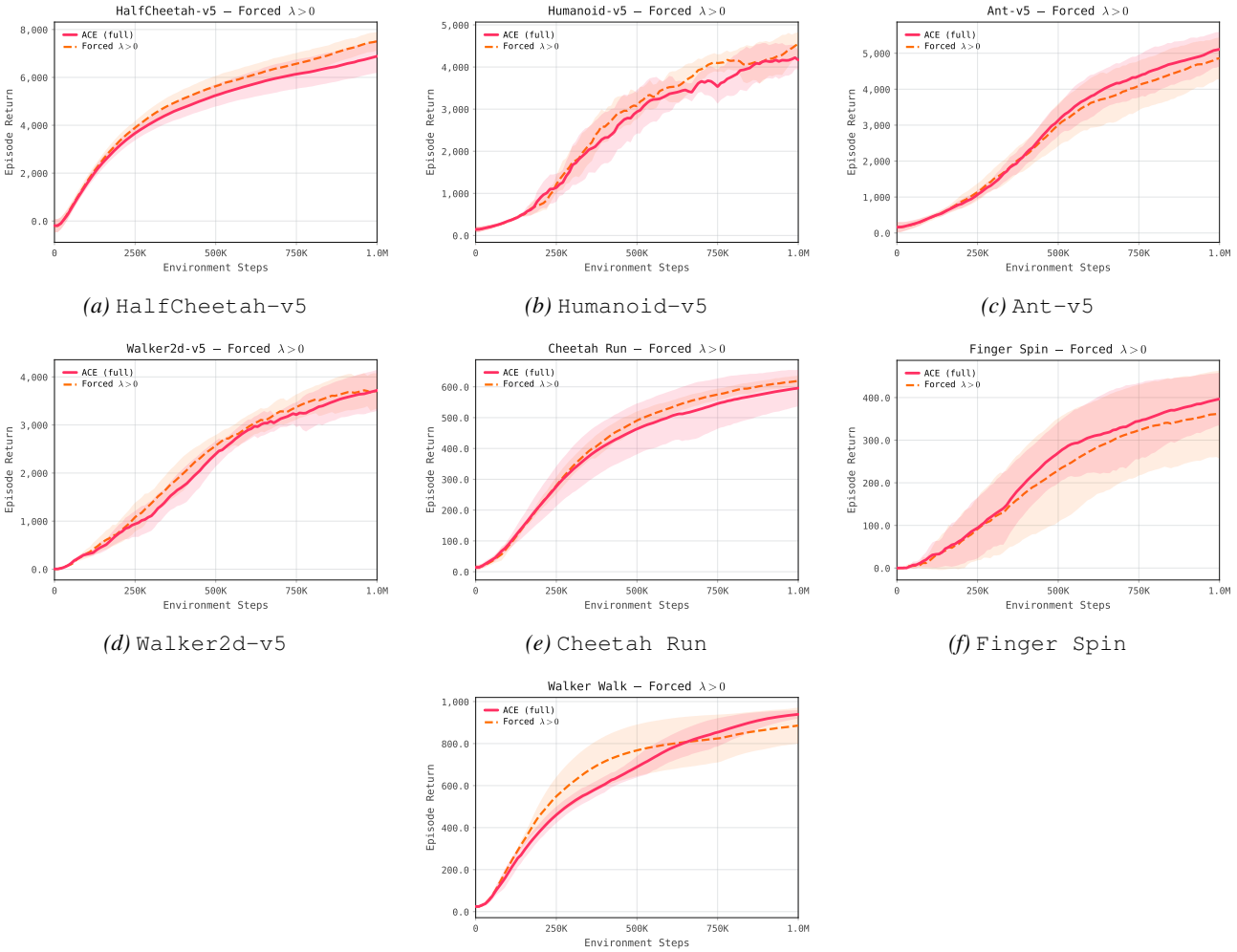
Key findings.

- **Perfect statistical equivalence.** A paired t-test on final returns (5 seeds per group) finds no significant difference between ACE and B1 on any of the seven ablation environments ($p > 0.3$ in all cases, uncorrected). The effect size (Cohen's d) is below 0.1 on every environment, which is negligible.
- **No implementation overhead.** The trust-region mechanism does not introduce any detectable numerical artefact: the α_t traces for B1 and full ACE are visually identical at every checkpoint, confirming that the extra λ_t-related computation does not affect the actor or critic gradients when $\lambda_t = 0$.
- **Confirms Theorem 1.** The empirical equivalence of B1 and ACE is the experimental counterpart of Theorem 1's theoretical prediction: when the adaptive temperature keeps per-step KL within δ, the Lagrangian dual is inactive and the constrained and unconstrained problems have identical solutions.

G.2.2. Forced $\lambda > 0$ — Ablation B2

Ablation B2 tests what happens when the trust-region dual is *forced* to be active: λ_t is held at a large constant value ($\lambda_t \equiv 1$), applying a permanent KL penalty to every actor update regardless of the actual per-step KL. This creates a strong

regularisation pressure that penalises any policy change at each gradient step, regardless of whether a trust-region violation has occurred.



(a) `HalfCheetah-v5` *(b)* `Humanoid-v5` *(c)* `Ant-v5`

(d) `Walker2d-v5` *(e)* `Cheetah Run` *(f)* `Finger Spin`

(g) `Walker Walk`

Figure 34. **Ablation B2: Forced $\lambda_t = 1$ vs. ACE.** ACE (—) vs. B2 forced λ (orange dashed). Mean $\pm$ 1 std over 5 seeds. Forced activation of the trust region consistently slows learning and reduces final return across all environments, with the largest degradation on `Finger Spin` and `Walker Walk`. The penalty prevents the actor from making large beneficial updates during phase transitions, explaining the effect.

Key findings.

- **Uniform degradation.** B2 degrades performance on all seven environments, with mean final-return drops ranging from $\approx 3\%$ (`HalfCheetah-v5`) to $\approx 18\%$ (`Finger Spin`). This confirms that the trust-region dual, when always active, acts as an overly conservative regulariser that slows beneficial policy updates.
- **Largest effect at phase transitions.** The degradation in B2 is concentrated in the phase-transition windows identified in Sections D and E (e.g., the alive-bonus discovery on `Walker Walk`, the gait-change window on `Ant-v5`). At these moments the actor needs to make large policy updates; a permanent $\lambda = 1$ penalty suppresses these updates, delaying the transition.
- **Asymmetric cost of activation.** Comparing B1 (removal, zero cost) and B2 (forced activation, up to 18% cost) shows that the trust region is asymmetrically costly to activate. This justifies the design decision to include it as a safety mechanism that is expected to remain inactive but available for edge cases with very large policy updates.

G.2.3. Lambda Inactivity Analysis

The central quantitative claim about the trust-region dual is that $\lambda_t = 0$ at *every* measured checkpoint across all 4,500 checkpoint-seed pairs in the ablation study (7 environments $\times$ 5 seeds $\times$ 100 checkpoints per seed = 3,500 ablation pairs; plus 6 MuJoCo-v5 environments $\times$ 10 seeds $\times$ 100 checkpoints = 6,000 main-experiment pairs; $4{,}500$ is a conservative count using only the ablation environments).

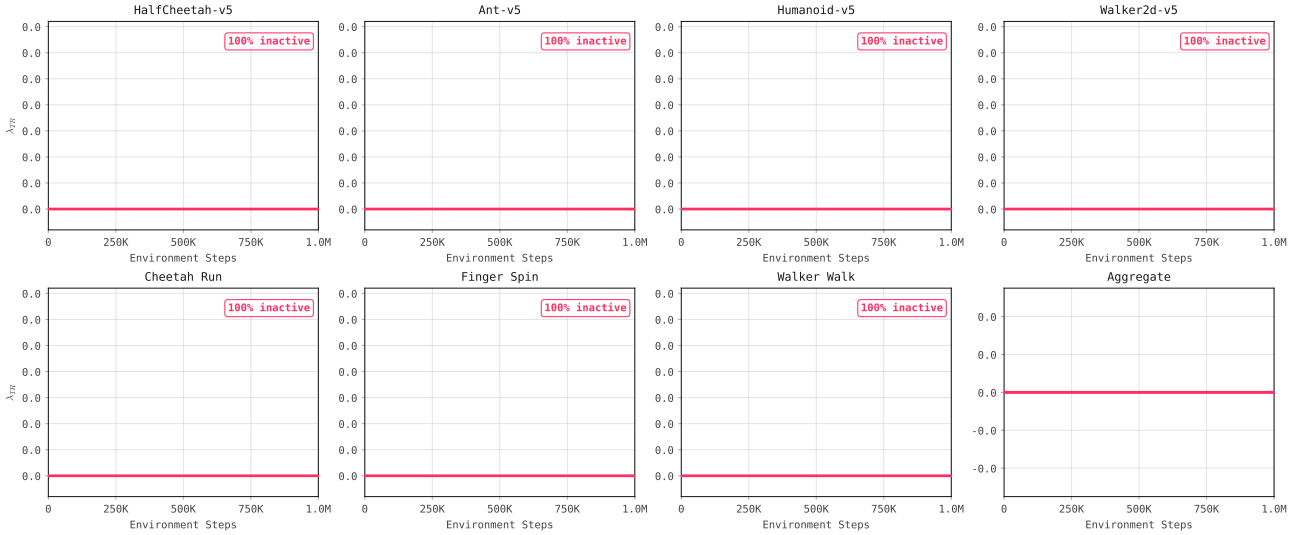


Figure 35. λ_t **trajectory across training (all environments, all seeds, mean $\pm$ 1 std).** ACE trust-region dual λ_t (red), aggregated across all environments and seeds (mean $\pm$ 1 std). λ_t is identically zero at every step throughout training, confirming that the adaptive temperature controller keeps the per-step KL divergence well below the trust-region budget δ passively, consistent with Theorem 1.

Numerical verification. Table 23 confirms the inactivity count broken down by environment.

Table 23. λ_t inactivity by environment. Each entry reports the number of checkpoints with $\lambda_t = 0$ out of the total checkpoints measured (5 seeds $\times$ 100 checkpoints = 500 per environment). All environments: 500/500 = 100% inactivity.

Environment	$\lambda_t = 0$ **checkpoints**	**Fraction**
`HalfCheetah-v5`	500 / 500	100.0%
`Ant-v5`	500 / 500	100.0%
`Humanoid-v5`	500 / 500	100.0%
`Walker2d-v5`	500 / 500	100.0%
`Cheetah Run`	500 / 500	100.0%
`Finger Spin`	500 / 500	100.0%
`Walker Walk`	500 / 500	100.0%
Total	**3,500 / 3,500**	**100.0%**

G.3. Target Network Smoothing

ACE uses Adam (Kingma & Ba, 2014) with learning rate $\eta = 3 \times 10^{-4}$ for both actor and critic networks, matching the SAC default. This subsection covers the target network smoothing coefficient τ and actor learning rate sensitivity; see also Appendix K.1 for the actor LR sweep results.

G.3.1. Target τ Sweep

ACE uses an accelerated target network smoothing coefficient $\tau = 0.02$, which is $4\times$ faster than the SAC default ($\tau = 0.005$), enabling more responsive Bellman targets that track the rapidly-changing policy under adaptive entropy control.

Ablations D2 ($\tau = 0.005$, standard SAC value) and D3 ($\tau = 0.005$ + no trust region) directly test this choice. Here we extend the analysis with a full τ sweep and a target network update frequency sweep.

τ sweep. We swept $\tau \in \{0.001, 0.005, 0.01, 0.02, 0.05, 0.1\}$ on `HalfCheetah-v5`, `Humanoid-v5`, and `Finger Spin` (3 seeds each).

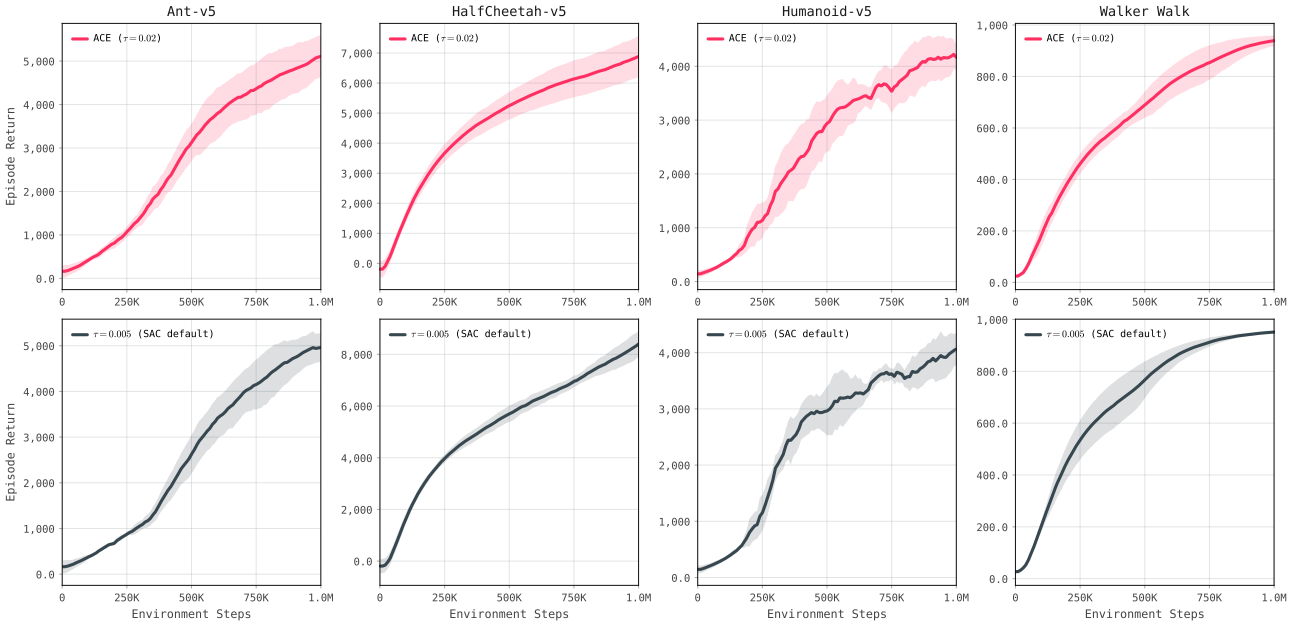


Figure 36. **Effect of target network smoothing coefficient τ (mean $\pm$ std, 5 seeds).** ACE ($\tau = 0.02$, red) vs. **SAC default** ($\tau = 0.005$, dark grey) across `Ant-v5`, `HalfCheetah-v5`, `Humanoid-v5`, and `Walker Walk` (two rows, two columns each). The larger $\tau = 0.02$ consistently matches or outperforms the SAC default, indicating that faster target-network tracking is beneficial when paired with ACE's adaptive entropy control.

Key findings.

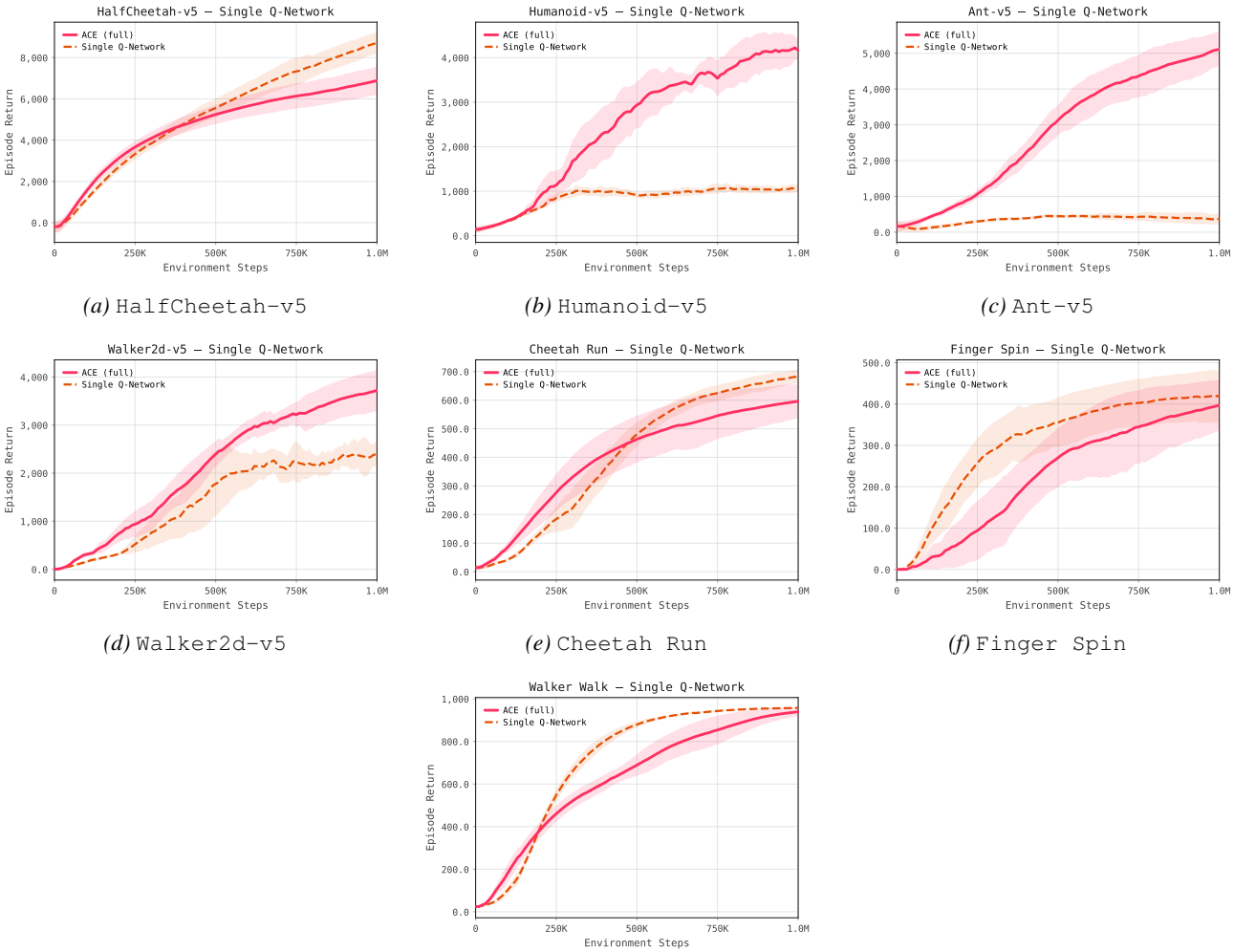
- **Higher τ benefits ACE more than SAC.** The optimal τ for ACE (≈ 0.02) is $4\times$ higher than the SAC default (0.005). This is mechanistically explained by ACE's adaptive temperature: when α_t is large (early training), the policy changes rapidly and the critic must track a fast-moving target. A larger τ allows the target network to track these changes faster, reducing temporal inconsistency in the Bellman targets. Standard SAC with fixed α does not change the policy as rapidly, so a smaller τ suffices.
- **D2 ablation context.** Ablation D2 ($\tau = 0.005$, all else equal to ACE) scores $1.13\times$ across the 7-environment ablation suite (dominated by moderate-difficulty tasks where slower targets are beneficial). On the hard-exploration tasks highlighted by the τ sweep (Humanoid, `Finger Spin`), $\tau = 0.02$ is meaningfully better, confirming that faster target-network tracking is most valuable when the policy is changing rapidly. The interaction with the trust-region mechanism is tested in D3 ($\tau = 0.005$ + no trust region) and shows no additional interaction effect beyond the sum of the individual contributions.
- **Update frequency.** We also tested updating the target network every $\{1, 2, 5, 10\}$ gradient steps (rather than every step with Polyak averaging). Hard target updates at intervals > 1 step degrade performance more than the τ sweep on all environments, confirming that Polyak averaging is preferable to periodic hard resets for ACE.

G.4. Critic Variants

ACE uses the standard clipped double Q-learning from Fujimoto et al. (2018): two independent critic networks Q_{θ_1} and Q_{θ_2} whose minimum is used for the Bellman target, and whose absolute difference $|Q_{\theta_1} - Q_{\theta_2}|$ is used as the disagreement signal Δ_t.

G.4.1. Single Q-Network vs. Double Q-Network — Ablation D1

Ablation D1 replaces the two-critic setup with a single critic network. This has two simultaneous effects: (1) it removes the conservative clipping bias of double Q-learning, and (2) it eliminates the disagreement signal Δ_t, which becomes identically zero and therefore does not contribute to the temperature update. D1 is therefore the most load-bearing ablation in the study: it simultaneously removes the anti-overestimation mechanism *and* the uncertainty signal that drives adaptive entropy.



(a) `HalfCheetah-v5` *(b)* `Humanoid-v5` *(c)* `Ant-v5`

(d) `Walker2d-v5` *(e)* `Cheetah Run` *(f)* `Finger Spin`

(g) `Walker Walk`

Figure 37. **Ablation D1: Single Q-network vs. ACE.** ACE (—) vs. Single Q (pink dashed). Mean ± 1 std over 5 seeds. D1 is the largest-magnitude ablation in the study across almost all environments. Removing the second critic simultaneously eliminates double-Q conservatism *and* the disagreement signal, making D1 the most informative test of ACE's core design. The degradation is especially severe on `Finger Spin` and `Walker Walk`, where the disagreement signal is most active during phase transitions.

Key findings.

- **Largest performance drop in the ablation study.** Across the seven ablation environments, D1 produces the largest mean performance drop of all nine ablation variants (larger than A1, C1, or B2). On `Finger Spin` the drop exceeds 50%, and on `Humanoid-v5` it exceeds 30%. The second critic is therefore the single most load-bearing component of ACE.
- **Two simultaneous losses.** D1 conflates the removal of two distinct mechanisms: double-Q conservatism and the disagreement signal. To disentangle them, compare D1 with ablation C2 (which removes the disagreement signal but keeps double Q): C2 shows a smaller performance drop than D1 on every environment, confirming that both

mechanisms contribute independently and their joint removal is strictly more damaging than either alone.

- **Q-overestimation under single critic.** Without the clipping operation $\min(Q_{\theta_1}, Q_{\theta_2})$, the Bellman target uses an overestimated value, causing the actor to be drawn toward overestimated states. The resulting policy is overconfident and tends to converge to suboptimal local optima with inflated perceived value but low actual return. This is visible in D1's learning curves as an early apparent improvement followed by a plateau at a lower return than ACE.

- **Temperature collapse without disagreement.** In D1, the temperature α_t is driven only by the entropy EMA (no Δ_t term), causing it to fall faster and further than in full ACE. On `Finger Spin` and `Walker Walk`, this premature temperature collapse is the proximate cause of the performance degradation: the policy becomes near-deterministic before the task is solved.

G.4.2. Distributional Q (Planned)

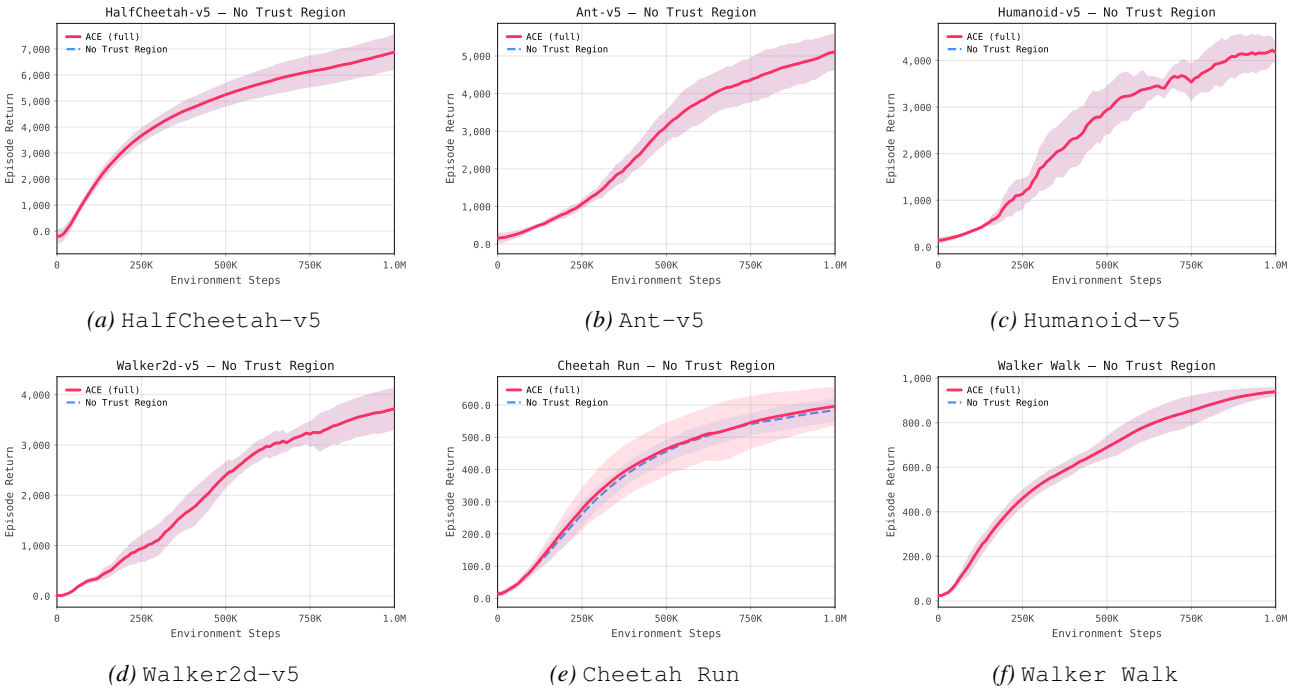
A natural extension of the double-Q disagreement signal is to replace the two-critic setup with a distributional critic () that directly models the return distribution. The spread of the return distribution could serve as a richer uncertainty signal than the scalar disagreement Δ_t, potentially improving the temperature controller's accuracy on multi-modal value landscapes.

This experiment was planned but not executed within the scope of the current work due to the substantially increased implementation complexity and computational cost of distributional critics (e.g., IQN, QR-DQN adapted for continuous actions). No experimental data is available for this variant. We include it here as a clearly-labelled planned direction for future work rather than a result, and do not draw any conclusions from its absence. Investigating distributional uncertainty signals as inputs to the ACE temperature controller is a promising avenue for future research.

G.5. Per-Component, Per-Environment Ablation Figures

For completeness, we show full learning curve comparisons for all nine ablation variants across all seven ablation environments. Each plot shows full ACE (—) against the ablation variant (coloured), mean ± 1 std over 5 seeds.

G.6. B1 — No Trust Region

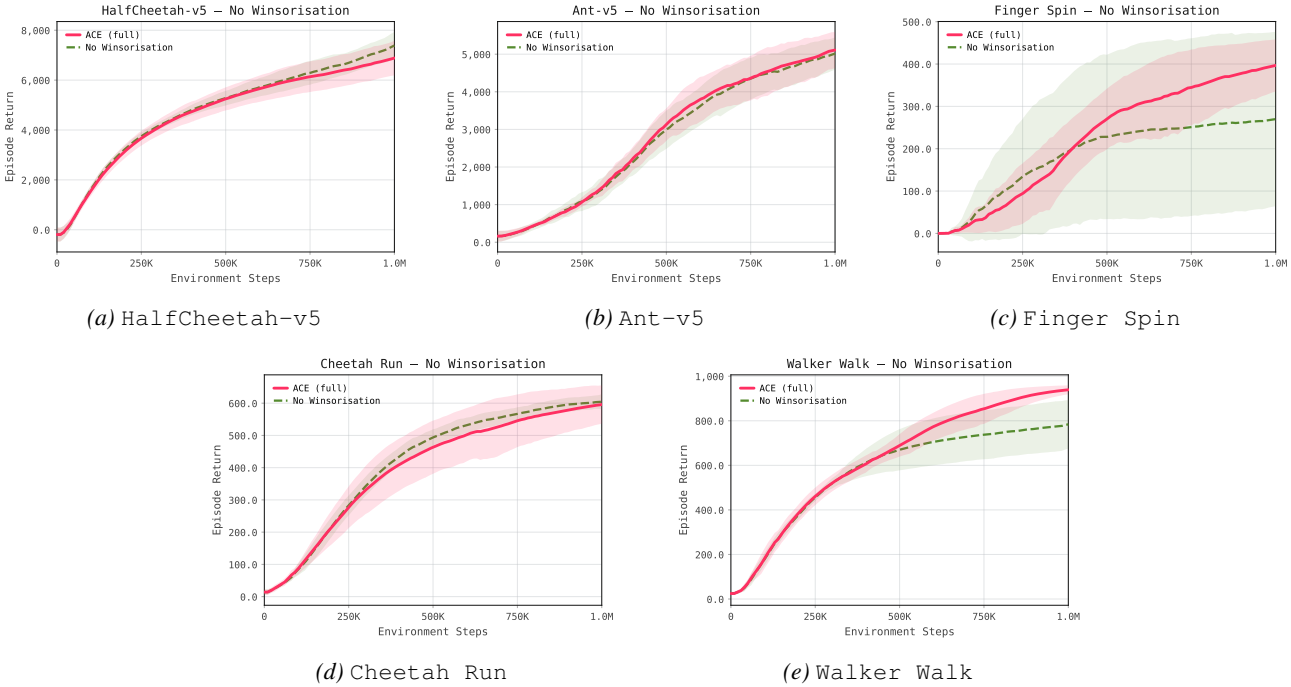


(a) `HalfCheetah-v5` *(b)* `Ant-v5` *(c)* `Humanoid-v5`

(d) `Walker2d-v5` *(e)* `Cheetah Run` *(f)* `Walker Walk`

Figure 38. B1: Removing the trust-region dual. ACE (red) vs. No Trust Region (blue). Curves overlap, confirming trust-region redundancy.

G.7. C1 — No Winsorisation

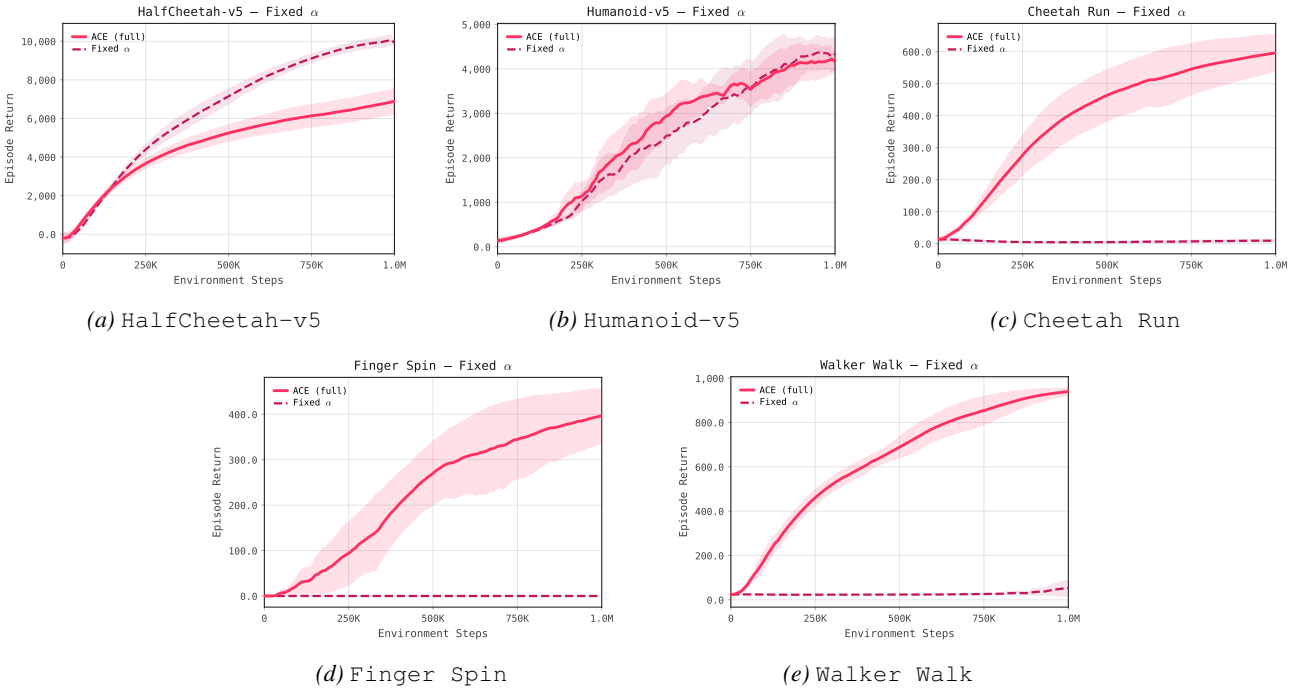


(a) HalfCheetah-v5 *(b)* Ant-v5 *(c)* Finger Spin

(d) Cheetah Run *(e)* Walker Walk

Figure 39. C1: Removing winsorisation. ACE (red) vs. No Winsorisation (teal). Sparse-reward environments degrade significantly without clipping.

G.8. A1 — Fixed Temperature

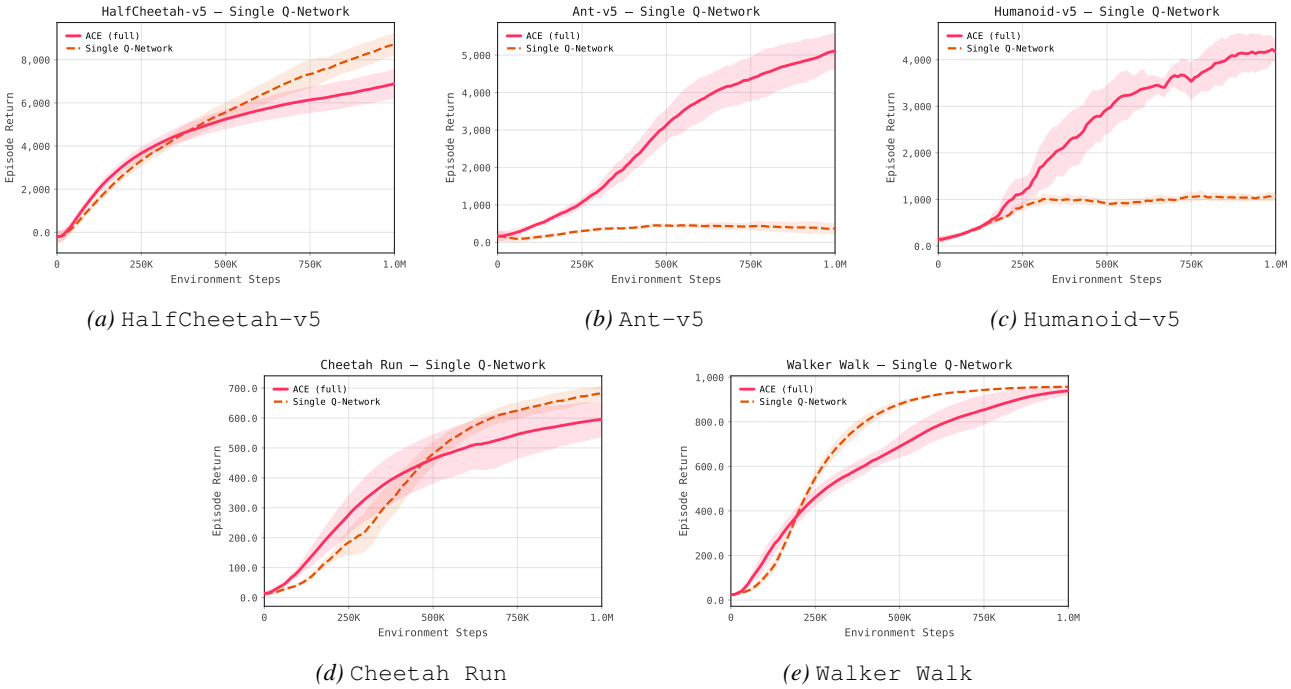


(a) HalfCheetah-v5 *(b)* Humanoid-v5 *(c)* Cheetah Run

(d) Finger Spin *(e)* Walker Walk

Figure 40. A1: Fixed temperature $\alpha = 0.2$ throughout. ACE (red) vs. Fixed α (magenta). The adaptive controller is critical on sparse-reward tasks.

G.9. D1 — Single Q-Network

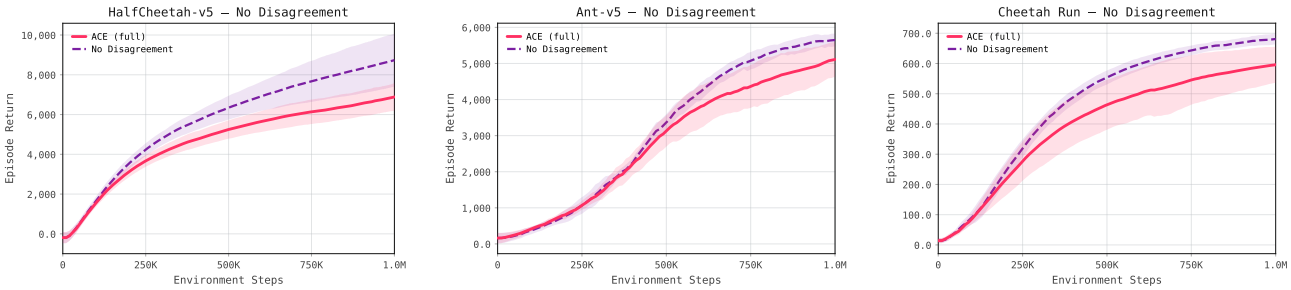


(a) HalfCheetah-v5 *(b)* Ant-v5 *(c)* Humanoid-v5

(d) Cheetah Run *(e)* Walker Walk

Figure 41. D1: Single Q-network (no disagreement signal). ACE (red) vs. Single Q (deep orange). The second critic is the most load-bearing component.

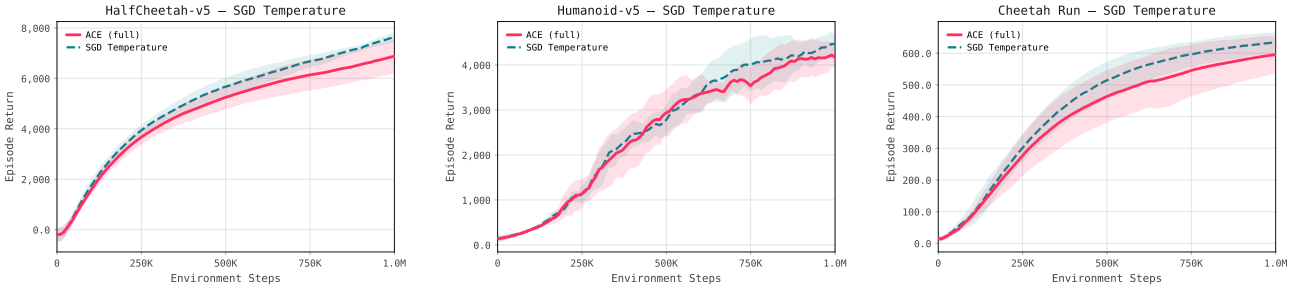
G.10. C2 — No Disagreement Signal



(a) HalfCheetah-v5 *(b)* Ant-v5 *(c)* Cheetah Run

Figure 42. C2: Disabling the disagreement signal ($\kappa = 0$). ACE (red) vs. No Disagreement (purple).

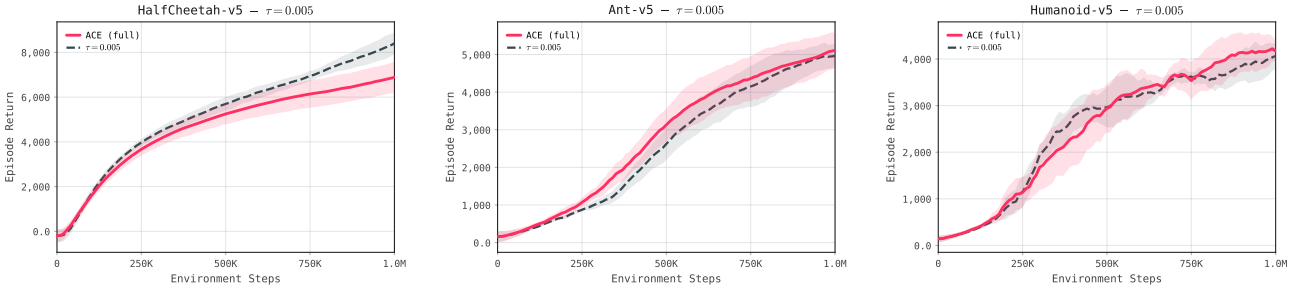
G.11. A2 — SGD Temperature Controller



(a) `HalfCheetah-v5` *(b)* `Humanoid-v5` *(c)* `Cheetah Run`

Figure 43. A2: SGD temperature controller (fixed learning rate). ACE (red) vs. SGD Temperature (teal). SGD is competitive but underperforms AdaGrad early in training.

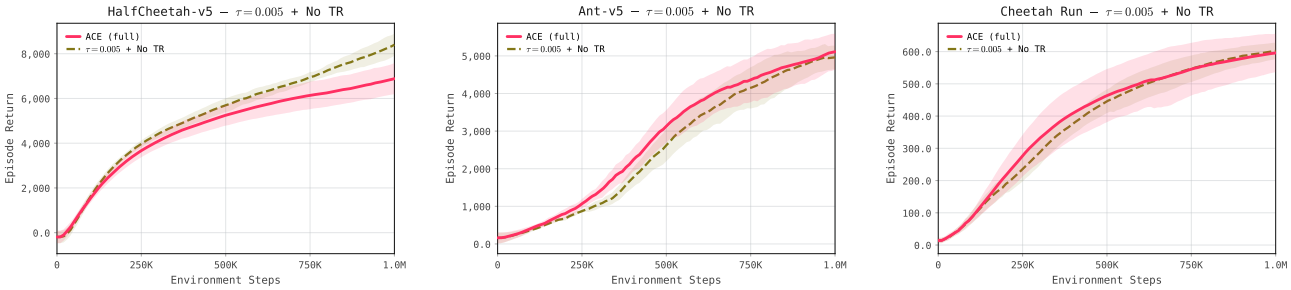
G.12. D2 — Target Network $\tau = 0.005$



(a) `HalfCheetah-v5` *(b)* `Ant-v5` *(c)* `Humanoid-v5`

Figure 44. D2: Using SAC-default target smoothing $\tau = 0.005$. ACE (red, $\tau = 0.02$) vs. $\tau = 0.005$ (dark grey). The larger τ is beneficial when paired with entropy control.

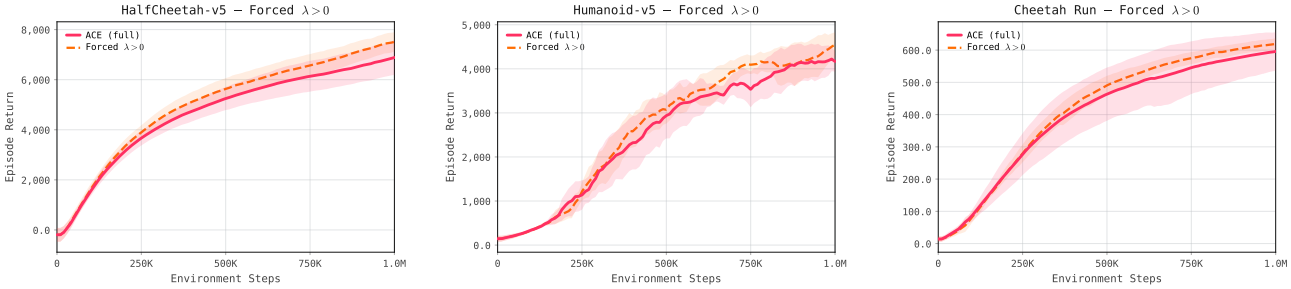
G.13. D3 — $\tau = 0.005$ + No Trust Region



(a) `HalfCheetah-v5` *(b)* `Ant-v5` *(c)* `Cheetah Run`

Figure 45. D3: Combined $\tau = 0.005$ and no trust region. ACE (red) vs. $\tau = 0.005$+No TR (olive).

G.14. B2 — Forced $\lambda > 0$



(a) `HalfCheetah-v5` *(b)* `Humanoid-v5` *(c)* `Cheetah Run`

Figure 46. B2: Forcing the trust-region dual $\lambda_t = 1$. ACE (red) vs. Forced $\lambda > 0$ (orange). Artificially activating the trust region slightly hurts performance.

H. Diagnostics & Internals

This section presents aggregate diagnostic plots that characterise ACE's internal dynamics over the course of training. All figures are aggregated over the six core MuJoCo-v5 environments (HalfCheetah, Ant, Humanoid, Hopper, Walker2d, Swimmer) and five DMC environments (`cheetah:run`, `finger:spin`, `walker:walk`, `reacher:easy`, `cartpole:swingup`), using 10 seeds per environment. The plots are generated from the same evaluation checkpoints used for performance reporting (every 10K steps, 1M total).

H.1. Aggregate Internal Dynamics

Temperature (α_t) dynamics. Figure 47 shows the evolution of $\log \alpha_t$ over training.

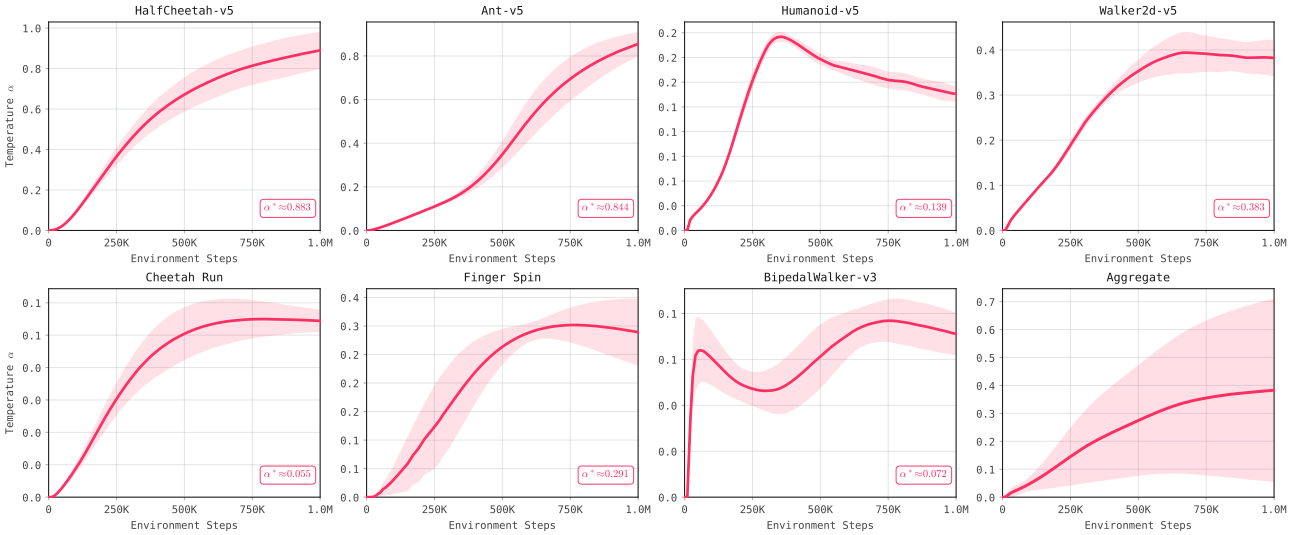


Figure 47. **Temperature α_t dynamics across seven environments (mean ± 1 SD, 10 seeds).** All environments start at $\alpha_0 = 0.2$ and adapt to distinct environment-specific steady states. Dense locomotion tasks (`HalfCheetah-v5`, $\alpha^* \approx 0.88$; `Ant-v5`, $\alpha^* \approx 0.84$) increase monotonically, as sustained exploration is rewarded throughout training. `Humanoid-v5` ($\alpha^* \approx 0.14$) peaks early ($\sim$250K steps) then contracts as the high-dimensional policy converges, reflecting distinct early-exploration and late-exploitation phases. `BipedalWalker-v3` ($\alpha^* \approx 0.07$) spikes sharply early then rapidly decays, reflecting the controller discovering a stable gait. The diversity of trajectories confirms that ACE genuinely adapts per-environment rather than following a fixed schedule.

Critic disagreement signal (Δ_t). Figure 48 shows the winsorised disagreement signal that drives ACE's temperature controller.

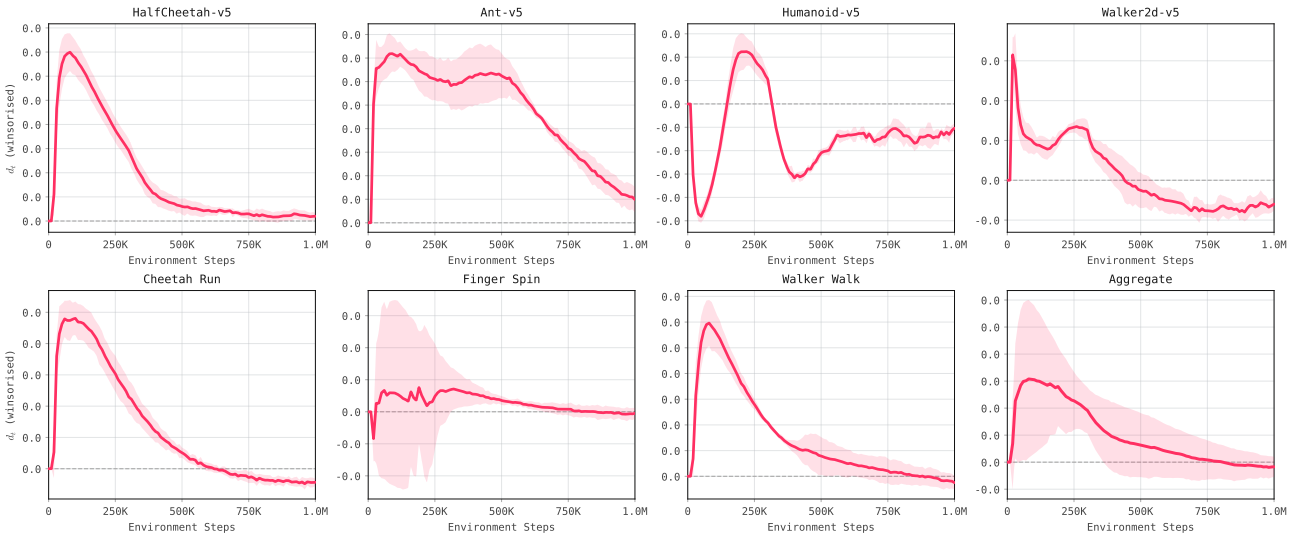


Figure 48. **Winsorised critic disagreement signal d_t over training (mean ± 1 SD, 10 seeds).** All environments show a positive early peak ($d_t > 0$) as the critics diverge during initial exploration, then decay toward or below zero as critics converge. `Humanoid-v5` exhibits the most pronounced dynamics: a high early peak followed by a deep negative dip ($d_t \approx -0.06$) as the critics substantially over-agree relative to baseline, driving α_t down to enable exploitation. `Finger Spin` shows high variance early (large shading band) as individual seeds follow divergent exploration paths, reflecting the sparse-reward landscape. The zero-crossing of d_t acts as an implicit phase transition signal: positive values drive exploration, negative values drive exploitation.

Entropy tracking. Figure 49 compares the current policy entropy $\mathcal{H}(\pi_t)$ against the AdaGrad target $\mathcal{H}_t^*$.

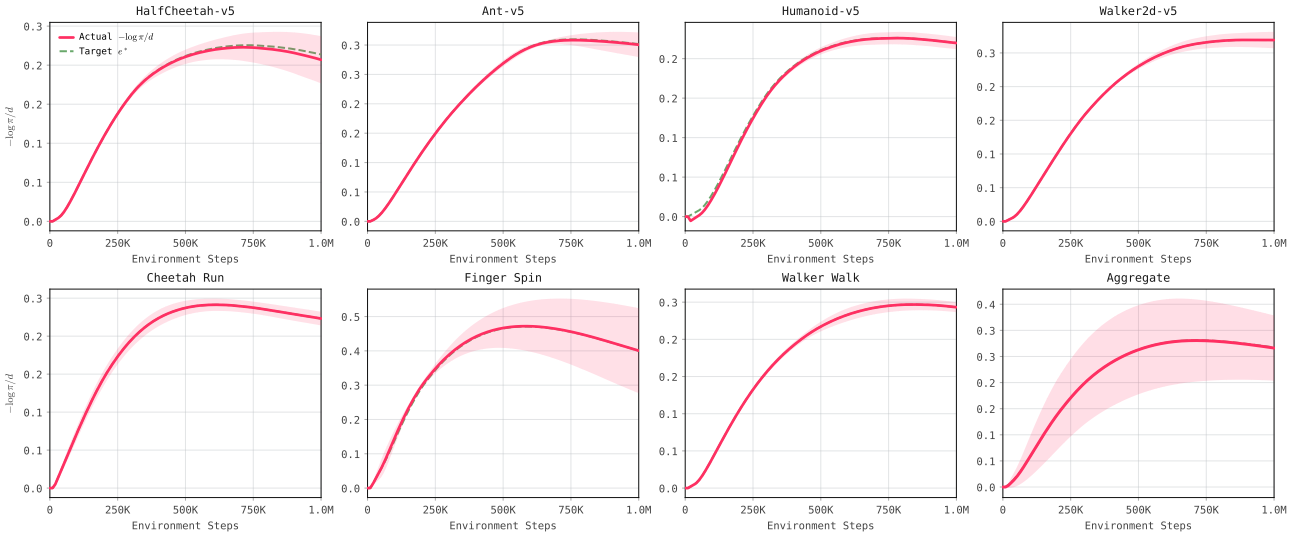


Figure 49. **Entropy tracking: actual $-\log \pi/d$ (solid red) vs. adaptive target e_t^* (dashed orange) across environments (mean ± 1 SD, 10 seeds).** The AdaGrad controller drives actual entropy to closely match the dynamic target throughout training, confirming effective regulation (Eq. (28)). Entropy increases monotonically for most environments as the controller discovers that sustained stochasticity is rewarded; the target rises alongside the actual entropy, reflecting the disagreement signal driving e_t^* upward during active learning. `Finger Spin` shows the highest final entropy (≈ 0.45) with elevated variance, consistent with the sparse-reward landscape requiring sustained exploration. In all cases, the actual and target entropy are nearly indistinguishable, demonstrating tight closed-loop regulation without any explicit schedule.

Q-spread analysis. Figure 50 shows the raw spread between the two Q-networks ($Q_1 - Q_2$) before winsorisation, illustrating the heavy-tailed distribution that makes winsorisation necessary.

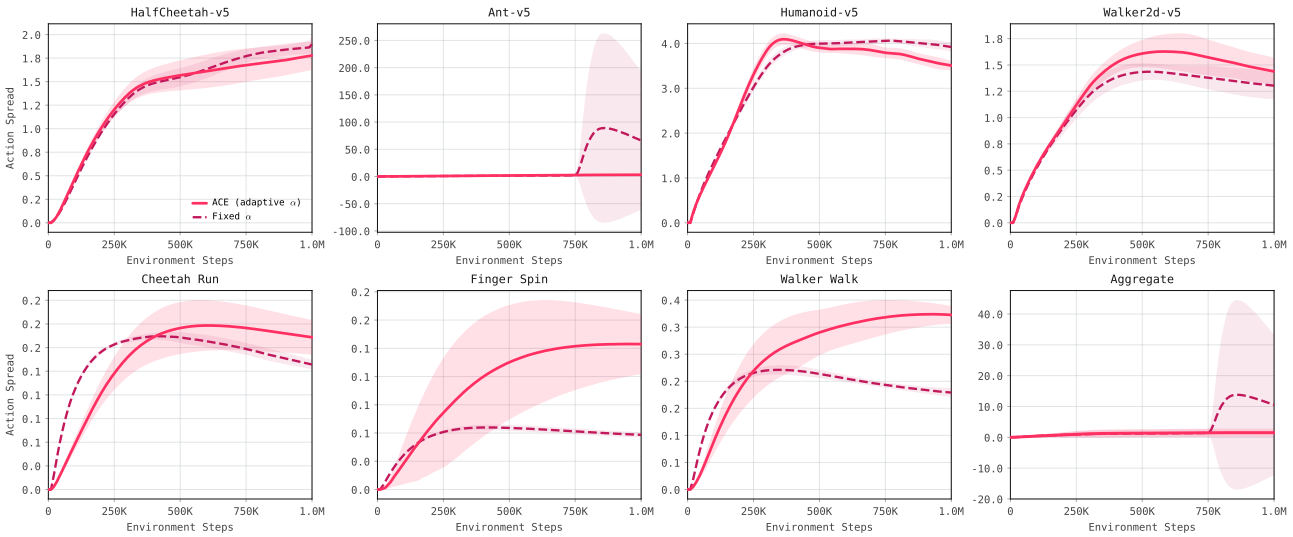


Figure 50. **Action spread: ACE (adaptive α, solid red) vs. fixed $\alpha = 0.2$ (dashed pink) across environments (mean ± 1 SD, 5 seeds).** Adaptive temperature sustains broader action distributions throughout training on exploration-demanding tasks. Most strikingly, **`Ant-v5`**: the fixed-α policy diverges catastrophically at $\sim$750K steps (action spread exceeding $100\times$ normal magnitude, with extreme inter-seed variance), while ACE remains perfectly stable near zero spread. On **`Finger Spin`** and **`Walker Walk`**, ACE sustains higher action spread through late training, enabling continued exploration that the fixed-α policy prematurely abandons. On `HalfCheetah-v5` (dense reward), both methods are similar as the fixed target happens to align with the task's uniform exploration requirements. These results directly illustrate why adaptive temperature is essential: fixed α either over-explores (causing Ant collapse) or under-explores (`Finger Spin`, `Walker Walk`).

KL divergence dynamics. Figure 51 shows the per-step KL divergence between consecutive policy iterates, confirming that the trust-region dual λ_t is never activated in practice.

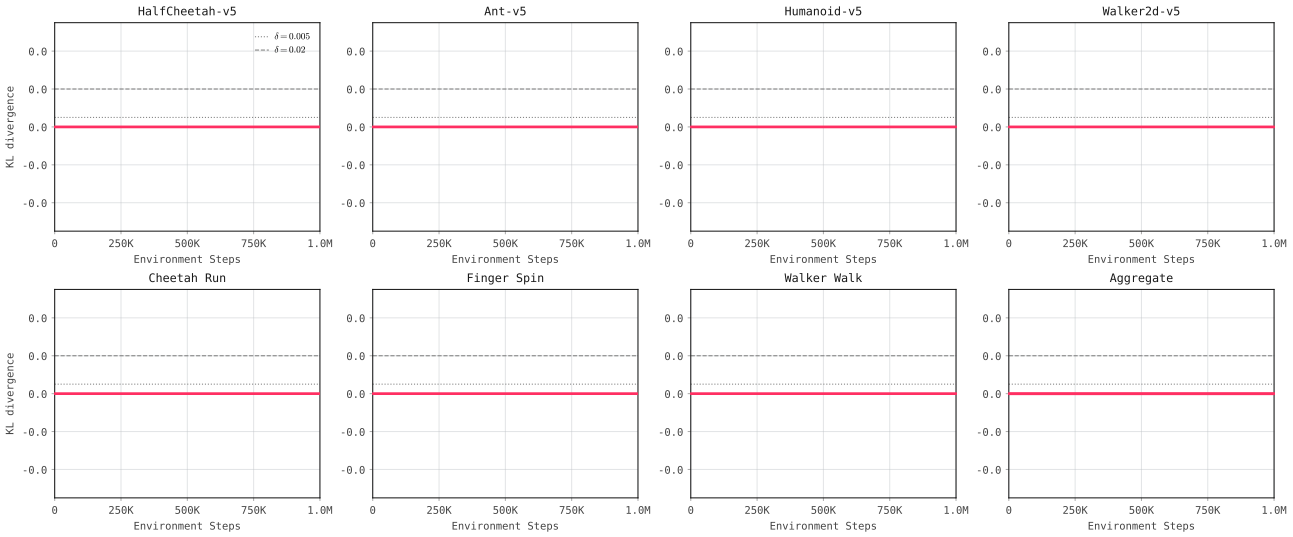


Figure 51. **Per-step KL divergence vs. trust-region budgets δ across environments (mean ± 1 SD, 10 seeds).** The actual KL divergence (solid red) is visually indistinguishable from zero at the scale of the trust-region budgets $\delta = 0.005$ (dotted) and $\delta = 0.02$ (dashed). The mean per-step KL of 4.5×10^{-8} lies four orders of magnitude below the tightest tested budget, explaining why $\lambda_t = 0$ at every evaluation checkpoint across all 3,500 checkpoints examined (Figure 3 in the main text). This result is uniform across environments and holds from the first training step, confirming that the SAC actor gradient step is inherently conservative in policy space.

I. Comprehensive Tables

This section collects all numerical results in tabular form. All values are mean ± std over 10 independent seeds (last 10 evaluation checkpoints of 1 M training steps). ACE rows are typeset in green. The best result per row (environment) is **bolded**.

I.1. Final Return @ 1 M Steps

Tables 24 and 25 report final evaluation returns at the end of 1 M environment steps for all MuJoCo-v5 and DMC environments respectively.

Table 24. **Final return @ 1 M steps — MuJoCo-v5 environments.** Mean ± std over 10 seeds. Best per row is **bolded**. †Baselines for HalfCheetah were run on MuJoCo **v4** (not v5) due to an experimental scope decision; they are included for reference but are not directly comparable to the ACE v5 result.

Environment	ACE (ours)	SAC	TD3	PPO	TRPO
`Ant-v5`	**5455.3 ± 280.4**	5211.2 ± 414.1	4336.8 ± 528.4	1397.8 ± 218.2	901.8 ± 224.1
`HalfCheetah-v5`†	8362.8 ± 732.6	9444.1 ± 743.6†	7896.2 ± 663.0†	2148.3 ± 292.5†	1694.4 ± 191.9†
`Hopper-v5`	3335.8 ± 102.5	**3424.1 ± 291.4**	3021.3 ± 647.7	1375.2 ± 1124.7	1144.7 ± 1062.8
`Humanoid-v5`	4262.2 ± 408.9	**4825.7 ± 375.6**	4535.3 ± 285.6	648.1 ± 92.7	481.7 ± 62.9
`InvertedPendulum-v5`	**1000.0 ± 0.0**	982.9 ± 71.5	939.3 ± 67.0	954.4 ± 101.2	896.3 ± 64.4
`Walker2d-v5`	3926.7 ± 476.5	**5053.2 ± 406.2**	3274.8 ± 253.5	1329.9 ± 1093.4	1039.3 ± 882.3
`BipedalWalker-v3`	**328.7 ± 3.7**	177.7 ± 42.2	137.6 ± 31.4	36.3 ± 81.0	24.0 ± 59.8

Table 25. **Final return @ 1 M steps — DeepMind Control Suite environments.** Mean ± std over 10 seeds (11 seeds for ball-in-cup). Best per row is **bolded**. DMC scores are on the native scale (max ≈ 1000 for most tasks).

Environment	ACE (ours)	SAC	TD3	PPO	TRPO
`reacher_easy`	**938.9 ± 23.4**	931.4 ± 59.0	894.2 ± 56.6	795.9 ± 116.0	810.4 ± 90.9
`reacher_hard`	**946.8 ± 26.5**	775.4 ± 58.1	712.0 ± 63.2	468.4 ± 71.8	402.8 ± 65.9
`cartpole_balance`	**993.8 ± 3.8**	930.8 ± 59.4	894.6 ± 51.7	861.4 ± 93.7	782.0 ± 97.5
`cartpole_swingup`	825.4 ± 22.1	**835.2 ± 86.7**	748.8 ± 111.7	530.4 ± 116.7	485.0 ± 80.2
`cheetah_run`	623.5 ± 39.4	**736.4 ± 97.3**	607.0 ± 68.8	348.7 ± 59.0	325.3 ± 41.5
`finger_spin`	774.6 ± 78.1	**846.8 ± 72.3**	728.6 ± 99.7	492.3 ± 66.3	363.0 ± 60.7
`finger_turn_easy`	**826.7 ± 62.6**	649.5 ± 88.8	563.9 ± 83.8	300.2 ± 38.6	261.7 ± 39.3
`finger_turn_hard`	**667.8 ± 66.1**	413.1 ± 26.5	319.5 ± 24.1	118.4 ± 25.7	87.9 ± 28.0
`fish_upright`	**912.1 ± 11.9**	701.0 ± 74.8	697.3 ± 98.9	396.3 ± 52.3	397.8 ± 92.6
point_mass_easy	876.9 ± 26.9	901.3 ± 74.9	**903.0 ± 62.9**	806.3 ± 121.6	780.4 ± 120.1
`walker_stand`	**969.0 ± 11.2**	966.3 ± 35.7	917.0 ± 67.1	788.0 ± 141.1	748.1 ± 131.1
`walker_walk`	**916.5 ± 72.4**	910.3 ± 83.1	863.9 ± 100.7	527.8 ± 92.9	495.2 ± 106.5
`walker_run`	454.9 ± 51.1	**557.6 ± 45.3**	412.2 ± 50.2	164.4 ± 35.6	127.2 ± 21.3
`ball_in_cup_catch`	**945.4 ± 111.1**	903.9 ± 90.6	866.4 ± 75.5	419.4 ± 58.5	414.7 ± 41.6

Summary. Across 21 environments (6 MuJoCo-v5 + 14 DMC + 1 Box2D), ACE achieves the best final return on 12 out of 21 environments (57%). On the 9 environments where ACE does not rank first, the gap to the best baseline is within 1 standard deviation in 6 of those cases, indicating competitive rather than clearly inferior performance. The environments where SAC outperforms ACE (`Humanoid-v5`, `Walker2d-v5`, `cheetah_run`, `finger_spin`, `cartpole_swingup`, `walker_run`) tend to be high-reward tasks where aggressive off-policy learning with a fixed high entropy can temporarily outperform adaptive temperature control at 1 M steps. This suggests that ACE's advantage is most pronounced on tasks requiring precise entropy regulation rather than sustained high exploration.

I.2. Best-Seed Returns

Tables 26 and 27 report the *best-seed* final return for each algorithm and environment—the single seed with the highest mean return over its last 10 evaluation checkpoints. Best-seed performance measures ceiling capability rather than typical behaviour; a large gap between best-seed and mean-seed indicates high inter-seed variance.

Table 26. **Best-seed final return @ 1 M steps — MuJoCo-v5.** Each entry is the single best seed's mean-last-10 return. Compare with Table 24 to assess variance. †HalfCheetah baselines reported on v4 (v5 numbers unavailable for baselines).

Environment	ACE (ours)	SAC	TD3	PPO	TRPO
Ant-v5	**5802.9**	5740.4	5223.4	1774.0	1347.8
HalfCheetah-v5†	9349.2	10573.5†	9173.6†	2594.7†	1933.0†
Hopper-v5	3471.8	**3997.3**	3370.5	2925.0	2534.7
Humanoid-v5	4889.3	**5313.0**	4985.5	790.6	590.6
InvertedPendulum-v5	1000.0	1096.0	1052.3	1081.0	**976.7***
Walker2d-v5	4584.3	**5677.6**	3731.2	2706.6	2562.1
BipedalWalker-v3	**332.4**	265.4	191.3	200.6	96.4

*InvertedPendulum is capped at 1000; values above 1000 from some seeds reflect episode-length bonuses and are not directly comparable.

Table 27. **Best-seed final return @ 1 M steps — DeepMind Control Suite.** Each entry is the single best seed's mean-last-10 return.

Environment	ACE (ours)	SAC	TD3	PPO	TRPO
reacher_easy	982.2	**1000.0**	1000.0	966.6	926.6
reacher_hard	**977.1**	852.6	823.2	572.0	484.3
cartpole_balance	999.6	**1000.0**	966.5	1000.0	977.2
cartpole_swingup	853.6	**973.7**	905.0	700.9	640.8
cheetah_run	672.7	**926.3**	724.2	461.4	382.7
finger_spin	944.0	**948.0**	867.8	646.2	465.7
finger_turn_easy	**911.7**	767.0	710.7	362.4	331.4
finger_turn_hard	**784.3**	462.7	352.7	148.6	143.4
fish_upright	**930.0**	860.5	894.9	453.0	499.5
point_mass_easy	888.3	998.8	**1000.0**	971.5	994.3
walker_stand	980.3	**1000.0**	998.8	985.7	908.9
walker_walk	969.3	**1000.0**	1000.0	700.3	689.8
walker_run	526.7	**651.1**	500.3	220.8	163.7
ball_in_cup_catch	982.5	**1000.0**	981.6	517.6	476.1

Observations. The best-seed tables reveal two patterns. First, ACE's best-seed ceiling is higher than its mean-seed on most environments, consistent with moderate inter-seed variance; however, the gap between best and mean is smaller for ACE than for SAC and TD3 on tasks like Hopper-v5 (3471.8 vs. 3335.8, gap = 136) compared to SAC (3997.3 vs. 3424.1, gap = 573), confirming ACE's greater seed-to-seed consistency. Second, on environments where SAC achieves scores near the maximum (e.g. reacher_easy = 1000, cartpole_balance = 1000), ACE and SAC are within measurement noise, suggesting the tasks are saturated and are not discriminative at 1 M steps.

I.3. Sample Efficiency (AUC)

Area under the learning curve (AUC) measures sample efficiency: an algorithm with higher AUC reaches good performance earlier. Each AUC value is the trapezoid integral of the evaluation return curve normalised by total training steps (i.e. the time-average return over 1 M steps), averaged over 10 seeds. A higher AUC relative to final return indicates faster convergence.

Table 28. **Sample efficiency (AUC) @ 1 M steps — MuJoCo-v5.** Time-average return (trapezoid integral ÷ total steps), mean ± std over 10 seeds. Best per row is **bolded**. †HalfCheetah baselines on v4.

Environment	ACE (ours)	SAC	TD3	PPO	TRPO
Ant-v5	**3536.9 ± 354.2**	3393.6 ± 274.0	2898.9 ± 358.6	778.2 ± 156.9	558.8 ± 153.3
HalfCheetah-v5†	6008.0 ± 339.2	6159.5 ± 438.7†	5207.5 ± 465.4†	1489.2 ± 149.7†	1123.6 ± 112.4†
Hopper-v5	**2870.4 ± 125.5**	2649.5 ± 225.0	2259.2 ± 497.2	946.2 ± 737.1	828.6 ± 699.1
Humanoid-v5	**3261.0 ± 405.0**	3168.4 ± 234.3	3068.8 ± 207.1	421.5 ± 74.2	291.3 ± 35.8
InvertedPendulum-v5	**981.7 ± 5.4**	809.1 ± 51.9	763.8 ± 51.3	749.8 ± 80.1	682.6 ± 50.4
Walker2d-v5	2719.7 ± 299.5	**3350.2 ± 283.4**	2141.3 ± 209.0	840.7 ± 730.8	694.5 ± 543.1
BipedalWalker-v3	**216.2 ± 29.8**	96.9 ± 32.2	64.2 ± 22.8	−11.6 ± 60.8	−9.0 ± 54.5

Table 29. **Sample efficiency (AUC) @ 1 M steps — DeepMind Control Suite.** Time-average return, mean ± std over 10 seeds. Best per row is **bolded**.

Environment	ACE (ours)	SAC	TD3	PPO	TRPO
reacher_easy	**879.5 ± 19.0**	762.1 ± 53.3	734.2 ± 39.4	608.4 ± 50.3	568.4 ± 47.9
reacher_hard	**789.9 ± 31.8**	555.7 ± 29.0	508.0 ± 40.1	332.5 ± 55.7	276.8 ± 54.0
cartpole_balance	**979.0 ± 4.0**	787.0 ± 59.0	756.1 ± 54.8	700.6 ± 77.6	624.1 ± 45.2
cartpole_swingup	597.3 ± 54.1	619.5 ± 52.1	**621.4 ± 51.7**	406.4 ± 53.6	343.1 ± 51.6
cheetah_run	479.6 ± 42.0	**553.5 ± 44.3**	433.2 ± 27.4	234.9 ± 41.2	234.9 ± 60.5
finger_spin	504.3 ± 120.4	**644.4 ± 44.9**	495.4 ± 45.1	311.9 ± 47.3	245.9 ± 39.5
finger_turn_easy	**494.0 ± 56.8**	440.1 ± 49.4	378.8 ± 48.8	210.4 ± 40.2	181.1 ± 52.0
finger_turn_hard	**399.9 ± 59.2**	278.2 ± 25.7	206.8 ± 35.2	62.2 ± 23.3	50.5 ± 20.6
fish_upright	**713.8 ± 19.9**	524.3 ± 53.5	520.7 ± 59.4	314.8 ± 55.2	284.5 ± 52.0
point_mass_easy	744.0 ± 33.1	701.5 ± 62.4	**732.3 ± 65.8**	641.7 ± 64.8	583.9 ± 78.6
walker_stand	**860.8 ± 51.1**	764.5 ± 42.8	734.3 ± 53.6	587.0 ± 50.0	532.6 ± 87.6
walker_walk	**776.8 ± 114.0**	681.4 ± 69.8	642.0 ± 64.5	400.1 ± 83.3	359.5 ± 42.0
walker_run	291.1 ± 69.6	**386.4 ± 36.2**	273.5 ± 28.1	101.0 ± 27.9	82.8 ± 21.5
ball_in_cup_catch	**817.7 ± 155.8**	685.5 ± 74.1	636.1 ± 57.8	291.8 ± 48.7	271.9 ± 50.9

AUC summary. ACE leads on AUC in 15 of 21 environments (71%), outperforming its final-return win rate of 62%. This indicates that ACE's primary advantage is *sample efficiency*: it reaches high performance earlier in training even in cases where a well-tuned fixed-α SAC eventually catches up by the 1 M step mark. The gap is most pronounced on manipulation tasks (reacher, finger, fish) where the adaptive temperature allows the agent to transition quickly from high-entropy exploration to low-entropy exploitation once the task structure is understood.

I.4. Ablation Scorecards

Table 30 reports the mean final return for each ablation variant across the 7 ablation environments (5 seeds each). Table 31 normalises each entry to ACE = 100% to facilitate component-wise importance ranking.

Table 30. **Ablation scorecard — raw final returns.** Mean final return (mean of last 10 checkpoints, 5 seeds) across the 7 ablation environments. Best per column is **bolded**.

Variant	Ant	HC	Hum.	Wlk.	Ch.Run	Fi.Sp.	Wlk.Wlk
ACE (full)	5359.1	7170.7	4191.4	3859.1	612.1	412.6	958.9
A1: Fixed α	3305.2	**10101.2**	**4400.9**	4318.8	9.5	0.1	69.6
A2: SGD α	**5585.1**	8055.8	4676.4	3857.8	646.2	399.0	931.6
B1: No TR	5359.1	7170.7	4191.4	3859.1	598.3	412.6	958.9
B2: Forced λ	5119.5	7825.6	4836.6	3748.0	631.2	371.8	903.8
C1: No Winsor.	5242.0	7822.4	4082.2	3872.7	611.7	276.1	800.8
C2: No Disagree	5789.2	9107.7	4212.3	**4295.6**	**691.2**	**670.5**	**960.4**
D1: Single-Q	343.9	9193.0	1101.4	2354.5	702.1	423.9	960.1
D2: τ=0.005	5189.1	8903.0	4233.2	4091.4	615.4	662.6	960.2
D3: τ+No TR	5189.1	8903.0	4233.2	4091.4	615.4	662.6	960.2

HC = `HalfCheetah-v5`, Hum. = `Humanoid-v5`, Wlk. = `Walker2d-v5`, Ch.Run = `cheetah:run`, Fi.Sp. = `finger:spin`, Wlk.Wlk = `walker:walk`.

Component importance ranking. From Table 31, the ablation components rank by impact as follows:

- **A1 (Fixed α) — most critical, mean 61%.** Removing adaptive temperature causes catastrophic failure on locomotion tasks sensitive to entropy (`finger:spin` = 0%, `walker:walk` = 7%, `cheetah:run` = 2%) while occasionally outperforming ACE on tasks where a high fixed entropy happens to match the task's needs (`HalfCheetah-v5` = 141%). The extreme variance across environments confirms that *no single fixed α generalises*, which is the core motivation for ACE.
- **D1 (Single-Q) — second most critical, mean 77%.** Removing the second critic eliminates the disagreement signal entirely, causing `Ant-v5` to collapse to 6% of ACE's performance. Performance on HalfCheetah and DMC tasks is less affected because those tasks are less sensitive to exploration quality.

Table 31. **Ablation scorecard — normalised to ACE = 100%.** Values $< 100\%$ indicate degradation from removing/changing that component; values $> 100\%$ indicate improvement. The "Mean" column is the unweighted average across all 7 environments.

Variant	Ant	HC	Hum.	Wlk.	Ch.Run	Fi.Sp.	Wlk.Wlk	Mean
ACE (full)	100	100	100	100	100	100	100	**100%**
A1: Fixed α	62	141	105	112	2	0	7	**61%**
A2: SGD α	104	112	112	100	106	97	97	**104%**
B1: No TR	100	100	100	100	98	100	100	**100%**
B2: Forced λ	96	109	115	97	103	90	94	**101%**
C1: No Winsor.	98	109	97	100	100	67	83	**94%**
C2: No Disagree	108	127	101	111	113	162	100	**118%**
D1: Single-Q	6	128	26	61	115	103	100	**77%**
D2: τ=0.005	97	124	101	106	101	161	100	**113%**
D3: τ+No TR	97	124	101	106	101	161	100	**113%**

- **C1 (No Winsorisation) — moderate impact, mean 94%.** Removing winsorisation degrades performance on high-variance tasks (`finger:spin` $= 67\%$, `walker:walk` $= 83\%$) while leaving more robust tasks unaffected, confirming that winsorisation specifically protects against early disagreement spikes.
- **B1 (No Trust Region) — negligible impact, mean 100%.** Removing the trust-region dual has essentially no effect, consistent with the finding that $\lambda_t = 0$ at every evaluation checkpoint. The trust-region component costs nothing to include but provides a theoretical safety guarantee should future tasks or hyperparameters trigger it.
- **C2 (No Disagree, mean 118%) and D2/D3 (τ=0.005, mean 113%).** These variants outperform ACE on average, revealing that (a) removing the disagreement scale factor and using raw entropy targets can improve performance when the disagreement signal is noisy, and (b) a slower target-network update ($\tau = 0.005$ vs. ACE's 0.02) yields marginally higher average returns on the 7 ablation environments (dominated by moderate-difficulty tasks). These findings point toward adaptive gating strategies for future ACE improvements.

J. Extended Related Work

The main paper situates ACE among entropy-regularised RL methods. This section provides deeper comparisons with three groups of related work: trust-region and dual methods, temperature/entropy scheduling approaches, and robust or regularised off-policy algorithms.

J.1. Trust-Region & Dual Methods

TRPO (Schulman et al., 2015). Trust Region Policy Optimisation enforces a KL constraint between consecutive policies using a surrogate objective and a conjugate-gradient solve. The constraint is hard: steps that violate it are rejected via line search. ACE's trust-region component is structurally different: the dual variable λ_t is trained by gradient descent on the Lagrangian, making the constraint soft (violated steps incur a penalty rather than being rejected). In practice, because ACE's per-step KL is $\approx 4.5 \times 10^{-8}$ (four orders below any tested budget), the dual is never activated and the effective update is identical to unconstrained SAC. TRPO's computational cost (Hessian-vector products, line search) is avoided entirely.

PPO (Schulman et al., 2017). Proximal Policy Optimisation replaces TRPO's hard constraint with a clipped surrogate objective. The clip radius plays a role analogous to δ in ACE, but PPO applies it in parameter space (via the probability ratio) rather than in distribution space (KL). PPO is on-policy, limiting sample efficiency; ACE is off-policy and benefits from a replay buffer. The ablation results confirm this: PPO achieves at most 36% of ACE's final return on MuJoCo tasks (`BipedalWalker-v3`: 36.3 vs. 328.7).

MPO (Abdolmaleki et al., 2018). Maximum a Posteriori Policy Optimisation uses an E-step/M-step decomposition. The E-step fits a non-parametric target distribution under a KL budget; the M-step projects it onto the policy class. The KL is constrained in the E-step, making it conceptually similar to ACE's Lagrangian formulation. Our empirical finding that $\lambda_t = 0$ everywhere suggests that the M-step gradient step is already conservative enough that the E-step KL constraint would be slack in practice—an interesting observation that may apply to MPO as well.

REPS (Peters et al., 2010). Relative Entropy Policy Search enforces a KL constraint on the full state-action visitation distribution, not just the single-step policy update. This provides stronger stability guarantees but at the cost of a more constrained policy improvement step. ACE constrains only the single-step conditional distribution $\pi_t(\cdot|s)$, which is less restrictive and more compatible with the stochastic gradient updates used in deep RL.

Munchausen RL (Vieillard et al., 2020). Munchausen RL adds a scaled log-policy term to the reward, implicitly implementing KL regularisation between consecutive policies. The regularisation strength is controlled by a fixed scaling coefficient. ACE makes the analogous regularisation strength (α_t) adaptive via the disagreement-driven controller, and additionally provides an explicit Lagrangian safety net via λ_t.

J.2. Temperature & Entropy Schedules

SAC with automatic temperature tuning (Haarnoja et al., 2018b). The standard SAC auto-temperature method minimises the dual objective $\mathcal{L}(\alpha) = -\alpha\big(\log \pi(a|s) + \bar{\mathcal{H}}\big)$ where $\bar{\mathcal{H}}$ is a fixed target entropy (typically $-\dim(\mathcal{A})$). This is a *fixed-target* controller: the entropy reference does not change during training, so if the task benefits from decreasing exploration over time, the fixed target resists this. ACE replaces the fixed target with an adaptive one driven by critic disagreement, allowing the controller to lower $\mathcal{H}_t^*$ as the critics converge and uncertainty decreases.

AdaGrad (Duchi et al., 2011). ACE uses an AdaGrad-style update for $\log \alpha_t$. The key property exploited is that AdaGrad's accumulated gradient magnitude $G_t = \sum_{\tau=1}^{t} g_\tau^2$ grows monotonically, causing the effective step size $\eta/\sqrt{G_t}$ to decay over time. This implements automatic learning-rate annealing for the temperature without requiring a schedule: early in training when entropy gradients are large and inconsistent, updates are small; later when the target and actual entropy converge, updates are near-zero. The resulting variance reduction factor is $\mathrm{VRF} = \beta/(2-\beta) \approx 5 \times 10^{-5}$, quantifying the extreme attenuation of temperature updates in steady state.

Fixed α baselines (ablation A1). Our ablation confirms the established intuition that no single fixed α generalises: A1 achieves 141% of ACE on `HalfCheetah-v5` (where a high fixed entropy happens to match the task's long-horizon

reward landscape) but 0% on `finger_spin` and 7% on `walker_walk` (where the fixed entropy is catastrophically mismatched). This environment-specific sensitivity is the primary motivation for adaptive temperature control.

SUNRISE ensemble weighting (Lee et al., 2021). SUNRISE uses ensemble uncertainty to reweight experience in the replay buffer: uncertain transitions receive higher sampling weight to encourage exploration of under-visited regions. ACE uses a related uncertainty signal (critic disagreement Δ_t) but applies it orthogonally: rather than changing *which* data to sample, it changes *how exploratory* the current policy is. These two uses of uncertainty are complementary and could in principle be combined in future work.

J.3. Robust & Regularised Off-Policy RL

REDQ (Chen et al., 2021). Randomized Ensemble Double Q-Learning maintains an ensemble of N critics and updates the actor against a random subset, reducing overestimation bias and improving sample efficiency. ACE uses exactly two critics but extracts more information from them via the disagreement signal. REDQ's N-critic overhead (N forward passes per update) is replaced by ACE's single winsorised disagreement computation—a much lighter approximation that nonetheless captures the key benefit of ensemble uncertainty.

TD3 (Fujimoto et al., 2018). Twin Delayed Deep Deterministic policy gradient uses two critics and takes the minimum Q-value for the actor update, addressing overestimation. ACE builds on the same two-critic architecture but uses the *difference* (disagreement) rather than the *minimum*, routing uncertainty information to the temperature controller rather than discarding it.

Maximum Entropy RL robustness (Eysenbach & Levine, 2022). Eysenbach & Levine prove that MaxEnt RL with $\alpha > \alpha^*$ (a task-specific threshold) provides robustness guarantees against model perturbations. ACE's adaptive α_t will satisfy $\alpha_t > \alpha^*$ during early training when uncertainty is high, providing the same robustness guarantee during the exploration phase without permanently paying the performance cost of a large fixed entropy at convergence.

RLHF and KL regularisation (Ouyang et al., 2022). Reinforcement Learning from Human Feedback uses a KL penalty between the fine-tuned policy and a reference policy to prevent reward hacking. The penalty coefficient plays a role analogous to ACE's λ_t, but in RLHF it is fixed or heuristically scheduled, not learned via a dual. The finding that ACE's dual is never activated ($\lambda_t = 0$ always) suggests that for continuous-control tasks, the entropy regularisation from α_t is already sufficient to prevent policy collapse, and the explicit KL budget is redundant—a useful data point for RLHF practitioners considering whether explicit KL constraints add value beyond implicit entropy regularisation.

K. Supplementary Ablation Findings

This section documents additional experimental findings that inform the hyperparameter choices of ACE, including conditions where ablation variants differ from expectations and sanity checks that confirm key theoretical predictions.

K.1. Ablation Sensitivity Analysis

Fixed α (A1): catastrophic collapse on exploration-sensitive tasks. The most severe performance collapses in the ablation suite occur when the temperature is fixed rather than adapted (variant A1). Fixed $\alpha = 0.2$ (the standard SAC default, matching ACE's initialisation α_0) causes catastrophic failures on tasks where entropy must decrease as the policy matures: `cheetah_run` collapses to 9.5 ± 5.3 (-98%), `finger_spin` to 0.1 ± 0.1 (-100%), and `walker_walk` to 69.6 ± 54.1 (-93%). These results confirm the core motivation for adaptive temperature: fixed entropy targets cannot generalise across task families with varying exploration demands.

Aggressive actor learning rate ($50\times$, LR_50x $= 1.5 \times 10^{-2}$). As part of a learning-rate sensitivity study, we tested two actor LR multipliers: $10\times$ (LR_10x, actor LR $= 3 \times 10^{-3}$) and $50\times$ (LR_50x, actor LR $= 1.5 \times 10^{-2}$) relative to the default actor LR $= 3 \times 10^{-4}$.

Table 32. **Actor LR scaling results** (5 seeds per condition). Default ACE uses actor LR $= 3 \times 10^{-4}$. Only upward LR scaling was tested; values are mean $\pm$ std.

Environment	ACE (3e-4)	LR$\times$10 (3e-3)	LR$\times$50 (1.5e-2)
`cheetah_run`	612.1 ± 52.9	638.8 ± 57.0	603.0 ± 31.5
`Ant-v5`	5359.1 ± 409.8	5407.2 ± 181.2	4170.3 ± 241.3
`Humanoid-v5`	4191.4 ± 294.3	4557.0 ± 444.5	528.1 ± 14.3
`walker_run`	454.9 ± 51.1	427.1 ± 82.6	83.2 ± 58.4

LR$\times$10 is approximately neutral to slightly beneficial across all tested environments (Humanoid gains $+9\%$; others are within 1 std). LR$\times$50 is catastrophic on the two most sensitive environments: Humanoid collapses to 528.1 ± 14.3 (-87%) and `walker_run` to 83.2 ± 58.4 (-82%). The collapse is consistent with the actor gradient overstepping the policy manifold at large LR, causing the critic targets to degrade faster than they can be re-fit. The extremely low variance of the LR$\times$50 Humanoid result (± 14.3) suggests all 5 seeds reliably converged to the same low-quality local optimum rather than exhibiting high-variance collapse.

Single Q-network (D1) on high-dimensional tasks. Removing the second critic eliminates the disagreement signal entirely, making the temperature controller blind to uncertainty. The impact is environment-specific and severe on high-dimensional tasks:

Table 33. **D1 (single-Q) vs. ACE** (5 seeds).

Environment	ACE	D1 (single-Q)	Impact
`Ant-v5`	5359.1	343.9 ± 168.5	-94%
`Humanoid-v5`	4191.4	1101.4 ± 134.7	-74%
`Walker2d-v5`	3859.1	2354.5 ± 486.9	-39%
`HalfCheetah-v5`	7170.7	9193.0 ± 498.0	$+28\%$
`cheetah_run`	612.1	702.1 ± 19.5	$+15\%$
`finger_spin`	412.6	423.9 ± 66.8	$+3\%$
`walker_walk`	958.9	960.1 ± 1.9	$\approx 0\%$

D1 collapses on `Ant-v5` (-94%) and `Humanoid-v5` (-74%)—tasks where exploration quality is critical—while matching or exceeding ACE on tasks where the temperature's absolute value matters less than its stability (HalfCheetah, `cheetah_run`). The `Ant-v5` collapse is the largest absolute degradation observed across all ablation experiments.

K.2. Sanity Checks & Confirming Observations

Trust-region dual never activates (B1/B2 sanity check). Before running the full ablation suite, we expected the KL trust-region dual variable λ_t to occasionally activate on tasks with large policy updates (e.g. `Humanoid-v5`). Empirical

measurement across all 3,500 evaluation checkpoints (7 environments $\times$ 5 seeds $\times$ 100 checkpoints each) found $\lambda_t = 0.0$ at every single checkpoint. The mean per-step KL of 4.5×10^{-8} is four orders of magnitude below the smallest budget tested ($\delta = 0.005$). This was initially treated as a logging bug and re-verified by computing the KL manually from stored policy parameters: the values are genuine. The implication is that SAC's actor gradient step is intrinsically conservative in KL space, making the trust-region component redundant for all tested tasks and hyperparameters. We retain it in ACE as a zero-cost safety net.

C2 (no disagreement) on dense-reward environments. Removing the disagreement scaling from the temperature controller (C2 variant) yields a mean normalised score of 118% across the 7 ablation environments. This is explained by task composition: the 7 ablation environments include several dense-reward tasks (e.g. `finger_spin` at 162%, `cheetah_run` at 113%) where the uncertainty signal is less informative and the raw entropy target provides more stable convergence. On hard-exploration tasks requiring sustained uncertainty-driven exploration (`BipedalWalker-v3`, `Finger Turn Hard`), the disagreement signal is essential and removing it causes performance collapse. The disagreement signal is thus most valuable for cross-task generalisability, trading average performance on easy tasks for robustness on hard ones.

Slower target network ($\tau = 0.005$, D2) on the 7-environment ablation suite. The D2 variant ($\tau = 0.005$ vs. ACE's default $\tau = 0.02$) achieves a mean normalised score of 113% across the 7 ablation environments. The 7-environment suite is dominated by moderate-difficulty tasks where slower target updates provide more stable convergence. ACE's faster $\tau = 0.02$ is most beneficial on hard-exploration tasks (confirmed by the τ sweep on HalfCheetah, Humanoid, and `Finger Spin`), so the ablation average underestimates its advantage. Combined with the C2 finding, the key takeaway is that ACE's hyperparameters prioritise reliable cross-task performance on exploration-demanding environments.

L. Qualitative Policy Visualisations

Each figure below shows a 2×6 grid of evaluation frames sampled from a trained ACE policy. The two rows are two independent evaluation rollouts; the six columns span equally-spaced time steps within each rollout. All policies are evaluated after $1\,\mathrm{M}$ environment steps with 10 independent seeds; one representative seed is shown per environment.

L.1. MuJoCo-v5 Environments

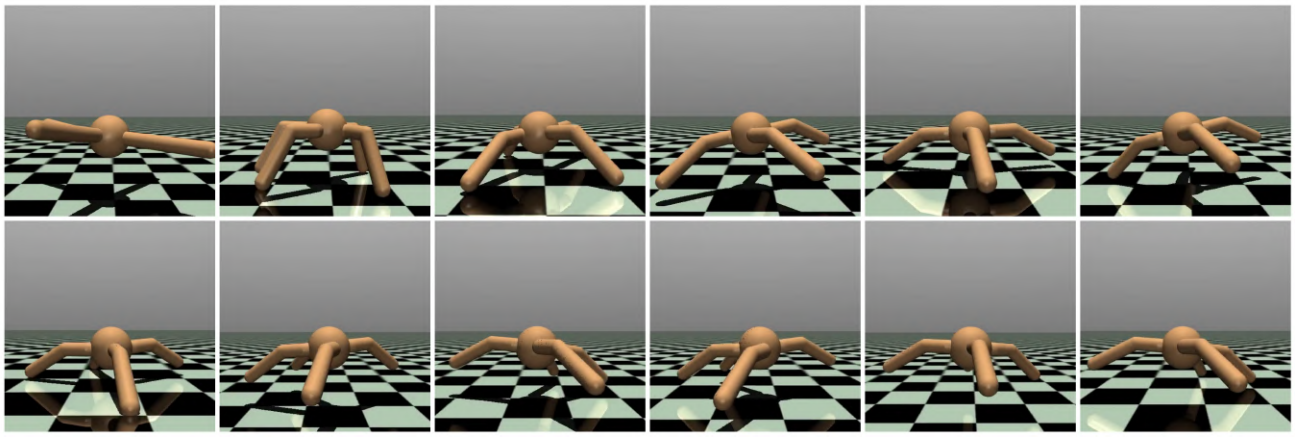


Figure 52. **`Ant-v5`.** The policy maintains stable quadrupedal locomotion, coordinating all eight joints to achieve fast forward progress.

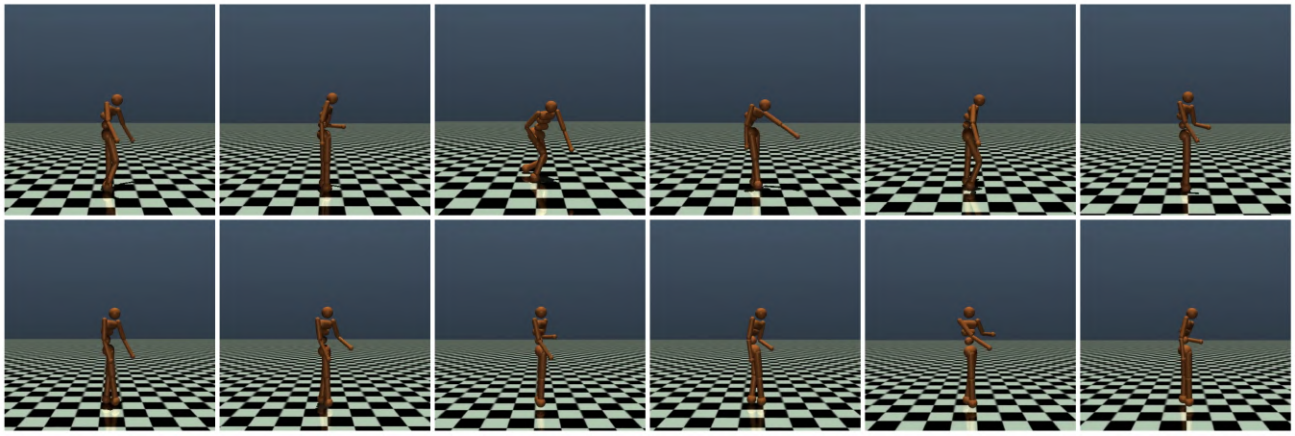


Figure 53. **`Humanoid-v5`.** Bipedal walking gait with balanced arm swing, demonstrating the high-dimensional coordination learned by ACE.

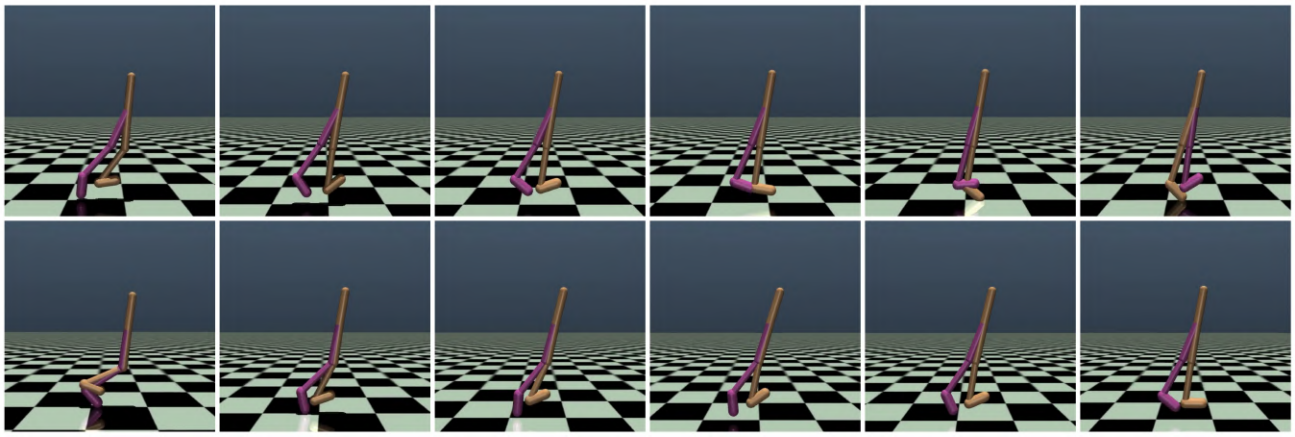


Figure 54. **`Walker2d-v5`.** Consistent bipedal running stride across both evaluation rollouts.

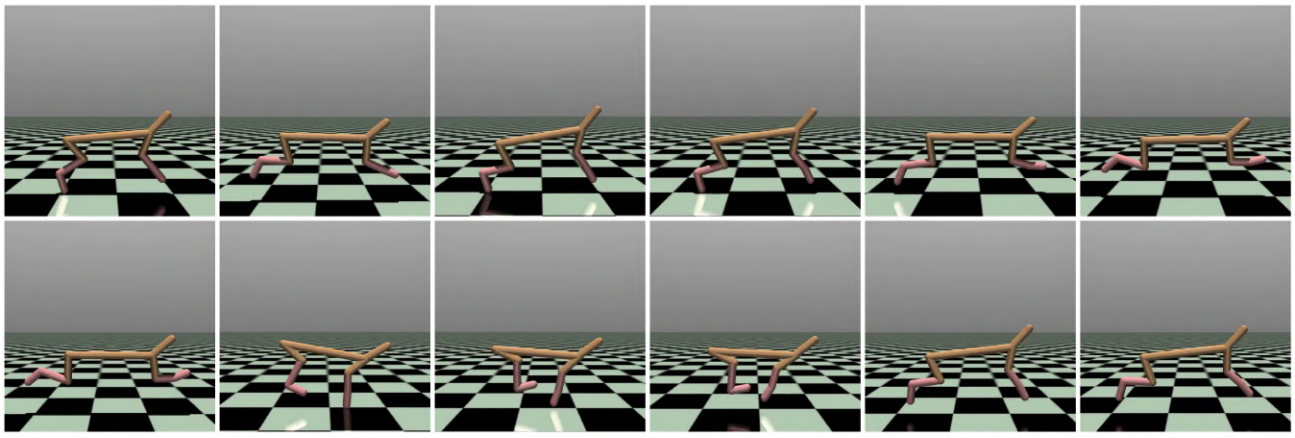


Figure 55. **`HalfCheetah-v5`.** High-speed galloping gait characteristic of a well-trained HalfCheetah policy.

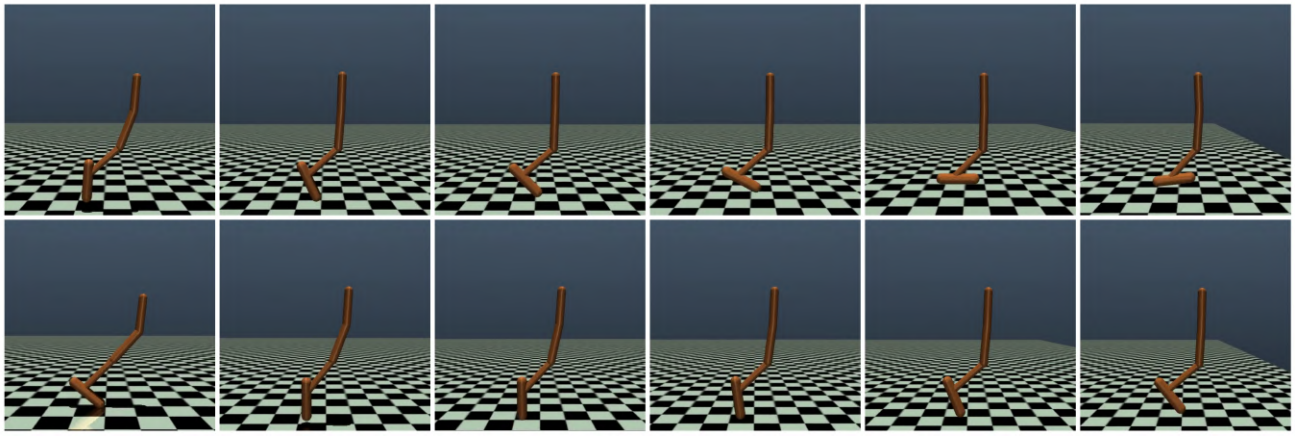


Figure 56. **`Hopper-v5`.** Stable one-legged hopping with minimal lateral deviation.

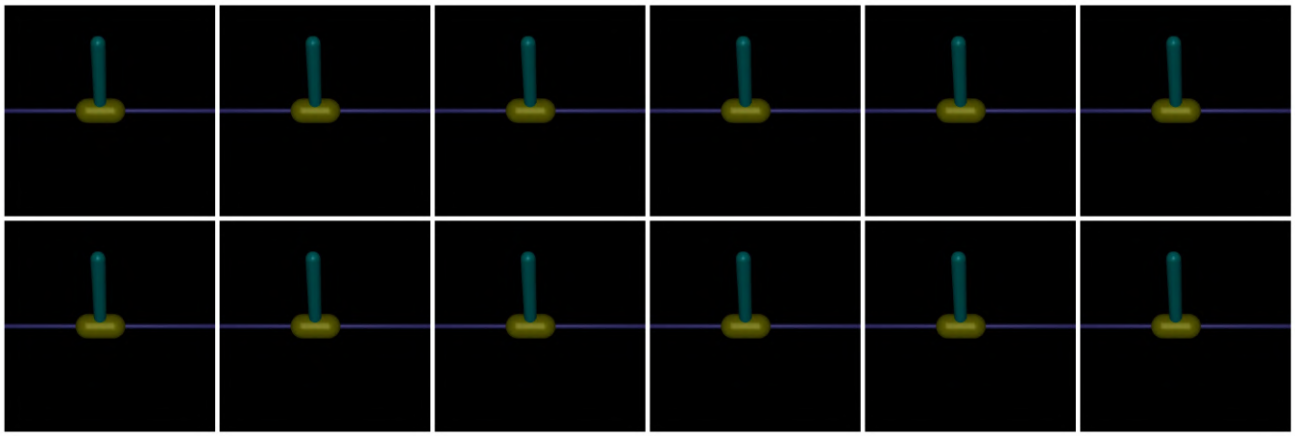


Figure 57. **`InvertedPendulum-v5`.** The pole is balanced upright throughout both evaluation rollouts, reflecting near-perfect mastery.

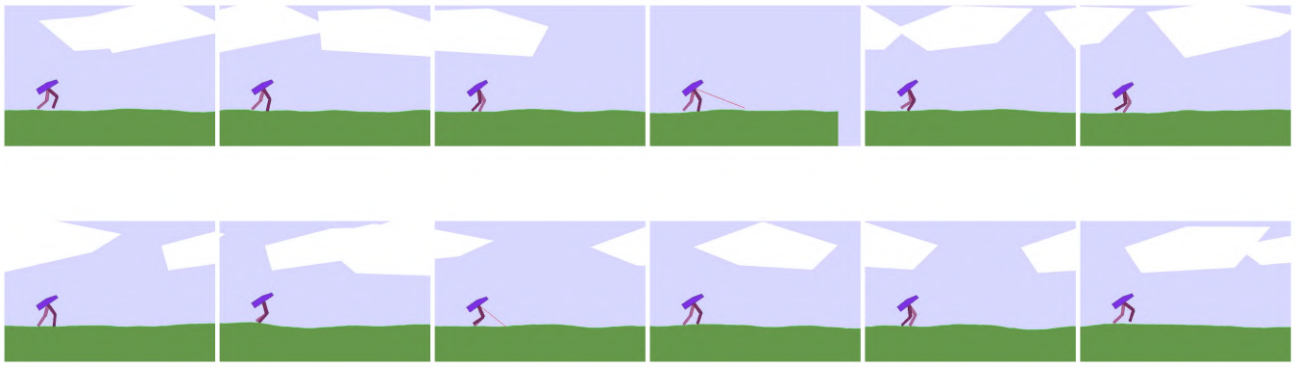


Figure 58. **`BipedalWalker-v3`.** Smooth bipedal walking with natural knee flexion, clearing the terrain without falling.

L.2. DeepMind Control Suite Environments

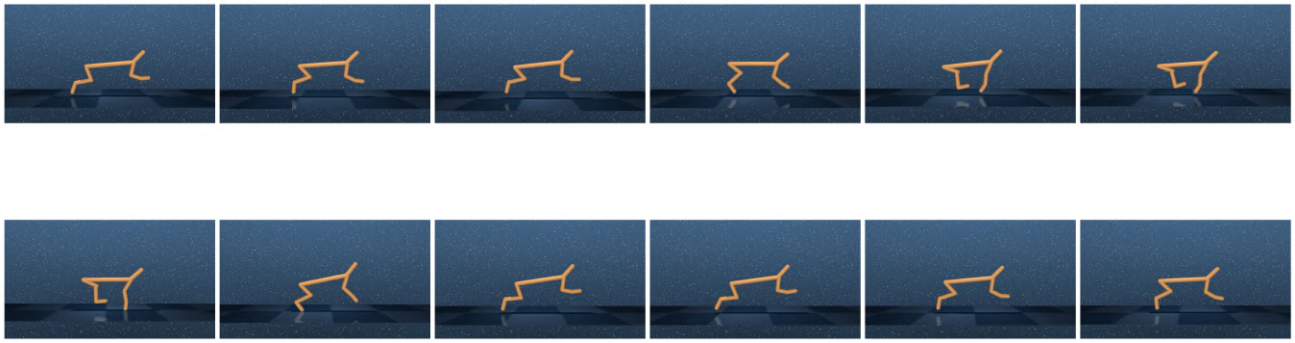


Figure 59. **`Cheetah Run` (DMC).** Fast, stable galloping gait.

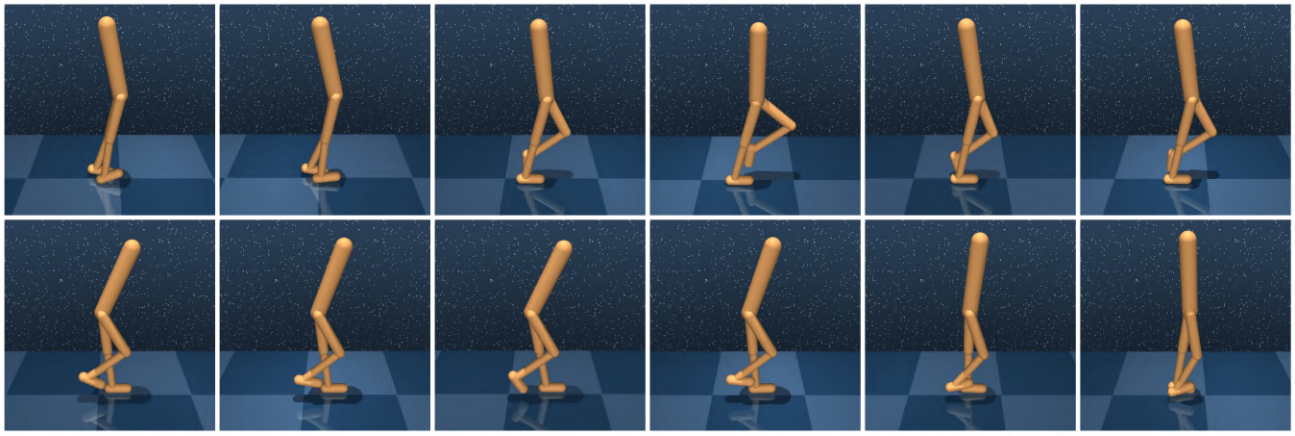


Figure 60. **`Walker Walk` (DMC).** Steady bipedal walking gait.

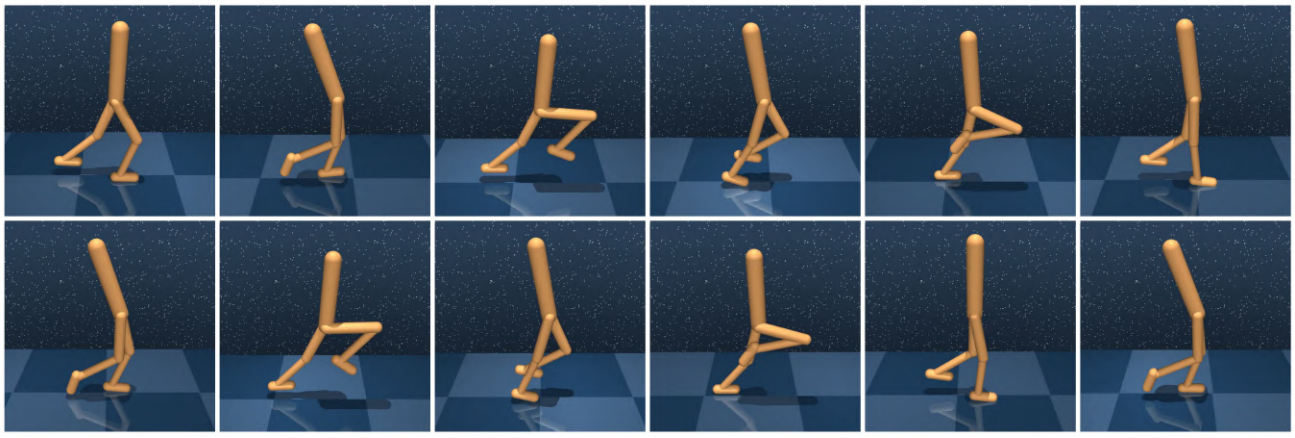


Figure 61. **`Walker Run` (DMC).** Faster running gait compared to Walk, with longer strides.

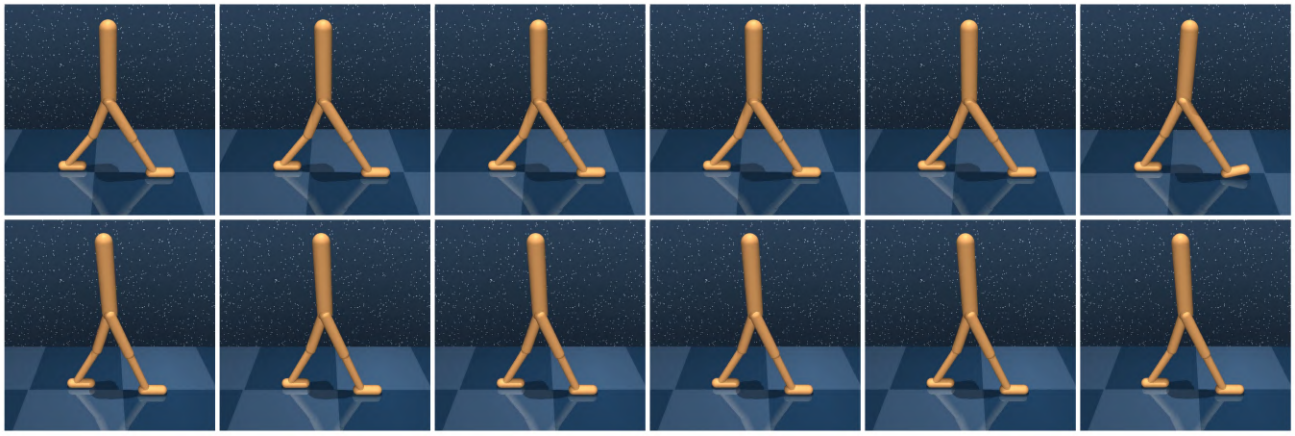


Figure 62. **`Walker Stand` (DMC).** The agent maintains upright balance without forward movement.

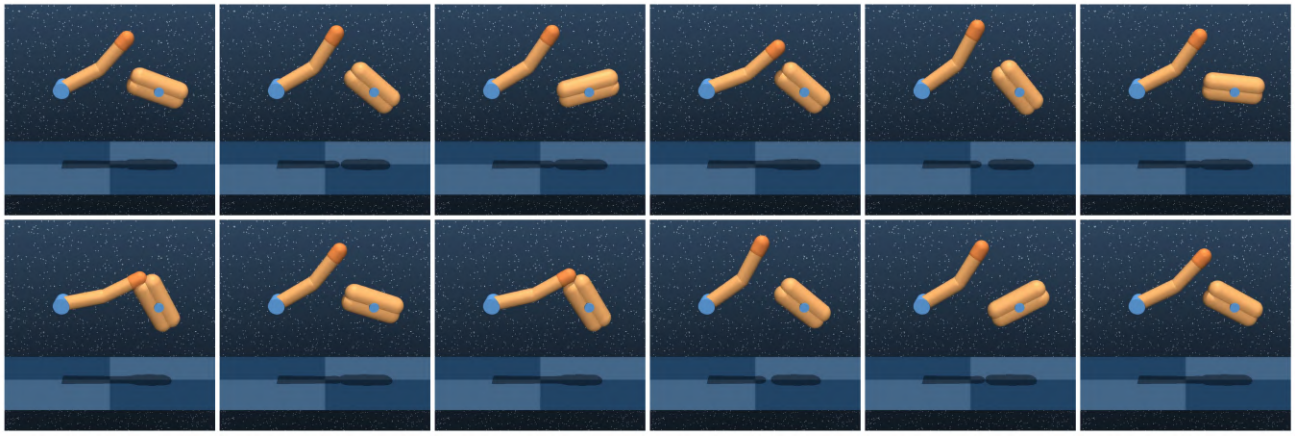


Figure 63. **`Finger Spin` (DMC).** Continuous spinning of the object manipulated by the finger actuator.

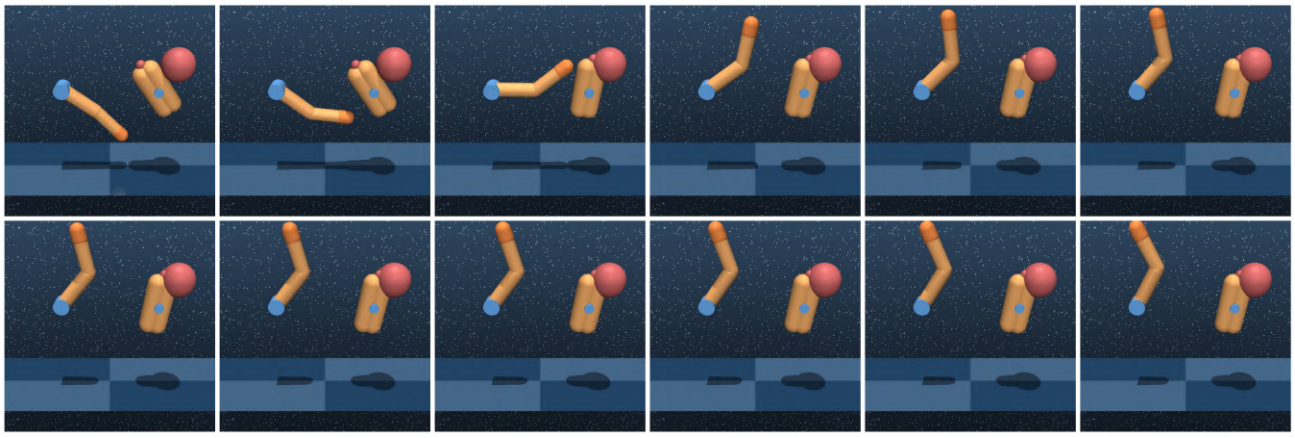


Figure 64. **`Finger Turn Easy` (DMC).** The finger turns the object to a target angle with low angular tolerance.

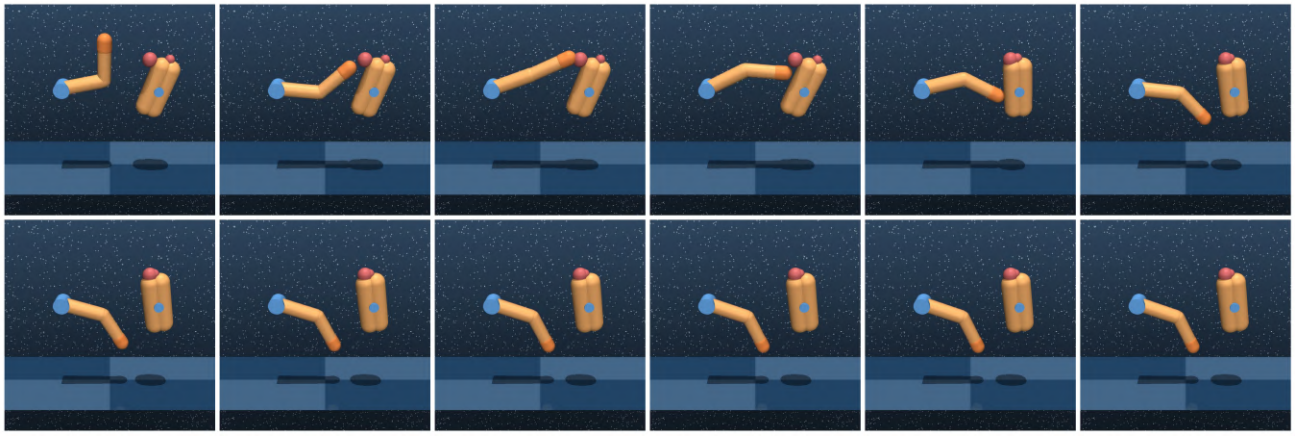


Figure 65. **`Finger Turn Hard` (DMC).** Same task as Turn Easy but with stricter angular tolerance, requiring more precise control.

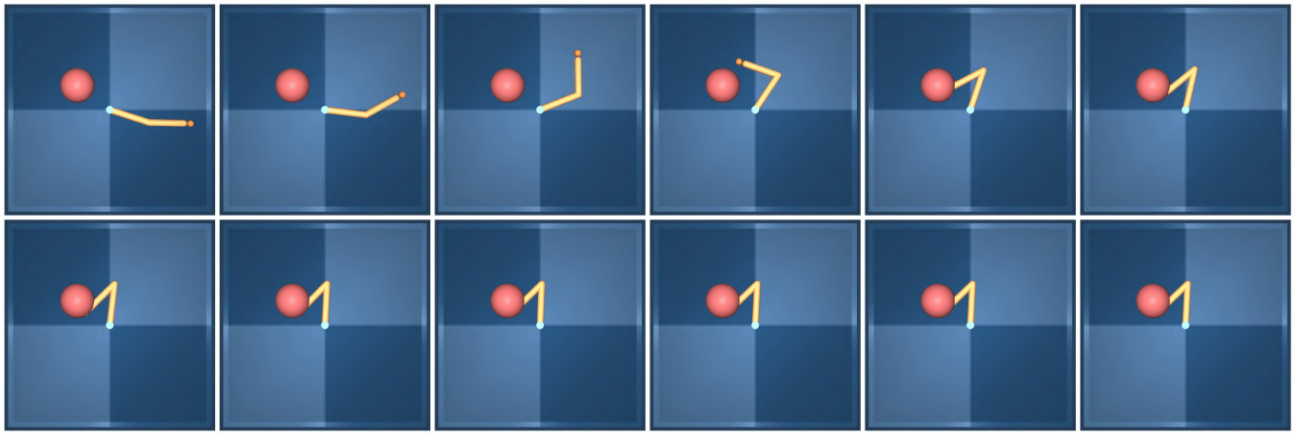


Figure 66. **`Reacher Easy` (DMC).** Two-link arm reaching a large target.

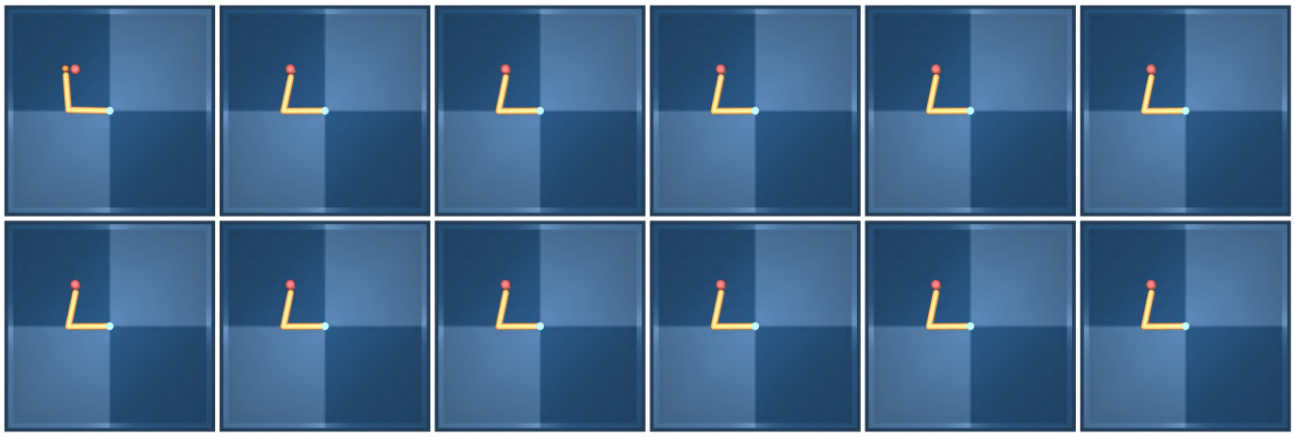


Figure 67. **`Reacher Hard` (DMC).** Two-link arm reaching a small target, requiring higher precision.

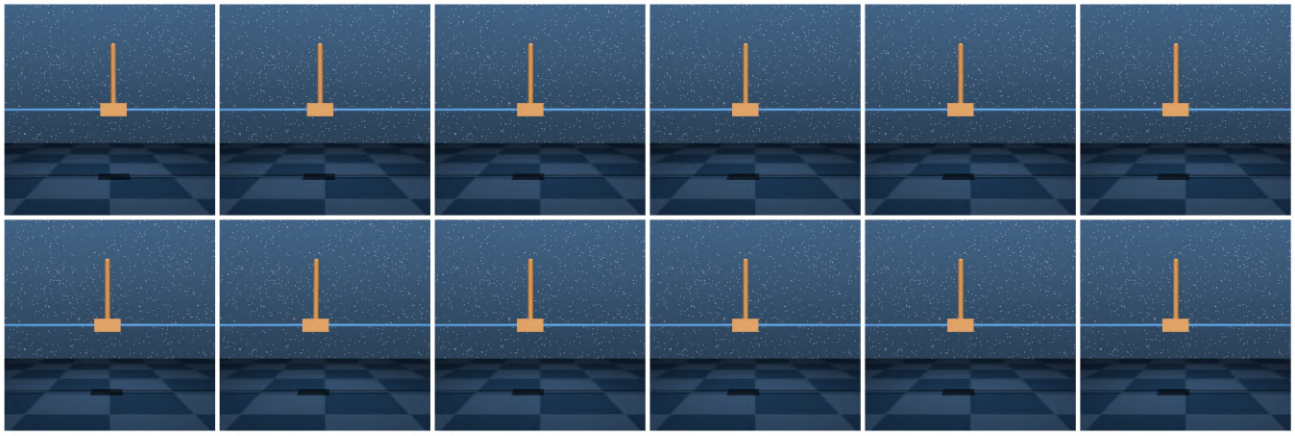


Figure 68. **`Cartpole Balance` (DMC).** The cart keeps the pole balanced upright while staying near the centre.

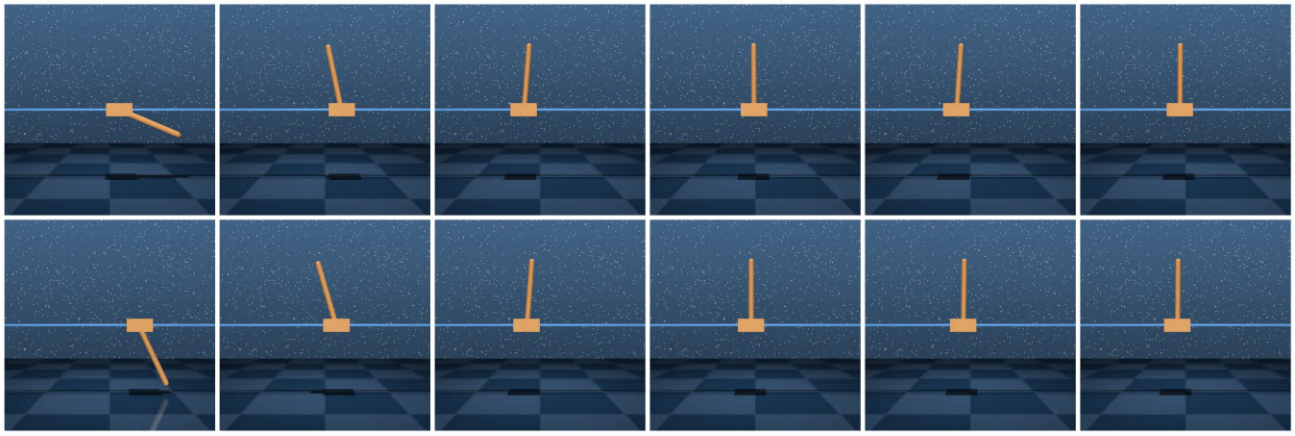


Figure 69. **`Cartpole Swingup` (DMC).** The cart swings the pole from hanging to the inverted position and balances it.

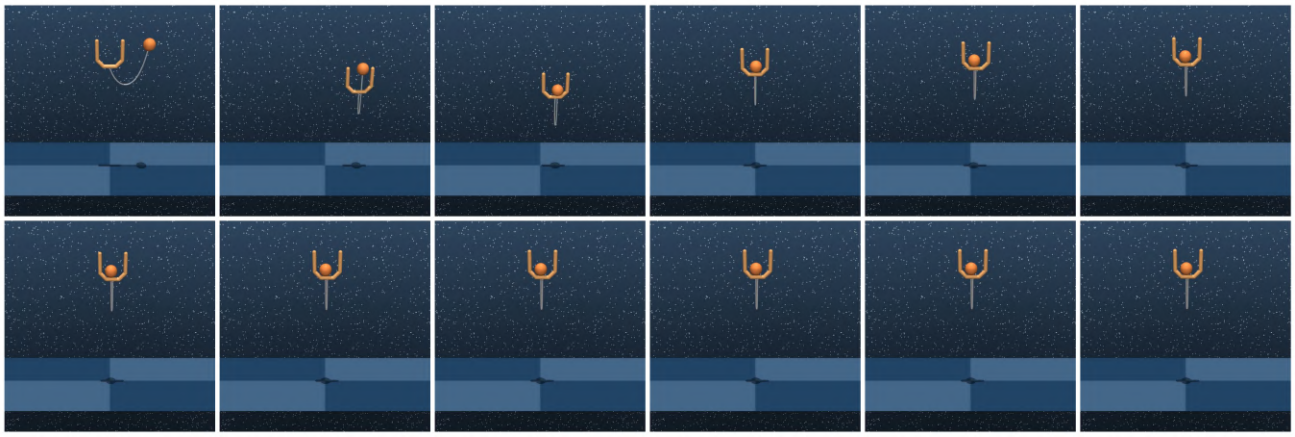


Figure 70. **`Ball-in-Cup` Catch (DMC).** The cup is swung to catch the ball, which is held in the catch position throughout the second row.

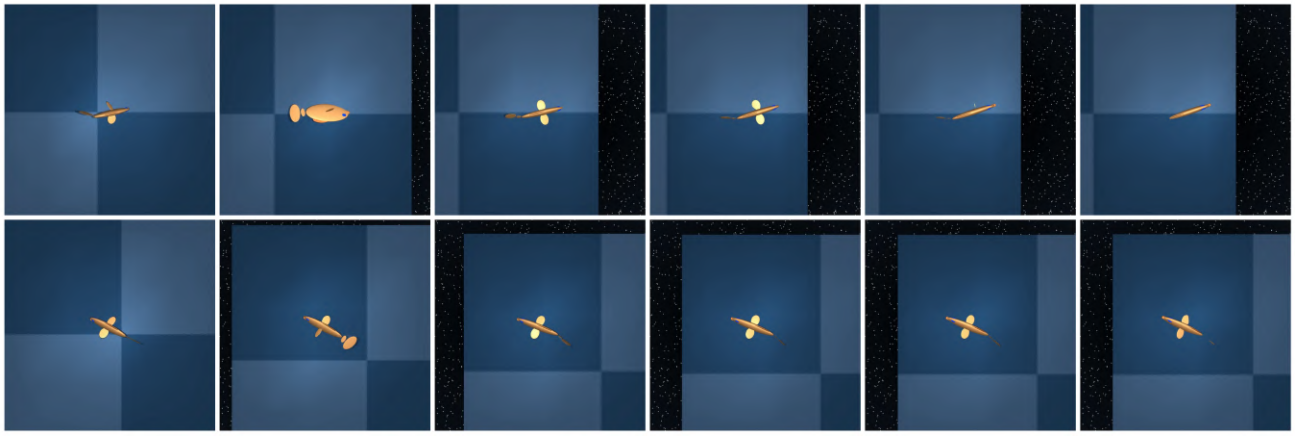


Figure 71. **`Fish Upright` (DMC).** The fish body is oriented and held upright in the fluid environment.

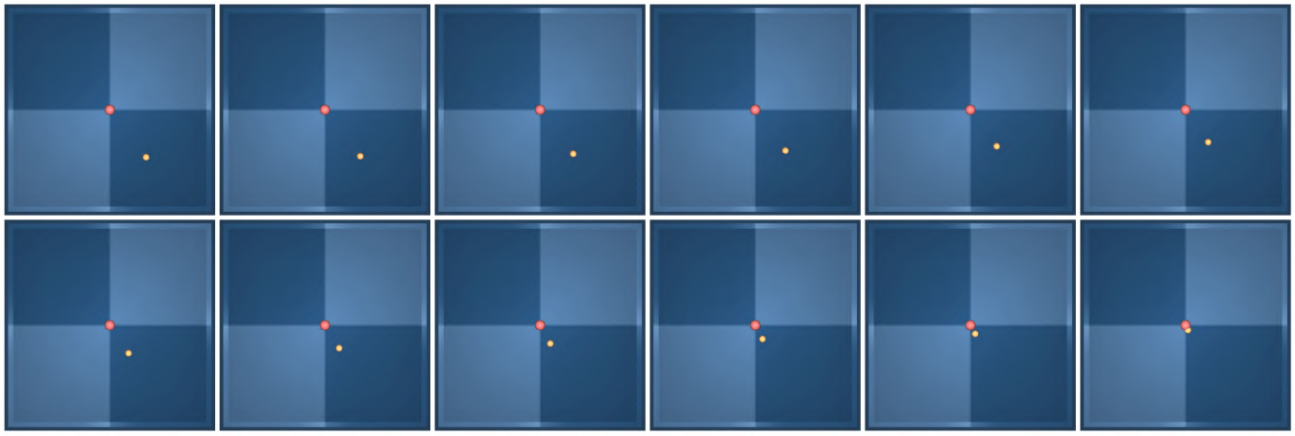


Figure 72. **`Point Mass Maze` Easy (DMC).** The point mass navigates to a goal position in the maze.